DEPOSITION OF A SPRAY ON THE WALLS OF A STRAIGHT DUCT

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Abstract

The rate of deposition of a spray on the walls of a straight duct under various flow conditions was determined and the data were correlated by simple diffusion theory. The average apparent diffusivity of the spray for predicting net deposition on non-wetted walls was found to be of the order of 20 to 30 in.^{2}/sec. and to increase proportionally with the 0.6 power of the air velocity.

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Introduction -

The purpose of this investigation was to study the deposition, on the walls of a straight duct, of a spray generated by the injection of a jet of water into an air stream flowing at moderate velocities in the duct.

The characteristics of sprays formed by air atomization have received considerable attention (2, 7, 12). Most of the research has involved studies of the frequency distribution in the drop size spectrum and its relation to the independent variables in the systems employed and to the combustion characteristics of fuel sprays (4, 5, 7, 9, 12). Several attempts have been made to elucidate the mechanism of the break-up of liquid jets and droplets in air streams (1, 3, 6, 8, 10, 13, 15).

Theory -

Consider a spray moving with axially symmetrical flow in a straight duct. Assume that the liquid droplets move with the same velocity as the air stream and that the velocity parallel to the axis of the duct is constant over any cross section. Postulate that the statistical behavior of the water droplets is governed by the laws of diffusion, whence

$$\left(\frac{\partial M}{\partial \theta}\right)_r = -\alpha A'\left(\frac{\partial c}{\partial r}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $\left(\frac{\partial M}{\partial \theta}\right)_r$ is the rate of transport of liquid through a cylindrical surface A' normal to the radius of the duct, α is the diffusivity, and $\left(\frac{\partial c}{\partial r}\right)$ is the radial concentration gradient of liquid. Taking a mass balance on a volume element having a length 1, a diameter r, and a thickness dr, the following differential equation is readily obtained.

$$\left(\frac{\partial c}{\partial \theta}\right)_r = \alpha\left[\frac{1}{r}\frac{\partial c}{\partial r} + \frac{\partial^2 c}{\partial r^2}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The element of volume is considered to move with the air stream, and diffusion of liquid drops through the ends of the element is neglected. Let the boundary conditions be

(a) $C = f(r)$ when $\theta = \theta_0$

(b) $C = 0$ $\qquad \theta = \infty$

(c) $C = 0$ $\qquad r = \frac{D}{2}$

Note also that

$$\theta = x/V \tag{3}$$

where x is the distance the volume element has traveled from the origin and V is the velocity of travel, equal to air velocity.

Solving Equation 2

$$C = \sum_{1}^{\infty} A_n e^{-a_n^2 x \theta} J_0(a_n r) \quad . \; . \; . \; . \; . \tag{4}$$

From boundary condition (c)

$$J_0\left(a_n \frac{D}{2}\right) = 0 \quad . \; . \; . \; . \; . \; . \tag{5}$$

and from condition (a)

$$f(r) = \sum_{1}^{\infty} A_n J_0(a_n r) \; . \; . \; . \; . \tag{6}$$

Now the mass of the liquid still in suspension at any section along the length of the duct is given by the integral

$$m = \int_0^{\frac{D}{2}} C V 2 \pi r dr \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \tag{7}$$

Substituting from Equation 4, and integrating

$$m = \pi V D \sum_{1}^{\infty} A_n e^{-a_n^2 \alpha \theta} J_1\left(a_n \frac{D}{2}\right) \; . \; . \; . \; . \tag{8}$$

If all terms in this series except the first are negligible,

$$m \doteq \pi v D A_1 e^{-a_1^2 \alpha \theta} J_1\left(\frac{a_1 D}{2}\right) \quad . \; . \; . \; . \quad (9)$$

Differentiating with respect to θ

$$\frac{dm}{d\theta} = -\pi v D A_1 a_1^2 \alpha e^{-a_1^2 \alpha \theta} J_1\left(\frac{a_1 D}{2}\right) \quad . \; . \quad (10)$$

Substituting from Equation 9

$$\frac{dm}{d\theta} = -a_1^2 \alpha \, m \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

But

$$\frac{dm}{d\theta} = V \frac{dm}{dX} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (12)$$

whence

$$\frac{d\left(\ln \frac{m}{m_0}\right)}{dX} = -\frac{a_1^2 \alpha}{V} \quad (13)$$

where m_0 is the rate at which water is injected.

If Equation 13 holds, then a plot of the logarithm of the percentage of liquid in suspension versus duct length will give a straight line having a slope of $-a_1^2\alpha/V$.

The assumption that all terms except the first in Equation 8 are negligible is based on the expectation that the spray distribution approaches, after a reasonable length of test section and regardless of the original distribution, a form describable by a single term of the Bessel series. This condition was realized experimentally, as seen in Fig. 5.

The fact that α is to be evaluated from the straight portions of the graphs in Fig. 5 does not mean that the value found does not apply to the curved portions of the graphs. The same value can be used to correlate the data in the curved portions by taking higher terms in Equation 8. This involves evaluating the function in Equation 6.

Apparatus and Procedure -

The apparatus shown in Fig. 1 consisted of a seamless steel tube, a traversing water nozzle, a collection chamber, and auxilliary flow control and measurement equipment.

The seamless tube was 1.81 inches in internal diameter and 72 inches long, and provided a calming section 37 inches long and a test section 35 inches long. The tapered brass water nozzle was attached to the end of a section of 1/8 inch pipe 86 inches long which was centered on the axis of the test section by two sets of radial prongs. The 1/8 inch pipe extended through a packing gland in the up-stream end of the calming section in such a manner that the nozzle could be traversed along the axis of the test section. The end of the test section was chamfered and discharged into the collection chamber which consisted of an outer annular chamber, housing and positioning a tube of internal diameter of 1.75 inches on the axis of the test section as shown in Fig. 1. This tube was chamfered, and the whole collection chamber could be moved relative to the test section so that the width of the slot formed between the end of the test section and the inner tube of the collection chamber could be varied.

Air, supplied from a 500 cfm. source, flowed through the test section toward the collection chamber. Water, injected through the brass nozzle on the axis of flow as a solid jet, was atomized by the air stream. Part of the resulting spray deposited on the walls of the test section and flowed toward the collection chamber where it and a thin layer of the air stream passed into the annular chamber; the remainder of the undeposited spray passed into the inner tube of the collection chamber and was discharged to the atmosphere. The rate at which water flowed out of the collection chamber was measured and related to the air velocity, water velocity and rate, position of the nozzle relative to the collection chamber, and to the turbulence of the flow in the test section.

A negligible portion of the undeposited spray was carried into the collection chamber by the boundary layer of the air stream. Further, all the water flowing over the chamfered lip of the test section entered the collection chamber. Under conditions of constant nozzle position and air and water velocity, the width of the slot was varied through ten mm. The rate of collection rose rapidly and became sensibly constant at a slot width of three mm. All experiments were conducted with a slot width of eight mm.

Two other possibilities exist regarding the assumption that the water flowing out of the collection chamber represented all of the water depositing on the walls of the test section. It is possible that some of the water flowing down the walls of the duct was blown off and re-entered the air stream. From visual observation of a similar flow system in a glass tube, it was concluded that this effect was not important. The second possibility concerns atomization of drops impinging at high velocities on the wall. Great difficulty was encountered in the early part of the work in obtaining a steady state. Under constant operating conditions the collection rate sometimes rose steadily over a period of two hours or more. It was conjectured that lubricating oil, carried over as a fine mist from the air compressor, deposited on the walls of the test section in the interval between the starting of the air flow and the beginning of water injection, and as the oil was washed away by the water, the wall was progressively wetted, whereupon drops striking the growing liquid film were collected more efficiently than those striking the unwetted portions.

The choice of operating under wetted or non-wetted conditions was presented. The non-wetted condition was chosen. The test section was cleaned and swabbed with oleic acid at frequent intervals, and this resulted in stability of the collection rate at the lower values. The conclusions drawn from this research, therefore, are strictly valid for only the deposition of sprays on non-wetted surfaces.

A moderate level of turbulence was obtained by inserting two suitably supported 40 mesh copper screens into the calming section immediately following the air inlet, and attaching a brass plate drilled with 60 holes 0.140 inches in diameter to the 1/8 inch water line 3.33 inches above the tip of the nozzle. This plate moved with the nozzle and its position relative to the spray remained constant. When both the plate and screens were removed, the flow in the test section became highly turbulent because of the disturbance in the flow created by the right angle entry of the air into the duct

Provision for heating or cooling part of the air was made so that the temperature could be maintained at any value between outdoor temperatures and 50°C. Most of the experiments were performed at 30°C.

The assumptions of (1) uniform air velocity across the test section, (2) a concentration of liquid at the walls of zero, and (3) neglect of the spattering effect, blow-off, or other wall phenomenon, make the diffusivity, α, an average, apparent diffusivity for predicting net deposition on non-wetted walls. Further, α is also average with respect to drop size; if large drops diffuse to the wall more rapidly than small, the value of α would be expected to decrease with increasing distance from the nozzle.

In the experiments reported here, it was not possible to change the velocity of the air in the test section without also changing the velocity of the air relative to the water jet. It is possible that as the air velocity relative to the nozzle was increased the mean drop size of the spray was decreased, and that this may have caused changes in the apparent diffusivity not correctly attributable to the change of air velocity in the test section. Also, a number of other possible effects of increasing air velocity exist, viz.: (1) an increase in spattering effect, (2) an increase in blow-off, (3) a decrease in collection chamber efficiency, (4) a decrease in relative intensity of turbulence in the air stream, (5) the existence of a finite resistance to deposition at the walls, etc. Further research, in which the depositional process is isolated from the atomization process, is under way to determine which, if any, of these possibilities is important.

Results -

Six experiments in which the test section length was held at 35.4 inches were performed. In each experiment, the air velocity was held constant while the water rate was varied from 2 to 12 feet per second. The results are shown in Fig. 2, where the percent of water injected ~~that was~~ which deposited on the walls is plotted versus water velocity. It is observed that the deposition rate, at each air velocity, passed through a maximum as the water velocity was increased, and then became constant at a lower value when the water velocity was in excess of 10 feet per second. Increasing the water velocity through this range (2 - 10 feet per second) decreased the velocity of the air relative to the water by only six percent (at the lowest air velocity, 170 feet per second). It does not seem that this small change in the relative velocity can account for the large changes in the rate of deposition. Rather, the behavior illustrated in Fig. 2 appears to be associated with the air and water flow conditions at the nozzle as determined by the geometry of the nozzle. Observation revealed the existence of a stable, turbulent region (probably a vortex ring) in the air stream seated on the rim of the nozzle adjacent to the water jet. This eddy region interacted with the water stream and caused it to break up. As the water velocity was increased, the point at which the water stream was disintegrated moved away from the nozzle and then approached it again until the water velocity reached 10 feet per second, after which its position remained constant.

In these experiments the water was injected into the test section through a nozzle having an internal diameter of 0.0620 inches. It is not possible to deduce with certainty whether the behavior illustrated in Fig. 2 is a function of water rate or of the water velocity. Accordingly, a nozzle having an internal diameter of 0.0978 inches was substituted and two experiments were performed. In one case the water rates were matched at different velocities and in the other the water velocities were matched at different rates. In the experiments reported in Fig. 3, the air velocity was approximately 300 feet per second. The two upper curves represent data obtained in experiments in which the water rates were 550 and 275 ml. per minute and the water velocity was 6.2 feet per second. In the experiment represented by the lower curve, the water rate was 550 ml. per minute and the velocity was 12.4 feet per second. It is seen that, regardless of water rate, nearly the same percentage deposition rate is obtained when velocities are matched.

Obviously, the apparent diffusivity of the spray varies greatly and in no simple manner when the water velocity is increased. However, the theory of eddy diffusivity provides no method for correlating rates of deposition with water velocity. Hence, no further investigation of this effect of water velocity was made. The effect was observed and isolated with respect to the eddy-diffusional process, as follows.

The percentage rate of deposition in Fig. 2 was observed to be independent of the velocity of the water at the nozzle when that velocity was in excess of 10 feet per second. It was expected that the deposition data could be correlated by the simple eddy diffusion theory if the water velocity were maintained above this value. This expectation was realized.

The use of the multiple-orifice plate in the system gave a maximum deposition in a test section 35.4 inches long of less than 30 percent. It is obvious that the curves in Fig. 3 must, if extended by the use of longer test sections, eventually bend over and approach asymptotically some horizontal line corresponding to a percentage deposition not greater than 100 percent. In order that this bending over might be observed, the turbulence level in the test section was raised by removing both the screens and the multiple-orifice plate. This resulted in flow in the test section at a high level of turbulence generated in the entrance to the calming section. The flow was accompanied by pressure fluctuations amounting to one-half inch of water of one-fifth to one-half seconds duration. This high level of turbulence resulted in a percentage deposition rate of 75 to 80 percent in a test section 35.4 inches long, and the rate, when plotted versus test section length, gave curves such as that shown in Fig. 4. A decrease in the rate of deposition with increasing test section length was observed, as expected, for lengths greater than 15 inches. The characteristic inflection of the curves in Fig. 3 is not evident.

A series of experiments at five different air velocities and a constant water velocity of 12 feet per second was performed in which the test section length was varied from zero to 35 inches. At this water velocity, the rate of deposition, and hence the diffusivity also, is independent of the water velocity, as seen in Fig. 2. The data were plotted according to Equation 13, and the results are shown in Fig. 5. It is seen that the plots, after initially curving, become straight, and that the slopes of the straight portions decrease with increasing air velocity. The data for the air velocity of 263 feet per second break away sharply from the straight line in the region of test section length of 75 to 90 cm. This effect was reproducible, but the cause of it is not known.

The values of the diffusivities corresponding to the slopes of the curves were calculated according to Equation 13 and are shown graphically in Fig. 6, where the diffusivity is plotted against air velocity on log-log coordinates. It is seen that a straight line having a slope of 0.6 was obtained so that

$$\alpha = 0.11 V^{0.6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where α is in the range of 20 to 30 in.^{2}/sec.

Sherwood, et al. (16) carried out a series of experiments similar to those reported here except that CO_2 gas was injected instead of water, and the measurements were made close to the nozzle in the region where the walls of the duct had negligible effect on the spread of the CO_2 stream. They found diffusivities of CO_2 of the order of 1 to 2 in.^{2}/sec.; moreover, they found the diffusivity to be proportional to the air velocity instead of the 0.6 power of the velocity as found here.

Conclusions

It was concluded that the average apparent diffusivity for the prediction of the net deposition of a spray on the non-wetted walls of a straight duct under the conditions of high turbulence and method of spray generation used is of the order of 20 to 30 in.^{2}/sec., and is proportional to the 0.6 power of the velocity of the air stream.

Symbols

A' = cylindrical surface co-axial

A_n = nth. coefficient in Bessel series

a_n = nth. root of Bessel function

C = concentration of liquid

D = diameter of test section

J_n = nth. order Bessel function of first kind

M = mass of liquid

m = mass rate of flow of liquid through any section of test section (not including liquid on walls of duct)

m_0 = mass rate of liquid injection

r = cylindrical coordinate, distance from axis of test section

V = average velocity of air stream in test section

x = linear coordinate, distance from spray nozzle along axis of test section

α = average apparent diffusivity of liquid in spray

θ = time

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Fig. 1

Apparatus for the Study of the Deposition of Sprays on the Walls of a Straight Duct

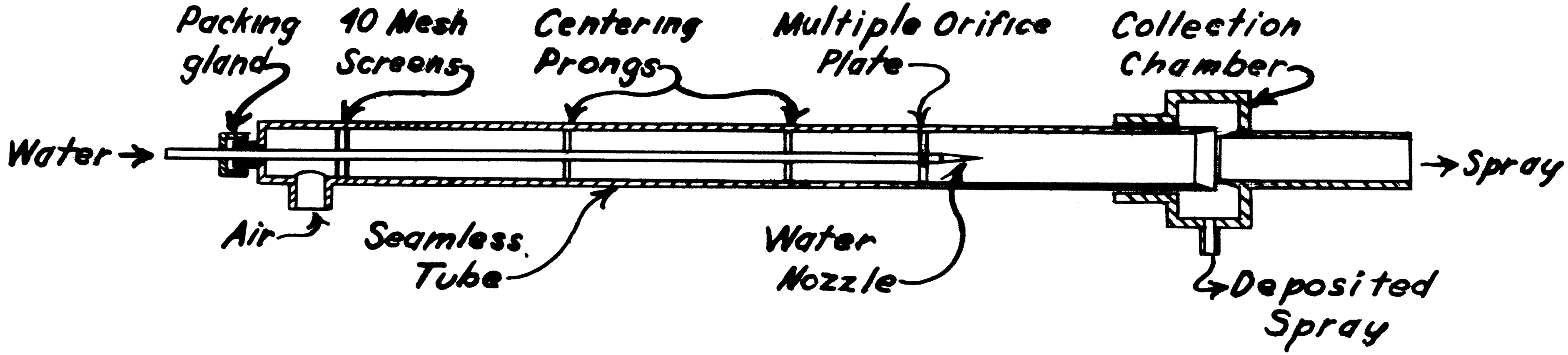

Fig. 2

Deposition of a Spray on the Walls of a Duct at Moderate Levels of Turbulence and High Water Velocity

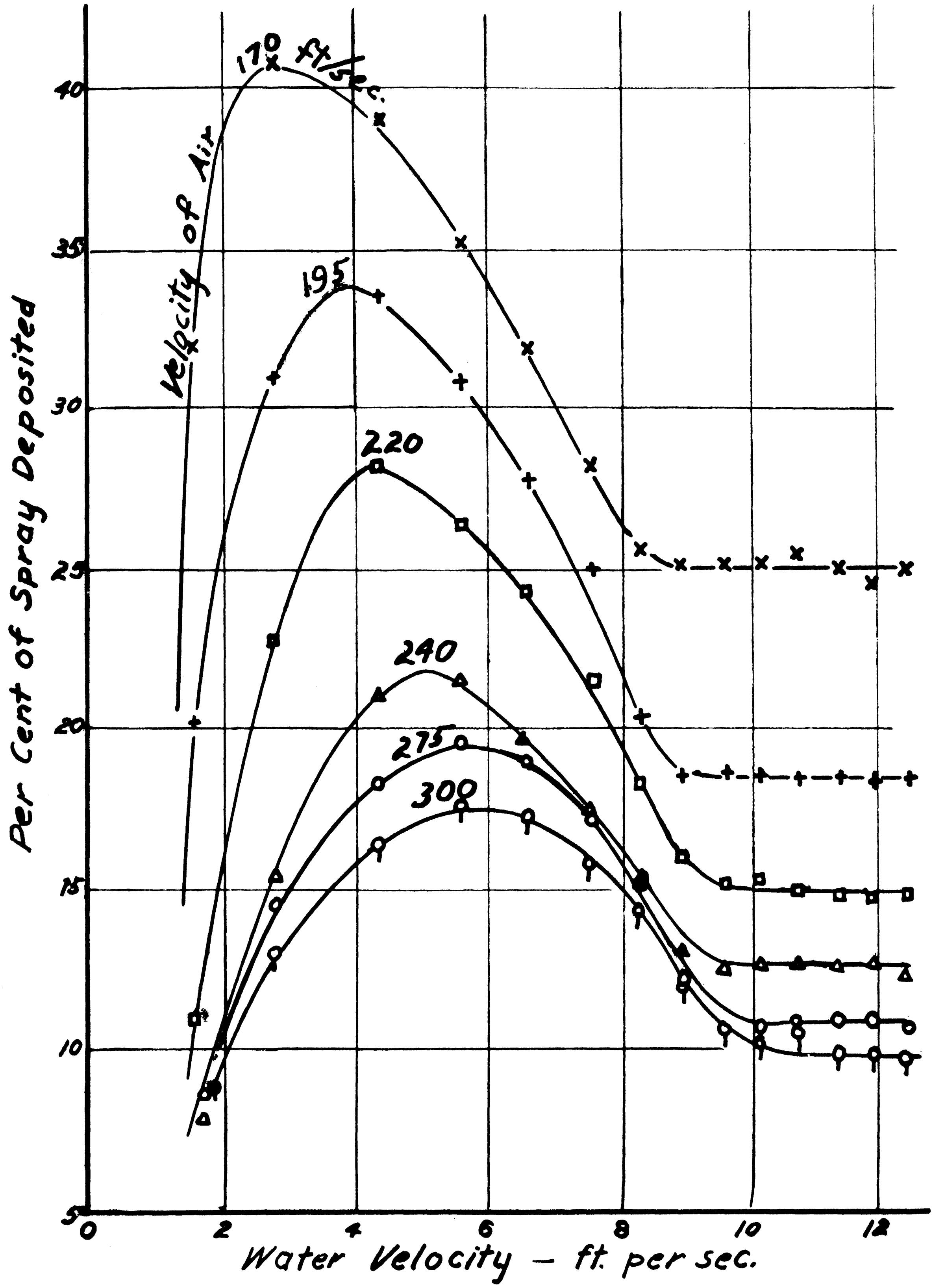

Fig. 3

Deposition of a Spray on the Walls of a Straight Duct: Relation to Water Rate and Velocity
Air Velocity 300 ft./sec.

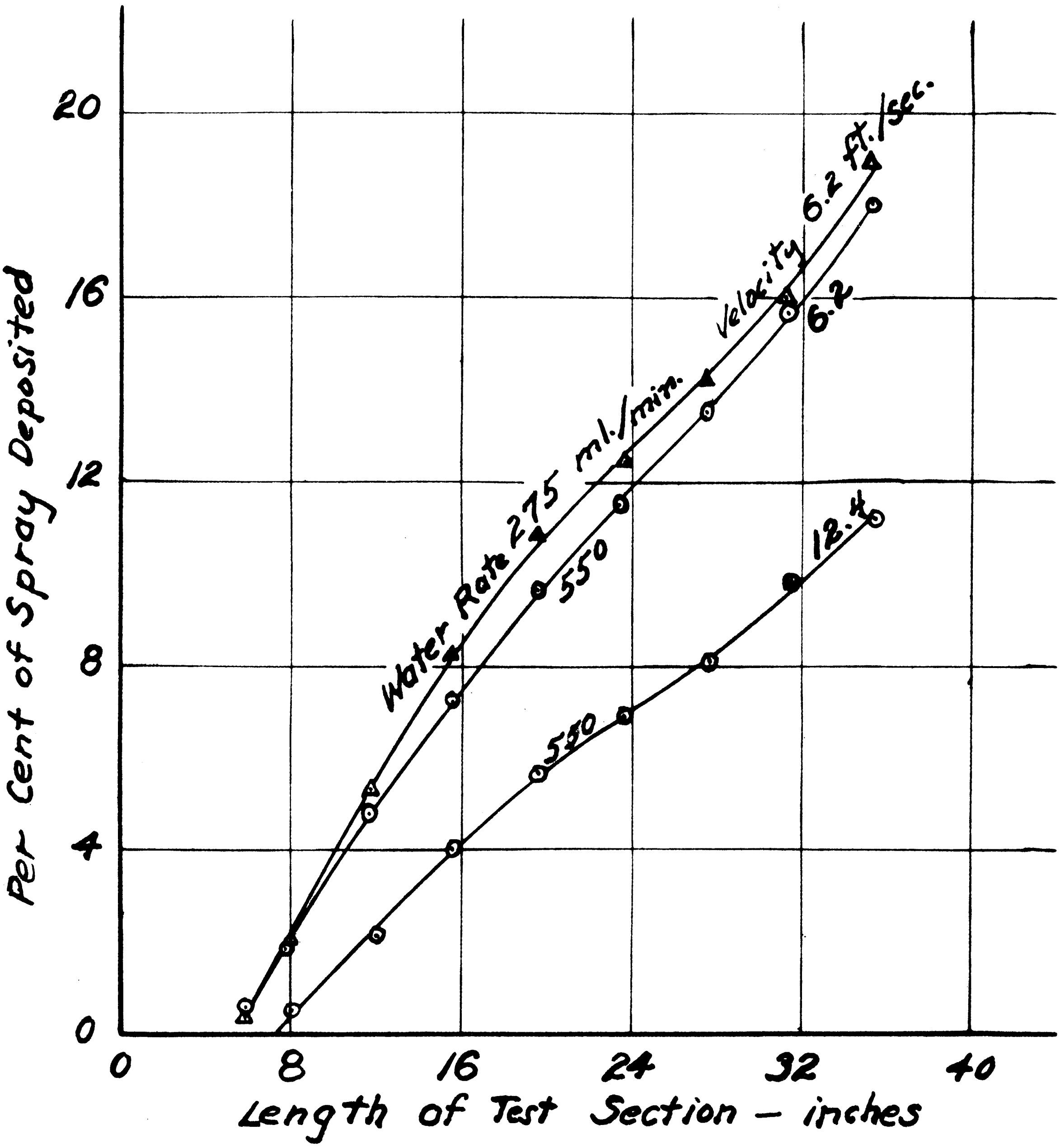

Fig. 4

Deposition of a Spray on the Walls of a Straight Duct at High Levels of Turbulence. Air Velocity 180 ft./sec.

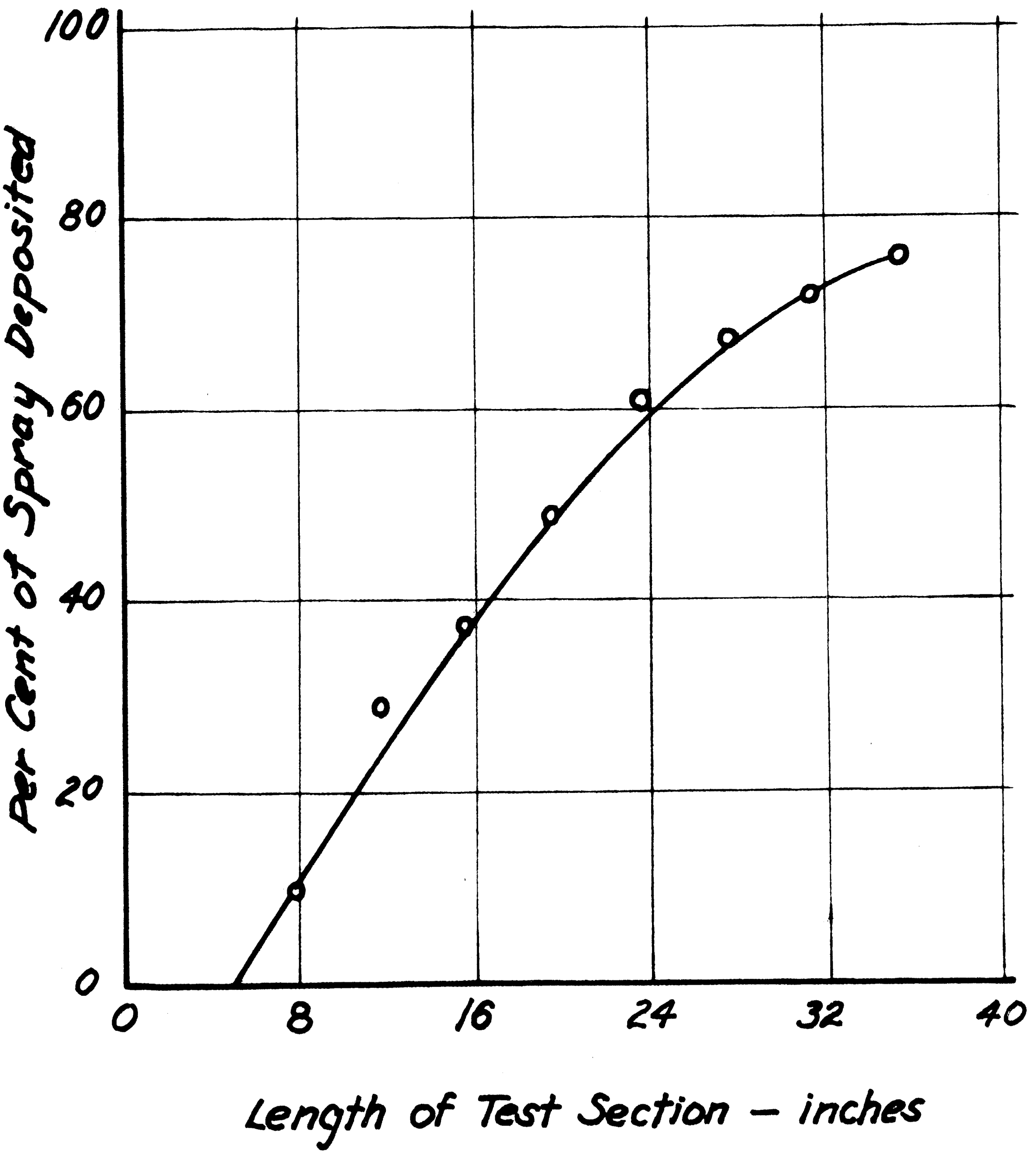

Fig. 5

Deposition of Spray on the Walls of a Straight Duct at High Levels of Turbulence

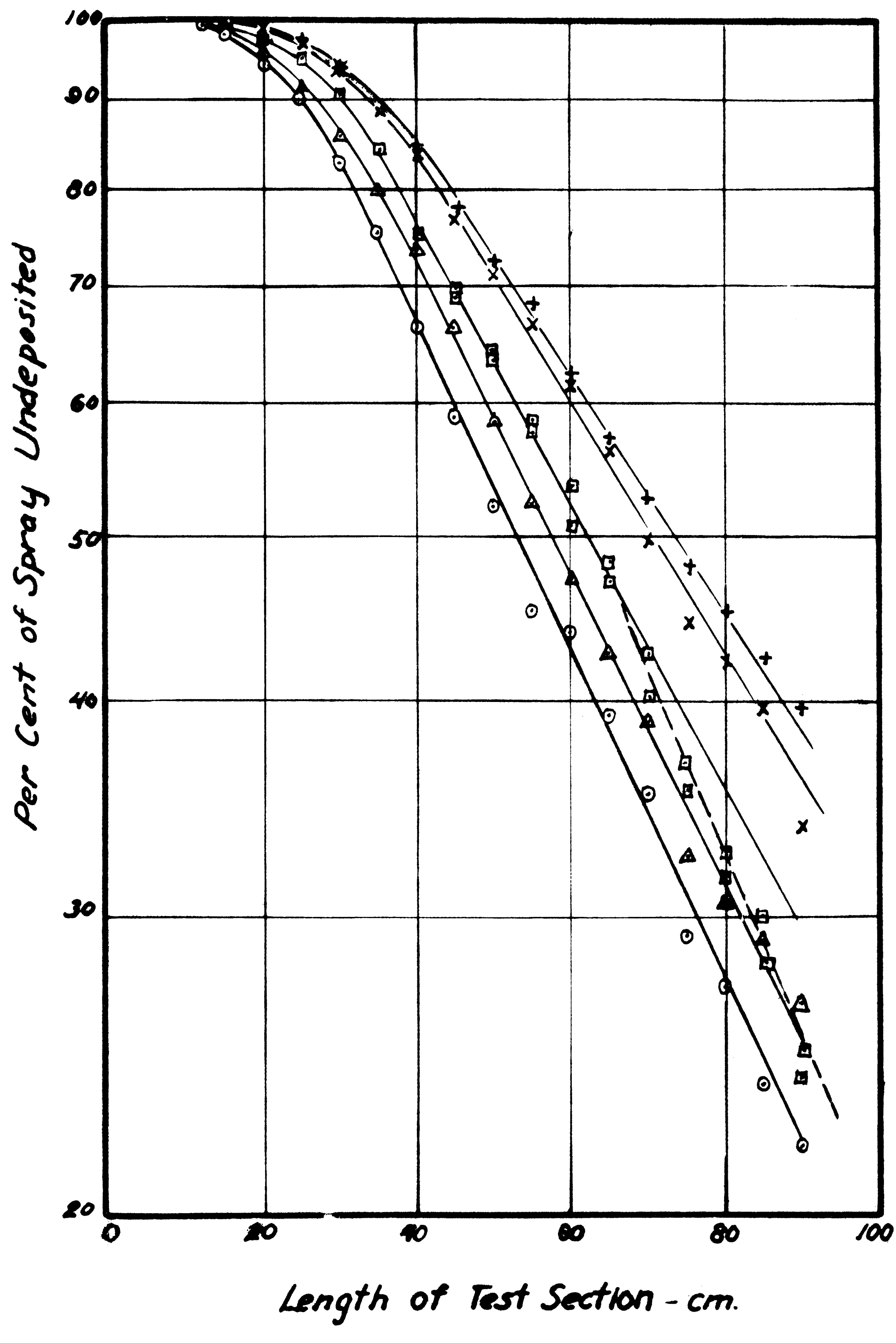

Fig. 6

Deposition of a Spray on the Walls of a Straight Duct; Diffusivity of Water Droplets

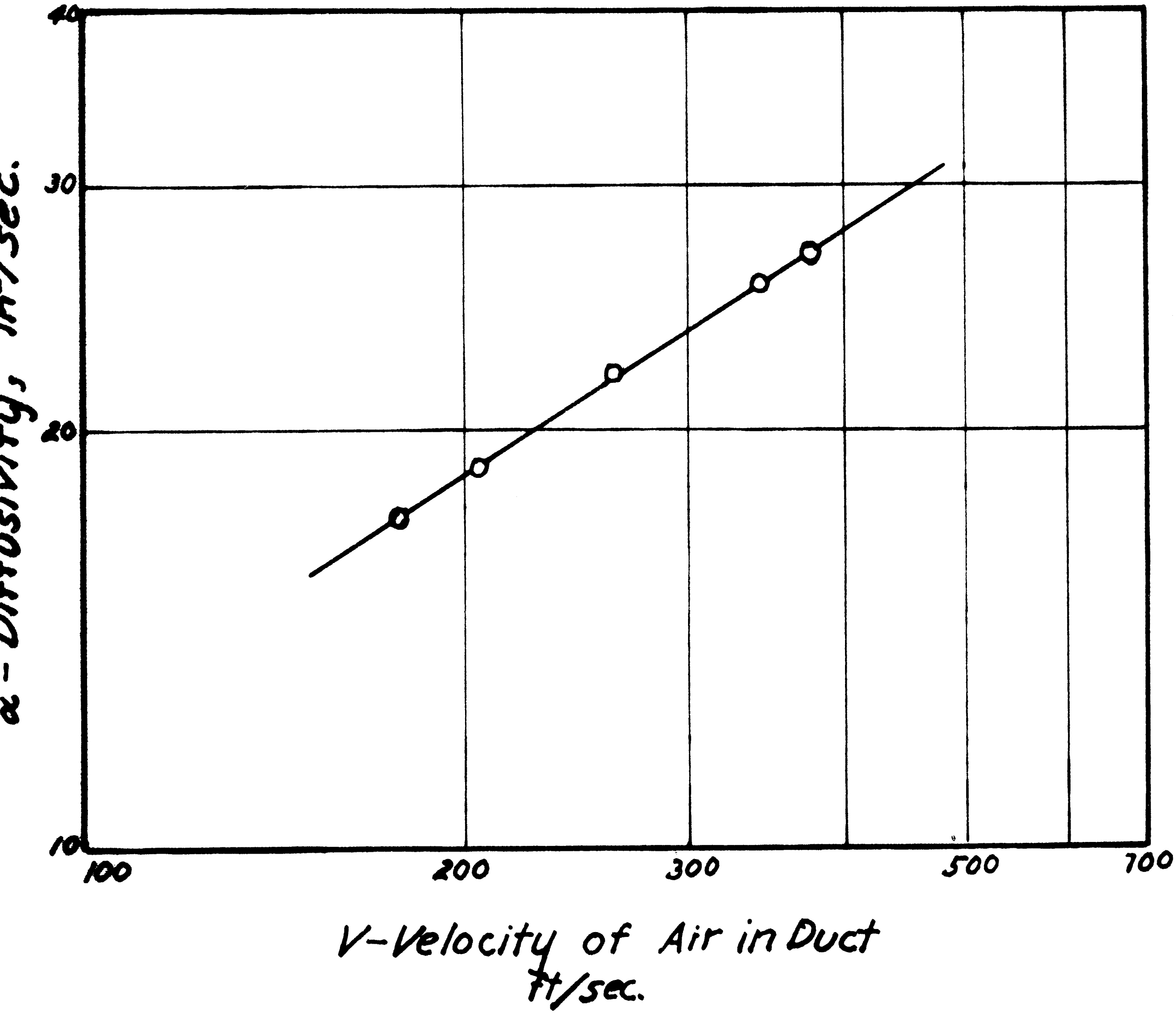

Fig. 6

Deposition of a Spray on the Walls of a Straight Duct; Diffusivity of Water Droplets