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LEVEL STRUCTURE OF ^{32}P FROM ^{31}P(d, p)^{32}P REACTIONS

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Abstract: States up to an excitation energy of 6.8 MeV of the nucleus ^{32}P are investigated by studying the angular distribution of protons in the reaction ^{31}P(d, p)^{32}P. One new state is found at 1.51 MeV. A state previously reported at 6.56 MeV is found to be a doublet with excitation energies 6.58 MeV and 6.53 MeV. The angular distributions for 28 of the stronger levels are fitted with Butler curves, and probable l_n values are given for these levels. Relative reduced widths are given for the same levels. The results are discussed in terms of the Nilsson rotational model and the shell model.

1. Introduction

In view of the success with which the nuclear collective model accounts for the energy levels of some of the even nuclei and odd-mass nuclei in the region of the (1d) shell, this study was undertaken as a part of a larger investigation [1-4]) done with the University of Michigan cyclotron in order to see if the model also would give a simple description of the odd nuclei in this region.

The proton spectrum for the ^{31}P(d, p)^{32}P reaction is well known up to an excitation energy of 6.8 MeV [5,6]). Also the proton angular distributions are measured previously in the same energy region [5,7,8]), but with the exception of the investigation by W.C. Parkinson [7]) in this laboratory, the proton analysing systems used for the angular distribution measurements have not been good enough to resolve the proton groups, thus making an interpretation of the experimental results difficult. In the present work it has been possible to identify and resolve most proton groups leading to the stronger stripping levels below 6.8 MeV excitation energy.

2. Experimental Procedure

This study has been performed using essentially the same procedure as Parkinson [7]) used for the ground state doublet: Targets of Li_3PO_4 evaporated on backing of gold leaves were bombarded with 7.8 MeV deuterons from the University of Michigan cyclotron. The protons were analysed by a high resolution magnetic spectrometer [9]) and detected by nuclear emulsion plates. In some cases, when the proton groups from the ^{31}P(d, p)^{32}P reaction were interferred by proton groups from reactions in Li,

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or when a check for possible impurities in the target was needed, targets of P_2O_5 on gold leaves were employed. As these targets were exposed to air on the way from the evaporator to the target chamber, they absorbed some water, which they partly gave off again during the first half hour of bombardment. From then on they appeared fairly unchanging. Changes in the target were easily noticed by comparison of the bombarding current as measured by the current integrator and the number of monitor counts from protons emitted at 90°. The P_2O_5 targets, however, still always contained relatively more oxygen than the Li_3PO_4, and were not well suited for high resolution work on weak transitions. As impurities in the Li_3PO_4 targets were found carbon and small amounts of nitrogen and sodium. Proton groups from these impurities were identified by their kinematic shift in energy by angle, and by comparing the measured energies with energies of known strong proton groups from (d, p) reactions in the elements in question. At excitation energies above 6.5 MeV elastic scattered protons from the d+H reaction would come in at small angles.

The energy resolution of the cyclotron and analysing system is 15-20 keV, but in order to get high enough proton intensity, the targets used were mostly somewhat too thick to utilize the highest resolution. In most cases an energy spread of 30-40 keV of protons from one group was sufficient to identify the stronger levels. A weak level too near a strong one, could not be well measured, but in these cases an increase of the resolving power by a decrease in the target thickness, would have led to a corresponding increase in running time in order to get a sufficient number of protons from the weaker transition. This again would rise difficulties in keeping the instrumentation stable over a long time, and little would have been gained. Usually running time for one exposure was between 25 and 50 min.

In one exposure around 10% of the energy spectrum could be covered. The magnetic field was varied with angle in order to keep the peak corresponding to the same level in the same position on the plates, thus making identification simple and avoiding correction for solid angle due to a shift in position. As the resolution decreased a little towards the edges of the plates, the stronger peaks, which were considered the most interesting, were usually placed near the centre of the plates. Plates from different runs always overlapped. Usually the agreement between different runs was good when compared with the statistical errors due to the number of counts. But in some cases, especially at low angles, the agreement was not as good as expected from pure statistical consideration. At these angles scattered protons or protons produced by scattered deuterons caused a considerable background in the plates. Corrections for this background were estimated from the number of registered tracks between the peaks. However, in regions where the level density was high, only few regions on each plate could be considered as suitable for background readings. The background varied very much as well within one plate, as from one plate to another. It also depended severely on the judgement of the plate reader, who was supposed to count tracks coming in right direction and with right length only, a judgement that easily could vary from time to time. The corrections used were consequently rather uncertain.

The angular distributions were measured in steps of 5° between 0° and 50° and in steps of 10° between 50° and 90°.

Absolute energy measurements were not attempted. The values used for the excitation energies are the ones given by Piraino *et al.* [6]), or for the levels not reported by them, based upon the value for their nearest level and the known dispersion of the analyser.

3. Experimental Results

During the experiments two new peaks were identified as due to the ^{31}P(d, p)^{32}P reaction, one from a transition to a level at (1.51 ± 0.02) MeV (fig. 1), and one due to the resolving of a doublet given by Dalton *et al.* [5]). These authors identified a level at 6.56 MeV, which is now determined to be one level at (6.53 ± 0.02) MeV and one at (6.58 ± 0.02) MeV (fig. 2).

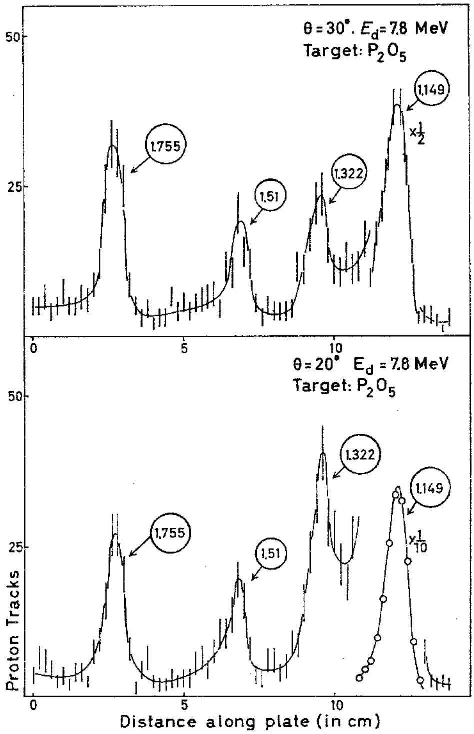


Fig. 1. Parts of spectra obtained with a P_2O_5 target showing the proton peak from transitions to the 1.51 MeV level.

Some of the weaker peaks reported previously were not resolved from adjacent stronger peaks in our experiments. However, the position of the peaks usually gave a clear indication of which transition was the dominating in the reaction. An example of this is shown in fig. 3. Three levels at 3.994 MeV, 4.010 MeV and 4.040 MeV are known to exist [6]). The positions of proton groups from transitions to these levels, calculated from the 4.412 MeV level, are shown in the figure. Dalton *et al.* with a resolution too low to resolve these groups, report a strong $l_n = 1$ transition to a level at 4.032 MeV. From fig. 3 it is obvious that the $l_n = 1$ transition mainly goes to the 4.040 MeV level. An estimate of the intensities and angular distributions for the transitions to the two other levels is uncertain.

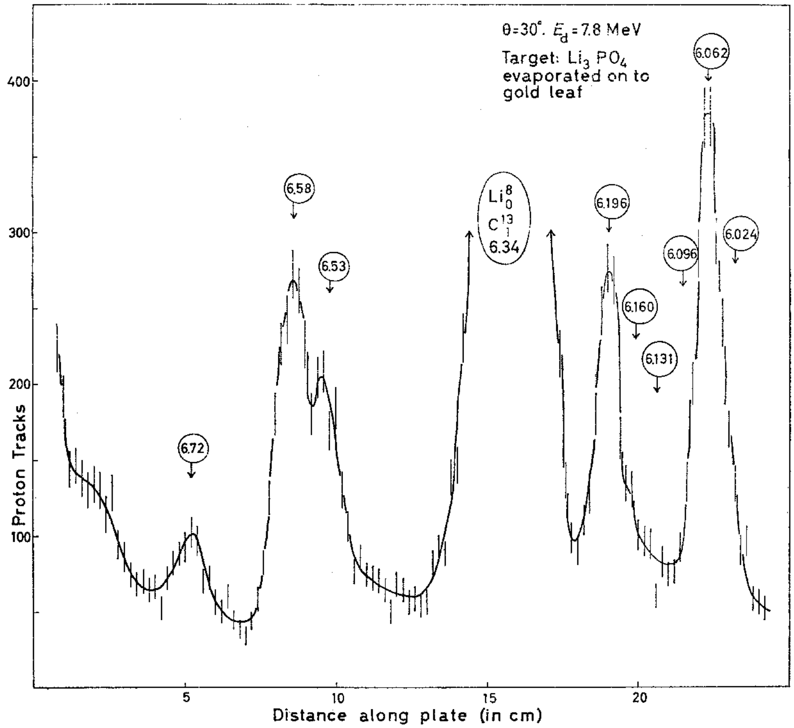


Fig. 2. Part of spectrum obtained with a Li_3PO_4 target showing the levels at 6.53 MeV and 6.58 MeV. There are indications of several more weak levels in this energy region.

For all the stronger transitions attempts were made to fit the measured angular distributions with theoretical Butler curves [10]) as obtained from the tables calculated by Lubitz [11]) with simple Butler-Born approximation theory. In this way values were obtained for the relative reduced width θ^2 multiplied by $[I] = 2I+1$, and for the orbital angular momentum l_n of the captured neutron. The reliability of the reduced widths obtained by this procedure, and their application when seeking information about the nuclear structure is discussed by M. H. Macfarlane and J. B. French in their review article about stripping reactions [12]). As is pointed out in the same article, an ambiguity often exists in the determination of l_n by this procedure, due to the uncertainty in the choice of the cut-off radius r_0 in the calculation.

For the $l_n = 2$ transitions to the ground state and the first excited state in ^{32}P, good fit is obtained with the Gamow Gritschfield value $r_G = (1.7+1.22\,A^{\frac{1}{3}})$fm $=$ 5.7 fm (ref. [7])). Most of the higher excited states are, however, fitted only with a considerably larger ($r_0 \approx 7$ fm) or smaller ($r_0 \approx 4$ fm) value, thus leaving open the question of choosing between the two corresponding l_n values.

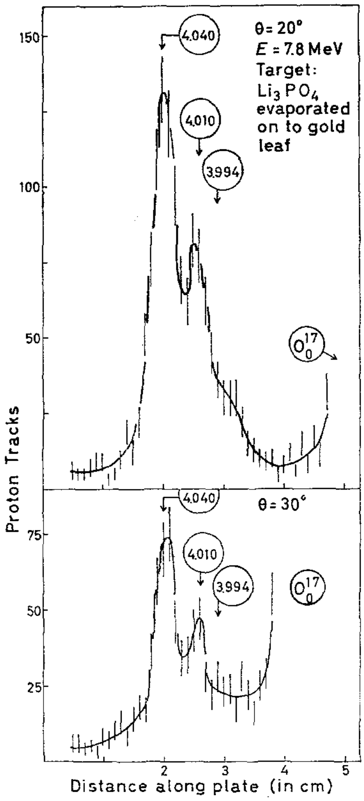


Fig. 3. Spectra obtained at 20° and 30° showing transitions to the three levels at 3.994 MeV, 4010 MeV and 4.040 MeV.

The results of the measurements are summarized in table 1. In fig. 4 are shown the angular distributions obtained, together with the calculated "best fitted" Butler curve (or curves) for each level. "Best fit" was considered only around the peak of the distribution. In most cases a value of r_0 more than 10% different from the value used, would disagree with the experimental points.

In several cases a complete angular distribution from 0° to 90° was not obtained.

TABLE 1

Relative reduced widths θ^2 and orbital angular momentum l_n of the captured neutron for the stronger ^{31}P(d, p)^{32}P transitions

Exitation energy (MeV)	$J\pi$	$l_n = 0$		$l_n = 1$		$l_n = 2$		$l_n = 3$		From ref. [5]	
		r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	l_n	$[J]\theta^2$
0	1^+	7.5	0.20			5.7	3			2	7.2
0.077	2^+					5.7	4.25				
0.516	$0,1^+$	7	0.69							0	0.7
1.149	1^+	7	1.23							0	1.2
1.322											
1.51 ± 0.02											
1.755											
2.177										0,1	0.6
2.223	1^+	7	0.34			5	1.20				
2.657						(4.25)	(0.40)	(6.5)	(0.60)		
2.743						5	0.18				
3.007						(4.25)	(0.46)	(6.5)	(0.70)		
3.148											
3.265	2^-			4.5	4.8					1	8
3.324						(4.25)	(3.4)	6.5	4.9		
3.447						(4.25)	(6.2)	6.5	9.2		
3.798											
3.890											
3.994											
4.010				4.25	[1.0]					1	9
4.040				4.25	4.3	(7.25)	(9.6)				
4.158											
4.209		7	0.41							0	0.8
4.280											
4.316											
4.412				4.5	0.19	(7.5)	(0.40)			1	1
4.560											
4.615											
4.664				4	[1.5]	(7)	(3.2)				
4.878				4	1.6	(7)	(3.3)			1	5
4.944											
5.010											
5.077						(4)	(1.6)	6	2.0		
5.129											
5.237											
5.346				4	2.7	(7)	(6.0)			1	7
5.394											
5.510				4	0.9	(7)	(2.0)			1	2
5.550											
5.657											
5.700				4	0.3			6	3.4		
5.724											
5.775				4	2.8	(7)	(6.3)			1	8
5.813											
5.835											
5.858											

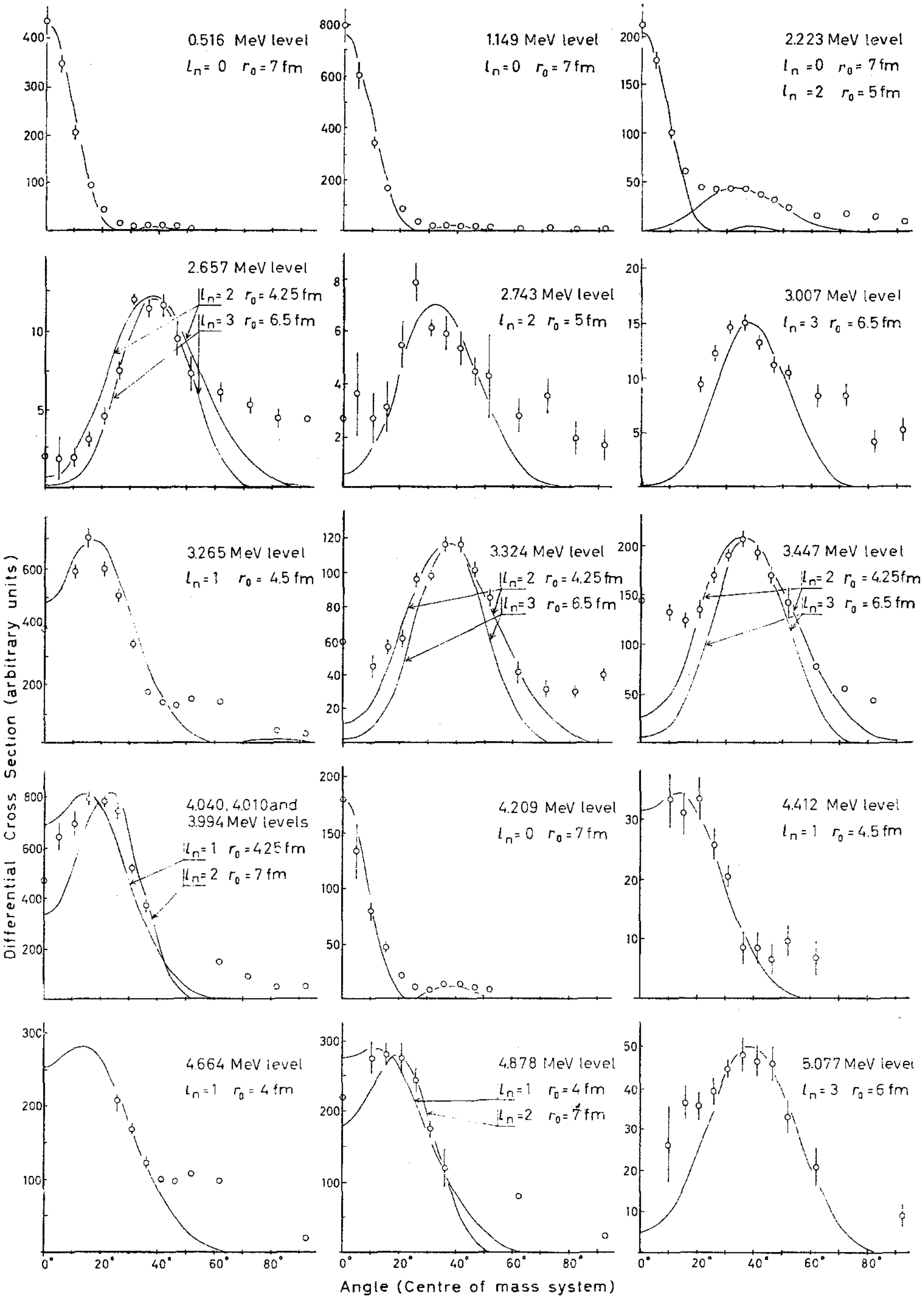
TABLE 1 (continued)

Exitation energy (MeV)	$J\pi$	$l_n = 0$		$l_n = 1$		$l_n = 2$		$l_n = 3$		From ref. [5])	
		r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	r_0(fm)	$[J]\theta^2$	l_n	$[J]\theta^2$
5.964											
5.989											
6.024											
6.062				4	[1.0]	(6)	(2.4)			1	3
6.096											
6.131											
6.160											
6.196				4	[0.8]	(6)	(1.9)				
6.34±0.02		5.5	0.70							1	1
6.53±0.02								6.5	1.3	} 1,2	
6.58±0.02											
6.71±0.02								6	0.8		

In cases where no definite assignment can be made from the Butler curves alone, the two alternative sets of values for l_n, r_0 and $[J]\theta^2$ are given in paranthesis. In cases where one value is to be preferred from other reasons the least probable is placed in paranthesis. In brackets are placed the reduced widths in the cases where the values are uncertain due to lack of data around the peak. For completeness all known levels are included in the first column. The uncertainties in the energy determinations in this experiment are given for the levels not reported in ref. [6]). The values of $[J]\theta^2$ for the $l_n = 0$ part of the ground state and for the 0.077 MeV level are taken from ref. [7]).

At low angles a large amount of scattered protons or protons from scattered deuterons would come in, especially when working on the low energy side of one of the strong ^{7}Li, ^{13}C or ^{17}O peaks. This explains the lack of data at 0° and (or) 5° for some levels. At larger angles the lack of data is due to coincidences with one of the unwanted peaks. In most cases, however, this is of little importance with regard to interpretation. Only for the levels at 4.664 MeV and 6.196 MeV the observations are insufficient to give a reliable idea about the distribution at angles below 25°. However, in both these cases we have a transition to an adjacent level (the 4.878 MeV level and the 6.062 MeV level) of about the same strength and showing the same angular dependence at higher angles, indicating that they may be pair of transitions with same l_n value, and also making it possible to assume a value for the cross-section. The assumed values of θ^2 for these levels are placed in brackets in table 1.

To the three levels around 4 MeV (see fig. 3) there seems to be two rather strong transitions, namely to the $l_n = 1$ level at 4.040 MeV and to the level at 4.010 MeV. The ratio between the transitions to the 4.010 MeV and the 4.040 MeV level are from the peak heights estimated to be about 1 : 4 at all angles below 30°. Between 40° and 50° they are covered by the ^{17}O peak and at higher angles they are both rather weak. Although nothing definite can be said about the transition to the 4.010 MeV level, the angular distribution obtained indicates $l_n = 1$ and an intensity of about 25% relative to the 4.040 MeV level.


0.516 MeV level
$l_n = 0$ $r_0 = 7$ fm
1.149 MeV level
$l_n = 0$ $r_0 = 7$ fm
2.223 MeV level
$l_n = 0$ $r_0 = 7$ fm
$l_n = 2$ $r_0 = 5$ fm
2.657 MeV level
$l_n = 2$ $r_0 = 4.25$ fm
$l_n = 3$ $r_0 = 6.5$ fm
2.743 MeV level
$l_n = 2$ $r_0 = 5$ fm
3.007 MeV level
$l_n = 3$ $r_0 = 6.5$ fm
3.265 MeV level
$l_n = 1$ $r_0 = 4.5$ fm
3.324 MeV level
$l_n = 2$ $r_0 = 4.25$ fm
$l_n = 3$ $r_0 = 6.5$ fm
3.447 MeV level
$l_n = 2$ $r_0 = 4.25$ fm
$l_n = 3$ $r_0 = 6.5$ fm
4.040, 4.010 and 3.994 MeV levels
$l_n = 1$ $r_0 = 4.25$ fm
$l_n = 2$ $r_0 = 7$ fm
4.209 MeV level
$l_n = 0$ $r_0 = 7$ fm
4.412 MeV level
$l_n = 1$ $r_0 = 4.5$ fm
4.664 MeV level
$l_n = 1$ $r_0 = 4$ fm
4.878 MeV level
$l_n = 1$ $r_0 = 4$ fm
$l_n = 2$ $r_0 = 7$ fm
5.077 MeV level
$l_n = 3$ $r_0 = 6$ fm
Differential Cross Section (arbitrary units)
Angle (Centre of mass system)
0° 20° 40° 60° 80°

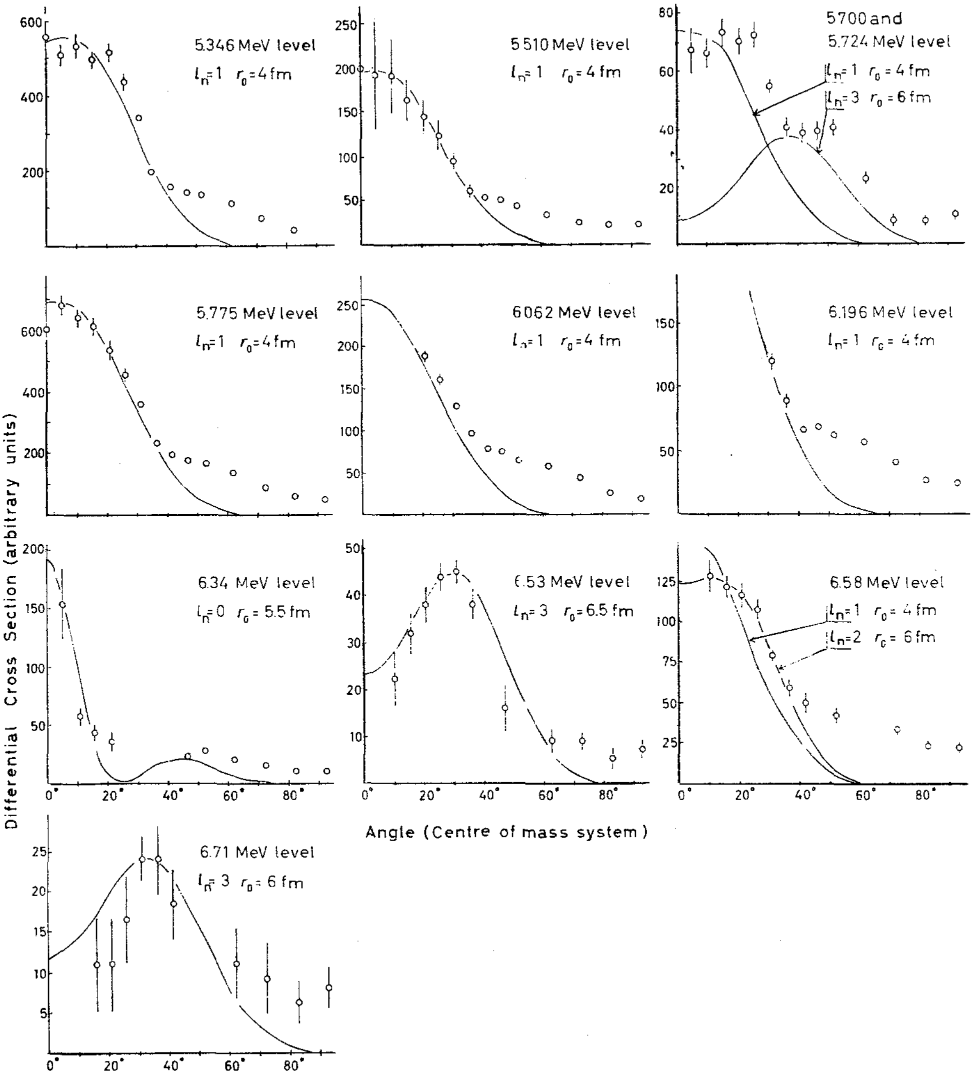


Fig. 4. Angular distributions for the stronger proton groups in the ^{31}P(d, p)^{32}P reaction. The cross sections for all distributions are given in same arbitrary units. "Best fitted" Butler curve(s) is shown for each distribution.

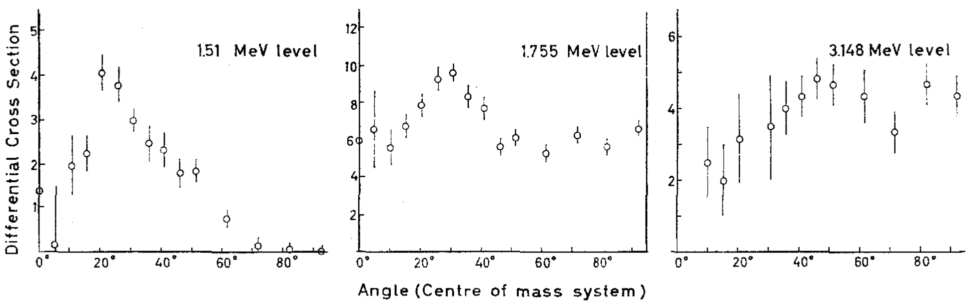


Fig. 5. Angular distribution for a few of the weaker proton groups. The units are the same as in fig. 4.

Angular distribution measurements for some transitions not listed in table 1 were also carried out. Only in a few cases the corresponding levels were so well separated that a significant result could be obtained for a single transition. Some examples of distributions obtained for the weakest transitions are shown in fig. 5. In most cases, adjacent strong proton groups interferred, and no reliable distribution was obtained. There are no indication of transitions noticeably stronger than the weaker ones listed in the table.

4. Comparison with Other Data

The angular distributions obtained are in good agreement with the results obtained by Dalton *et al.* [5]), but the higher resolution used here clarify the situation in cases where there previously has been doubt whether the distribution obtained was due to one single or a group of levels. The new results may be specified as follows:

The distribution from the 2.223 MeV level is explained as a mixture of $l_n = 0$ and $l_n = 2$ components. The transition to the lower lying 2.177 MeV level is at all angles much weaker, and does not contribute noticeably to the angular distribution curve, even if not resolved.

The transitions to the levels at 3.265 MeV and 3.324 MeV, previously unresolved in angular distribution measurements, are both strong stripping transitions with $l_n = 1$ and $l_n = 3$ (or 2) respectively.

To the 6.34 MeV level Dalton *et al.* report a weak $l_n = 1$ transition. The angular distribution, now extended from 10° to 5°, indicates that this is a $l_n = 0$ transition.

In agreement with Dalton *et al.* we find that their $l_n = 1$ distributions show a drop below the Butler curves at low angles if their value of $r_0 = 5.5$ fm is used. By using $r_0 = 4$ fm much better fits are obtained. Reasonable fits are also obtained with $l_n=2$ and $r_0 = 7$ fm and the possibility that these transitions are $l_n = 2$ transitions could not be completely ruled out.

Whereas Dalton *et al.* suggest $l_n = 1$ or $l_n = 2$ for the transition to the 6.56 MeV level, we find that the angular distribution for the transition to the lower level in this doublet is best described by $l_n = 3$ and $r_0 \approx 6.5$ fm. To the upper level we find $l_n = 2$ and $r_0 \approx 6$ fm. If $l_n = 1$ is assumed for this transition, an unreasonably low value ($r_0 \approx 3.5$ fm) is required to obtain fit between the angular distribution and the Butler curve.

A large number of transitions, including these to the levels at 3.447 MeV and 5.077 MeV (3.45 MeV and 5.11 MeV in ref. [5])) give rise to angular distributions of very similar shape. Around the peak the angular distributions may be described by Butler curves assuming either $l_n = 2$, $r_0 \approx 4$ fm or $l_n = 3$, $r_0 \approx 6.5$ fm. Any experimental evidence for preferring either of these assumptions does not exist. It may be noticed that whereas the stronger of these transitions all seem to be described by the same l_n and r_0 value, the uncertainties in the experimental points for the weaker transitions are too large to place them definitely in the same group.

In one case, for transitions to levels around 5.70 MeV we have a distribution of quite different shape. This may be interpreted as a mixture of $l_n = 1$ and $l_n = 3$ transitions. The position of the proton peak at larger angles indicates that the assumed $l_n = 3$ part of the transition goes to the 5.700 MeV level. At angles below 30° the proton peak interferes with the peak from the strong $l_n = 1$ transition to the 5.775 MeV level. After correcting for the contribution from this transition, it is not possible to say if the remaining part belongs to the 5.700 MeV or the 5.724 MeV level.

Whereas the angular distributions obtained in this work generally are in good agreement with what is previously reported, the values given for the relative reduced widths differ considerably. This might be explained as due to the better resolution used in our experiment, whereby contributions from adjacent levels were avoided in the measured cross-section. It may also partly be a result of different procedures used by Dalton *et al.* and by us when fitting Bulter curves to the experimental points. The discrepancies are, however, too large to be explained by these two factors alone. As the two experiments are carried out with somewhat different deuteron energies (8.9 MeV by Dalton *et al.*, 7.8 MeV in our experiment) the results may indicate that the energy dependence of the reduced widths as obtained from the Butler-Born approximation theory, is not negligible.

5. Discussion

5.1. GENERAL

The stripping process gives no unambiguous assignment of the spin value for the final state of the residual nucleus, but has to be supplemented with other data. In this case other data available are limited to γ-spectroscopy [13]) and γ-γ angular correlation measurements [14]). From these works it is suggested that the 0.516 MeV level has $J = 0$ or 1, the 1.149 MeV level has $J = 1$ and a level at 3.27 MeV has $J^\pi = 2^-$.

In the stripping measurements six distributions show contributions from $l_n = 0$ transitions. The corresponding levels have $J^\pi = 0^+$ or 1^+. The 1.149 MeV level, belonging also to this group, is thus a 1^+ state. The ground state and the 2.223 MeV level are reached also through $l_n = 2$ transitions and are therefore 1^+ states. For the levels at 0.516 MeV, 4.209 MeV and 6.34 MeV no further information exists.

The 2^- level at 3.27 MeV is obviously the same as the strong stripping level at 3.265 MeV, which is in agreement with the assumption of a $l_n = 1$ transition to this level.

The rest of the stronger stripping levels may be divided into two groups, one described with $l_n = 1$, $r_0 \approx 4$ fm or $l_n = 2$, $r_0 \approx 7$ fm and the other with $l_n = 2$, $r_0 \approx 4$ fm or $l_n = 3$, $r_0 \approx 6.5$ fm. From the simple Butler-Born approximation stripping theory it is not possible to make any conclusions about which of these values are to be preferred. It is known from similar cases in other nuclei [15]) that a distorted wave calculation will be a better way of describing the stripping process, and probably lead to an unambigious assignment of l_n values.

However, it seems necessary to assume that at least one of the above mentioned groups has odd parity. It is known that a competition in energy exists between the ($1f_{\frac{7}{2}}$) and (2p) levels. In other nuclei in this mass region (^{29}Si, ^{33}S, ^{36}Cl) the lowest (1f) and (2p) levels are not very different in energy. Since these levels are reached by $l_n = 3$ and $l_n = 1$ transitions respectively, it seems likely that if the $l_n = 1$ transitions are found, also the $l_n = 3$ transitions should be seen. It will be discussed later whether it is possible to explain one of the groups mentioned as due to $l_n = 2$ transitions.

5.2. THE NILSSON MODEL

The purpose of this work was to make a test of the application of the Nilsson rotational model [16]. To make such a test we ought to have some information about the spin of the states in question. Since the stripping reaction gives us knowledge only about the l_n value, and usually allows a choice between 3 different values of the final spin, no reliable test can be carried out based on this information only. For the odd parity states we know the J value for the 3.265 MeV level only and do not have any possibility of identifying rotational bands. For the even parity levels the situation is simpler, and an attempt is made to interpret the results in terms of the Nilsson model. This model has previously been applied with some success on the ^{31}P nucleus [17].

We assume, using the enumeration of levels from the Nilsson scheme, that the ^{31}P ground state corresponds to the configuration $(1-7)^4(9)^3$. The even parity states of ^{32}P formed in the (d, p) stripping reaction will then have an additional neutron in level 8 or 11. In the following discussion we think of the nucleus ^{32}P as one neutron and one proton added to a core with the configuration $(1-7)^4(9)^2$, which is the same as used for the ^{31}P nucleus. It is then reasonable to expect that the basic parameters involved, i.e. the nuclear deformation and the moment of inertia parameter $\hbar^2/2\mathscr{I}$, are about the same for the two nuclei. Further information about the values to be used for these parameters may be obtained from the magnetic moment and from the ft value for the β-decay of the ^{32}P ground state.

The levels of ^{32}P are in the simple rotational model description characterized by the total angular momentum I and its projection $K = \Omega_n \pm \Omega_p$ on the nuclear symmetry axis. Ω_n and Ω_p are the projections on this axis of the odd neutron and odd proton angular momentum respectively. For fixed values of K and Ω_n(neglecting rotational particle coupling) we get rotational bands with the spin sequences $I = K, K+1, \ldots$, and energies $E_I = E_0 + I(I+1)\hbar^2/2\mathscr{I}$. The constant E_0 is different for each rotational band. It includes the individual particle energies and effects of residual interactions. Since we have $\Omega_p = \frac{1}{2}$, and also may have $\Omega_n = \frac{1}{2}$ the rotational particle or Kerman coupling (R.P.C.) [18] will play an important role, giving rise to interactions between states with the same I. From these considerations a level scheme as shown in fig. 6 may be drawn. Arrows indicate the levels which interact through rotational particle coupling. In the stripping reaction the $\Omega_n = \frac{3}{2}$ levels will be reached through $l_n = 2$ transitions only, whereas $\Omega_n = \frac{1}{2}$ levels are reached through $l_n = 0$ as well as $l_n = 2$ transitions, when the spin of the final state allows both values.

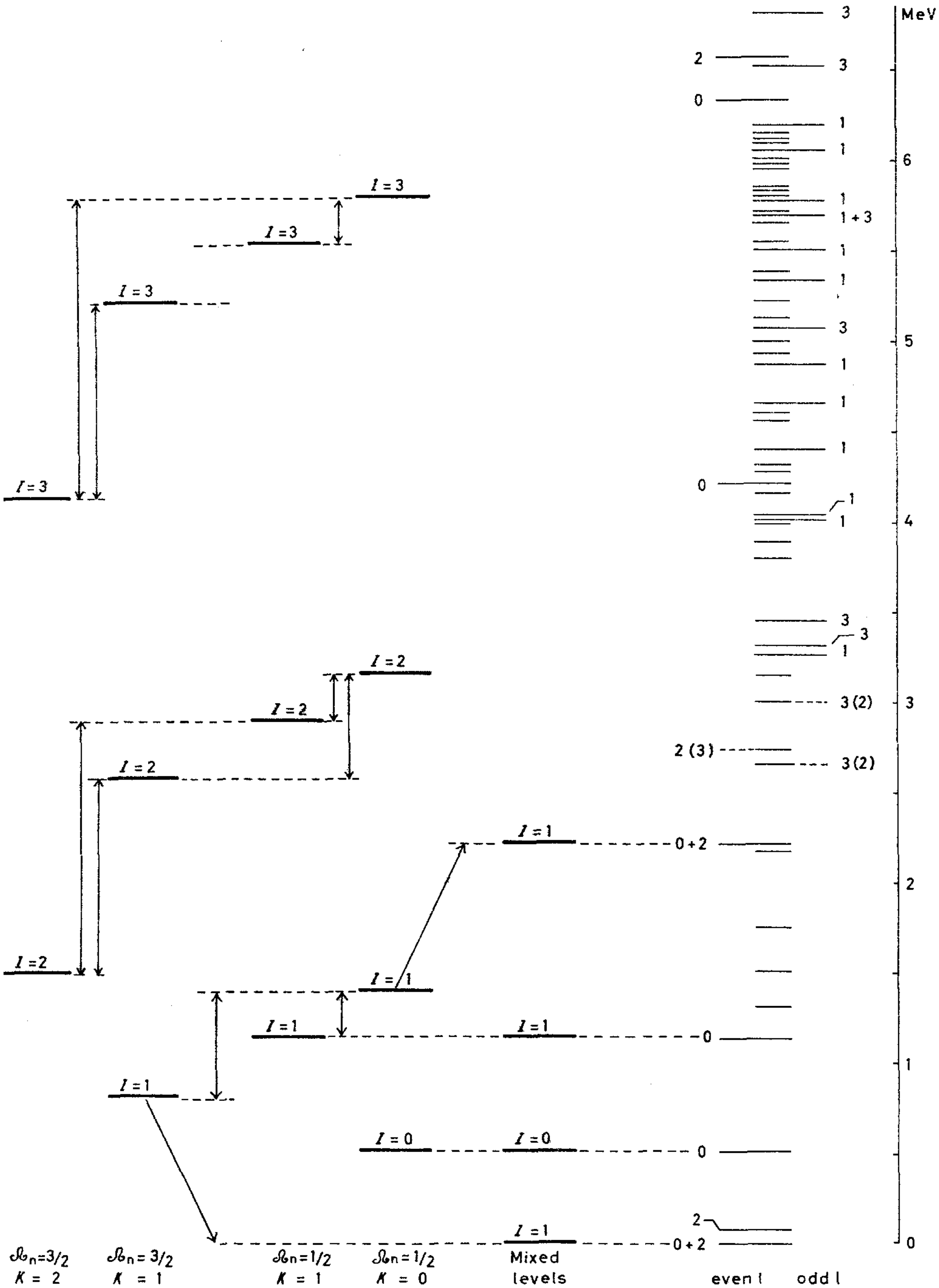


Fig. 6. Diagram of the even parity levels drawn on the basis of the Nilsson model. On the right is shown the experimentally found level diagram, divided in groups of even and odd parity levels. The numbers given are the assumed l_n values.

It is seen that we have one $I = 0$ and three $I = 1$ levels, in agreement with the four levels below 2.5 MeV showing $l_n = 0$ transitions. Of these it is known that only the 0.516 MeV level may have $I = 0$. The ground state, reached mainly through a $l_n = 2$ transition, must correspond to the $I = 1$, $\Omega_n = \frac{3}{2}$ level. The two $l_n = 0$ transitions to the levels at 4.209 MeV and 6.34 MeV are not accounted for.

To check the assumptions made here, and to see if also the $I = 2$ levels can be accounted for, more careful calculations have to be done. We need then information about the nuclear deformation and the moment of inertia.

The nuclear deformation may be determined by plotting the total particle energy for a given configuration as a function of deformation (using formula (C 3) in Nilssons's article). Such a plot is shown in fig. 7 for the case with 2 neutrons and one proton in level 9 and one neutron in level 8 or level 11. The deformation parameter δ is the one defined by Nilsson. The plot is made, using different values for the ratio κ between spin-orbit coupling and well depth. The plot indicates a small negative deformation.

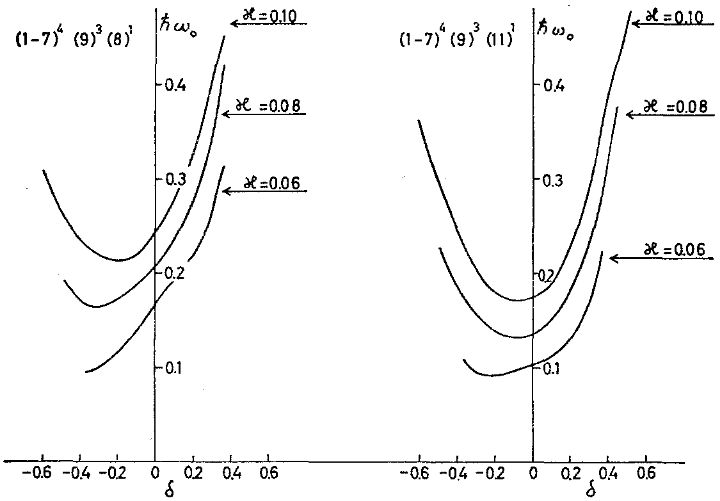


Fig. 7. Plot of the total binding energy of ^{32}P as a function of the nuclear deformation, assuming the levels 1-7 completely filled, 2 neutrons and 1 proton in level 9 and the last neutron either in level 8 or 11.

The β-decay of ^{32}P is characterized by an extremely high ft value ($\log ft = 7.9$) for an allowed transition. This is usually explained as due to l-forbiddenness. Calculation of the transition rate based on the Nilsson model is carried out assuming the ground state of ^{32}P to be a mixture of the pure rotational states $|\Omega_n = \frac{3}{2}, K = 1\rangle$ and $|\Omega_n = \frac{1}{2}, K = 0\rangle$ and the ground state configuration of ^{32}S to be $(1-7)^4\ (9)^4$. Only for a nearly spherical nucleus the high ft value is obtained. This corresponds to the odd neutron being in a pure ($1d_{\frac{3}{2}}$) state, and does not agree with the fact that the ^{32}P ground state is formed by a considerable amount of s-wave neutrons.

The magnetic moment can be calculated by the method suggested by Nilsson. It is again assumed that the ground state is a mixture of $|\Omega_n = \frac{3}{2}, K = 1\rangle$ and $|\Omega_n = \frac{1}{2}, K = 0\rangle$ states:

$$|(^{32}\mathrm{P})_{\mathrm{g\,st}}\rangle = \alpha_1|\Omega_n = \tfrac{3}{2}, K = 1\rangle + \alpha_0|\Omega_n = \tfrac{1}{2}, K = 0\rangle. \tag{1}$$

The magnetic moment may then be written as

$$\mu = \alpha_1^2 \mu_{K=1} + 2\alpha_1\alpha_0\mu_{10} + \alpha_0^2\mu_{K=0}. \tag{2}$$

Fig. 8 shows a plot of the three quantities $\mu_{K=1}$, μ_{10} and $\mu_{K=0}$ as a function of the deformation parameter $\eta = \delta/\kappa$. For the ground state $\alpha_1\alpha_0$ has to be positive. It is seen that the experimental value $\mu_{\mathrm{exp}} = -0.24$ n.m. corresponds to a small deformation, $-2 < \eta < 1$, and little admixture of the $|\Omega_n = \frac{1}{2}, K = 0\rangle$ state.

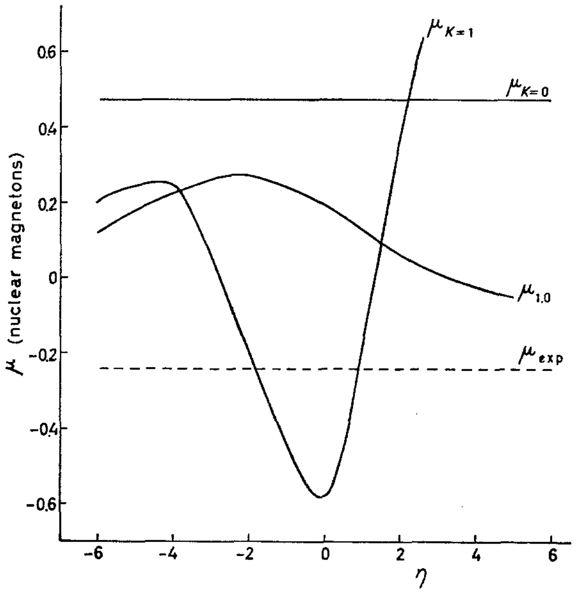


Fig. 8. Magnetic moment of ^{32}P calculated as a function of the nuclear deformation, assuming the ground state to be a mixture of $\Omega_n = \frac{3}{2}$, $K = 1$ and $\Omega_n = \frac{1}{2}$, $K = 0$ states. The coefficient $2\alpha_1\alpha_0$ for the term μ_{10} is positive for the ground state. The experimental value $\mu_{\mathrm{exp}} = -0.24$ n.m. is consistent only with a small deformation and little contribution from the $\Omega_n = \frac{1}{2}$ part.

To determine the energy levels we have to diagonalize the matrix:

$$|\langle\Omega_n, IK|\mathscr{H}_0 - \sum_i (I'_+ j'_{i+} + I'_- j'_{i-})|\Omega'_n, IK'\rangle|. \tag{3}$$

The values of the matrix elements are given explicitly in table 2.

The matrix elements are for simplicity written in units of $\hbar^2/2\mathscr{I}$ in the table. E_2, E_1, E'_1 and E_0 are the eigenvalues of the operator $\mathscr{H}'_0 = \mathscr{H}_0 - I^2 + K(K+1)$.

They give the position of the lowest ($I = K$) state for each band, and will be regarded as empirically determined parameters. The parameters A_{nm} and a_n are given by:

$$A_{nm} = \sum_j C_j^n C_j^m (j+\tfrac{3}{2})^{\frac{1}{2}} (j-\tfrac{1}{2})^{\frac{1}{2}}, \tag{4}$$

$$a_n = \sum_j (C_j^n)^2 (j+\tfrac{1}{2})(-1)^{j-\frac{1}{2}}, \tag{5}$$

where C_j^n are the linear expansion coefficients in the j-Ω-representation, and may be calculated from Nilsson's tables.

TABLE 2

Matrix elements of the rotational Hamiltonian

Ω_n, K	$\frac{3}{2}$, 2	$\frac{3}{2}$, 1	$\frac{1}{2}$, 1	$\frac{1}{2}$, 0
$\frac{3}{2}$, 2	$E_2+I(I+1)-6$	$-a_9[(I-1)(I+2)]^{\frac{1}{2}}$	$-A_{8,11}[(I-1)(I+2)]^{\frac{1}{2}}$	0
$\frac{3}{2}$, 1	$-a_9[(I-1)(I+2)]^{\frac{1}{2}}$	$E_1+I(I+1)-2$	0	$(-)^I A_{8,11}[I(I+1)]^{\frac{1}{2}}$
$\frac{1}{2}$, 1	$-A_{8,11}[(I-1)(I+2)]^{\frac{1}{2}}$	0	$E'_1+I(I+1)-2$	$[(-)^I a_9-a_{11}][I(I+1)]^{\frac{1}{2}}$
$\frac{1}{2}$, 0	0	$(-)^I A_{8,11}[I(I+1)]^{\frac{1}{2}}$	$[(-)^I a_9-a_{11}][I(I+1)]^{\frac{1}{2}}$	$E_0+I(I+1)$

For $I < 2$ the terms with $K > I$ vanish. Thus E_0 is the energy of the $I = 0$ state. The three $I = 1$ levels give then, for known values of a_9, a_{11} and $A_{8,11}$ enough information to determine E_1, E'_1 and $\hbar^2/2\mathscr{I}$. The constants a_9, a_{11} and $A_{8,11}$ have been calculated from Nilsson's tables as functions of the deformation parameter. Fit with the experimental data are obtained with $\hbar^2/2\mathscr{I} = 440$ keV and η in the interval -4 to -2, which agrees with the values found for ^{31}P($\hbar^2/2\mathscr{I} = 430$ keV, $\eta = -3$). With $\eta = 2$ one gets $\hbar^2/2\mathscr{I} = 300$ keV.

The final wave function corresponding to the energy E_i and the total angular momentum I may be written as:

$$\begin{aligned} |(^{32}\mathrm{P})_i^I\rangle &= \sum_k \alpha_{ik}^I |(\Omega_n)_k, IK_k\rangle \\ &= \alpha_{i1}^I|\tfrac{3}{2}, I2\rangle + \alpha_{i2}^I|\tfrac{3}{2}, I1\rangle + \alpha_{i3}^I|\tfrac{1}{2}, I1\rangle + \alpha_{i4}^I|\tfrac{1}{2}, I0\rangle. \end{aligned} \tag{6}$$

For the three $I = 1$ states we have $\alpha_{i1} = 0$, and get, using $\eta = -3$,

for the ground state: $\alpha_{02} = 0.80$, $\alpha_{03} \approx 0$, $\alpha_{04} = 0.60$,

for the 1.149 MeV level: $\alpha_{12} \approx 0$, $\alpha_{12} = 1$, $\alpha_{14} \approx 0$,

for the 2.223 MeV level: $\alpha_{22} = -0.60$, $\alpha_{23} \approx 0$, $\alpha_{24} = 0.80$.

With the wave function known, the reduced width for the neutron capture may be calculated. Following the method given by Macfarlane and French in their review article and using their notation together with Nilsson's, we get for the spectroscopic factor S in the reduced width (corresponding to formula (III, 204) in ref. [12])

$$S^I_{ij} = \frac{2I_0+1}{2I+1}\langle \mathrm{f}|\mathrm{i}\rangle^2[\langle I_0 j \tfrac{1}{2}\tfrac{3}{2}|I2\rangle\alpha_{i1}C_j + \langle I_0 j -\tfrac{1}{2}\tfrac{3}{2}|I1\rangle\alpha_{i2}C_j$$
$$+\langle I_0 j \tfrac{1}{2}\tfrac{1}{2}|I1\rangle\alpha_{i3}C'_j + \langle I_0 j \tfrac{1}{2} -\tfrac{1}{2}|I0\rangle\alpha_{i4}C'_j(-1)^{j-\frac{1}{2}}]^2. \quad (7)$$

Here C_j refers to level 8 and C'_j to level 11 in the Nilsson diagram. By evaluation of the Clebsch-Gordan coefficients we get (when taking the core overlap integral $\langle \mathrm{f}|\mathrm{i}\rangle = 1$):

$$\begin{aligned}
S^0_{\frac{1}{2}} &= 2(\tfrac{1}{2}\sqrt{2}C'_{\frac{1}{2}})^2, \\
S^1_{i\frac{1}{2}} &= \tfrac{2}{3}[(\alpha^1_{i3}+\tfrac{1}{2}\sqrt{2}\alpha^1_{i4})C'_{\frac{1}{2}}]^2, \\
S^1_{i\frac{3}{2}} &= \tfrac{2}{3}[\tfrac{1}{2}\sqrt{3}\alpha^1_{i2}C_{\frac{3}{2}}+(\tfrac{1}{2}\alpha^1_{i3}-\tfrac{1}{2}\sqrt{2}\alpha^1_{i4})C'_{\frac{3}{2}}]^2, \\
S^2_{i\frac{3}{2}} &= \tfrac{2}{5}[(\alpha^2_{i1}+\tfrac{1}{2}\alpha^2_{i2})C_{\frac{3}{2}}+(\tfrac{1}{2}\sqrt{3}\alpha^2_{i3}-\tfrac{1}{2}\sqrt{2}\alpha^2_{i4})C'_{\frac{3}{2}}]^2, \\
S^2_{i\frac{5}{2}} &= \tfrac{2}{5}[(\tfrac{1}{6}\sqrt{6}\alpha^2_{i1}-\tfrac{2}{6}\sqrt{6}\alpha^2_{i2})C_{\frac{5}{2}}+(\tfrac{1}{3}\sqrt{3}\alpha^2_{i3}+\tfrac{1}{2}\sqrt{2}\alpha^2_{i4})C'_{\frac{5}{2}}]^2.
\end{aligned}$$

Due to orthogonality of as well the intrinsic as the final wave functions the α coefficients for each I form an orthogonal matrix and we have $\sum_i \alpha^I_{ik}\alpha^I_{ij} = \delta_{kj}$, thus making it possible to calculate the sum of the reduced widths for all levels with same I without knowing the α coefficients of the individual levels.

The ratio between the $l_\mathrm{n} = 0$ transitions to the $I = 1$ and $I = 0$ levels is found to be

$$\sum_i S^1_{i\frac{1}{2}}/S^0_{\frac{1}{2}} = 1. \quad (8)$$

The observed ratio between the reduced widths for the $l_\mathrm{n} = 0$ transitions to the three assumed $I = 1$ levels and the 0.516 MeV ($I = 0$) level is 0.87.

Under the assumption that the 1.149 MeV level is a pure $K = 1$, $\Omega_\mathrm{n} = \frac{1}{2}$ state, we expect the ratio between the $l_\mathrm{n} = 0$ reduced width of this level and the other two $I = 1$ levels to be 2 : 1. An observed ratio of 2.3 indicates that this assumption is fairly good. The ratio of the reduced widths between the ground state and the 2.223 MeV level gives the amplitudes in the wave functions for these two levels as $\alpha^2_{02} = \alpha^2_{24} = 0.36$; $\alpha^2_{04} = \alpha^2_{22} = 0.64$, which happens to be the same values as was calculated from the level spacing with $\eta = -3$.

When comparing the $l_\mathrm{n} = 2$ parts of the transitions to the $I = 1$ levels one may notice that the ratio between the $C_{\frac{3}{2}}$ coefficients which are involved here, are not very sensitive functions of the nuclear deformation ($C_{\frac{3}{2}}/C'_{\frac{3}{2}} = 0.97 \pm 0.03$ for $2 > \eta > -6$). Summation over the three $I = 1$ levels gives

$$\frac{\sum_i S^1_{i\frac{3}{2}}}{\sum_i S^1_{i\frac{1}{2}}} = \frac{\frac{3}{4}(C^2_{\frac{3}{2}}+C'^2_{\frac{3}{2}})}{\frac{3}{2}C'^2_{\frac{1}{2}}} \approx \left(\frac{C'_{\frac{3}{2}}}{C'_{\frac{1}{2}}}\right)^2. \quad (9)$$

If this is set equal to the observed ratio between the $l_\mathrm{n} = 2$ and $l_\mathrm{n} = 0$ reduced widths for the three known $I = 1$ levels,

$$\sum \theta^2(l_\mathrm{n} = 2)/\sum \theta^2(l_\mathrm{n} = 0) = 2.4, \quad (10)$$

one gets a deformation $\eta \approx -4$ or $\eta \approx +2$. The experimental value of this ratio is not too well established since the $l_n = 2$ parts are more or less covered by the $l_n = 0$ contributions to the angular distribution curves. For instance may the $l_n = 2$ contribution in the 1.149 MeV level, although not seen, still be of the same order of magnitude as the $l_n = 0$ part. Furthermore the single particle reduced width for the $l_n = 0$ and $l_n = 2$ part may differ considerably. However, the ratio $C'_{\frac{3}{2}}/C'_{\frac{1}{2}}$ increases rapidly with decreasing deformation and the values found for $|\eta|$ are therefore not very much too large. It thus seems to be an inconsistency between the deformation needed to account for the relative reduced widths, and what one gets from the magnetic moment.

A further test on the model is to see if it accounts for the $I = 2$ levels. The only level known to have $I = 2$ is the 0.077 MeV level. This gives enough information to determine the unperturbed position of the lowest level in the $K = 2$ band. However, it turns out that the energy matrix for any choice of the parameters involved made to fit with the $I = 0$ and $I = 1$ levels, has one solution for the $I = 2$ levels more than 0.5 MeV below the ground state. This is mainly due to the strong interaction between the $I = 2$ levels in the $K = 0$ and $K = 1$, $\Omega_n = \frac{1}{2}$ bands.

The reduced widths of the $I = 2$ levels cannot be calculated separately without knowing the coefficients for the wave functions, but also here the sum of the reduced widths for the predicted four $I = 2$ states is easily obtained due to the orthogonality of the α_{ij} matrix.

One gets

$$\sum_{ij} S_{ij}^2 = \tfrac{1}{2}(C_{\frac{3}{2}}^2 + C_{\frac{3}{2}}'^2) + \tfrac{1}{3}(C_{\frac{5}{2}}^2 + C_{\frac{5}{2}}'^2). \tag{11}$$

For any reasonable deformation one has $C_{\frac{3}{2}}/C_{\frac{5}{2}} > 2$ for as well level 8 as level 11, and one may as a good approximation neglect the $C_{\frac{5}{2}}$ terms.

For the $l_n = 2$ part of the $I = 1$ levels one gets

$$\sum_{i} S_{i\frac{3}{2}}^1 = \tfrac{1}{2}(C_{\frac{3}{2}}^2 + C_{\frac{3}{2}}'^2). \tag{12}$$

The sum of the reduced widths for the $l_n = 2$ transitions to the $I = 2$ levels and to the $I = 1$ levels thus is expected to be approximately equal. From the values of the reduced widths given in table 1 for the $I = 1$ levels and the $I = 2$ level at 0.077 MeV, one finds that the latter accounts for about 60 % of the total strength of the transitions to the $I = 2$ levels. Levels corresponding to the remaining 40 % of the strength are not identifiable from these experiments. The possibility that the strong transitions which could be described with $l_n = 1$ or 2 or with $l_n = 2$ or 3 Butler curves, are $l_n = 2$ transitions, disagrees with the predictions from this model.

5.3 COMPARISON WITH THE SHELL MODEL

The nucleus ^{32}P is included in the discussion by Macfarlane and French [12]) in their review article. It may be of some interest to see if the data presented here affect their discussion.

Following the same procedure as used by them we discuss first the odd parity states. One expects to find such states arising from the coupling of a (2p) or ($1f_{\frac{7}{2}}$) neutron to the ground state of ^{31}P, namely the states: $[\phi_0 \dot{\times} 2p]_{J^-}$ and $[\phi_0 \dot{\times} 1f_{\frac{7}{2}}]_{J^-}$. These states may be formed by $l_n = 1$ transitions and $l_n = 3$ transitions respectively. At least four strong $l_n = 1$ transitions and two strong $l_n = 3$ transitions are predicted. Interaction between these states and states arising from excited levels in ^{31}P may cause a fragmentation of the single particle states, thus increasing the number of odd parity levels reached by stripping reactions.

Applying the sum rule given by eq. (III.185) in ref. [12]) one gets the total strength of the $l_n = 1$ and $l_n = 3$ transitions as:

$$\sum_i (2J_i+1)\theta_i^2 = C \sum_{l_n=1} [J]\theta^2 = 12\theta_0^2(2p), \tag{14}$$

$$\sum_j (2J_j+1)\theta_j^2 = C \sum_{l_n=3} [J]\theta^2 = 16\theta_0^2(1f), \tag{15}$$

where $i(j)$ labels all levels reached by $l_n = 1 (l_n = 3)$ transitions, and C is a factor normalizing the observed relative reduced widths given in table 1 as $[J]\ \theta^2$.

If one assumes that all (or at least all the stronger) transitions listed in table 1 as possible $l_n = 1$ and $l_n = 3$ transitions, really go to odd partity states, one finds an observed ratio:

$$\sum_{l_n=1} [J]\theta^2 / \sum_{l_n=3} [J]\theta^2 = 1.0. \tag{16}$$

If our measurements include all levels containing components of the states $[\phi_0 \dot{\times} 2p]_{J^-}$ or $[\phi_0 \dot{\times} 1f_{\frac{7}{2}}]_{J^-}$, one has from eqs. (14)-(16):

$$\theta_0^2(1f)/\theta_0^2(2p) = 0.75. \tag{17}$$

From comparison with other reactions (see ref. [12]), figs. 58 and 59), a value $\theta_0^2(1f)/\theta_0^2(2p) \approx 0.5$ would be expected. The value found here, although somewhat larger, seems not unreasonably large.

No absolute measurement of the differential cross-section is carried out in this investigation, and the normalization factor C in eqs. (14) and (15) is undetermined. Information about C may be obtained from the single particle reduced widths $\theta_0^2(2p)$ and $\theta_0^2(1f)$. From other nuclei (see ref. [12])) the values $\theta_0^2(1f) = 0.012 \pm 0.002$ and $\theta_0^2(2p) = 0.025 \pm 0.005$ seems rather well established. If one takes into account that the $l_n = 1$ transitions compared with the $l_n = 2$ transition to the ground state appear weaker in this investigation than in the investigation by Dalton *et al.*, it seems reasonable to assume a rather low value of $\theta_0^2(2p)$. Taking $\theta_0^2(2p) = 0.020$ and $\theta_0^2(1f) = 0.012$ one obtains a mean value $C = 0.010$, which is to be compared with the value 0.0074 used in ref. [12]).

The odd parity states in ^{32}P are expected to give information about the energy split occurring when coupling a single particle in the ($1f_{\frac{7}{2}}$) or (2p) shell with a single (2s) particle (or hole). These levels are expected to show up as doublets with approximately the same reduced widths for the two members.

Of the levels reached by $l_n = 1$ transitions the only one that can be identified is the lowest at 3.265 MeV, which is assumed to have $J^\pi = 2^-$ corresponding to the state $[\phi_0 \dot{\times} 2p_{\frac{3}{2}}]_{2^-}$. The reduced width is, however, only about half of what one would expect for an unperturbed state.

The two levels at 3.324 MeV and 3.447 MeV may be the main components of the $[\phi_0 \dot{\times} 1f_{\frac{7}{2}}]_{J^-}$ states with $J = 3$ and 4 respectively. The 3^- state is, however, considerably weaker than expected, and information about the energy split cannot be obtained unless the other components of this state are identified.

For the even parity levels the experimental results presented here are also slightly different from those on which Macfarlane and French based their discussion. Attempts are made by them and by Parkinson [7]) from information about the $l_n = 0$ and $l_n = 2$ transitions to determine the wave function for the ^{31}P ground state. In the following we are going to see how the data presented here change their results.

It is assumed that the ground state of ^{31}P is described by a completely filled $(1d_{\frac{5}{2}})$ shell and three particles in the $(2s_{\frac{1}{2}})$ or $(1d_{\frac{3}{2}})$ state. The wave function may be written in the following way:

$$\psi(^{31}\mathrm{P}) = \alpha_1(2s_{\frac{1}{2}}^3)_{T=\frac{1}{2},\, J=\frac{1}{2}} + \alpha_2[(2s_{\frac{1}{2}}^2)_{01}(1d_{\frac{3}{2}})]_{\frac{1}{2}\frac{1}{2}}$$
$$+\alpha_3[(2s_{\frac{1}{2}})(1d_{\frac{3}{2}}^2)_{10}]_{\frac{1}{2}\frac{1}{2}} + \alpha_4[(2s_{\frac{1}{2}})(1d_{\frac{3}{2}}^2)_{01}]_{\frac{1}{2}\frac{1}{2}} + \alpha_5(1d_{\frac{3}{2}}^3)_{\frac{1}{2}\frac{1}{2}}. \tag{18}$$

The absolute reduced width for a stripping transition is given as $\theta^2 = S\theta_0^2$ where θ_0^2 is the single particle reduced width. The spectroscopical factor S is essentially determined by a fractional parentage coefficient and two Racah coefficients. It is given explicitly by formula (III. 65) in ref. [12]).

We take account of all possible ways that a $(2s_{\frac{1}{2}})$ or $(1d_{\frac{3}{2}})$ particle can be added to the assumed ground state configuration of ^{31}P to form configurations in ^{32}P. By summing S over all possible transitions through the same channel l_n and leading to states with same J, we obtain the following expressions for $\sum S_{l_n}^J$:

$$\sum S_0^0 = \tfrac{4}{3}\alpha_3^2 + \alpha_5^2, \tag{19}$$

$$\sum S_0^1 = \tfrac{2}{3}(\alpha_2^2 + \alpha_3^2 + \alpha_4^2) + \alpha_5^2, \tag{20}$$

$$\sum S_2^1 = \alpha_1^2 + \tfrac{7}{6}\alpha_2^2 + \tfrac{7}{12}\alpha_3^2 + \tfrac{11}{12}\alpha_4^2 + \tfrac{1}{6}\sqrt{5}\alpha_3\alpha_4 + \alpha_5^2, \tag{21}$$

$$\sum S_2^2 = \alpha_1^2 + \tfrac{1}{2}\alpha_2^2 + \tfrac{7}{12}\alpha_3^2 + \tfrac{13}{20}\alpha_4^2 + \tfrac{1}{10}\sqrt{5}\alpha_3\alpha_4 + \tfrac{1}{5}\alpha_5^2, \tag{22}$$

where the $\alpha_3\alpha_4$-terms arises from cases where one pure ^{32}P configuration is formed coherently from different parts of the ^{31}P ground state configuration.

We want to use the observed reduced widths as given in table 1 to obtain values for the single particle reduced widths $\theta_0^2(2s)$ and $\theta_0^2(1d)$, and to determine the amplitudes in the ground state wave function of ^{31}P. Following the procedure used in the discussion of ^{32}P in ref. [12]), we normalize the relative reduced widths given in table 1 by multiplying with $C = 0.010$ which was estimated in the discussion of the odd parity levels.

In the ground state wave function of ^{31}P the amplitude α_1 is presumably of the order 1, the state being mainly $(2s_{\frac{1}{2}}^3)$. The admixture of a configuration which differs in as much as three particles from the dominating part, is probably small. Therefore in first approximation we may put $\alpha_5 = 0$, which leaves us with the wave function suggested by Macfarlane and French (eq. (V. 38) in ref. [12]).

The reduced widths for the $l_n = 2$ transitions, except to the ground state and the 0.077 MeV level, are not too well known. We assume these two states to have mainly the configurations $[(2s_{\frac{1}{2}}^3)(1d_{\frac{3}{2}})]_{1J}$ with $J = 1$ and $J = 2$ respectively [7,12]). By comparison with eqs. (20) and (21) one gets from the normalized observed reduced width of the ground state transition:

$$\alpha_1^2\theta_0^2(1d) = 0.010, \tag{23}$$

$$\tfrac{2}{3}\alpha_2^2\theta_0^2(2s) = 0.0007. \tag{24}$$

Taking the 0.516 MeV level as having $J = 0$ one gets in the same way, using eq. (19):

$$\tfrac{4}{3}\alpha_3^2\theta_0^2(2s) = 0.007. \tag{25}$$

The 1.149 MeV level and the 2.223 MeV level are both determined as 1^+ levels. The other two levels reached through $l_n = 0$ transitions may be either 0^+ or 1^+ states. For the 6.34 MeV level, which is not previously reported, the neutron separation energy is small, and the reduced width, as extracted by the procedure used here, is very sensitive to the value used for r_0, and not too reliable. If this level is neglected, and the one at 4.209 MeV is assumed to have $J = 1$, we get from eq. (20) (omitting the α_2^2 part which accounted for the $l_n = 0$ transition to the ground state):

$$\tfrac{2}{3}(\alpha_3^2+\alpha_4^2)\theta_0^2(2s) = 0.007. \tag{26}$$

Corresponding values between $\theta_0^2(2s)$, $\theta_0^2(1d)$, α_1^2 and α_3^2 arising from relations (23) to (26) together with the normalization of the amplitudes, are plotted in fig. 9. By compar-

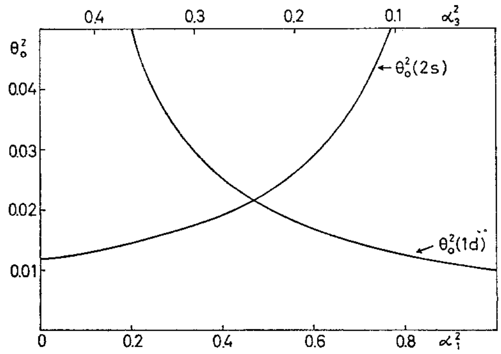


Fig. 9. Corresponding values of $\theta_0{}^2(2s)$ and $\theta_0{}^2(1d)$ as a function of the amplitudes α_1 and α_3 in the ^{31}P wave function. Values $\theta_0{}^2(2p) = 0.020$ and $\theta_0{}^2(1f) = 0.012$ are used to normalize the reduced widths.

ison with figs. 56 and 57 in ref. [12]) we may take as a reasonable set of values:

$$\theta_0^2(2s) = 0.030, \qquad \theta_0^2(1d) = 0.016, \tag{27}$$

$$\alpha_1^2 = 0.64, \qquad \alpha_2^2 = 0.04, \qquad \alpha_3^2 = 0.16, \qquad \alpha_4^2 = 0.16. \tag{28}$$

These values correspond well with those given by the expressions (V. 42) and (V. 43) in ref. [12]) although some different assumptions are made here. The large difference in the ratio α_3^2/α_4^2 is obviously because the statististical factor $(2J+1)$ was set equal to 3 also when inserting for the reduced width of the 0.516 MeV level. The different value used for $\theta_0^2(2p)$ to obtain the normalization factor C is compensated by the different ratio between the reduced widths for the $l_n = 1$ and $l_n = 2$ transitions. Finally, in ref. [12]) the assumption of the ^{32}P ground state as having the configuration $[(2s_{\frac{1}{2}}^3)(1d_{\frac{3}{2}})]_{11}$ was used to obtain the ratio $\alpha_1^2\, \theta_0^2(1d)/\alpha_2^2\, \theta_0^2(2s)$, whereas the same assumption was not used to determine the absolute reduced widths for the ground state. In this way the values of $\theta_0^2(1d)$ and $\theta_0^2(2s)$ were undetermined, and had to be estimated from other sources. In this discussion the reduced widths for the ground state are also used to obtain values for $\alpha_1^2\theta_0^2(1d)$ and $\alpha_2^2\theta_0^2(2s)$. One gets thus, as is seen in fig. 9, a relationship between the two single particle reduced widths.

Including the 6.34 MeV level amongst the $J = 1$ states with the strength given in table 1, would lead to somewhat larger values for α_4^2 and the single particle reduced widths. A small admixture in the ^{31}P wave function of the $(1d_{\frac{3}{2}}^3)$ state will, on the other hand, decrease the values found for the reduced widths.

The reduced widths for the rest of the $l_n = 2$ transitions may be estimated by inserting the suggested values for the amplitudes in the expressions (21) and (22) for $\sum S_2^1$ and $\sum S_2^2$. This gives as result a total strength $\sum(2J+1)\,\theta^2$ for the $l_n = 2$ transitions to the $J = 1$ levels, the ground state excepted, equal to the strength of the $l_n = 2$ transition to the ground state. Also the strength of the transitions to the $J = 2$ levels (the 0.077 MeV state excepted) will be about the same. A number of possible configurations exists for as well the $J = 1$ as the $J = 2$ states. One does therefore expect the rest of the $l_n = 2$ transitions to be considerably weaker than the ground state transition. Thus it seems reasonable to assume some of the weak stripping levels to be $J^\pi = 1^+$ or 2^+ states, whereas the stronger stripping levels, which from the angular distribution curves could be interpreted as due to either $l_n = 1$ or 3 or $l_n = 2$ transitions, are odd parity levels.

6. Conclusion

The present information about the level structure of the ^{32}P nucleus, based mainly on l_n values and reduced widths from (d, p) stripping reactions, is insufficient for a comparison with predictions based on current models. It is at present possible to describe the relative positions of four low-lying states reached by $l_n = 0$ transitions with the Nilsson rotational model. In this description the values used for the deformation and the moment of inertia are the same as those applied by Broude *et al.* [17]) in their rotational model description of the ^{31}P nucleus ($\eta = -3$, $\hbar^2/2\mathscr{I} =$ 430 keV). The observed relative reduced widths of the same states are in good agreement with the predictions from the model. The model predicts the lowest 2^+ state to be more than 0.5 MeV below the lowest 1^+ state whereas 1^+ is observed for the ground state. Agreement between the calculated and observed value of the magnetic moment is obtained only with a smaller deformation and less mixture between the rotational bands than needed to obtain agreement between the observed and predicted relative reduced widths. The high *ft* value found for the β-decay is not accounted for by this model.

When the *j-j* coupling shell model is applied, it is seen that a considerable deviation from a single particle state exists in the ^{31}P ground state. From the relative reduced widths of the even parity stripping transitions we estimate the $(2s_{\frac{1}{2}}^{3})$ part of the ^{31}P ground state to be between 0.6 and 0.7, the rest being explained as admixtures from the $(1d_{\frac{3}{2}})$ shell. An estimate of the single particle reduced widths gives $\theta_0^2(1d) = 0.016$ and $\theta_0^2(2s) = 0.030$ when determined relatively to $\theta_0^2(2p) = 0.020$ or $\theta_0^2(1f) = 0.012$.

Strong stripping transitions where the l_n values obtained from the "best fitted" Butler curves are not unambiguous, can be explained only as transitions to odd parity states. This is so whether we use the rotational model or the shell model picture. We therefore assume these levels to have $l_n = 1$ or $l_n = 3$. The ratio of 1.33 between $\theta_0^2(2p)$ and $\theta_0^2(1f)$ thus obtained is in reasonable agreement with what is expected.

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