THE DECAY OF Ge\textsuperscript{77}\
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Abstract: Directional correlation and polarization-direction correlation measurements have been done on gamma rays following the decay of Ge\textsuperscript{77}. The results are found to be consistent with previous spin assignments and a spin of \( \frac{5}{2}^+ \) can be assigned to the 1.19 MeV level. The quadrupole content of several gamma transitions is given.

1. Introduction

The decay of 12 h Ge\textsuperscript{77} to excited states of As\textsuperscript{77} has been studied by several investigators\textsuperscript{1 - 5}). From their results on the beta spectrum, the relative gamma ray intensities and gamma coincidence measurements, the level scheme given in fig. 1 has been proposed.

The present investigation is concerned with gamma-gamma directional correlation and gamma-gamma polarization-direction correlation. Such measurements should give confirmation of the assigned spins and information about the multipole mixing ratios of the gamma rays. Directional correlation measurements alone are not always sufficient in the cases where the two gamma rays are mixtures, if the spin assignment is not complete for the three levels of a cascade.

\[ \text{Fig. 1. Level scheme of As}^{77}, \text{low energy part.} \]
2. Sources and Apparatus

2.1. PREPARATION OF THE SOURCES

Samples of GeO₂ enriched to 84.7% in Ge⁷⁶ were irradiated for 7 h in a flux of \(2 \times 10^{12}\) neutrons/sec \(\cdot\) cm² in the Ford Nuclear Reactor. The germanium oxide was then dissolved in a concentrated solution of sodium hydroxide at a temperature of 90°–100° C, placed in a lucite holder, diluted and sealed. Because of the 82 min activity of Ge⁷⁵ and of the build-up of As⁷⁷ which decays to Se⁷⁷ with a half-life of 39 h, the sources were used only for 20 h, starting 3 h after the end of irradiation.

2.2. DIRECTIONAL CORRELATION APPARATUS

The directional correlation apparatus used here has been described previously ⁶). The detectors were two 5 cm \(\times\) 5 cm NaI(T1) crystals mounted on 6342A RCA photomultipliers. The fast coincidence circuit had a resolving time of 20 nsec.

2.3. POLARIZATION-DIRECTION CORRELATION APPARATUS

The polarization-sensitive detector was of the type first used by Metzger and Deutsch ⁷). Its principle is based on the fact that the Compton scattering cross section depends on the polarization direction of the incident gamma ray. It consists of two 5 cm \(\times\) 5 cm NaI(T1) detectors, shielded from the source, placed symmetrically on opposite sides of a NaI(T1) scattering detector, the axes of the three detectors being in a common plane. The source is on the axis of the scatterer. The geometry of the instrument is shown in fig. 2.

![Fig. 2. Geometry of the polarization-direction detector. S: scatterer; F₁ and F₂: detectors of the scattered gamma rays; L: lead shield; M: moving detector.](image)

The polarization-insensitive detector was a 5 cm \(\times\) 5 cm NaI(T1) crystal, which could be moved to four positions, two parallel to the plane of the polarization analyser and two perpendicular to it. The angle between the two gamma rays of the cascade was kept at 90°.
The coincidence circuit which was used for the polarization-direction correlation measurements is of the fast-slow type. A block diagram is given in fig. 3. The fast coincidence circuit had a resolving time of 50 nsec. The slow side consists of two linear amplifiers with discriminators, one of which is fed by the moving detector, the other by a linear adder whose output is proportional to the sum of the pulses given by the scatterer and the side crystals. In this way, energy discrimination is achieved on both gamma rays.


2.4. CALIBRATION OF THE POLARIZATION ANALYSER

The state of linear polarization of the analyzed gamma ray can be specified by the polarization ratio J defined as follows:

\[ J = \frac{W(\frac{1}{2}\pi) - W(0)}{W(\frac{1}{2}\pi) + W(0)}, \]

where \( W(\phi) \) is the polarization-direction correlation function given in ref. 8, specialized to the case where the two photons are emitted at 90° to each other. Here, \( \phi \) is the angle between the polarization vector and a normal to the plane of the two gamma rays.

We define the experimental polarization ratio \( \rho \) as

\[ \rho = \frac{N_\perp - N_\parallel}{N_\perp + N_\parallel}, \]

where \( N_\perp \) and \( N_\parallel \) are the number of fast-slow coincidences for the moving detector perpendicular and parallel to the plane of the analyser, respectively. As Compton scattering occurs predominantly in a plane perpendicular to the polarization vector of the incident gamma ray, \( J \) and \( \rho \) have the same sign.

We define the efficiency \( \epsilon \) of the polarization analyser by

\[ \rho = \epsilon J. \]
The quantity \( \varepsilon \) depends on the geometry of the detectors and on the energy of the analysed gamma ray. The dependence on the spin sequence and multipolarity of the gamma transition through the polarization-direction correlation function is assumed to be negligible.

An estimate of the value of \( \varepsilon \) versus energy can be made by integrating the Compton cross section over the polar and azimuthal ranges which represent the crystals solid angles. Several graphs of \( \varepsilon \) versus energy were drawn, corresponding to different angular spreads of the crystals, and the curve which fitted best the experimental determination of \( \varepsilon \) using known cascades was chosen. The cascades used for this calibration were the \( 0^+(513 \text{ keV})2^+(631 \text{ keV})0^+ \) cascade in Pd\(^{106}\) and the \( 4^+(1.17 \text{ MeV})2^+(1.33 \text{ MeV})0^+ \) cascade in Ni\(^{60}\).

3. Results

3.1. GENERAL

Directional correlation and polarization-direction correlation experiments were done on cascades involving the lowest energy gamma rays of As\(^{77}\). The relevant part of the energy levels is given in fig. 1. The gamma ray spectrum is shown in fig. 4.

The following cascades were considered:

\[
\begin{align*}
367 \text{ keV} - 265 \text{ keV}, \\
210 \text{ keV} - 265 \text{ keV}, \\
417 \text{ keV} - 215 \text{ keV}, \\
561 \text{ keV} - 632 \text{ keV}, \\
561 \text{ keV} - 417 \text{ keV}, \\
561 \text{ keV} - 367 \text{ keV}.
\end{align*}
\]

![Fig. 4. Gamma ray spectrum of Ge\(^{77}\).](image-url)
Directional correlation experiments were done on the six cascades, and the results, after correction for accidental coincidences, were used to make a least-squares fit to the function

\[ W(\theta) = \alpha_0 + \alpha_2 P_2(\cos \theta) + \alpha_4 P_4(\cos \theta). \]

The coefficients \( \alpha_0, \alpha_2 \) and \( \alpha_4 \) were then corrected for solid angle of the detectors and for interferences from other cascades; this, after normalization, gives the experimental values of the coefficients \( A_2 \) and \( A_4 \) in the function

\[ W(\theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta), \]

which is predicted by the theory of angular correlation. The experimental values of \( A_2 \) and \( A_4 \) are summarized in table 1.

Polarization-direction correlation measurements were made when the intensities were sufficient. From the known efficiency of the polarization analyser, the value of the polarization ratio \( J \) was then obtained for seven transitions. The results of the measurements are given in table 1. Here, \( J^1 \) and \( J^2 \) refer to polarization of the first and second gamma ray of the cascade, respectively.

<table>
<thead>
<tr>
<th>Cascade energies (keV)</th>
<th>( A_2 )</th>
<th>( A_4 )</th>
<th>( J^1 )</th>
<th>( J^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>367-265</td>
<td>-0.321±0.009</td>
<td>-0.007±0.012</td>
<td>0.53±0.06</td>
<td>0.075±0.036</td>
</tr>
<tr>
<td>210-265</td>
<td>-0.190±0.004</td>
<td>-0.009±0.004</td>
<td>0.2±0.2</td>
<td>-0.024±0.028</td>
</tr>
<tr>
<td>417-215</td>
<td>-0.031±0.007</td>
<td>0.017±0.009</td>
<td>0.033±0.036</td>
<td></td>
</tr>
<tr>
<td>561-367</td>
<td>-0.035±0.020</td>
<td>0.068±0.020</td>
<td></td>
<td></td>
</tr>
<tr>
<td>561-417</td>
<td>0.163±0.016</td>
<td>0.052±0.024</td>
<td></td>
<td></td>
</tr>
<tr>
<td>561-632</td>
<td>0.085±0.027</td>
<td>0.065±0.030</td>
<td>-0.04±0.35</td>
<td>-0.37±0.23</td>
</tr>
</tbody>
</table>

3.2. METHOD OF ANALYSIS OF THE RESULTS

The directional correlation data were analysed according to the method given in ref. 10). A brief outline will be given here.

The coefficients \( A_2 \) and \( A_4 \) which appear in the directional correlation function can be written as a product of two terms, each of which is a function of one of the transitions only. Thus:

\[ A_v = A_v^{(1)} A_v^{(2)}, \]

where the superscript (1), and (2) denote the first and second transitions. A plot of \( A_v^{(1)} \) versus \( A_v^{(2)} \) by making use of the experimental value \( A_v \) is an equilateral hyperbola, or because of the statistical uncertainty, a region between two hyperbolae. Moreover,
it can be shown \(^{(10)}\) that a plot of \(A_2^{(k)}\) versus the quadrupole content \(Q^{(k)}\) (the fraction of the energy radiated as a quadrupole) is an ellipse, and that \(A_2^{(k)}\) versus \(Q^{(k)}\) is a straight line. The three graphs i.e. \(A_2^{(1)}\) versus \(A_2^{(2)} - A_2^{(1)}\) and \(A_2^{(1)}\) versus \(Q^{(1)} - A_2^{(2)}\) and \(A_2^{(2)}\) versus \(Q^{(2)}\) can be combined, and restrictions on the allowed values of \(Q^{(k)}\) can be found, as shown in fig. 5.

The polarization-direction correlation data were analysed by the method of ref. \(^{(11)}\). The polarization ratio for dipole-quadrupole mixtures, for the case where the angle between the two gamma rays is 90° can be written

\[
J^{(1)} = (-1)^{\sigma(L_1)} \frac{A_2 - 4A_2^{(2)} - \frac{5}{4}A_4}{2 - A_2 + \frac{3}{4}A_4}
\]

This corresponds to the polarization of the first radiation measured; a similar expression is obtained for \(J^{(2)}\). Here, \((-1)^{\sigma(L)}\) is equal to \(+1\)\((-1)\) of the dipole radiation is electric (magnetic). The coefficients \(A_2, A_4\) and \(A_2^{(2)}\) have the same value as in the directional correlation experiment, and a graph of \(A_2^{(1)}\) versus \(Q^{(1)}\) is a straight line, determined by the condition \(A_2^{(k)} = A_2^{(k)}\) for \(Q^{(k)} = 0\) or \(Q^{(k)} = 1\). From the experimental value of \(A_2, A_4\) and \(J\), the value of the product \(A_2^{(1)}A_2^{(2)}\) can be obtained and represented as an area between equilateral hyperbolae in the plane \(A_2^{(1)}; A_2^{(2)}\). This graph can be combined with the graph used for directional correlation. (See fig. 5.)

3.3. THE 367 keV-265 keV CASCADE

From the experimental values of \(A_2, A_4, J^1\) and \(J^2\) (see table 1), the following values for the products \(A_2^{(1)}A_2^{(2)}\) and \(A_2^{(2)}A_2^{(1)}\) are obtained:

- \(A_2^{(1)}A_2^{(2)} = -0.38 \pm 0.03\),
- \(A_2^{(2)}A_2^{(1)} = -0.032 \pm 0.018\).

![Fig. 5. Graphical analysis of the 367 keV-265 keV cascade.](image)
From these values and with the values of $A_2$ and $A_4$, the graphs given in fig. 5, can be constructed. By examination of these graphs, it is possible to see that the quadrupole contents are limited to the ranges $Q^{(1)} < 0.01$ and $0.06 < Q^{(2)} < 0.15$ or $0.60 < Q^{(2)} < 0.65$. The value found for the product $A_2^{(2)} A_2^{(1)}$ favours the second of these ranges. However, it should be mentioned that the 265 keV gamma ray is in an energy region where the counting rate efficiency of the polarization analyser increases very rapidly with the energy. Because of this, interferences from other cascades might be important and difficult to evaluate. Thus, we consider the two ranges given for $Q^{(2)}$ as permissible.

3.4. THE 210 keV-265 keV CASCADE

The analysis of this cascade is given in fig. 6. It is assumed that the first transition was a mixture of electric octupole and magnetic quadrupole. Using the values of $A_2^{(2)}$ found in the analysis of the 367 keV-265 keV cascade, the octupole content $O$ can be restricted to the values: $O^{(1)} < 0.002$ or $0.76 < O^{(1)} < 0.79$. A calculation done using the value of $J^1$ (here relation (1) does not apply) shows that the first range of value is favoured. The experimental value of $J^2$ seems to favour the region $0.06 < Q^{(2)} < 0.15$. However, as was mentioned in the analysis of the previous cascade, this measurement of $J^2$ might suffer from strong interferences.

3.5. THE 561 keV-367 keV, 561 keV-417 keV AND 561 keV-632 keV CASCADES

From the relative intensities of the 928 keV, 718 keV and 651 keV transitions, it can be assumed that the spin of the 1.19 MeV level is restricted to the possible values $\frac{9}{2}$, $\frac{7}{2}$ or $\frac{5}{2}$. 

![Graphical analysis of the 210 keV-265 keV cascade.](image-url)
The values of $A_4$ for the three cascades considered are all positive and of the same order of magnitude. But the value of the quadrupole content of the 367 keV transition has been shown to be smaller than 1%; this would lead to a value of $A_4$ equal to zero, at the accuracy of the measurements. We are then lead to assume that the non-zero values of $A_4$ for these three cascades are produced by interferences from high energy gamma rays.

The three possible spin assignments for the 1.19 MeV level are now considered.

3.5.1. The 1.19 MeV level has a spin $\frac{9}{2}$. The 561 keV transition can be assumed to be pure quadrupole and $A_2^{(1)}$ is equal to $-0.19$. Using the value of $Q(367)$ found previously, $A_2$ would be equal to $0.076 \pm 0.019$ for the 561 keV-367 keV cascade. The value found experimentally is $-0.035 \pm 0.020$; the case of a spin $\frac{9}{2}$ can then be ruled out.

3.5.2. The 1.19 MeV level has a spin $\frac{7}{2}$. From the experimental value of $A_2$ for the 561 keV-367 keV cascade, and using the value of $Q(367)$ found previously, a graphical analysis of the data shows that the value of $Q(561)$ is restricted to the ranges $Q(561) < 0.02$ or $0.95 < Q(561) < 0.99$.

Using the values of $Q(561)$ found above, the graphical analysis of the 561 keV-417 keV gives the results: $A_2^{(417)} > 0.7$ or $0.03 < Q(417) < 0.80$. It is assumed here that the spin of the 215 keV level is $\frac{3}{2}$, as will be justified later.

Similarly, the analysis of the 563 keV-632 keV can be made graphically, using the value of $Q(563)$ given above. The directional correlation gives $Q(632) < 0.10$. The polarization-direction correlation measurement made on the first transition of the 561 keV-632 keV cascade gives no other information.

3.5.3. The 1.19 MeV level has a spin $\frac{5}{2}$. In the case of an assignment $\frac{5}{2}^+$, the 561 keV and 718 keV transitions are M1 + E2 and E2, respectively. A transition to the ground state from the 1.19 MeV level would be E1 and strongly favoured. Its absence seems then to rule out the assignment of $\frac{5}{2}^+$.

An assignment of $\frac{5}{2}^-$ for the 1.19 MeV level gives a multipolarity E1 + M2 to the 561 keV transition. The quadrupole content should then be very small. The graphical analysis of the data shows that the quadrupole content of this transition is larger than 0.14.

3.6. THE 417 keV-215 keV CASCADE

The log ft value of the beta transitions from the $\frac{1}{2}^-$ isomeric level of Ge$^{77}$ at 159 keV allows the spin and parity values of $\frac{1}{2}^-$ or $\frac{1}{2}^+$ to the 215 keV level in As$^{77}$.

A spin of $\frac{1}{2}^+$ would make the 417 keV-215 keV cascade isotropic. Experimentally it is found that $A_2$ equals $-0.031 \pm 0.007$. The case $\frac{1}{2}^+$ is then ruled out.

Using the value of $Q(417)$ found previously, a graphical analysis of the cascade gives the values of $Q(215)$: $0.02 < Q(215) < 0.05$ or $0.99 < Q(215)$. The polarization-direction correlation experiment is consistent with both ranges.
3.7. SUMMARY OF THE ANALYSIS OF THE EXPERIMENTS

The spins and parities of the first five levels in As$^{77}$ have been confirmed, including a spin of $\frac{3}{2}$ for the 215 keV level. The experiment indicates that the spin of the 1.19 MeV level is $\frac{7}{2}$. The quadrupole and octupole contents listed in table 2 are consistent with the experiment. In table 2 is given another quantity of interest i.e. the sign of the mixing ratio $\delta$, as defined in ref. 8).

<table>
<thead>
<tr>
<th>Gamma transition (keV)</th>
<th>Quadrupole or octupole content</th>
<th>Sign of $\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>$Q^{(210)} &lt; 0.002$ (octupole content)</td>
<td></td>
</tr>
<tr>
<td>215</td>
<td>$0.02 &lt; Q^{(215)} &lt; 0.05$ or $0.99 &lt; Q^{(215)}$</td>
<td>$(-)$</td>
</tr>
<tr>
<td>265</td>
<td>$0.06 &lt; Q^{(265)} &lt; 0.15$ or $0.60 &lt; Q^{(265)} &lt; 0.65$</td>
<td>$(-)$</td>
</tr>
<tr>
<td>367</td>
<td>$Q^{(367)} &lt; 0.01$</td>
<td></td>
</tr>
<tr>
<td>417</td>
<td>$0.03 &lt; Q^{(417)} &lt; 0.80$</td>
<td>$(-)$ ($\ast$)</td>
</tr>
<tr>
<td>561</td>
<td>$Q^{(561)} &lt; 0.02$</td>
<td></td>
</tr>
<tr>
<td>632</td>
<td>$0.95 &lt; Q^{(632)} &lt; 0.99$</td>
<td>$(-)$</td>
</tr>
</tbody>
</table>

$\ast$) Measured in the 561 keV-417 keV cascade.

4. Discussion

The value of the quadrupole content of the 265 keV transition is more than 500 times the estimates for a single proton transition. It seems then likely that this transition involves a collective excitation. However, an attempt to describe the nucleus by a simple unified model calculation, based on the independent-particle spectrum given by Nilsson$^{12}$) is unsuccessful. Indeed, for 33 particles, the presence of a low-lying $\frac{9}{2}^+$ level suggests a large negative deformation with $\Omega = \frac{1}{2}$ for the ground state. The rotational energy is then given by

$$E_{rot} = \frac{\hbar^2}{2J} \left\{ I(I+1) + a(-1)^{I+\frac{1}{2}}(I+\frac{1}{2})\delta_{\Omega,\frac{1}{2}} \right\},$$

where $a$ is the decoupling parameter. However, there is no value of the decoupling parameter which gives the correct spin sequence. Moreover, the absence of the $\frac{1}{2}^-$ level in the observed spectrum is not explained. The introduction of rotation-particle coupling cannot explain the absence of this level either, as the effect of such an interaction would be to push apart the two $I = \frac{3}{2}$ levels of the $\Omega = \frac{1}{2}$ and $\Omega = \frac{3}{2}$ bands.

A pairing model calculation for the 33 neutrons, based on the spherical shell model is unsuccessful too. It seems then necessary to include pairing energy for both protons and neutrons.
References

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