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THE NUCLEAR STRUCTURE OF ^{24}Na AND SOME REMARKS ON THE NUCLEAR STRUCTURE OF ^{25}Mg

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Abstract: The experimental data on the ^{23}Na(d, p)^{24}Na reaction at $E_d = 7.8$ MeV reported in a previous paper [1]) (referred to as I) are used for a tentative interpretation of the even parity levels of ^{24}Na at low excitation. Comparison with the interpretation of the nuclear structure of ^{25}Mg leads to assignment of rotational bands in ^{24}Na (table 5 and fig. 6). The spectroscopic data of the (d, p) reaction on ^{23}Na and ^{24}Mg and the position of the intrinsic states of the rotational bands in ^{24}Na and ^{25}Mg yield a reasonable fit to the unified model for $\beta \approx 0.2$ at $\kappa = 0.08$, to be contrasted with a value of $\beta \approx 0.4$ to 0.5 at $\kappa = 0.08$ deduced from the magnetic moment, the total energy of the nucleus in the Nilsson model and the quadrupole moments of nuclei in this region. The rotation-particle coupling distorts the rotational structure of ^{24}Na and ^{25}Mg considerably. The unified model alone and this model including a residual central two-body interaction between the two odd nucleons predict a ground state spin 1^+ instead of 4^+ for ^{24}Na. Taking into account a two-body spin-orbit force and a tensor force may yield a correct prediction of the ground state spin.

1. Introduction

The structure of several nuclei in the region around mass number 25 has been described with more or less success in terms of the unified model. Therefore, we also try to use this model for the description of the nucleus ^{24}Na. A calculation in terms of the shell model is prohibited by the complexity of the problem viz. the construction of a properly antisymmetric wave function which describes all possible configurations of a system of eight particles, three protons and five neutrons, in the 1d-2s shell is too difficult to be handled.

2. Useful Formulae

We summarize the formalism of the unified model in as far as necessary for the present paper. The Hamiltonian is

$$H = H_{\text{single particle}} + H_{\text{collective}} + H_{\text{interaction}} + H_{\text{residual}}, \tag{1}$$

which in the case of ^{24}Na can be simplified to

$$H = H_{\text{Nilsson}} + H_{\text{rotational}} + H_{\text{rotation-particle coupling}} + H_{\text{residual}}. \tag{2}$$

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In the case that the matrix elements of H_{RPC} and H_{residual} vanish, the basic set of wave functions for odd-mass nuclei (^{23}Na, ^{25}Mg) is

$$\psi = \left(\frac{2J+1}{16\pi^2}\right)^{\frac{1}{2}} \sum_j c_{j\Omega}\{\chi_{j\Omega} D^J_{MK} + (-1)^{J-j}\chi_{j-\Omega} D^J_{M-K}\}$$
$$J = K,\ K+1,\ K+2, \ldots, \quad (3)$$

and for odd nuclei (^{24}Na)

$$\psi = \left(\frac{2J+1}{16\pi^2}\right)^{\frac{1}{2}} \sum_{j_1 j_2} c_{j_1\Omega_1} c_{j_2\Omega_2}\{\chi_{j_1\Omega_1}\chi_{j_2\Omega_2} D^J_{MK} + (-1)^{J-j_1-j_2}\chi_{j_1-\Omega_1}\chi_{j_2-\Omega_2} D^J_{M-K}\} \quad (4)$$

if $K = \Omega_1 + \Omega_2$, $J = K, K+1, K+2, \ldots$,

$$\psi = \left(\frac{2J+1}{16\pi^2}\right)^{\frac{1}{2}} \sum_{j_1 j_2} c_{j_1\Omega_1} c_{j_2\Omega_2}\{\chi_{j_1\Omega_1}\chi_{j_2-\Omega_2}(-1)^{j_2-\frac{1}{2}} D^J_{MK} + (-1)^{J-j_1-\frac{1}{2}}\chi_{j_1-\Omega_1}$$
$$\chi_{j_2\Omega_2} D^J_{M-K}\} \quad (5)$$

if $K = \Omega_1 - \Omega_2$, $J = |K|, |K|+1, |K|+2, \ldots$. Here, $\boldsymbol{J}$ is the spin of the nucleus, M the projection of $\boldsymbol{J}$ on the space-fixed z axis, K the projection of $\boldsymbol{J}$ on the body-fixed z' axis, the symmetry axis of the axially symmetric nucleus, $\boldsymbol{j}_i$ is the spin of an odd particle and Ω_i the projection of $\boldsymbol{j}_i$ on the z' axis, $\chi_{j_i\Omega_i}$ is a shell model wave function for the state of an odd particle, D^J_{MK} is the rotational wave function and $c_{j_i\Omega_i}$ is the expansion coefficient of the Nilsson wave functions in terms of shell model wave functions. These coefficients are listed by Newton [2]).

The spectroscopic factor [3]) of the (d, p) reaction in this model is

$$S_l = \sum_j S(lj), \quad (6)$$

where

$$S(lj) = \rho^2 \langle \mathrm{f}|\mathrm{i}\rangle^2 \frac{2J_\mathrm{i}+1}{2J_\mathrm{f}+1} (J_\mathrm{i} j K_\mathrm{i}\Omega | J_\mathrm{f} K_\mathrm{f})^2 c^2_{(l)j\Omega}. \quad (7)$$

A convenient quantity is

$$\frac{2J_\mathrm{f}+1}{2J_\mathrm{i}+1} S_l = \sum_j \rho^2 \langle \mathrm{f}|\mathrm{i}\rangle^2 (J_\mathrm{i} j K_\mathrm{i}\Omega | J_\mathrm{f} K_\mathrm{f})^2 c^2_{(l)j\Omega}, \quad (8)$$

which is actually the number derived from the experiment as the ratio between the experimental differential cross section and the theoretical differential cross section for capture of a neutron into a single particle state. We proposed to call this quantity the *spectroscopic ratio* (see I eqs. (5) and (7)). Further, $\boldsymbol{J}_\mathrm{i}$ and $\boldsymbol{J}_\mathrm{f}$ are the spins of the initial and final nucleus with projections K_i and K_f on the z' axis, $\boldsymbol{l}$ is the orbital angular momentum transferred by the neutron to the final nucleus, $\boldsymbol{j} = \boldsymbol{l} \pm \frac{1}{2}$, the total angular momentum of the neutron in the final nucleus with projection Ω on the z' axis, $c_{(l)j\Omega}$ is the expansion coefficient of the Nilsson state, into which the neutron has been captured, in terms of shell model wave functions, $\langle \mathrm{f}|\mathrm{i}\rangle$ is the core overlap as discussed

by Macfarlane and French [3]). In many cases $\langle f|i\rangle^2 = 1$ (see ref. [3])). These authors state that $\rho = \sqrt{2}$ if either K_f or K_i equals zero, and $\rho = 1$ otherwise. This rule applies if either the initial or final nucleus is an even nucleus, for which $K = 0$. However, this statement is not complete, because $\rho = 1$ also for an odd nucleus with $K = 0$ as can be seen from the fact that states in odd nuclei with $K = 0$ are not degenerate.

A convenient sum rule, assuming $\langle f|i\rangle^2 = 1$, is

$$\sum_{J_f} \frac{2J_f+1}{2J_i+1} S(lj) = c^2_{(l)j\Omega}. \tag{9}$$

This sum is over all J_f of a rotational band. The rule is due to the normalization of the Clebsch-Gordan coefficients. The sum over all l components of all members of a rotational band is

$$\sum_{J_f l} \frac{2J_f+1}{2J_i+1} S_l = \sum_{J_f lj} \frac{2J_f+1}{2J_i+1} S(lj) = 1, \tag{10}$$

due to the normalization of the single particle wave functions, viz. $\sum_{lj} c^2_{(l)j\Omega} = 1$. From eqs. (8)-(10) it is evident that the condition stated in eq. (8) of I is correct.

The energy eigenvalues of the Hamiltonian (2) for an odd nucleus are the following: The expectation value of the single particle Hamiltonian has been tabulated by Newton [2]) as a function of the deformation β. Newton's table also lists the coefficients $c_{j\Omega}$.

$$\langle H_{rot}\rangle = \frac{\hbar^2}{2\mathscr{I}} \left(\{J(J+1) - 2K^2 + 2\Omega_1\Omega_2\} + \langle j_1\rangle^2 + \langle j_2\rangle^2\right) \tag{11}$$

where

$$\langle j^2\rangle = \sum_j c^2_{j\Omega} j(j+1).$$

Here, $\mathscr{I}$ is the moment of inertia. For a rigid body [4])

$$\mathscr{I}_{rigid} = \tfrac{2}{5} mAR_0^2(1+0.31\beta), \tag{12}$$

where $R_0 \approx 1.2 \times A^{\frac{1}{3}}$ fm. Actually $\mathscr{I} \leqq \mathscr{I}_{rigid}$. The rotation-particle coupling [5])

$$H_{RPC} = \frac{\hbar^2}{2\mathscr{I}} [(j_{1+}j_{2-} + j_{1-}j_{2+}) - (J_+ j_{1-} + J_- j_{1+}) - (J_+ j_{2-} + J_- j_{2+})] \tag{13}$$

has matrix elements between states of different K bands, e.g.,

$$\langle JK \pm 1\Omega_1\Omega_2 \pm 1|H_{RPC}|JK\Omega_1\Omega_2\rangle = \frac{\hbar^2}{2\mathscr{I}} \{(J \mp K)(J \pm K + 1)\}^{\frac{1}{2}} \sum_{j'_2 j_2} c_{j'_2\Omega_2 \pm 1} c_{j_2\Omega_2} \{(j_2 \mp \Omega_2)(j_2 \pm \Omega_2 + 1)\}^{\frac{1}{2}} \delta_{j'_2 j_2}. \tag{14}$$

The matrix elements of $H_{residual}$ are discussed in sect. 7.

The intrinsic quadrupole moment in the unified model [4]) is

$$Q_{intr} = \frac{3}{(5\pi)^{\frac{1}{2}}} ZR_0^2 \beta(1+0.16\beta \ldots), \tag{15}$$

with $R_0 \approx 1.2A^{\frac{1}{3}}$ fm. The relation between Q_{intr} and the spectroscopic quadrupole moment is

$$Q_{\text{sp}} = \frac{3K^2 - J(J+1)}{(J+1)(2J+3)} Q_{\text{intr}}. \tag{16}$$

The magnetic dipole moment in the unified model [6]) is

$$\mu = \frac{1}{J+1} \sum_{\text{odd particles}} \{(g_s - g_l)\langle \boldsymbol{s} \cdot \boldsymbol{J} \rangle + (g_l - g_R)\langle \boldsymbol{j} \cdot \boldsymbol{J} \rangle + g_R \langle \boldsymbol{J}^2 \rangle\}]. \tag{17}$$

Here, $\boldsymbol{j}$ and $\boldsymbol{s}$ are total angular momentum and intrinsic spin of the odd particles, $\boldsymbol{J}$ is the spin of the nucleus and g_s, g_l and g_R are the gyromagnetic ratios for the intrinsic motion and the orbital motion of the odd particles and for the motion of core, respectively. The last is usually estimated to be $g_R \approx Z/A$.

3. Comparison of Data on ^{24}Na and ^{25}Mg

Litherland, McManus, Paul, Bromley and Gove [7]) have interpreted the structure of ^{25}Mg and ^{25}Al. They recognized rotational bands built on intrinsic states of the odd neutron and proton, respectively, in Nilsson orbitals 5, 9 and 11. From the experimental data they derive values of $\hbar^2/2\mathcal{I}$ for these bands from 150 to 230 keV in the case of ^{25}Mg. A reasonable fit to several data is obtained for a deformation parameter η = 3-6 and a spin-orbit coupling parameter $\kappa = 0.08$. These values correspond to a deformation $\beta \approx \kappa\eta \approx 0.24$ to 0.48.

Recent spectroscopic data on the ^{24}Mg(d, p)^{25}Mg reaction at $E_d = 10.1$ MeV have been published by Middleton and Hinds [8]). We have made a crude DWBA analysis of these data for those levels which are of interest for the comparison between ^{25}Mg and ^{24}Na, using the theoretical DWBA curves for ^{24}Na at $E_d = 7.8$ MeV as presented in I. The results of this analysis do not check completely with those of the DWBA analysis by Buck and Hodgson [9]). These authors obtain the optical model parameters from a best distorted wave fit to the (d, p) transition to the 0.581 MeV level. These parameters do not give a best fit to the elastic scattering of 10.1 MeV deuterons on ^{24}Mg. A best fit to these data is also reported in ref. [9]). The difference in such sets of optical model parameters and their fits has been discussed in sect. 6 of I. For the ^{23}Na(d, p)^{24}Na reaction Satchler has used the optical model parameters from a best fit to the elastic scattering data. This difference in approach for the DWBA analysis causes probably that both sets do not agree completely. In addition, the difference between the two nuclei and the bombarding energies do have some effect. However, the author thinks that using the same method of analysis of the data on both nuclei is better for the present comparison of the data. Table 1 shows the result.

Information about the deformation of ^{24}Na can be obtained from the magnetic moment and the quadrupole moment. The measured value [10, 11]) of the magnetic moment is 1.688 n.m. Fig. 1 shows μ as a function of β for $\kappa = 0.08$ (from eq. (17)).

The experimental value leads to an estimated $\beta = 0.52$. In this region of nuclei the quadrupole moments of ^{23}Na and ^{27}Al have been measured [12]), $Q_{sp} = 0.10$ b and $Q_{sp} = 0.15$ b, respectively. With eqs. (15) and (16) we obtain $\beta = 0.48$ for ^{23}Na and $\beta = 0.32$ for ^{27}Al. These values of β indicate that ^{24}Na and ^{25}Mg may have a similar deformation. Anyway it is useful to look for similarities in the structure of these nuclei as reflected in the spectroscopic ratios of the (d, p) transitions to levels in these nuclei.

TABLE 1

A crude DWBA analysis of some transitions of the ^{24}Mg(d, p)^{25}Mg reaction, analogous to the analysis of the ^{23}Na(d, p)^{24}Na reaction [1]) (data from ref. [8]))

Level MeV	Spectroscopic ratio		$J\pi$	Nilsson orbital	Theoretical [a]) spectroscopic rato at $\beta \approx 0.20$	
	$l = 0$	$l = 2$			$l = 0$ [b])	$l = 2$
0.000		1.60	$\frac{5}{2}^+$	5		2.00
0.581	0.80		$\frac{1}{2}^+$	9	1.20	
0.976		0.76	$\frac{3}{2}^+$	9		0.42
1.960		0.58	$\frac{5}{2}^+$	9		0.38
2.564	0.33		$\frac{1}{2}^+$	11	0.48	
2.798		1.05	$(\frac{3}{2}^+)$	11		1.48
3.901			$(\frac{5}{2}^+)$	11		0.04

[a]) See fig. 4.

[b]) For a discussion of the transition to the 5.465 MeV level see sect. 3.

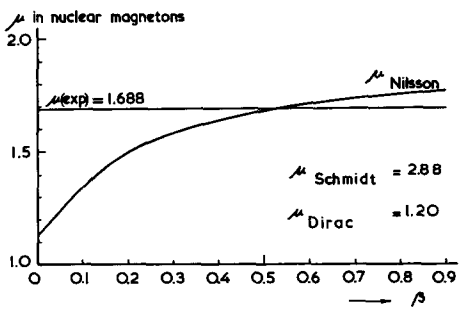

Fig. 1. The magnetic dipole moment μ of ^{24}Na in the Nilsson model as a function of the deformation β at $\kappa = 0.08$.

Fig. 2 shows the Nilsson diagram for positive values of the deformation using $\kappa = 0.08$. The occupation of levels by protons and neutrons is indicated for the case of ^{23}Na. ^{24}Mg should have an additional proton in orbital 7. For both nuclei the neutron from the (d, p) reaction can be captured into the orbitals 5, 9, 11 and 8 of the 1d-2s shell and into orbitals of higher shells. Table 2 shows the rotational bands occurring in the 1d-2s shell for ^{24}Na. Two rotational bands can be constructed upon

each Nilsson state, i.e., one with $K_+ = \Omega_p + \Omega_n$ and one with $K_- = |\Omega_p - \Omega_n|$ (see eqs. (4) and (5)).

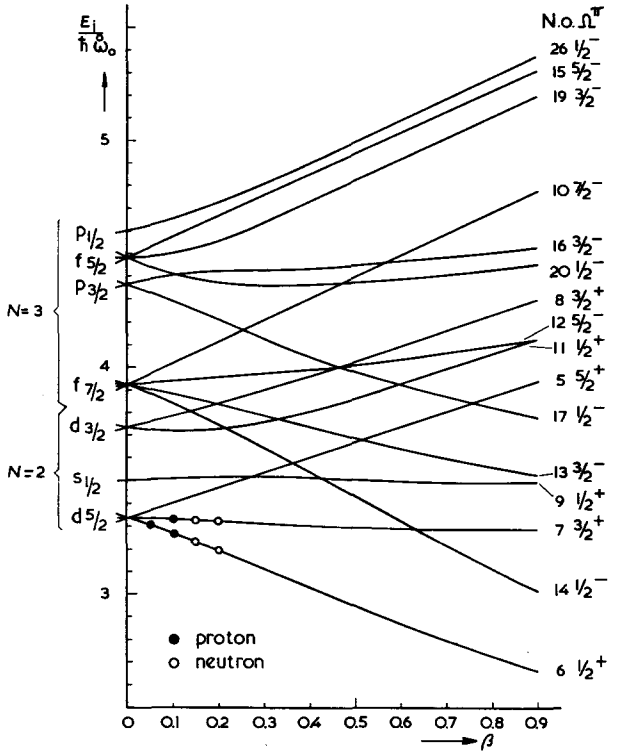

Fig. 2. The Nilsson diagram and the assumed occupation of the Nilsson orbitals by protons and neutrons for $^{23}Na_{g.s.}$ at $\kappa=0.08$. The occupation schemes for ^{24}Na and ^{25}Mg are that for ^{23}Na with one neutron in any of the unfilled orbitals and for ^{25}Mg an additional proton in orbital 7 (N.o. means odd neutron in Nilsson orbital).

TABLE 2

Intrinsic states and rotational bands of ^{24}Na in the (1d-2s) shell

Neutron in orbital		5		9		11		8	
Ω_n		$\frac{5}{2}$		$\frac{1}{2}$		$\frac{1}{2}$		$\frac{3}{2}$	
Intrinsic states [a])	$J=$	4	1	2	1	2	1	3	0
K bands	$K=$	4	1	2	1	2	1	3	0

[a]) Proton in orbital 7 with $\Omega_p = \frac{3}{2}$.

The Nilsson model also provides an estimate of the deformation of the nucleus. The value of β at the minimum in the total energy of the nucleus as a function of the deformation is defined as the equilibrium deformation [6]). Fig. 3 shows the result which is $\beta \approx 0.4$ to 0.5 if the odd neutron of ^{24}Na occupies an orbital of the 1d-2s shell.

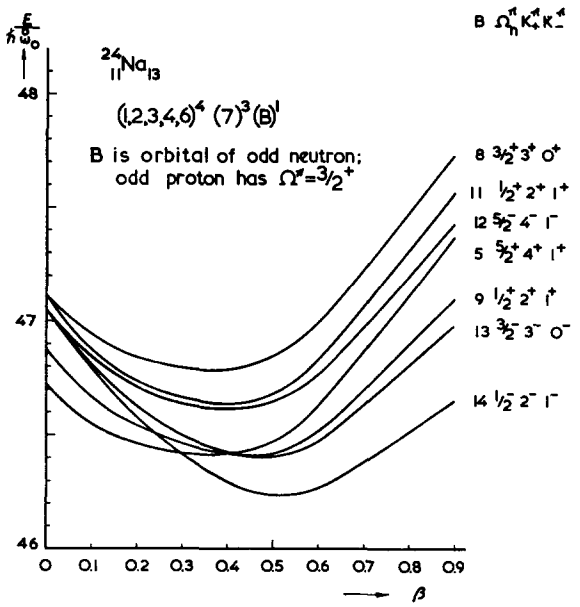

Fig. 3. The total energy of ^{24}Na in the Nilsson model for the configuration of fig. 2 with the additional neutron in orbital B as a function of the deformation β at $\kappa = 0.08$ ($K_+ = \Omega_p + \Omega_n$, $K_- = |\Omega_p - \Omega_n|$).

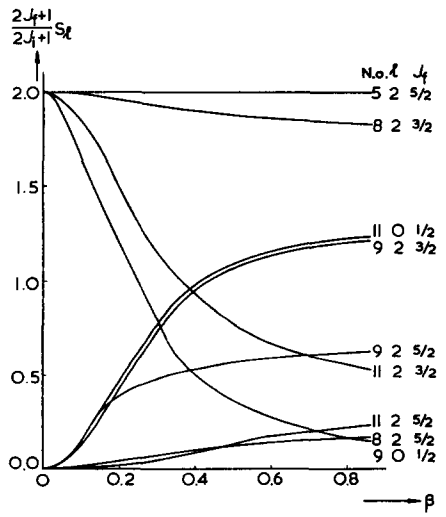

Fig. 4. The value of the spectroscopic ratio of the (d, p) reaction (eq. (8)) in the unified model for the members of the rotational bands on the states in the (1d-2s) shell for ^{25}Mg as a function of the deformation β at $\kappa = 0.08$ (N. o. means odd neutron in Nilsson orbital).

With the help of eqs. (6) and (7) and the table of Newton [2]) we can plot the spectroscopic ratio (8) of the (d, p) transition to levels of ^{25}Mg and ^{24}Na as a function of the deformation. Figs. 4 and 5 show these plots for the capture of the odd neutron into states of the 1d-2s shell. Again a value of $\kappa = 0.08$ has been used. The figures show that the spectroscopic ratio in the cases of orbitals 5 and 8 is independent or almost

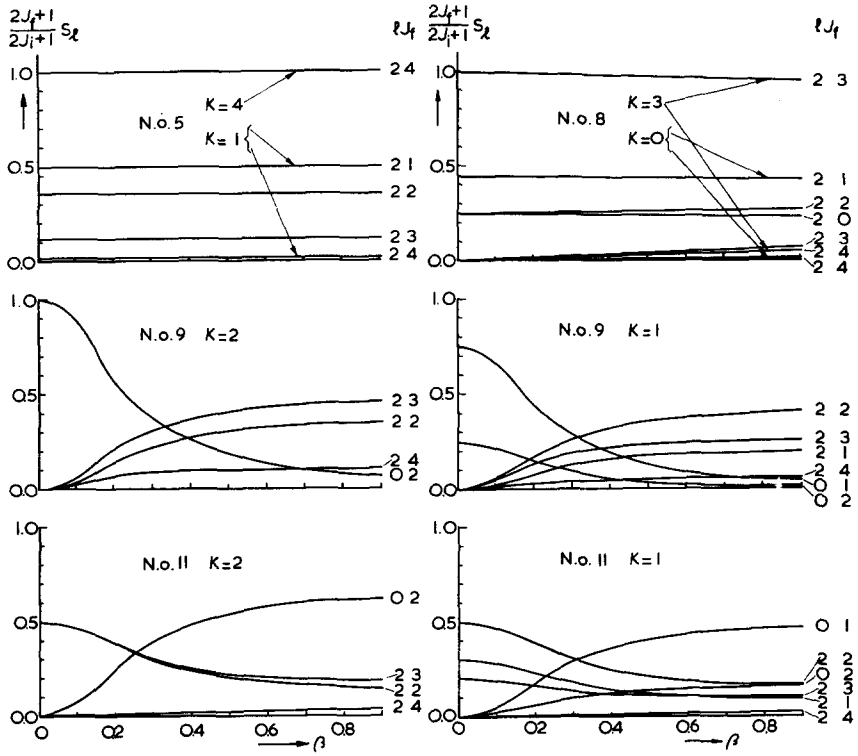

Fig. 5. The value of the spectroscopic ratio of the (d, p) reaction (eq. (8)) in the unified model for the members of the rotational bands on the states in the (1d-2s) shell for ^{24}Na as a function of the deformation β at $\kappa = 0.08$ (N.o. means odd neutron in Nilsson orbital).

independent of the deformation. Therefore, such cases can serve as a check either on the assumption that the core overlap $\langle f|i\rangle^2$ is complete or on the correct prediction of the absolute value of the single particle cross section by the DWBA analysis. On the other hand, the spectroscopic ratio in the cases of orbitals 9 and 11 is strongly dependent on the deformation and it may then provide an estimate of the deformation in addition to the information obtained from the magnetic moment and the quadrupole moment.

If we use the identification of Litherland *et al.* [7]) that the ground state of ^{25}Mg is the intrinsic state of the $K = \frac{5}{2}$ band on orbital 5, we see that the experimental value of the spectroscopic ratio 1.60 (table 1) is only 0.8 of the theoretical value 2 (fig. 4).

In the case of ^{24}Na we are led to identify the 4^+ ground state and the 1^+ excited state at 0.472 MeV as the intrinsic states of the $K = 4$ and the $K = 1$ band, respectively, on orbital 5. Here, the experimental values of the spectroscopic ratio are 0.69 and 0.39 (table 4 of I), viz. 0.7 and 0.8 times the theoretical values of 1 and 0.5, respectively. We may conclude that for ^{25}Mg as well as for ^{24}Na the core overlap or the DWBA analysis or a mixture of both is responsible for this factor 0.7 to 0.8. We may expect that the same will be the case for the transitions to other levels in ^{25}Mg and ^{24}Na. On this basis we find for ^{25}Mg a fit to the spectroscopic ratios of the transitions to the intrinsic states of the $K = \frac{1}{2}$ bands on orbitals 9 and 11, viz. to the 0.581 MeV and the 2.564 MeV level, respectively, at a deformation $\beta \approx 0.20$. At this value of β the theoretical spectroscopic ratio is 1.20 for orbital 9 and 0.48 for orbital 11 to be compared with the experimental values 0.80 and 0.33, respectively. In both cases the experimental values are again about 70% of the theoretical values. This value of β is just below the lower limit of the range of values $\beta \approx 0.24$ to 0.48 of Litherland *et al.* [7]). We can give some suggestions for the understanding of this problem. E.g., the deformation of the nucleus in states for which the last neutron is in orbitals 9 or 11, may be smaller than for states for which the neutron is in orbital 5. Or, the strength of the spin-orbit coupling parameter may be larger than 0.08, making $\beta \approx \kappa\eta$ larger. However, we do not have an independent measurement of this parameter, and we know from the analysis of nuclei in the 1p shell [12]) that this parameter is not unique for all nuclei in this shell.

In this discussion we have neglected the existence of a strong $l = 0$ (d, p) transition to the level at 5.465 MeV in ^{25}Mg, reported by Middleton and Hinds [8]). These authors list a value $(2J_f+1)\theta^2$ for this transition which is 1.16 times that of the transition to the 0.581 MeV level. Hinds, Middleton and Parry [13]) have previously obtained a value which is 2.22 times that for the latter transition at $E_d = 8.9$ MeV. Finally, Parkinson [14]) can make a crude estimate from the comparison of the angular distributions of these two transitions at 10°, 15° and 20° yielding a factor $\frac{2}{3} \pm 50\%$ between the two at $E_d = 7.8$ MeV.

Macfarlane and French [3]) remark: "It is very hard to account for a very large $2s_{\frac{1}{2}}$ component as high as 5.5 MeV in ^{25}Mg. The Q value of the transition in question is within 10 keV of that of the intense $l = 0$ transition to the first excited level of ^{13}C. In separating these groups, Hinds *et al.* [13]) may have underestimated the C contribution".

Sheline and Harlan [16]) think that the 5.465 MeV level might be the $K = \frac{1}{2}^+$ one phonon gamma-vibrational state on the $K = \frac{5}{2}^+$ band. We agree with Middleton and Hinds [8]) that this interpretation is not favoured since the reduced width would be unusually large for such a case of core excitation. Another possibility would be that the $\frac{1}{2}^+$ spin is determined by a hole in orbital 6 (see fig. 2), but again a poor core overlap (eq. (8)) would make the spectroscopic ratio small. Therefore, we agree with the general remark of Macfarlane and French.

However, their argument that the Q values of the considered transitions differ by

only 10 keV is not adequate. At an angle of 0° in the laboratory system the energies of the protons corresponding to these transitions differ by 0.06 MeV at E_d = 7.8 MeV and E_d = 8.9 MeV and by 0.08 MeV at E_d = 10.1 MeV due to the kinematics of the reaction. Table 3 shows these proton energies.

TABLE 3

The energy of the protons emerging from the (d, p) reaction at an angle of 0° in the laboratory system for some (d, p) transitions in connection with the discussion on the 5.465 MeV level in ^{25}Mg

Transition	Q value (MeV)	E_d = 7.8 MeV	8.9 MeV	10.1 MeV
		E_p(MeV)	E_p(MeV)	E_p(MeV)
^{12}C(d, p)^{13}C$_{3.090\ MeV}$	−0.37	7.32	8.41	9.59
^{13}C(d, p)^{14}C$_{6.091\ MeV}$	−0.14	7.55	8.64	9.82
^{23}Na(d, p)^{24}Na$_{4.94\ MeV}$	−0.21	7.54		
^{24}Mg(d, p)^{25}Mg$_{5.465\ MeV}$	−0.37	7.38	8.47	9.67

We suggest considering the possibility of the effect of another contaminant group. In natural C a 1.11 % admixture of ^{13}C is present which gives rise to the ^{13}C(d, p)^{14}C transition to the level at 6.091 MeV in ^{14}C. This contaminant caused trouble in the l assignment of the (d, p) transition to the 4.94 MeV level in ^{24}Na. In a preliminary examination [15]) we identified an s wave transition to this level. Closer investigation revealed that the forward peak in the angular distribution was due to this contaminant (see table 3). Its presence escapes observation at larger angles easily due to the fact that a sharp forward peak is characteristic for an l = 0 stripping distribution.

Table 3 shows that the energy of the protons of the ^{13}C(d, p)^{14}C$_{6.091\ MeV}$ transition differs by 0.18 MeV at E_d = 7.8 MeV and E_d = 8.9 MeV and by 0.15 MeV at E_d = 10.1 MeV from that of the protons of the ^{24}Mg(d, p)^{25}Mg$_{5.465\ MeV}$ transition. We think that the presence of this peak and that of the ^{12}C(d, p)^{13}C$_{3.090\ MeV}$ transition makes a background correction at small angles very uncertain yielding too high points for the transition to ^{25}Mg$_{5.465\ MeV}$. This uncertainty may explain the widely differing values of the reduced width [8, 13, 14]). We conclude that a closer examination of the 5.465 MeV level of ^{25}Mg may be necessary.

Turning our attention to ^{24}Na we find in table 4 of I, six levels, which are reached by s wave transitions in the (d, p) reaction, viz. the levels at 0.564, 1.341, 1.844, 3.409, 3.582 and 4.16 MeV. In table 4 we list the spectroscopic ratios for these levels and a tentative assignment of these levels to rotational bands. The spectroscopic ratios for these levels are found in fig. 5 at a deformation $\beta \approx 0.2$. The question arises again, as for ^{25}Mg, whether the states based on orbitals 9 and 11 are associated with a smaller deformation of the nucleus than for the states on orbital 5, or whether the spin-orbit coupling parameter should be chosen larger to make correspondence with the value $\beta \approx 0.5$ previously obtained from the magnetic moment, the quadrupole moment of

the nucleus in this region and the calculation of the equilibrium deformation in the Nilsson model.

We have now recognized certain similarities between the structure of ^{25}Mg and ^{24}Na. Another important point, which is also basic in table 4 for the assignment of levels in ^{24}Na to rotational bands, is that the sum of all spectroscopic ratios with

TABLE 4

Analysis of (d, p) transitions with s wave components to members of rotational bands in ^{24}Na

Level (MeV)	Spectroscopic ratio $l = 0$	J^π	K band	Nilsson orbital
0.564	0.10	$(2)^+$	2	9
1.341	0.62	1^+	1	9
1.844	0.20	$(2)^+$	1	9
Sum	0.92[a])			
3.409	0.22	$(2)^+$	2	11
3.582	0.027	$(1)^+$	1	11
4.16	0.007	$(2)^+$	1	11
Sum	0.254[b])			

[a]) To be compared with 0.80 for 0.581 MeV level in ^{25}Mg (see table 1).
[b]) To be compared with 0.33 for 2.564 MeV level in ^{25}Mg (see table 1).

$l = 0$ for the 0.564, 1.341 and 1.844 levels is about equal to the spectroscopic ratio for the 0.581 MeV level in ^{25}Mg, viz. 0.92 compared with 0.80. The same sum for the 3.409, 3.582 and 4.16 MeV levels is about equal to the spectroscopic ratio for the 2.564 MeV level in ^{25}Mg, viz. 0.254 compared with 0.33. This means that the amount of s wave in orbitals 9 and 11 is similar for ^{25}Mg and ^{24}Na, indicating that these two nuclei have a similar deformation.

However, if we accept the interpretation of s wave levels of ^{24}Na indicated in table 4, we notice that the values of the spectroscopic ratio (8) do not follow from eqs. (6) and (7) for a single value of the deformation for all bands. The ratio of the experimental spectroscopic ratio for the $J = 1$ and $J = 2$ members of the $K = 1$ bands is approximately 3 : 1 as predicted by eqs. (6) and (7) due to the Clebsch-Gordan coefficients. But, the ratio of the experimental spectroscopic ratios of the intrinsic states of the $K = 1$ and the $K = 2$ band on orbitals 9 and 11 is 6.2 and 0.12, respectively, instead of 0.75 for both as predicted by eq. (8), viz. the ratio is not as determined by the C. G. coefficient only. Other effects must be present if the above interpretation is assumed to be correct, e.g. a two-body interaction between the odd proton and the odd neutron. We shall come back to the possible influence of such an interaction in sect. 7. For further discussion of the similarities of ^{25}Mg and ^{24}Na the other members of rotational bands in ^{24}Na will have to be identified.

4. The Rotational Bands

Litherland *et al.* [7]) have already discussed the rotational structure of ^{25}Mg in detail. The new data of Middleton and Hinds [8]) show a poor fit for the spectroscopic ratios for the $\frac{3}{2}^+$ state at 0.976 MeV and $\frac{5}{2}^+$ state at 1.960 MeV of the $K = \frac{1}{2}$ band on orbital 9, viz. 0.76 and 0.58 experimentally compared with 0.43 and 0.39 theoretically at $\beta \approx 0.2$ in fig. 3. On the other hand the choice of the 2.798 MeV level as the $\frac{3}{2}^+$ state of the $K = \frac{1}{2}$ band on orbital 11 leads to good agreement between experiment and theory for the spectroscopic ratio of 1.05, which is 70% of the theoretical value of 1.48, which percentage is the same as that for the intrinsic states on orbitals 9 and 11.

TABLE 5

Tentative interpretation of ^{24}Na in terms of rotational bands

Nilsson orbital	K band	Level	$J\pi$	Spectroscopic ratio					$\hbar^2/2\mathscr{I}$ [c]) (MeV)
				experiment [a])		theory at →		β [b])	
		(MeV)		$l=0$	$l=2$	$l=0$	$l=2$		
5	4	0.000	4^+		0.69		1.00	any	0.322
		3.22	(5^+)						
5	1	0.472	1^+	(0.014)	0.39		0.50	any	0.510
		2.51	$(2)^+$		(0.08)		0.36		
9	2	0.564	$(2)^+$	0.10	(0.30)	0.14	0.43	0.55	0.220
		1.884	$(3)^+$		0.29		0.33		
		3.648	$(4)^+$		0.13		0.10		
9	1	1.341	1^+	0.62	(0.37)	0.63	0.02	0.10	0.125
		1.844	$(2)^+$	0.20	(0.39)	0.21	0.06		
		2.56	$(3)^+$		0.068		0.05		
11	2	3.409	$(2)^+$	0.22	(0.24)	0.31	0.36	0.23	
11	1	3.582	$(1)^+$	0.027	(0.09)	0.04	0.27	0.10	0.145
		4.16	$(2)^+$	0.007	(0.05)	0.013	0.47		
8	3	2.99	$(3)^+$		0.42		0.98	0.1-0.5	
8	0	3.623	$(0)^+$		0.27		0.27	0.1-0.5	

[a]) See table 4 of 1.

[b]) See fig. 5.

[c]) Rotation particle coupling and residual interactions have not been taken into account. For a rigid rotator $\hbar^2/2\mathscr{I}_{\text{rigid}} \approx 0.150$ MeV (from eq. (12)).

The comparison between the (d, p) data on ^{25}Mg and ^{24}Na leads to a tentative interpretation of the rotational structure of ^{24}Na. Table 5 shows the result of the analysis in terms of rotational bands. Fig. 6 shows the level diagram of ^{24}Na and these rotational bands. Fig. 3 shows that a rotational band on orbital 5, 9, 13 or 14 will

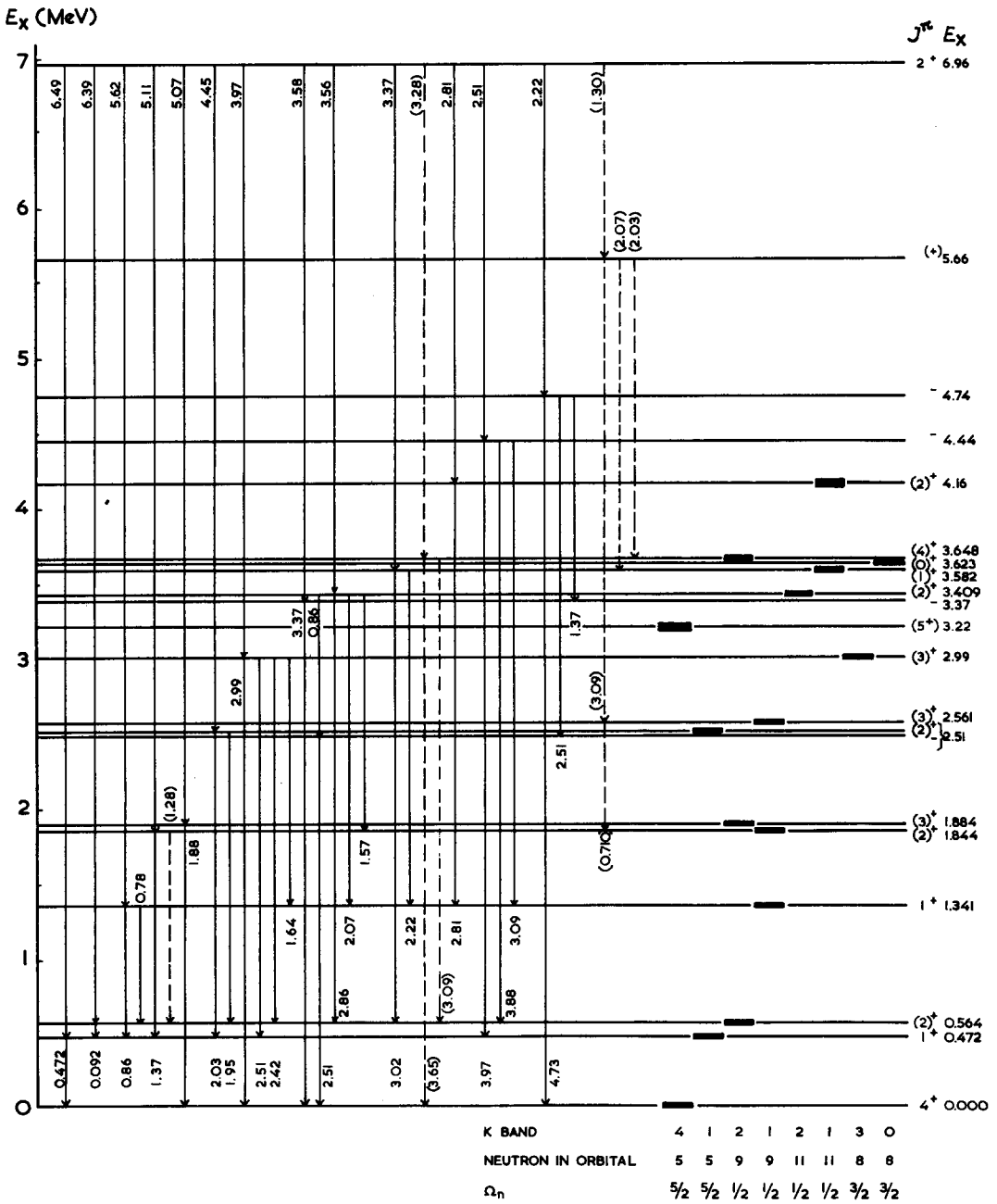

Fig. 6. A diagram of the tentative interpretation of the even parity levels of ^{24}Na. Some of the γ transitions of the (n, γ) reaction have been fitted tentatively (see table 5 of I). A few of the most important negative parity levels are also shown. The doublet at 2.51 MeV has been separated artifically. The separation does not imply an indication of the relative order of the odd and even parity state.

be the ground state band. Apparently, we have to identify the 4^+ ground state as the intrinsic state of the $K = 4$ band on orbital 5. The experimental spectroscopic ratio of the (d, p) transition to this state is 0.69 compared to 1.00 theoretically. The ratio of these two numbers checks well with the ratio of 0.7 to 0.8 between experimental and theoretical spectroscopic ratios for the intrinsic states of the rotational bands on orbitals 5, 9 and 11 in ^{25}Mg (see previous section). In that section we identified some members of the rotational bands on orbitals 9 and 11 (table 4).

The 1^+ level at 0.472 MeV is a likely candidate for the intrinsic state of the $K = 1$ band on orbital 5. The spectroscopic ratio of 0.39 for this level is about half of that for the ground state as expected in the Nilsson model. The 0.472 MeV state is de-excited by a 0.472 MeV γ transition of M3 character to the ground state and it has a life time [15]) of 19.6 msec. This life time is in reasonable agreement with the Moszkowski estimate [17]), $\approx$ 11 msec, for a single particle transition of M3 type, viz. the relative orientation of the angular momenta of the odd neutron and the odd proton is reversed involving a change in magnetic quantum number of the odd proton only.

We now try to find the next member of the $K = 1$ band on orbital 5 and the $K = 2$ band on orbital 9. The level at 1.884 MeV is a potential candidate for both bands. The experimental spectroscopic ratio of 0.29 for this level agrees well with the theoretical values of 0.36 and 0.33 for these possible assignments. If we choose this level to be the 2^+ member of the $K = 1$ band on orbital 5, then both the 1.844 MeV and the 1.884 MeV level would have spin 2^+. But residual interactions would probably not allow two levels with the same spin only 40 keV apart. Therefore, we identify the 1.884 MeV level tentatively as the 3^+ member of the $K = 2$ band on orbital 9. The 4^+ member of this band is now identified with the level at 3.648 MeV. The experimental spectroscopic ratio of 0.13 also agrees with the theoretical value of 0.10. This combination of levels yields $\hbar^2/2\mathscr{I} = 0.220$ MeV. The gamma ray of 3.65 MeV (table 5 of I; for all gamma rays we refer to this table; all gamma transitions are shown in fig. 6) may be due to the de-excitation of this level to the ground state, supporting this choice of the high spin of this state. The gamma ray of 3.09 MeV may be partly due to the de-excitation of this state to the 0.564 MeV level, but which part of the intensity of this gamma ray is due to this transition is not known. One should also expect to observe a 1.76 MeV γ transition between the 3.65 MeV and the 1.884 MeV states, a 1.32 MeV transition between the 1.884 MeV and the 0.564 MeV levels and probably a 1.884 MeV and a 0.564 transition from the last two levels to the ground state. However, the data on relative intensities and positioning of the gamma transitions are too scarce for a check of the consistency of the present analysis with the (n, γ) data. We can only guess that a 1.32 MeV gamma ray may not have been observed in the presence of the strong transition at 1.35 MeV following the ^{24}Na(β^-)^{24}Mg decay. The gamma ray at 1.88 MeV may de-excite the 1.884 MeV level to the ground state supporting the choice of a spin 3^+ for this level.

The presence or absence of a gamma ray also depends on the feeding of the level which it de-excites in the (n, γ) reaction. Coincidence measurements between the gam-

ma rays of this reaction or between protons and gamma ray(s) of the (d, pγ) reaction would be very useful in this context.

Finally, about the 1.844 MeV and 1.884 MeV states, the spins 2^+ and 3^+ of these levels, respectively, are also in agreement with the absence of β^- branches to these levels in the ^{24}Ne(β^-)^{24}Na decay [15]).

The next candidates for the 2^+ member of the $K = 1$ band on orbital 5 are the levels at 2.51 MeV and 2.56 MeV. The gamma transition of 2.03 MeV leads to an identification of the 2.51 MeV level as this member. Higher members of this band are probably too weak to be observed among the strong $l = 1$ transitions above 3 MeV excitation. Coincidence measurements between the gamma rays from the (n, γ) reaction may reveal these states. The inertia parameter of this band is very high for this choice, $\hbar^2/2\mathscr{I} = 0.510$ MeV. On the other hand for the $K = 1$ band on orbit 9 we find $\hbar^2/2\mathscr{I} = 0.125$ MeV, smaller than $\hbar^2/2\mathscr{I}_{\text{rigid}} \approx 0.150$ MeV (eq. (12)). The members of these bands do not have matrix elements of the rotation-particle coupling between them (eq. (14)), but these states may interact because of residual two-body interaction. The 1^+ and 2^+ member of the $K = 1$ band on orbital 5 embrace the 1^+ and 2^+ members of the $K = 1$ band on orbital 9. If there exists such an interaction, this will increase the distance between the 1^+ and 2^+ members of the $K = 1$ band on orbital 5 and decrease that between the 1^+ and 2^+ members of the $K = 1$ band on orbital 9, resulting in an increased value of $\hbar^2/2\mathscr{I}$ for the first and a lowered value of $\hbar^2/2\mathscr{I}$ for the last band, compared to the values estimated from only the two lowest members of the bands.

In addition the states of the $K = 1$ band on orbital 9 are affected by the rotation-particle coupling to the $K = 2$ bands on orbitals 9 and 11 and to the $K = 0$ band on orbital 8. Therefore, a detailed discussion of the moment of inertia is extremely difficult.

We may assign now the 2.56 MeV level as the 3^+ member of the $K = 1$ band on orbital 9. This choice is supported by the γ ray of 0.710 MeV, which may be a transition between the 2.56 MeV level and the 1.844 MeV level.

The $K = 4$ band on orbital 5 may have the level at 3.22 MeV as 5^+ member, yielding $\hbar^2/2\mathscr{I} = 0.322$ MeV for this band. Actually, one should expect to find a 5^+ state in a shell model analysis from a $(\pi d_{\frac{5}{2}})^3(\nu d_{\frac{5}{2}})^5$ configuration (cf. also the 5^+ ground state of ^{26}Al). In the Nilsson model such a state can only be found for a configuration with the odd neutron and the odd proton both in orbital 5 ($\Omega = \frac{5}{2}$), which configuration can only be reached by core excitation in the (d, p) reaction and moreover it can only be reached by an $l = 4$ (d, p) transition. The presence of a g wave function can only be explained by coupling of the states in the $N = 2$ shell to those in the $N = 4$ shell, which coupling has been ruled to be negligibly small in the Nilsson model. Therefore, a closer identification of the tentative assignment of the 3.22 MeV level in the present analysis is not possible.

From the $K = 1$ band on orbital 11 we have only identified the 1^+ and 2^+ members at 3.582 MeV and 4.16 MeV, respectively, yielding $\hbar^2/2\mathscr{I} = 0.145$ MeV, smaller than

$\hbar^2/2\mathscr{I}_{\text{rigid}}$. The many possibilities of rotation-particle coupling of this band to other bands may explain the low value of $\hbar^2/2\mathscr{I}$. Higher members of this band are probably overshadowed by strong $l = 1$ transitions above 4 MeV. The same situation is probably the case for higher members of the $K = 2$ band on orbital 11.

The 2.99 MeV level may de-excite with the 2.99 MeV γ ray to the ground state, indicating that this level has a high spin. Therefore, this level may be the 3^+ intrinsic state of the $K = 3$ band on orbital 8. The predicted spectroscopic ratio of the (d, p) transition to this state is 0.98, more than twice as large as the measured value 0.42, indicating that the core overlap is poor for this transition, or the reduction is due to rotation-particle coupling. The 4^+ member of this band is too weak to be observed.

Finally, the 3.623 MeV level may be the 0^+ intrinsic state of the $K = 0$ band on orbital 8. The experimental and theoretical spectroscopic ratio are both 0.27, suggesting a complete core overlap. Unfortunately, we cannot identify higher members of this band among the strong $l = 1$ transitions.

We may conclude that with the help of the information on ^{25}Mg we can make a tentative interpretation of the rotational structure of ^{24}Na. In the next section we discuss the distortion of this structure by the rotation-particle coupling.

In view of the difficulties in the interpretation of the even parity states no effort has been made to analyse the odd parity levels. Low lying p and f wave states are in agreement with the general trend of the Nilsson model (see figs. 2 and 3).

5. The Rotation-Particle Coupling (RPC)

We have calculated the quantity indicated by the sum in eq. (14) for the RPC between bands in the 1d-2s shell. Table 6 shows the results as a function of the deformation for values of $\kappa = 0.05$ and 0.08 and for $\mu = 0$ (μ is the coefficient of the l^2 term in the Nilsson Hamiltonian [6]). This table also lists the decoupling parameter for states in this shell.

For ^{25}Mg coupling occurs between the $K = \frac{1}{2}$ band on orbital 9 and the $K = \frac{1}{2}$ band on orbital 11, between the $K = \frac{1}{2}$ band on orbital 9 and the $K = \frac{3}{2}$ band on orbital 8 and between the $K = \frac{5}{2}$ band on orbital 5 and the $K = \frac{3}{2}$ band on orbital 8. In addition the decoupling parameters a for orbitals 9 and 11 are of interest. Only the first RPC matrix elements can be checked with the present experimental data.

Table 6 shows that for a deformation larger than $\beta \approx 0.2$ the RPC matrix elements between the $K = \frac{1}{2}$ bands on orbitals 9 and 11 are approximately independent of the deformation. Second order perturbation theory corrects the $\frac{1}{2}^+$ states by about 0.030 MeV, the $\frac{3}{2}^+$ states by about 0.130 MeV and the $\frac{5}{2}^+$ states by about 0.260 MeV. The $\frac{1}{2}^+$ states exchange about 1.5% of the spectroscopic ratios, the $\frac{3}{2}^+$ states about 6% and the $\frac{5}{2}^+$ states about 12%. Then, for the $K = \frac{1}{2}$ band on orbital 9 we obtain $\hbar^2/2\mathscr{I} = 0.180$ MeV and $a = -0.25$ and for that on orbital 11 we find $\hbar^2/2\mathscr{I} = 0.120$ MeV and $a = -0.61$. Mottelson and Nilsson [18]) list for the $K = \frac{1}{2}$ band on orbital 9 the values $\hbar^2/2\mathscr{I} = 0.165$ MeV and $a = -0.20$ and for that on orbital 11

TABLE 6

Value of the sum in the RPC matrix elements (eq. (14)) $\Sigma_{j'j}c_{j'\Omega-1}c_{j\Omega}((j+\Omega)(j-\Omega+1))^{\frac{1}{2}}\delta_{j'j}$ as a function of the deformation β at $\kappa = 0.05$ and $\kappa = 0.08$

β at $\kappa =$	β at $\kappa =$	Value of the sum for coupling of Nilsson orbitals										a for Nilsson orbital		
0.05	0.08	5&7	5&8	6&7	6&8	6&9	6&11	7&9	7&11	8&9	8&11	9&11	6	9	11
+0.5	+0.86	+2.12	+0.64	+2.22	+0.03	−1.69	+0.04	−1.13	−1.16	−1.75	+0.58	+1.58	+1.76	−0.19	+0.43
+0.4	+0.68	+2.14	+0.59	+2.26	+0.04	−1.61	+0.06	−1.15	−1.09	−1.68	+0.69	+1.58	+1.88	−0.15	+0.27
+0.3	+0.51	+2.16	+0.52	+2.23	+0.06	−1.49	+0.10	−1.18	−0.97	−1.59	+0.89	+1.65	+2.05	−0.05	+0.005
+0.2	+0.33	+2.18	+0.42	+2.42	+0.08	−1.28	+0.18	−1.18	−0.75	−1.36	+1.14	+1.57	+2.29	+0.25	−0.53
+0.1	+0.16	+2.20	+0.26	+2.60	+0.10	−0.87	+0.24	−1.03	−0.32	−0.80	+1.55	+1.10	+2.64	+0.85	−1.49
0.0	0.0	+2.22	0.00	+2.83	0.00	0.00	0.00	0.00	0.00	0.00	+1.73	0.00	+3.00	+1.00	−2.00
−0.1	−0.16	+2.20	−0.43	+2.18	−0.56	+1.02	−0.003	+1.76	−0.07	+0.22	+1.68	−1.07	+2.24	+1.40	−1.64
−0.2	−0.31	+2.00	−0.97	+1.51	−1.06	+1.17	+0.04	+2.18	−0.17	−0.06	+1.68	−1.73	+1.73	+1.19	−0.92
−0.3	−0.45	+1.74	−1.38	+1.02	−1.34	+1.24	+0.04	+2.21	−0.10	−0.40	+1.74	−1.98	+1.52	+0.84	−0.36
−0.4	−0.59	+1.54	−1.60	+0.74	−1.42	+1.28	+0.02	+2.20	−0.06	−0.61	+1.79	−2.08	+1.39	+0.60	+0.014
−0.5	−0.73	+1.40	−1.73	+0.55	−1.45	+1.31	+0.02	+2.16	−0.04	−0.73	+1.84	−2.08	+1.30	+0.44	+0.25

The decoupling parameter $a = \Sigma_j c_{j\frac{1}{2}}(j+\frac{1}{2})(-1)^{j+\frac{1}{2}}$ is also listed.

the values $\hbar^2/2\mathscr{I} = 0.150$ MeV and $a = -0.42$, where they do not take the rotation-particle coupling into account.

In the present analysis $\hbar^2/2\mathscr{I} = 0.120$ MeV for the band on orbital 11 is too low compared to the rigid rotator value of 0.150 MeV, indicating that other effects, e.g. rotation-particle coupling with the not observed band on orbital 8 are probably not negligible. We find the derived values of the decoupling parameter a for the bands on orbital 9 and 11 at the same deformation if we take $\mu = 0.5$ (see ref. [19])), i.e., $\eta = 1.2$ or for $\kappa = 0.08$ the deformation is about 0.1. However, in that case the orbitals 11 and 8 almost coincide. The rotation-particle coupling between the bands on orbitals 11 and 8 would destroy the result of the present analysis completely, so that the predicted values of a would be different, if we start a new analysis for $\mu = 0.5$. Also the fit to the spectroscopic ratios is poor for this value of μ and the predicted deformation is very small.

Independent information on the deformation of the nucleus in each state, e.g., by measuring magnetic moments and quadrupole moments of excited states, may lead to a closer analysis of the problem. We may conclude that it is still an open question whether a rotational description of the nuclear structure of ^{25}Mg is really adequate.

In ^{24}Na the situation is more complicated than in ^{25}Mg. The RPC is possible not only to states in which the odd neutron is excited, but also to states in which the odd proton is excited, which states can only be observed with great difficulty in the (d, p) reaction and, in fact, were not observed by us. Therefore, we can only estimate those RPC matrix elements, for which P stands for the odd neutron.

We calculate these matrix elements at a value of $\kappa = 0.08$ and $\beta = 0.2$. We present the following typical examples. Table 6 shows immediately that the coupling is negligible between the states of the $K = 4$ band on orbital 5 and the $K = 3$ band on orbital 8 and between those of the $K = 1$ band on orbital 5 and the $K = 0$ band on orbital 8. The RPC between the 2^+ states of the $K = 1$ band and the $K = 2$ band on orbital 9 (0.564 MeV and 1.844 MeV) shifts these states by about 0.070 MeV and exchanges about 5% of the spectroscopic ratios. The RPC between the analogous states on orbital 11 (3.409 MeV and 4.16 MeV) is too strong to be estimated with second-order perturbation theory. A more accurate calculation of this matrix element has no sense in view of the uncertainties in the identification of rotational bands. The possible presence of strong RPC matrix elements merely shows the weakness of this analysis.

The RPC between the 2^+ states of the $K = 2$ band on orbital 9 and the $K = 1$ band on orbital 11 (0.564 MeV and 4.16 MeV) is negligibly weak. The coupling between the 2^+ states of the $K = 1$ band on orbital 9 and the $K = 2$ band on orbital 11 (1.844 MeV and 3.409 MeV) shifts the levels by about 0.150 MeV and exchanges about 10% of the spectroscopic ratios. Incidentally, this result yields for the unperturbed $K = 1$ band on orbital 9 a value of $\hbar^2/2\mathscr{I} = 0.163$ MeV, larger than the rigid rotator value (cf. table 5).

The RPC matrix elements between states of bands on orbitals 9 and 8 and for bands on orbitals 11 and 8 are probably strong. We can make an estimate for only one case,

i.e. the coupling between the 3^+ states of the $K = 2$ band on orbital 9 and the $K = 3$ band on orbital 8 (1.884 MeV and 2.99 MeV). The energy shift is about 0.320 MeV and the exchange of spectroscopic ratios is about 30% from second-order perturbation theory which, therefore, is not applicable in this case of strong coupling.

The conclusion is that the analysis in terms of rotational bands in the previous section may be correct. However, the values of the moment of inertia, derived from that analysis, do not have much meaning in view of these strong or unknown RPC matrix elements. Only those values of the spectroscopic ratio of transitions to states of ^{24}Na can yield a reasonable comparison with the prediction by the unified model, if these states are not or only weakly affected by the RPC as indicated in above examples.

6. The Intrinsic States

In sect. 4 we have identified the rotational bands on orbitals 5, 9, 11 and 8 in ^{24}Na. We can now compare the position of the intrinsic states for ^{24}Na and ^{25}Mg with those predicted by the Nilsson model. For purposes of comparison we have first to correct

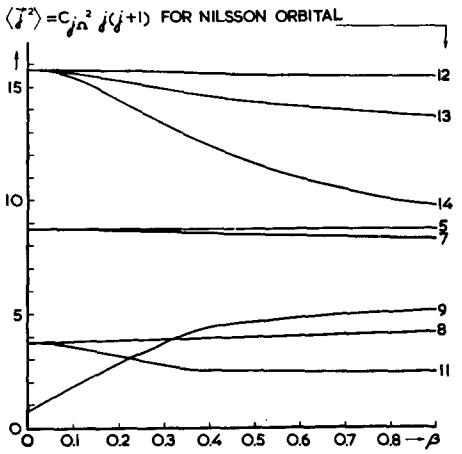

Fig. 7. Value of the expectation value $\langle j^2 \rangle$(eq. (11)) as a function of the deformation β at $\kappa = 0.08$.

for the rotational energy. For ^{24}Na the rotational energy is given by eq. (11). For ^{25}Mg we can apply the same formula minus the terms $\Omega_p \Omega_n$ and $\langle j_p^2 \rangle$, but adding a term $a(-1)^{J+\frac{1}{2}}(J+\frac{1}{2})\delta_{K,\frac{1}{2}}$, where a is the decoupling parameter. Fig. 7 shows the value of $\langle j^2 \rangle$ as a function of deformation, again for a value of $\kappa = 0.08$. Because we want to make only a crude estimate of the position of the intrinsic states with respect to each other, we take for the moment of inertia and the decoupling parameter the values used by Mottelson and Nilsson [18]) for ^{25}Mg, i.e., for the rotational band on orbital 5 we take $\hbar^2/2\mathscr{I} = 0.230$ MeV, for that on orbital 9 we take $\hbar^2/2\mathscr{I} = 0.150$ MeV and $a = -0.20$ and for that on orbital 11 we use $\hbar^2/2\mathscr{I} = 0.150$ MeV and $a =$

-0.42. We use the same values for the corresponding bands in ^{24}Na and for the rotational bands on orbital 8 we take $\hbar^2/2\mathscr{I} = 0.200$ MeV. We neglect the effect of the rotation-vibration interaction. We consider the energy differences between the intrinsic states after correction for the rotational energy. In ^{24}Na we have to do this separately for the $K_+ = \Omega_p + \Omega_n$ bands and the $K_- = |\Omega_p - \Omega_n|$ bands. Actually in the Nilsson model the intrinsic states of a K_+ and K_- band on the same orbit are degenerate, but we observe that additional terms in the Hamiltonian, e.g. residual interactions, destroy this degeneracy. These interactions are discussed in the next section.

TABLE 7

Relative position of the intrinsic states of rotational bands with respect to each other for ^{24}Na and ^{25}Mg

Energy difference	^{24}Na			^{25}Mg (MeV)	Theory [a] MeV
	K_+ bands (MeV)	K_- bands (MeV)	Average (MeV)		
$E_{11} - E_5$	3.98	3.14	3.56	3.13	3.5
$E_9 - E_5$	1.16	0.95	1.06	1.15	0.6
$E_{11} - E_9$	2.82	2.19	2.51	1.98	2.9
$E_8 - E_5$	3.49	3.58	3.54		5.2

[a]) Prediction of Nilsson model at $\beta \approx 0.20$ for $\kappa = 0.08$ and $\hbar\mathring{\omega}_0 = 41A^{-\frac{1}{3}}$ MeV.

N.B. E_1 is obtained from the experimental level position after subtraction of the rotational energy (eq. (11)). For $\hbar^2/2\mathscr{I}$ the experimental values [18]) of the rotational bands in ^{25}Mg have been used for both nuclei. For the rotational bands on orbital 8 a value $\hbar^2/2\mathscr{I} = 0.200$ MeV has been used.

For $\langle j^2 \rangle$ we have taken the value at $\beta \approx 0.20$. The flat dependence of $\langle j^2 \rangle$ on β makes the choice of β fortunately uncritical (see fig. 7).

Table 7 shows these energy differences. The first two columns show the result for the K_+ and K_- bands, respectively. The differences in the values may be due to residual interactions or to rotation-particle coupling. Therefore we list the average value in the third column for comparison with the values for ^{25}Mg in the fourth column. In the last column we list the prediction of the Nilsson model for these energy differences for $\kappa = 0.08$, $\hbar\mathring{\omega}_0 = 41A^{-\frac{1}{3}}$ MeV, $\mu = 0$ and $\beta = 0.2$. We choose this value of the deformation, because this value of β yields a reasonable fit to most spectroscopic data of the (d, p) reaction in these two nuclei. The agreement is reasonable within the accuracy of the approximations. It is important to notice that the order of orbital 9 and 11 is interchanged above $\beta \approx 0.26$ (see fig. 2). The largest deviation between experiment and theory occurs for orbital 8. The experimental values suggest that orbital 11 and orbital 8 coincide. In sect. 4 we discussed that this situation occurs for $\kappa = 0.08$ and $\mu = 0.5$, but these parameters spoiled the agreement with the spectroscopic ratios. In sect. 5 we have seen that strong RPC matrix elements are present for the rotational bands on orbital 8 possibly explaining the disagreement between experiment and

theory. The experimental value for orbital 8 should actually lead to a prediction for the position of a $K = \frac{3}{2}$ band on orbital 8 in ^{25}Mg. This prediction is that the intrinsic state of this band should be expected to be at about 2.73 MeV excitation in ^{25}Mg. In this nucleus the 2.798 MeV level was identified as the $\frac{3}{2}^+$ member of the $K = \frac{1}{2}$ band on orbital 11 (see table 1). Actually, the large value of the spectroscopic ratio for this level, 1.05, also makes this level a likely candidate for the intrinsic state of the $K = \frac{3}{2}$ band on orbital 8 (see fig. 4). Up to now no $\frac{3}{2}^+$ state at higher excitation has been observed.

In ^{25}Al the $\frac{3}{2}^+$ level at 2.74 MeV corresponds to the 2.798 MeV state of ^{25}Mg according to the analysis in ref. [7]). Another $\frac{3}{2}^+$ level in ^{25}Al has been identified at 4.22 MeV, which level, however, has not been identified as the intrinsic state of the $K = \frac{3}{2}^+$ band on orbital 8. Perhaps, we should assign this level to this rotational band and, of course, consider the possibility of RPC between these two $\frac{3}{2}^+$ states. A partner for the 4.22 MeV state may be present in ^{25}Mg at a similar excitation energy.

Hence, we may consider the possibility that the 2.798 MeV level is the lower partner of two $\frac{3}{2}^+$ states, viz. that of the $K = \frac{1}{2}$ band on orbital 11 and that of the $K = \frac{3}{2}$ band on orbital 8, which positions have been influenced by strong rotation-particle coupling. Such a possibility implies that the numerical value of the moment of inertia and the decoupling parameter for the $K = \frac{1}{2}$ band on orbital 11 are those of a perturbed band. Therefore, these numbers are not reliable for comparison with a model, making the interpretation of the structure of ^{25}Mg very tentative.

We can discuss again why the value of $\beta \approx 0.2$, which gives a reasonable fit to the spectroscopic ratios and the energy differences between intrinsic states, differs so much from the value $\beta \approx 0.5$ obtained from the magnetic moment, the equilibrium deformation derived from the total energy and the quadrupole moment in this region of nuclei. We can obtain a better agreement between the values of β by increasing the value of κ. For a value κ' this means multiplying horizontal and vertical scale in fig. 2 by approximately $\kappa'/0.08$. However, the change in vertical scale makes the energy differences in the last column of table 7 larger by the same factor, spoiling the agreement between the experimental values in the first four columns. To counter this effect one might reduce the value of $\hbar\mathring{\omega}_0$, but such a change is not in agreement with other estimates of $\hbar\mathring{\omega}_0$ (see ref. [4])). Choosing individual values of β for each K band would be rather arbitrary because of lack of experimental data, e.g. on magnetic moments and quadrupole moments of the nucleus in excited states. A large β for bands on orbital 5 and a small β for bands on orbitals 9 and 11 probably reverses the order of states on orbitals 5 and 9.

The author has considered several different choices of $\hbar\mathring{\omega}_0$, κ and μ, but no other choice gives an appreciably better fit to the data than the present one, $\hbar\mathring{\omega}_0 = 41A^{-\frac{1}{3}}$ MeV, $\kappa = 0.08$ and $\mu = 0$.

We have already discussed in sect. 5 that the data on electromagnetic transitions in ^{24}Na are scarce. These data are not available in a form which would be useful for comparison with calculations [7]) on branching ratios, etc., in ^{25}Mg.

7. Residual Interactions

In the preceding discussions we have mentioned the possibility of a strong influence of residual interactions. Particularly, we stated that in an odd nucleus the two-body interaction between the odd proton and the odd neutron may be important. In the following discussion we treat only this interaction, because the data on odd mass nuclei indicate that the two-body interaction between the odd particle and the core particles and that between the core particles themselves probably do not cause large irregularities in the rotational structure of these nuclei.

Three types of two-body interactions can be considered: a central force along the radius vector between the particles, a two-body spin-orbit force and a tensor force [12]). The first force acts as well in the singlet as in the triplet intrinsic spin state of the two particles, the last two only in the triplet state.

In the subsequent discussion we follow the treatment of Elliot and Flowers [20]). These authors use for the central force problem a potential

$$U_{12} = \tfrac{1}{3}\tau_1 \cdot \tau_2(0.3+0.7\sigma_1 \cdot \sigma_2)V\mathrm{e}^{-r_{12}/a}/(r_{12}/a) = A_{TS}V\mathrm{e}^{-r_{12}/a}/(r_{12}/a). \quad (18)$$

This potential has yielded good results in the explanation of several properties of low-lying states of ^{16}N, ^{16}O, ^{18}O, ^{18}F, ^{19}O, ^{19}F for $V = 40$ to 45 MeV and $a = 1.4$ fm (the Compton wave length of the π meson, the range of the Yukawa force).

For a level occupation of ^{24}Na as in fig. 2 the isobaric spin is $T = 1$. The isobaric spin is probably a reasonably good quantum number for the low excited states of this light nucleus and it is determined by the two odd particles.

Two particles in shell model states $|j_1 m_1\rangle$ and $|j_2 m_2\rangle$ coupling to a state $|(j_1 j_2)J_{12} M_{12}\rangle$ are partly in a singlet and partly in a triplet intrinsic spin state. The probability that the two particles are in a singlet state is

$$\tfrac{1}{2}\begin{Bmatrix} l_1 & j_1 & \frac{1}{2} \\ j_2 & l_2 & J_{12} \end{Bmatrix}^2 (2j_1+1)(2j_2+1) \quad (19)$$

independent of the magnetic quantum numbers. The energy due to the two-body interaction of the two particles is

$$E_{12} = A_{11}V\sum_{k=0}^{\infty} F^{(k)}(n_1 l_1 n_2 l_2) f_{1k}(j_1 j_2 J_{12})$$
$$-\tfrac{1}{2}(A_{11}-A_{10})V\sum_{k=0}^{\infty} F^{(k)}(n_1 l_1 n_2 l_2) f_{2k}(l_1 l_2 J_{12})\tfrac{1}{2}\begin{Bmatrix} l_1 & j_1 & \frac{1}{2} \\ j_2 & l_2 & J_{12} \end{Bmatrix}^2 (2j_1+1)(2j_2+1). \quad (20)$$

Here, $F^{(k)}$ is the Slater integral defined as

$$F^{(k)}(n_1 l_1 n_2 l_2) = \int_0^\infty \int_0^\infty u^2_{n_1 l_1}(\rho_1) u^2_{n_2 l_2}(\rho_2) V_k(r_1, r_2) \rho_1^2 \rho_2^2 \, \mathrm{d}\rho_1 \, \mathrm{d}\rho_2 . \quad (21)$$

Here $\rho^2 = r^2/b^2$ and $b = (\hbar/m\mathring{\omega}_0)^{\frac{1}{2}} = 1.7$ fm for ^{24}Na, characteristic for the harmonic oscillator potential well which generates the single particle wave functions

$u_{nl}(\rho)$. The potential $V_k(r_1, r_2)$ is defined by the expansion of the shape factor of the two-body potential.

$$\frac{e^{-r_{12}/a}}{r_{12}/a} = \frac{e^{-|r_1-r_2|/a}}{|r_1-r_2|/a} = \sum_{k=0}^{\infty} V_k(r_1-r_2)P_k(\cos\omega_{12}), \tag{22}$$

where ω_{12} is the angle between r_1 and r_2.

Further, in the 1d-2s shell

$$f_{1k} = (-1)^{J_{12}+1}(2j_1+1)(2j_2+1)\begin{pmatrix} k & j_1 & j_1 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} k & j_2 & j_2 \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{Bmatrix} j_1 & k & j_1 \\ j_2 & J_{12} & j_2 \end{Bmatrix}\delta_{k,\,\text{even}}, \tag{23}$$

$$f_{2k} = (-1)^{J_{12}}(2l_1+1)(2l_2+1)\begin{pmatrix} l_1 & k & l_1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l_2 & k & l_2 \\ 0 & 0 & 0 \end{pmatrix}\begin{Bmatrix} l_1 & k & l_1 \\ l_2 & J_{12} & l_2 \end{Bmatrix}\delta_{k,\,\text{even}}. \tag{24}$$

Let us consider only diagonal matrix elements of the two-body interaction. Then, for a pair of 2s particles and a pair consisting of a 1d and a 2s particle only $k = 0$ occurs, and for a pair of 1d particles only $k = 0$, 2 and 4 occur. Table 8 shows the value of $F^{(k)}$ for these pairs of particles for an interaction of Yukawa type [20]).

TABLE 8

Values of the Slater integral $F^{(k)}$ for a Yukawa well [20]) for particles in the 1d-2s shell in the case of ^{24}Na ($a/b = 0.82$)

Pair of particles	$F^{(0)}$	$F^{(2)}$	$F^{(4)}$
1d-1d	0.070	0.165	0.060
1d-2s	0.075		
2s-2s	0.100		

In the unified model the particle states are not shell model states, but linear combinations of shell model states (see eqs. (3)-(5)). The energy due to the two-body interaction has to be calculated for these Nilsson states. The first term in eq. (20), common to both singlet and triplet intrinsic spin states of the two odd particles, yields for all states of a K band

$$E_{12}(1) = A_{11}V\sum_k\sum_{J_{12}}\sum_{j_1 j_2} F^{(k)}f_{1k}(j_1 j_2 \Omega_2 \Omega_1|J_{12}K)^2 c^2_{j_1\Omega_1}c^2_{j_2\Omega_2}. \tag{25}$$

The second term yields for all states of a K band

$$E_{12}(2) = -\tfrac{1}{2}(A_{11}-A_{10})V\sum_k\sum_{J_{12}}\sum_{j_1 j_2} F^{(k)}f_{2k}\begin{Bmatrix} l_1 & j_1 & \frac{1}{2} \\ j_2 & l_2 & J_{12} \end{Bmatrix}^2 (2j_1+1)(2j_2+1) \times(j_1 j_2 \Omega_1 \Omega_2|J_{12}K)^2 c^2_{j_1\Omega_1}c^2_{j_2\Omega_2}. \tag{26}$$

These terms are different for each K band. Therefore, the relative position of K bands is changed and the degeneracy of K_+ and K_- bands on the same Nilsson orbital is removed. In addition, it is interesting to note that these terms depend on the deformation because of the β dependence of the $c_{j\Omega}$'s.

In eqs. (25) and (26) the factor $(j_1 j_2 \Omega_1 \Omega_2 | J_{12} K)^2$ represents the probability that the angular momenta of the two odd particles couple to $\boldsymbol{J}_{12}$ with a projection K on the symmetry axis of the nucleus. Then, $\boldsymbol{J}_{12}$ couples with the angular momentum $\boldsymbol{R}$ of the collective rotational motion yielding $\boldsymbol{J}$ the total angular momentum of the nucleus. The projection of $\boldsymbol{R}$ on the symmetry axis is zero. The probability for the coupling $\boldsymbol{J}-\boldsymbol{J}_{12} = \boldsymbol{R}$ is $(JJ_{12}K-K|R0)^2$. For each pair of numbers J and J_{12} the sum of the probabilities over all possible values of R is $\sum_R (JJ_{12}K-K|R0)^2 = 1$. This is the reason that a sum over R is not present in eqs. (25) and (26).

Analogously we can calculate the contribution due to a two-body spin-orbit force $(\sigma_1+\sigma_2)\cdot((r_1-r_2)\times(p_1-p_2))V(r_{12})$ and a tensor force $(((\sigma_1\cdot r_{12})(\sigma_2\cdot r_{12}))-\frac{1}{3}(\sigma_1\cdot\sigma_2))V_2(r_{12})$ which only contribute in the triplet intrinsic spin state of the two particles. In principle, the matrix elements of these interactions can be calculated using the results of Hope and Longdon [21, 22]). The energy due to each of these interactions is

$$E_{12} = P_T \mathscr{V} \sum_k \sum_{J_{12}} \sum_{j_1 j_2} \mathscr{F}^{(k)} f_k 3(2j_1+1)(2j_2+1)(2L+1) \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & 1 & J_{12} \end{Bmatrix}^2 \times (j_1 j_2 \Omega_1 \Omega_2 | J_{12} K)^2 c^2_{j_1\Omega_1} c^2_{j_2\Omega_2}. \quad (27)$$

Here, $\mathscr{F}^{(k)}$ is the Slater integral for these interactions, f_k is similar to f_{2k}, $\mathscr{V}$ is the strength of the interaction including the exchange parameters (cf. A_{TS}, eq. (18)). The factor

$$3(2j_1+1)(2j_2+1)(2L+1) \begin{Bmatrix} l_1 & \frac{1}{2} & j_1 \\ l_2 & \frac{1}{2} & j_2 \\ L & 1 & J_{12} \end{Bmatrix}^2 \quad (28)$$

represents the probability that the two particles are in the triplet intrinsic spin state.

TABLE 9

Energy due to the two-body interaction of the two odd particles for a central force as a function of the deformation (eqs. (25) and (26); $A_{11} = 0.33$, $A_{10} = -0.60$, $V = 40$ MeV, $F^{(k)}$ from table 8; odd proton in orbital 7; E_{12} in MeV; $\kappa = 0.08$)

Neutron in orbital	5	5	9	9	11	11	8	8
K band	4	1	2	1	2	1	3	0
β	E_{12}	E_{12}	E_{12}	E_{12}	E_{12}	E_{12}	E_{12}	E_{12}
0.00	0.59	0.28	0.71	−0.26	0.98	1.54	−0.41	1.09
0.16	0.60	0.42	0.88	0.14	1.10	1.23	−0.39	1.05
0.33	0.62	0.66	1.15	0.67	1.22	0.73	−0.35	0.99
0.51	0.64	0.85	1.38	0.92	1.33	0.48	−0.32	0.94
0.68	0.65	1.00	1.51	0.99	1.44	0.38	−0.29	0.91
0.86	0.65	1.14	1.61	1.04	1.50	0.33	−0.27	0.87

For the central two-body interaction we can use the numbers of Elliot and Flowers [20]), viz. $A_{11} = +0.33$, $A_{10} = -0.60$ and $V = 40$ MeV. For the other types of interactions no numerical values are available. Table 9 shows $E_{12} = E_{12}(1)+E_{12}(2)$ as a function of the deformation (eqs. (25) and (26)).

In sect. 3 we mentioned the influence of the two-body force on the spectroscopic ratios for the members of the $K = 1$ and the $K = 2$ bands on orbitals 9 and 11 for the neutron, in sect. 4 the influence on the position of the 0.472 MeV and 1.341 MeV levels. We can correct the energy differences in table 7 for the two-body interaction and we can discuss the influence on the relative position of the K_+ and the K_- band on the same Nilsson orbital.

TABLE 10

Relative position of the intrinsic states of the K_+ band and the K_- band on the same Nilsson orbital for ^{24}Na (same conditions as in table 7)

Nilsson orbital	$E_{K_+}-E_{K_-}$ (MeV)	Theory [a]) (MeV)
5	1.16	−0.13
9	0.95	−0.70
11	0.32	0.00
8	1.25	1.42

[a]) Prediction of central two-body interaction between the two odd particles (eqs. (25) and (26)) at $\beta \approx 0.2$ for $\kappa = 0.08$ (see table 9).

TABLE 11

Relative position of the intrinsic states of rotational bands with respect to each other for ^{24}Na and ^{25}Mg including a correction for the central two-body interaction between the two odd particles of ^{24}Na at $\beta \approx 0.2$ (cf. table 7)

Energy difference	^{24}Na			^{25}Mg (MeV)	Theory (MeV)
	K_+ bands (MeV)	K_- bands (MeV)	Average (MeV)		
$E_{11}-E_5$	3.45	2.48	2.99	3.13	3.5
E_9-E_5	0.81	1.17	0.99	1.15	0.6
$E_{11}-E_9$	2.64	1.29	1.97	1.98	2.9
E_8-E_5	4.47	3.51	3.99		5.2

We start with the last point. Table 10 shows the energy difference $E_{K_-}-E_{K_+}$, which is the difference between the energies of the intrinsic states of the K_- and the K_+ bands on a Nilsson orbital after correction for the rotational energy, using the same parameters as in table 7. The agreement between theory and experiment is not very good. In particular, the theory predicts the 1^+ intrinsic state of the $K = 1$ band on orbital 5 to be the ground state of ^{24}Na instead of the 4^+ intrinsic state of the $K=4$

band on this orbital. Apparently additional terms in the Hamiltonian have to be taken into account to explain the 4^+ ground state of ^{24}Na or we have to choose for the central two-body force parameters which differ from those of Elliott and Flowers [20]). The two-body spin-orbit force and the tensor force (eq. (27)) may be able to solve this problem. However, the number of parameters in eq. (27) is too large that these parameters can be determined even crudely from the present experimental data.

Table 11 is an improved version of table 7, viz. the effect of the central two-body interaction has been included. The agreement between the average energy differences for ^{24}Na and the energy differences for ^{25}Mg is better than in table 7. The value of $E_8 - E_5 = 3.99$ MeV is now definitely larger than the value of $E_{11} - E_5 = 2.99$ MeV (see discussion in sect. 6) and the number is closer to the theoretical value of 5.2 MeV. This energy difference yields a prediction of the position of the $\frac{3}{2}^+$ intrinsic state of the $K = \frac{3}{2}$ band on orbital 8 at 3.13 MeV in ^{25}Mg instead of 2.73 MeV without the central two-body interaction in sect. 6. Again, the RPC matrix elements affect probably the position of such a level strongly (see sect. 6).

The third point is the coupling between the two 1^+ levels at 0.472 MeV and the 1.341 MeV. An off-diagonal matrix element of the two-body interaction may cause the coupling. Indications for this coupling are the unexplainable values of $\hbar^2/2\mathscr{I}$ for the rotational bands on these states (see sect. 4), the presence of an s wave contribution in the (d, p) reaction to the 0.472 MeV level, and finally the ^{24}Ne$(\beta^-)^{24}$Na decay [23]) to both these levels. Both β^- branches have $\log ft = 4.4$. We expect for ^{24}Ne in the Nilsson model that the last neutron pair is either in orbital 5 or in orbital 9 (see fig. 2). The Gamow-Teller operator converts one of the neutrons into a proton in orbital 7 with $\Omega = \frac{3}{2}$. If the last neutron pair is completely in orbital 5 or in orbital 9 we expect a β^- transition to the 0.472 MeV or the 1.341 MeV level only, respectively. The presence of both branches is an indication for coupling between these levels, but it also indicates that the ground state of ^{22}Ne is probably a mixture of the two (or more) possible occupation schemes of the last neutron pair in the Nilsson model. Such a mixture can again be explained by residual forces which couple particles in Nilsson states.

The $\log ft$ values of the ^{24}Ne$(\beta^-)^{24}$Na decay modes are within the range of the values for allowed β^- transitions. Such low values occur for several β^- transitions between states of nuclei in this region [15]). It is surprising that these transitions are not retarded due to the apparent mixing of shell model states by the quadrupole force of the Nilsson model or by the residual interactions.

Finally, table 9 shows that the energy due to the central two-body interaction is deformation dependent. However, this dependence is not sufficiently strong to cause an appreciable change of the equilibrium deformation (fig. 3). One may hope that the other types of two-body interaction which operate only in triplet states cause such a change, e.g., in particular a reduction of the deformation in the case of the intrinsic state at 1.341 MeV of the $K = 1$ band on orbital 9 towards a value of $\beta \approx 0.10$ (see

table 5) where a better agreement can be obtained between the experimental and theoretical spectroscopic ratio.

In this context a remark of Banerjee, Levinson and Meshkov [24]) is interesting. These authors calculate the spectra of ^{20}Ne and ^{24}Mg with SU3 group techniques and they state: "One important conclusion is that the calculations of low-lying spectra using zero-order SU3 wave functions alone, neglecting mixing of higher SU3 states, yields moments of inertia which are larger than observed from experiment. Our physical interpretation of this result is that the zero-order states are too deformed and that less deformed states must be admixed into the wave functions in order to produce agreement with experiment". Although the wave functions in the unified model are not the same as the SU3 wave functions, one may conclude from this discussion that wave functions of lower symmetry, e.g., those of states of core excitation in terms of the unified model, have to be mixed into the unified model wave functions (eqs. (3)-(5)) for the low-lying states. This mixing can be produced with two-body interactions.

We may conclude that the four discussed cases show qualitatively the need for including a two-body interaction between the two odd particles. The central two-body interaction alone is not sufficient to describe the deviations from the Nilsson model, a two-body spin-orbit force and a tensor force may have to be included in the description.

8. Conclusion

In the present paper we have made a tentative interpretation in terms of rotational bands (table 5) of the even parity levels of ^{24}Na at low excitation from the comparison with the rotational structure of ^{25}Mg [7]), particularly by comparing the s wave states in both nuclei (tables 1 and 4). The data of the (d, p) reaction show a reasonable fit with the unified model for a value of the deformation $\beta \approx 0.2$ for a spin-orbit coupling constant $\kappa = 0.08$, whereas the magnetic dipole moment, the quadrupole moment of nuclei in this region and the equilibrium deformation as derived from the total energy (fig. 3) yield a value $\beta \approx 0.4$ to 0.5 at $\kappa = 0.08$. The relative positions of the intrinsic states of the rotational bands (table 7) on orbitals 5, 9 and 11 are in good agreement with the prediction of the Nilsson model for $\beta \approx 0.2$ at $\kappa = 0.08$, but this is not the case for the position of the intrinsic state of the rotational bands on orbital 8 with respect to the other states. Rotation-particle coupling matrix elements in ^{25}Mg and ^{24}Na and residual interactions between the two odd particles in ^{24}Na seem to distort the rotational bands considerably, making derivations of the inertial parameter $\hbar^2/2\mathscr{I}$ and the decoupling parameter a from the experimental data (for ^{25}Mg in refs. [7, 18])) difficult and unreliable (see the discussion about a $K = \frac{3}{2}$ band on orbital 8 and the 2.798 MeV level in ^{25}Mg in sects. 6 and 7).

The unified model does not predict the 4^+ intrinsic state of the $K = 4$ band on orbital 5 as the ground state of ^{24}Na. The central two-body interaction does not improve this situation. A two-body spin-orbit force and a tensor force may be able to predict the ground state spin correctly.

One may consider the possibility that the residual forces are much stronger than the quadrupole force of the Nilsson model. In that case it would be more appropriate to take a shell model Hamiltonian and residual interactions for the zeroth order problem and to add this quadrupole force as a perturbation. But, as stated in sect. 1, such a shell model calculation is prohibited by its complexity. On the other hand, the Nilsson model has the advantage of being a simple tool for the analysis of the nuclear structure because of its small number of free parameters.

The tentative interpretation of the structure of ^{24}Na in this paper differs from the speculations about this structure in ref. [25]). This is because after the completion of the latter paper the data of Middleton and Hinds [8]) became available.

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Note added in proof: It is a pleasure to the author to communicate that Dr. Hinds has pointed out to him, that they [8]) have carefully considered the $^{14}C_{6.091\ MeV}$ contamination and other impurities, e.g. ^{29}Si, ^{26}Mg and ^{27}Mg, in connection with the $l = 0$ assignment to the transition to the 5.465 MeV level in ^{25}Mg. They still think to have a genuine $l = 0$ distribution. The author still has the opinion that a careful investigation of the angular distribution of this transition compared with those to the levels at 0.581 MeV and 2.564 MeV is necessary. He suggests to measure the energy dependence of the spectroscopic ratios of these transitions in the considered energy region of the deuterons in order to see, whether the strongly different values (see sect. 3) of the spectroscopic ratio of the transition to the 5.465 MeV level can be reproduced. The result of such an investigation is of extreme importance for the understanding of the single particle character of nuclear structure. A strong s-wave level at 5.465 MeV in ^{25}Mg in addition to those at 0.581 MeV and 2.564 MeV is in disagreement with our present understanding of the nuclear shell model and related models.

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