$$F(i\beta) = \int_{-\infty}^{\infty} \frac{v^2\, \varphi\, dv}{v^2 + \beta^2} \,. \tag{7}$$

If k^2 is rewritten as

$$k^2 = \nu^2/\beta^2 \,, \tag{8}$$

where ν is the growth rate $\omega = i\nu$, then (3) appears as

$$\nu^2/\beta^2 = \int_{-\infty}^{\infty} \frac{v^2\, \varphi\, dv}{v^2 + \beta^2} \,. \tag{9}$$

Both sides of (9) are plotted in fig. 1.

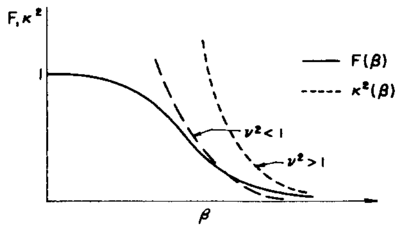

Fig. 1.

The inequalities which are depicted in fig. 1 are obtained by examining the asymptotic behaviour of F for large β, according to which

$$F \sim \frac{1}{\beta^2}\left(1 - \frac{3}{\beta^2} + \ldots\right) .$$

It follows that for large β both k^2 and the first derivative of k^2 are larger than the related values of F if $\nu^2 > 1$, which, in dimensional form, appears as

$$\nu > \omega_0 \,,$$

whence there are no solutions for $\nu > \omega_0$ and ω_0 represents the maximum growth rate of these instabilities.

As a concluding remark we note that all of the remaining modes are damped. The exact forms are obtained by continuing F analytically into the lower half z-plane. The only physically relevant fact about these damped modes is that some of them propagate (viz., the roots that lie off the imaginary axis).

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REGGE POLE HYPOTHESIS AND POLARISATIONS IN πp AND Kp SCATTERING

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Regge pole hypothesis [1] permits one to obtain the asymptotic behaviour of differential and total cross sections in the s channel for high energy, by assuming particles corresponding to the trajectories in the Chew-Frautschi diagram [2] being exchanged in the t channel of the same reaction. The high energy behaviour of various total cross sections was obtained in this way by Udgaonkar [3]. Gribov and Pomeranchuk [4] have pointed out that polarisations depend on interferences between various exchanged trajectories, thus providing additional information on the trajectories and their couplings. Polarisations were obtained by Hara [5] for NN scattering. In this paper, expressions for the polarisations are presented for πN and KN, $\bar{\text{K}}$N scattering, together with differential and total cross sections.

Using the general form of the scattering matrix for a zero-spin and a Dirac particle [6], we obtain for the differential cross section, asymptotically for $s \to \infty$

$$\frac{d\sigma}{dt} \approx \frac{1}{4\pi\delta}\left(\frac{m^2}{s}\,|A + \frac{s}{2m}B|^2 - \frac{t}{4s}\,|A|^2\right), \tag{1}$$

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and likewise for the polarisation

$$P \approx \tfrac{1}{2} \sqrt{-t}\, \mathrm{Im}\, B^* A \left(\frac{m^2}{s} \left|A + \frac{s}{2m} B\right|^2 - \frac{t}{4s} |A|^2\right) ; \qquad (2)$$

the optical theorem further gives

$$\sigma^{\mathrm{tot}} \approx \mathrm{Im} \left(\frac{2m}{s} A + B\right)_{t=0} . \qquad (3)$$

Here, m is the mass of the Dirac particle, s the squared total c.m. energy in the s channel, and t is the squared momentum transfer, related to the scattering angle by

$$\cos \theta \approx 1 + (2t/s) . \qquad (4)$$

The polarisation is given along the direction $\boldsymbol{k}' \times \boldsymbol{k}$, where $\boldsymbol{k}$ is the momentum of the incoming and $\boldsymbol{k}'$ that of the outgoing boson. The equations are correct if terms of order m^2/s are negligible; $|t|$ has been assumed to reach up to order m^2.

1. πp *scattering*. Here, the polarisation is already given implicitly in ref. [1]), the cross sections explicitly. For elastic scattering, $\pi^{\pm} + \mathrm{p} \to \pi^{\pm} + \mathrm{p}$, one has to replace

$$A \to A^+ \mp A^- , \qquad B \to B^+ \mp B^- , \qquad (5)$$

for charge exchange scattering, $\pi^- + \mathrm{p} \to \pi^0 + \mathrm{n}$,

$$A \to -\sqrt{2}\, A^- , \qquad B \to -\sqrt{2}\, B^- . \qquad (6)$$

From isotopic spin and G parity considerations, only the P, P' and ABC trajectories ($T = 0$, $G = +$) can contribute to A^+, B^+ and only ρ ($T = 1$, $G = +$) to A^-, B^-. One has

$$A^+ = -\frac{s_0}{2m} \left\{ \frac{1 + e^{-i\pi\alpha_P}}{\sin \pi\alpha_P} \left(\frac{s}{s_0}\right)^{\alpha_P} b_{AP} + \frac{1 + e^{-i\pi\alpha_{P'}}}{\sin \pi\alpha_{P'}} \left(\frac{s}{s_0}\right)^{\alpha_{P'}} b_{AP'} + \frac{1 + e^{-i\pi\alpha_{ABC}}}{\sin \pi\alpha_{ABC}} \left(\frac{s}{s_0}\right)^{\alpha_{ABC}} b_{A,ABC} \right\} , \qquad (7)$$

$$B^+ = -\frac{1 + e^{-i\pi\alpha_P}}{\sin \pi\alpha_P} \left(\frac{s}{s_0}\right)^{\alpha_P - 1} b_{BP} - \frac{1 + e^{-i\pi\alpha_{P'}}}{\sin \pi\alpha_{P'}} \left(\frac{s}{s_0}\right)^{\alpha_{P'} - 1} b_{BP'} - \frac{1 + e^{-i\pi\alpha_{ABC}}}{\sin \pi\alpha_{ABC}} \left(\frac{s}{s_0}\right)^{\alpha_{ABC} - 1} b_{B,ABC} , \qquad (8)$$

$$A^- = \frac{s_0}{2m} \frac{1 - e^{-i\pi\alpha_\rho}}{\sin \pi\alpha_\rho} \left(\frac{s}{s_0}\right)^{\alpha_\rho} b_{A\rho} , \qquad (9)$$

$$B^- = \frac{1 - e^{-i\pi\alpha_\rho}}{\sin \pi\alpha_\rho} \left(\frac{s}{s_0}\right)^{\alpha_\rho - 1} b_{B\rho} \qquad (10)$$

The $\alpha_P(t)$, $\alpha_{P'}(t)$ etc. are the Regge trajectories, and the $b(t)$ are unknown residue functions of t. All are real analytic functions [7]) with only right-hand cuts, thus are real for the physical values of $t < 0$. The quantity s_0 is an arbitrary parameter of dimension (mass)^2, usually taken as $2m^2$, or as $2m\mu$, with μ the boson mass.

The polarisations then depend only on interferences of different trajectories and behave as *

$$P \sim s^{\alpha_{P'} - \alpha_P} \qquad \text{or} \qquad \sim s^{\alpha_\rho - \alpha_P} , \qquad (11)$$

if the P' dominates the ρ trajectory, or vice versa; in the former case, $P_{\pi^+\mathrm{p}}$ and $P_{\pi^-\mathrm{p}}$ have the same sign, in the latter, opposite sign. Charge exchange scattering has $P = 0$, as it depends on one trajectory only. The total cross section is, with $b_{Ai} + b_{Bi} = \sigma_i(t)$

$$\sigma^{\mathrm{tot}}_{\pi^\pm \mathrm{p}} = \sigma_P(0) + \sigma_{P'}(0) \left(\frac{s}{s_0}\right)^{\alpha_{P'}(0)-1} + \sigma_{ABC}(0) \left(\frac{s}{s_0}\right)^{\alpha_{ABC}(0)-1} \mp \sigma_\rho(0) \left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1} ; \qquad (12)$$

* Calling $b_{Ai} + b_{Bi} = \sigma_i(t)$, $x_{ij} = [\epsilon_i \sin \pi\alpha_i - \epsilon_j \sin \pi\alpha_j + \epsilon_i \epsilon_j \sin \pi(\alpha_i - \alpha_j)]\, (\sin \pi\alpha_i \sin \pi\alpha_j)^{-1}$, where ϵ_i is the signature of the i-th trajectory, and

$$X_{ij} = x_{ij}\, \sigma_i \sigma_j \left(\frac{b_{Aj}}{\sigma_j} - \frac{b_{Ai}}{\sigma_i}\right) \left(\frac{s}{s_0}\right)^{\alpha_i + \alpha_j - 2} ,$$

the polarisation may be rewritten as

$$\left(\frac{d\sigma}{dt} P\right)_{\pi^\pm \mathrm{p}} = (\sqrt{-t}/16\pi m)\, [X_{PP'} + X_{P,ABC} + X_{P',ABC} \pm (X_{P\rho} + X_{P'\rho} + X_{ABC,\rho})] .$$

from the experimental behaviour of the high-energy cross sections, our $\sigma_i(0)$ are positive except perhaps for the ABC particle. Experiments give also [8] $\alpha_P(0) = 1$, $\alpha_{P'}(0) \approx \alpha_\omega(0) \approx 0.4$, $\alpha_\rho(0) \approx 0.3$ and $\alpha_{ABC}(0) \lesssim 0$; further Re $(\mathrm{d}\alpha/\mathrm{d}t) \approx m^{-2}$ for $t \geq 0$.

2. Kp *and* $\bar{\mathrm{K}}$p *scattering*. For elastic scattering, $\mathrm{K}^\pm + \mathrm{p} \to \mathrm{K}^\pm + \mathrm{p}$, one must put

$$A \to A_+^0 \mp A_-^0 \pm A' , \qquad B \to B_+^0 \mp B_-^0 \pm B' , \tag{13}$$

for charge exchange scattering, $\mathrm{K}^- + \mathrm{p} \to \bar{\mathrm{K}}^0 + \mathrm{n}$,

$$A \to -2A' , \qquad B \to -2B' . \tag{14}$$

From isotopic spin, parity, and G parity considerations, one has the trajectories P, P' and ABC contributing to A_+^0 ($T = 0$, $G = +$, parity = +), ω to A_-^0 ($T = 0$, $G = -$, parity = -) and ρ to A' ($T = 1$, $G = +$, parity = -). Accordingly,

$$A_+^0 = -\frac{s_0}{2m}\left\{\frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_P}}{\sin\pi\alpha_P}\left(\frac{s}{s_0}\right)^{\alpha_P} c_{AP} + \frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_{P'}}}{\sin\pi\alpha_{P'}}\left(\frac{s}{s_0}\right)^{\alpha_{P'}} c_{AP'} + \frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_{ABC}}}{\sin\pi\alpha_{ABC}}\left(\frac{s}{s_0}\right)^{\alpha_{ABC}} c_{A,ABC}\right\} , \tag{15}$$

$$B_+^0 = -\frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_P}}{\sin\pi\alpha_P}\left(\frac{s}{s_0}\right)^{\alpha_P-1} c_{BP} - \frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_{P'}}}{\sin\pi\alpha_{P'}}\left(\frac{s}{s_0}\right)^{\alpha_{P'}-1} c_{BP'} - \frac{1+\mathrm{e}^{-\mathrm{i}\pi\alpha_{ABC}}}{\sin\pi\alpha_{ABC}}\left(\frac{s}{s_0}\right)^{\alpha_{ABC}-1} c_{B,ABC} , \tag{16}$$

$$A_-^0 = \frac{s_0}{2m}\,\frac{1-\mathrm{e}^{-\mathrm{i}\pi\alpha_\omega}}{\sin\pi\alpha_\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega} c_{A\omega} , \qquad B_-^0 = \frac{1-\mathrm{e}^{-\mathrm{i}\pi\alpha_\omega}}{\sin\pi\alpha_\omega}\left(\frac{s}{s_0}\right)^{\alpha_\omega-1} c_{B\omega} , \tag{17}$$

$$A' = \frac{s_0}{2m}\,\frac{1-\mathrm{e}^{-\mathrm{i}\pi\alpha_\rho}}{\sin\pi\alpha_\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho} c_{A\rho} , \qquad B' = \frac{1-\mathrm{e}^{-\mathrm{i}\pi\alpha_\rho}}{\sin\pi\alpha_\rho}\left(\frac{s}{s_0}\right)^{\alpha_\rho-1} c_{B\rho} . \tag{18}$$

The polarisation, again depending only on interferences between different trajectories, behaves as

$$P \sim s^{\alpha_{P'}-\alpha_P} \quad \text{or} \sim s^{\alpha_\omega-\alpha_P} \quad \text{or} \sim s^{\alpha_\rho-\alpha_P} , \tag{19}$$

depending on whether P', ω or ρ lie closest to P. Again $P_{\mathrm{K}^+\mathrm{p}}$ and $P_{\mathrm{K}^-\mathrm{p}}$ have the same sign in the first case, opposite sign in the two latter cases, and there is no polarisation in charge exchange scattering. The total cross section is, with $c_{Ai} + c_{Bi} = \sigma_i^{\mathrm{K}}(t)$

$$\sigma_{\mathrm{K}^\pm\mathrm{p}}^{\mathrm{tot}} = \sigma_P^{\mathrm{K}}(0) + \sigma_{P'}^{\mathrm{K}}(0)\left(\frac{s}{s_0}\right)^{\alpha_{P'}(0)-1} + \sigma_{ABC}^{\mathrm{K}}(0)\left(\frac{s}{s_0}\right)^{\alpha_{ABC}(0)-1} \mp \sigma_\omega^{\mathrm{K}}(0)\left(\frac{s}{s_0}\right)^{\alpha_\omega(0)-1} \pm \sigma_\rho^{\mathrm{K}}(0)\left(\frac{s}{s_0}\right)^{\alpha_\rho(0)-1} \tag{20}$$

and the same remark applies as after eq. (12), except perhaps for the ABC and ρ particles.

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