STRUCTURE OF UNSTEADY FLOW AT THE OUTLET OF AN IMPELLER IN A CENTRIFUGAL PUMP

A. Saip Alpay

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c velocity vector
u, v, w components of the velocity vector in x, y, z directions
V velocity of flow
v_m relative velocity
v_m0 average relative velocity
\omega angular velocity
p pressure
\phi tensor of velocity vector C
\alpha angle between absolute and peripheral velocities
\beta angle between relative and peripheral velocities
\theta angle of rotation in a radial plane
r distance
n distance on the normal to the relative path
R radius of curvature at one point of the relative path
T period of oscillation
A, B, C constants
I INTRODUCTION

A. Objective

The primary objective of this research has been to determine the velocity profile in the axial and radial directions at the outlet of an impeller in a centrifugal pump, by a series of direct measurements.

The observed discrepancy between the classical theory and the measurements related to the velocity profile led to the proposal of a new theory considering the shock effect between the impeller discharge flow and the flow in the volute case. The transient flow was determined by kinematic considerations from the flow pattern produced by the shock effect. This flow pattern is renewed with every pressure wave.

B. Recent Development

The development in rotating fluid machinery in the last quarter of a century was mainly due to the adaptation of extensive research results made in the aeronautical engineering field, so that a comparatively rational analysis of axial turbo-machinery based upon airfoil and circulation theories, has been developed. But in the field of turbo-machinery with a substantial radial component of flow, no appreciable progress seems to have been attained. Undoubtedly the difficulties involved with the investigation of radial flow turbo-machinery are far greater than in the case of axial flow machines. These problems have to be attacked directly, because no theoretical "Similarity Solutions" can be borrowed from any better developed part of engineering. This kind of work has been carried out by researchers in some private industrial companies. Their investigations remain generally on an empirical basis, enabling them to develop
new machine units but without being able to establish an analysis of the prevailing flow conditions. The first theoretical and experimental investigations in a rational sense about the character of flow between rotating vanes were carried out by Thoms (25), Spannhake (80), Pfleiderer (66) and Hahn (30) in Germany and Oertli (61) in Switzerland, but research involving the transition zone between the impeller and volute casing flow has seldom been undertaken. At the beginning of the development, in most of all these investigations no genuine turbo-machine has been tested, but a simplified representative model of it has been used for qualitative studies of some flow effect, or to demonstrate some previously derived properties of the flow. Spannhake (81) succeeded in visualizing the channel eddy with a simplified experimental model about 15 years after its existence had been analytically established by Kucharski (49). The simplified model reduced undoubtedly the experimental difficulties and shut off some influences of secondary flow effects, leaving the investigator an opportunity to follow analytically the main effect. Many qualitative improvements have been reached and differences in opinions have been eliminated with the use of simplified models. Unfortunately the method is very restricted. With the oversimplification of experimental model some effects were restrained and some overstressed so that in no case a general functional agreement between experiments and the performance of genuine machine could be expected. Since a manufacturer is compelled to guarantee a definite efficiency under definite conditions, a routine rule of testing small models leads to the design of the genuine machine following the laws of similarity. The efficiency of the genuine
machine can be determined from the measured efficiency of the model allowing only some few percent to account for the fact that they both are operating in different ranges of Reynolds number. But this overall prediction, does not enlighten one about any facts concerning the structure of the flow inside the turbo-machine; it asserts only that similarity of flow in the sense of the Froude's number has been obtained in both of them. It avoids the question of the availability of a better design. It remains to the manufacturer to change arbitrarily, almost blindly, different dimensions of the model and to establish through various series of model tests, the best type to meet the demands of the specific purpose. These series of tests constitute a reliable empirical basis for the manufacturer but do not explain the mechanism between cause and effect, which can be determined only by investigating the functional changes in the nature of flow.

In the last few years the flow through rotating vanes has often been theoretically investigated by means of powerful analysis methods, like the solving of the Laplace equation for given boundary conditions through the relaxation method of Southwell. The results obtained are so far concordant. But the stand of the present knowledge about the flow leaving the impeller has not been improved through theoretical research, and only some few experimental investigations have been made about this part of the flow. To date methods with two different principles have been simultaneously employed. The first method considers the turbo-machine as a whole and investigates the flow conditions in different parts of it, taking in account automatically their mutual reactions. The representatives of
this method are J. F. Peck\(^{(65)}\), R. C. Binder and P. T. Knapp\(^{(9)}\). The last two researchers are the only ones, as far as the author is informed, who have investigated accurately and extensively the velocity profile in the axial direction. The second method is to deal first with individual parts of turbo-machinery and to leave the exploring of their interaction to future studies. This method has been successfully employed by W. C. Osborne and D. A. Morelli\(^{(62)}\) who investigated the velocity profile in a radial plane using a stereoscopic camera. Both methods have their own advantages and a preference of one of them seems to be intimately related to the employed measurement procedure. An outstanding contribution about the velocity profile in a radial plane has been made by F. F. Rashed\(^{(70)}\) in his doctoral thesis. He extended the theory of Spannake by conform mapping procedure and derived a theoretical solution for the velocity profile inside of the impeller, for the case of radial and infinitely thin vanes in limited numbers. Even for eight logarithmic spiral vanes A. J. Acosta\(^{(1)(2)(3)(62)}\) determined a velocity profile carrying out the solution of the Laplace Equation by the relaxation method of Southwell. The two last investigations accomplished by different theoretical means seem to have given at least in tendency, concordant results.

It should be underlined that the scope of theoretical results were limited to the periphery of the impeller and experimental observations referred to points outside of the impeller. The correlation between the two zones investigated by different means, remains open and the character
of mutual influences between the flow in the volute case and the flow discharged by the vanes at the immediate surroundings of the outlet remains to be correlated.

If the structure of flow could be entirely determined it would lead to the development of satisfactory analytical treatment of turbo-machinery, subsequently to the improvement of their design. The relative higher efficiency ranges of turbo-machinery is accepted "ipso facto" as a maximum, but there exists no rational proof for how far the limit can be extended. If the flow pattern were known, it should be possible to reach higher and stabler efficiency ranges with an appropriate design well adapted to the existent conditions.

C. Statement of the Problem

The statement of the conditions which determine a linear velocity vector function in space can be considered as the classical introduction in the analysis of the flow at the outlet of an impeller in a centrifugal pump.

Assuming that the velocity vector of the fluid \( \vec{v} \) in the considered outlet region is a regular function of position, the characteristic of the most general velocity field would be:

\[
\vec{v} = iu + jv + kw = r \times \vec{\phi}
\]

where \( \vec{\phi} \) is a tensor given by the equation:

\[
\vec{\phi} = \frac{\partial \vec{v}}{\partial x} + \frac{\partial \vec{v}}{\partial y} + k \frac{\partial \vec{v}}{\partial z} = \nabla \vec{v}
\]
The defined function of the velocity field determines one uniform domain of flow, and it does not include any singular points. Therefore if the uniformity should be maintained in the flow at the outlet of the impeller an infinite number of vanes with an infinite small vane thickness should be required. This, statement is the basis of the so called Euler's an or "Filament" or "Streamline" theory. These requirements can never be fulfilled practically by the design of an impeller. But when the number of vanes and their thickness are finite, different portions of the domain are limited and singularized so that the uniform linear velocity field will be undoubtedly distorted, and at a fixed point in space the velocity vector would be a time dependent function. The same effect can be formulated by stating that between two consecutive vanes the distribution of the velocity is a function of space and time, this function constitutes the velocity profile.

The correlation between a small model and the full scale design operating at the same specific speed with constant Froude's number, may at least raise a strong suspicion about the existence of a certain basic consistancy in the flow at the surroundings of the impeller. An attempt has been made to investigate the supposed consistancy, with some restrictions which were imposed by the nature of the problem. Only the substantial part of the flow has been considered, ignoring totally the shear forces in the fluid at the boundaries, so that changes due to Reynolds number or boundary layer effects are ignored. Although this simplification is not strictly correct, it is justifiable within certain limits in view of the fact that in a centrifugal pump the amount of energy transmitted is
roughly four times greater than the amount governed by shear forces and their derivative effects. Naturally even this justification holds only in the vicinity of maximum efficiency and at operating points away from the maximum efficiency zone the prevalence of shear force effects increases. However, the shear force effects should be neglected, for the main reason that no means are available to control them. This shortcoming is rooted in the well known contradiction between Froude's and Reynold's number which cannot be simultaneously controlled in any flow problem operating with the same fluid.

The second limitation is due to the fact that the velocity distribution was measured in an axial and radial plane, so that it constitutes strictly speaking only a two-dimensional method. Having both components in the radial and axial direction, the resultant could be theoretically computed, but even in this case the solution would remain a "quasi three dimensional" because no control over their mutual functional changes can be taken in account.\(^{(5)}\). The development and use of a three dimensional directional device is highly desirable, for accurate measurement of velocity and angle of outlet\(^{(5)}\). The author made an attempt to design and construct one using seven different stainless steel tubes with an inside diameter of 0.043 inches. The overall diameter of this instrument could not be reduced under 5/8-inch. But since the clear width of the impeller in the pump available for this research was only 1-inch, a device with 5/8-inch diameter located in the volute case at the outlet of the impeller, would have completely distorted the initial flow pattern. This specific condition led to the rejection of this instrument.
Figure I-1. Flow Circuits.
G. O. Ellis, and J. D. Stanitz\((23,24)\) have made a theoretical investigation for the comparison between two and three dimensional potential flow in an impeller using air as a fluid. They observed that the difference between the two solutions were not greatly different and so it can be stated that in the case of water with a much higher density, the difference between the two and three dimensional solutions would be substantially less.

D. Laboratory Facilities

The project was located on the first floor of the new Fluids Engineering Laboratory at the University of Michigan. The general purpose and facilities of the laboratory have been amply discussed in the recent paper of G. V. Edmonson\((22)\) so that a restricted description of the special test set-up will provide adequate understanding for the present investigation. Figure I-1 shows that water is supplied to the head tank from the sump by a pump running continuously during test and the head tank discharges through an 8-inch pipe. The 8-inch pipeline can be operated through a valve working with compressed air; on the same line there is located a venturi-meter \((D_1 = 7.981 \text{ inches and } D_2 = 3.461 \text{ inches})\). Water flowing in the 8-inch pipeline may be discharged to the weigh tanks in order to calibrate the venturi-meter. A flanged outlet on the 8-inch pipe is connected with a 4 1/2-inch flexible pipe which supplies a small tank and the pump with water.

The pump used for this investigation was an industrial Ingersoll-Rand pump. The impeller \((W-4RYL3AX1)\) has an outlet diameter of 10 7/8 inches and 7 vanes with three dimensional curvature in the inlet zone (Figure I-2). Pressure at the inlet and outlet were measured by means of two manifolds as prescribed by ASME regulations. Special care was taken to locate the
inlet and outlet manometers at the same level in order to avoid corrections. The capacity measurements were made by a calibrated flowmeter in the outlet line of the pump, and the signals of the flowmeter were transmitted to an electronic counter for direct readings. The variation of capacity at constant speed was obtained by operating a valve in the outlet line of the pump.

Constant speeds of the pump were maintained by a DC dynamometer and the readings from the scale of the dynomometer gave the torque and BHP measurements.

E. Preliminaries

The manufacturers of the pump supplied the design drawings of the pump and the impeller; this information was not adequate for deciding about the range of the flow in which the investigation should be carried out. In order to obtain the performance curves of the pump a series of tests were made with constant speeds of 800, 1000, 1200, 1400, 1600, 1700, 1750, 1800, 1900, 2000, 2100 RPM and changing the capacity. The results obtained are represented in diagrams in Figures I-3 and I-4. The measured values of the performance curves were very consistent on the right side of the maximum head parabola, but on the left side of this parabola the fluctuation of measured values, especially of capacity, were appreciably high.

The manufacturers rate the speed of 1750 RPM as normal operating speed for the pump and since the experimental performance curves verify this rating, this speed has been adopted as the basic speed for further investigations. The choice of the speed has a far reaching importance. If it should be operated in a lower range of speed the possibility of obtaining one percent higher maximum efficiency was offered, but a reduction in speed
Figure I-3. Efficiency Curves.
Figure I-4. Performance Curves.
will mean a reduction in the magnitude of the phenomenon to be observed. On the other hand a deliberate increase in speed will reduce the efficiency with the risk of obtaining values for a distorted flow. The speed of 1750 RPM was adopted as a compromise solution between the two opposing tendencies.

It was decided to use a cylindrical directional Pitot tube for the measurement of total, static pressure and angle of outlet. The size of the pressure hole was one of the essential decisions for the design of the Pitot tube. The requirements of an accurate measuring procedure of the Pitot tube was for one smallest possible tube diameter, which is intimately related with the size of pressure hole. The problem was to determine the size of the smallest hole which would give a consistent, not time dependent, value for the pressure, working in the ordinary laboratory water. As a preliminary the pressure response of different sizes of holes in 1/8 inch copper tubing, were observed in a weir, working with reduced velocity in order to observe their response characteristics. Holes of diameter listed below

<table>
<thead>
<tr>
<th>Drill Number</th>
<th>Hole Diameter Inches</th>
<th>Hole Diameter Inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>1/64 inch =</td>
<td>0.0135</td>
</tr>
<tr>
<td>79</td>
<td></td>
<td>0.0156</td>
</tr>
<tr>
<td>78</td>
<td></td>
<td>0.0160</td>
</tr>
<tr>
<td>76</td>
<td></td>
<td>0.0200</td>
</tr>
<tr>
<td>74</td>
<td></td>
<td>0.0225</td>
</tr>
<tr>
<td>72</td>
<td></td>
<td>0.0260</td>
</tr>
<tr>
<td>70</td>
<td></td>
<td>0.0280</td>
</tr>
<tr>
<td>68</td>
<td>1/32 inch =</td>
<td>0.0310</td>
</tr>
<tr>
<td>66</td>
<td></td>
<td>0.0312</td>
</tr>
</tbody>
</table>

were tried. Only the two last sizes (0.0310 and 0.0312) holes gave a lasting response independent from time. Having only the alternative between using 1/8 inch or 1/16 inch stainless steel tubes, the choice fell on 1/8 inch tube with 1/32 inch pressure hole. However, it
must be noticed that 1/32 inch hole operated in the laboratory water during the eight months of the investigation without failing to give a consistent pressure response, so the author would be inclined to use even reduced pressure hole diameters in future investigations.

The investigation about the structure of flow in axial and radial directions having been made by different instrumentation and different methods, it will be appropriate to handle them separately.
II. INVESTIGATION OF FLOW IN THE AXIAL DIRECTION

A. Instrumentation and Procedure

1. Directional Pitot Tube

The prospect of measuring the three dimensional flow having been abandoned as explained in I-C led to the use of a directional Pitot tube. It was necessary that the instrument be located as close as possible to the impeller outlet. Some investigators prefer to put the instrument a distance of about 1/2 inch from the impeller. It seems that at this distance only the characteristics of the full developed flow in the volute can be measured but not the structure of the transition zone. It was decided that the clear distance between the impeller outlet and the Pitot tube would be 1/8 inch: and this distance is one of the smallest realized clear distance in this sort of investigation. The Pitot tube was intended to measure first the total pressure and then, by turning it 90°, the static pressure, so that with one tube each measurement could be evaluated one after another in a single operation. Following this procedure, total and static pressure are, strictly speaking, measured in two different points with a distance equal to the radius of the tube in the direction of flow. In order to reduce this source of error the diameter of the tube must be as small as possible. Combining this condition with the requirement for the diameter of the pressure hole it was decided to use a stainless steel tube of 1/8 inch diameter. Obviously the distance between the two points where total and static pressure are measured remains 1/16 inch in the direction of flow; but this source of error can be completely
eliminated by employing two different tubes and changing them after every measurement.

The design and details of the Pitot tube can be seen from Figure II-1 and the Photographs 1 and 2. Briefly it is fitted with a protractor turning with the tube, and a stationary angle graduation scale which permits the direct reading of the outlet angle. Furthermore, it has two stops to adjust the tube parallel to the axis of the pump and one clamp to maintain it at a desired angle of deviation. The Pitot tube was connected with a mercury manometer.

Theoretically it seems to be an accurate procedure, to turn the Pitot tube until the maximum pressure is reached in order to obtain the direction of the outlet flow and the total pressure. But it must be noticed that at the maximum pressure, the Pitot tube does not give a sensible response to deviation angles of about 2° in both directions \(^{(8)}\)\(^{(29)}\). This error amounts to \(\frac{2 + 2}{15} = 25\) percent and could not be tolerated; therefore this source of error has been corrected with another reading procedure (Figure II-2). In the vicinity of maximum pressure an arbitrary reading \(H\) of the manometer and the corresponding reading for the angle \(\alpha_I\) were made. \(\alpha_I\) is the angle between the tangential direction of the impeller and the radius going through the pressure hole. By turning the Pitot tube through the maximum the proceeding mercury column height \(H\) can be attained at another angle \(\alpha_{II}\). It is obvious that the direction of the flow lays in \(\alpha = \frac{\alpha_{II} - \alpha_I}{2}\). After this operation adjusting the Pitot tube to the angle \(\alpha\) the total pressure can be read, and by turning it 90° the static pressure can be measured. The difference of these gives the dynamic pressure head, from which the velocity can be calculated.
Figure II-2. Reading Procedure for the Pitot Tube.
2. Mercury Monometer with Air Pressure Compensation

The Pitot tube was connected with a standard mercury manometer operating against adjustable compressed air back pressure in order to reduce the height of the mercury column to a reasonable value. As it was intended to evaluate the magnitude of velocities, only the difference in the height of mercury column for total and static pressure was to be considered. The readings of the manometer were always accompanied with some fluctuation in the mercury column, but working against the adjustable air pressure, (up to 30 p.s.i.), they were not very high and did not exceed 0.7 inch. Since the height of the mercury column itself was in the range of six or seven feet this constitutes an error of about 1/100. These fluctuations originated in the nature of the effect, and many different readings for the same position were made in order to evaluate an average.

3. Flowmeter

The capacity of the pump was measured by a turbine type flowmeter which has been calibrated (6.20.58) up to 700 GPM. following a linear functional variation (1 GPM = 1.2854 CPS). As the capacity range of the pump in the investigation area reached up to 1045 GPM, an extension of this calibration proceeded in two stages. The venturimeter on the eight inch pipe was connected with the weigh tanks and calibrated. The constant of the venturimeter was found to be 0.985. The flowmeter was then calibrated against the venturimeter. Theoretical consideration about turbine type flowmeters lead to the conservation of linearity between the revolution number of the flowmeter and the rate of flow.
Figure II-3. Calibration Curve for Flowmeter.
The calibration curve for the venturimeter with the constant 0.985 was made (Figure II-3) and the readings of the flowmeter, assuming linearity were compared to the calibration curve. This comparison confirmed that linearity was maintained in the flow range, 700 and 10,400 GPM.

4. Remarks About the Procedure

The velocity variation in the axial direction of the pump, determined by measuring pressure with a mercury manometer constitutes only a "ensemble average" value over time and number of vanes in the radial plane. The same applies to the angle of outlet. The instantaneous variations cannot be registered in the very short time between the passage of two vanes, since the inertia of the mercury column does not allow it to follow the rapid changes. From the assumption that the velocity and pressure distribution between two consecutive vanes are periodical functions, it can be concluded that the ensemble averages represent accurately, to some extent, the whole physical phenomenon. This statement cannot be generalized for every unsteady flow as in case of flow in the start or during a change of speed. For so far as the ensemble average can represent the whole effect in the flow, this might be called a "quasi steady" flow. The only attempt to obtain "instantaneous" values was made by Binder and Knapp (9) but the complicated measuring device used in this investigation did not register effective instantaneous values but only continuous values as usually appears in stroboscopic methods.

B. Observations and Measurements

The Pitot tube was located in the casing of the pump so that the radius going through the center of the tube made an angle of 94° with the radius of the cutoff of the volute. The clear distance between Pitot tube and impeller was 1/8 inch. From the performance curves the choice was made to investigate at a speed of 1750 RPM.
The impeller width of one inch was divided into ten equal parts and for different capacity values at least seven points were measured. In significant cases and where doubts arose the number of the measurement points were increased up to 17 (Figure II-4). Only in the vicinity of the wall has a narrower distance been taken (1/20 inch) to observe if some special effects were to be found. The above described procedure allowed the direct measurement of the effective outlet angle $\alpha_2$ and the magnitude of the absolute velocity by the measurement of total and static pressure by the derived formula

$$ V = \text{const} \sqrt{\Delta h}, \quad \Delta h = h_t - h $$

The radial velocity component areas $V_{2m} = V \sin \alpha_2$ and the angle of relative flow between two vanes at the outlet $\beta_2$ was given by the construction of the effective outlet triangle for each point.

The valve on the pressure line of the pump regulated the capacity range.

The results obtained from this systematical investigation are presented in form of diagrams as a function of the width of the impeller (Figure II-5 to II-12). Another graphical representation has been used for showing the variation of $\alpha_2$, $\beta_2$, $V_2$, $V_{2m}$, as a function of the capacity (Figure II-13 to II-20). The curves show a continuous variation and are consistent in their general trend of change. Care was taken that capacity should remain constant during each test. The highest change was seldom about one cycle per second corresponding to a capacity of 1.25 GPM, which
Figure II-9. Velocities and Angles for RPM 1750, GPM 600.
Figure II-11. Velocities and Angles for RPM 1750, GPM 200.
Figure II-13. Absolute Velocity $V_2$, Tank Side.
Figure II-14. Absolute Velocity $V_2$, Dynamometer Side.
Figure II-15. Effective Angle of Absolute Velocity, Tank Side.
Figure II-16. Effective Angle of Absolute Velocity, Dynamometer Side.
Figure II-19. Outlet Angle of Relative Velocity, Tank Side.
means an error between 0.125 - 0.35 percent. The exception was the case of 200 GPM, in which fluctuations of capacity were very high and sudden, so it was practically impossible to maintain it constant during the operation of one measurement. This instability amounted to about 15 percent of the nominal value.

Different attempts have been made to explore how a change in capacity and speed outside the range employed in this systematical investigation would effect the obtained results.

A rise in capacity was obtained by opening entirely the outlet valve. The capacity at this position was $10^{4.5}$ GPM and the different velocity and angle diagrams (Figure II-21) showed an undeniable tendency of forming some remarkable undulations $^9$.

A rise of the speed could not be obtained easily by the setup; but different lower speed ranges have been investigated and the characteristic tendency of the 1750 RPM curves were confirmed with attenuated magnitude (Figure II-22).

C. Theoretical Solutions Based on the Classical Conception of Mass Transport

The main function of a pump is mass transport, and continuous efforts have been made by prominent investigators relating kinematic and dynamic effects of the flow to the mass forces involved in mass transport. The investigation of Spannhake $^{80}$, Bauersfeld $^6$, Fluegel $^{26}$ and Rashid $^{70}$ are to be mentioned with this line of development. The two last authors have even outlined a graphical method for an approximate solution of the problem.
Figure II-21. Velocities and Angles, 1750 RPM, 1045 GPM.
The distribution of velocity in the axial and radial directions can be derived, from mass force consideration. This has been done in Appendix I, and it leads to the well known differential equation

\[
\frac{dV_m}{dn} - \frac{V_m}{R} + 2\omega = 0
\]

from the works of the preceding authors. But a special assumption has been used here in the case of the outlet point at the impeller for the integration of this equation. This assumption requires that the radius of curvature R should be constant on the normal of the curve at one point; as the constant R may change with every considered point the assumption does not imply any restriction. When the curvature changes rapidly and the distance between the guide walls is relatively long this assumption cannot be strictly valid. At the outlet of the impeller the radius of curvature being very big, even infinite, and the distance between the guide walls very small, the assumption of a constant radius of curvature can be maintained without distorting reality. The exact and approximate solutions of this equation are given in Appendix I and lead to the conclusion that in a radial plane the relative and absolute velocities of the flow increase from the inlet side following the monotonic exponential relation.

\[
V_m = Ce^{n/R} + 2\omega R
\]

In the same Appendix I a similar result has been derived for the velocity distribution in axial direction \( V_m' = C'e^{-n'/R'} \). The velocity increases from the inlet side to the backside. The two velocity distribution functions can be considered linear with a good approximation as it has been shown in the appendix.
In Appendix II the same analytical study has been made following closely the theory of Spannhake, with cylindrical coordinate system and taking in account the dynamic, kinematic and continuity relations. The basic assumption used was \( \frac{\partial \beta}{\partial \phi} = 0 \) or \( \beta \) is constant on streamlines which means that the relative flow pattern does not change. A mathematical deduction led to the relation

\[
\lambda = A e^{\chi \phi} + B \text{ where } \lambda = \frac{V_m}{V_{mo}}
\]

which states that the velocity following a monotonically increasing exponential relations. This would be linear in the case of logarithmic spiral blades.

The results of Appendix I and II confirmed and reinforced each other, in these two Appendices the employed mathematical deduction methods were different but the principle was the same. Both of these are based on mass forces. Classical and modern authors have referred implicitly to this character of the velocity distribution without giving the analysis of it. The more often referred case of linear distribution of velocity is considered sometimes as a pure assumption.

D. Discrepancy Between the Classical Theory and Observation

The results of the classical mass force theory which predicts a monotonically increasing velocity profile, and the experimental observation of a velocity profile with one or two pronounced dips in the middle zone are so far away from each other that no correlation between them seems to exist. This discrepancy in character of both profiles cannot
be attributed to a poor observation or to any incidental effect. Care was taken to eliminate any doubtfull cases by repeating the measurement and inserting new points between them. The influence of boundary layer effects can be admitted in vicinity of the walls but it does not explain the existence of the pronounced dip in the middle zone. On the diagram with 1750 RPM, 800GPM (Figure II-7) the absolute velocity distribution shows clearly a parabolic characteristic inside of the impeller. In this case the mass force theory predicts a constant or very small, almost linear increase from the point 05 on the tank side to point 05 on the dynamometer side. The discrepancy is consistent and can be followed at diagrams with different GPM. At 1750 RPM, 600 GPM (Figure II-9) a certain amount of increase from the tank side to the dynamometer side is to be observed, but even in this case a minimal value exists in the middle. Even this slight tendency disappears in the cases of 700 GPM and 400 GPM. In shut off (Figure II-12) and in some unreliable unstable case of 200 GPM (Figure II-11) no correspondence between theory and measurements can be traced.

The general tendency of experimental profiles indicate the existence of a dip up to 800 GPM and of a double dip above this value of capacity. However disappointing this result of finding a dip in the $V_2$ and $V_{2m}$ profile is, it is not the first time that it has been observed. Osborne and Morelli(62) found it with a photographic method in a radial plane and remarked that the "$V_{2m}$ profile is not yet substantiated by theory;" Rashed(70) found the same for strictly radial vanes through an extension of Spannhaeke's theory using complex functions and conform mapping. Peck(65),
Binder and Knapp\(^{(9)}\) observed this dip even in the axial direction. As the existence of a dip in the \(V_2\) profile is not to be denied. An attempt will be made to explain and justify its nature.

E. Proposal of a New Theory

In the classical mass force theory of centrifugal pumps, only the transport of material has been taken in account. An impeller converts mechanical energy, into different forms of energy in the flow so that the total energy in the flow leaving the impeller is increased. Continuous change of total energy in the flow is always accompanied by the formation of an unsteady flow pattern\(^{(18)(67)(70)(71)(23)}\), as is mathematically derived in Appendix III. An unsteady flow requires continuous pressure waves which are propagated in the fluid with the speed of sound. In this process the damping effect is exerted through viscosity and turbulence. But at each instant a new pressure wave is formed and a new instantaneous flow pattern must be established. As the generation of the flow is accomplished in a very short time, the viscous forces have no time to develop and the resulting pattern must be necessarily a potential flow. The generated potential flow may change in intensity and magnitude with every pressure wave. As the experiments show that a consistent ensemble average value exists, this value can be considered as representative for the general trend of the entire flow.

In the volute casing of the centrifugal pump there is a flow which consists as generally known, of two different vortex flows turning in opposite directions and advancing in the volute following a double spiral
path \((67)(65)\). When a solid body with a given shape is located in a flow, it is common practice in Fluid Mechanics to consider this solid body as a "bound" or "frozen" vortex, in order to evaluate mathematically the effect of its presence with a circulation function \((67)(18)\). In the pump the opposite of it should be assumed; the double vortex flow in the volute should be considered for a moment as a solid body so that the unsteady flow coming from the impeller produces a shock effect on this "artificial wall" before losing its flow identity (Figure II-24). The next moment the "shock flow" loses its identity and the material parts of it begins to be a part of the steadily enlarging double vortex flow. But then another pressure wave will send another "shock flow" so that the process will be intermittent but continuous. Since the physical effect is being determined by an instantenous potential flow against a "wall" formed by the double vortex flow of the volute, the possibility of using a calculation method for comparison is at hand \((67)(69)(71)\). In the following only the absolute velocity \(V_2\) in the axial plane \(x-x\) perpendicular (Figure II-24) to the plane of impeller will be considered and the velocity distribution in the axial direction will be investigated. For the performance at 800 GPM and 1750 RPM the structure of the double vortex flow and a first approximation of an arbitrarily chosen shock circle are represented in Figure II-24. The assumed distance of the shock circle will undoubtedly affect the numerical results, but it cannot affect the tendency of the velocity profile.

The conform mapping function for potential flow and velocity has been developed in Appendix IV and the location circle of the Pitot tube has been taken as a reference basis. For the performance at 1750 RPM
Figure II-23. Potential Flow Structure in Axial Direction.
Figure II-24. Double Vortex Flow and Shock Flow for RPM 1750, GPM 800.
and 800 GPM the potential flow pattern has been designed in Figure II-25 with the streamlines and the constant velocity curves from which the velocities corresponding to the position of the Pitot tube were calculated. On the upper left side of the same diagram the velocity distribution following the potential flow pattern has been traced and the values of effective measurements have been marked on it. It is remarkable how close the distribution of velocity following the shock flow theory and the measurement values approach each other. But the most interesting part remains that the theoretical and experimental velocity profiles show the same tendency disregarding the points in the vicinity of walls.

The same calculation has been repeated for another performance state of the pump. In order to avoid a change in the character of the double vortex flow in the volute casing, the choice of the second performance case fell on 1600 RPM and 800 GPM because this and the preceding state are on the same isoeficiency curve. The results for this case has been shown in diagram Figure II-26. The correspondance between the theoretical velocity profile and the measured values continues.

The flow mechanism in the performances GPM 1000, 900, 700, 600 and 400 with RPM 1750 can be explained with an unsymetrical change in the double vortex flow of the volute casing (Figure II-27). In particular the formation of the double dip in 1000 GPM could be predicted from it. Even the instable case of GPM 200 can be logically interpreted as it has been done in Figure II-28. In this case the capacity is reduced and consequently the double vortexes in the volute are smaller and cannot entirely fill the casing so that they can sometime "float" in the flow coming from the
Figure II-25. Velocity Distribution and Potential Flow, RPM 1750, GPM 800.

\[ N = 1750 \text{ RPM} \]
\[ Q = 800 \text{ GPM} \]

\[ \phi = a(x^4 - y^4) \]
\[ \psi = 2axy \]
\[ = 44.75xy \]

\[ v = \sqrt{\left(\frac{\partial \phi}{\partial x}\right)^2 + \left(\frac{\partial \phi}{\partial y}\right)^2} = 2ar \]
Figure II-26. Velocity Distribution and Potential Flow, RPM 1600, GPM 800.
impeller. This effect may explain the unsteadiness of this case. Furthermore, since the double vortices are small, they might be pressed against the wall of the casing so that the flow coming from the impeller has the same tendency in velocity profile as the flow in a pipe. In the case of Shutoff it must be remarked that the flow is returning in the impeller through the middle zone (Figure II-29). This fact is probably due to the lack of suction in the pump. The extreme case of GPM 1045 deserves special attention; the undulation in the velocity profile is not to be explained by admitting that the shock between the two mixing flows is produced on the surface of a simple cylinder as it has been assumed before. In order to obtain a first approximation for the case of maximum efficiency at high values of capacity, a certain undulation in the structure of the double vortex flow has been assumed. The flow coming from the impeller strikes this undulated "shock surface" (Figure II-30) and explains the form of the velocity profile. This conception of undulated shock surface will be followed further in the investigation of radial velocity distribution and a more conceivable explanation for the formation of double or more dips in the axial velocity profiles will be given in the following chapter.
Figure II-27. Unsymmetrical Double Vortex Flow.
Figure II-29. Case of Shut Off.
Figure II-30. Case of GPM 1045, Undulated Shock Surface.
III. INVESTIGATION OF FLOW IN THE RADIAL DIRECTION

The discrepancy between experiments and the predicted results by classical theory led to the proposal of a new theory which seemed to give concordant values with the measurements made in the axial direction. Should a transient shock flow of potential nature exist between the two mixing flows, it must necessarily exert its influence simultaneously in the radial direction. The presence of a dip in the velocity profile in the radial plane would be the best argument for the existence of the "shock flow". This dip has been calculated and observed by different investigators under special conditions. But reviewing the obtained results some doubts about its nature may arise. Osborne and Morrelli\(^{(62)}\) have observed its presence with a stereoscopic camera at the outlet of a "free" impeller, but the scattering of the obtained results were about \(\pm 50\) percent where experimental errors did not exceed \(\pm 2\) percent. Rashid\(^{(70)}\) predicted a dip in this profile, but for strictly radial vanes with no thickness, through an extension of Spannhake's theory using complex functions and conform mapping. A. J. Acosta\(^{(62)}\), solving the Laplace equation with special boundary conditions, predicted the same dip using the relaxation methods of Southwell, but his calculation was made for an idealized impeller consisting of eight logarithmic spiral vanes. Not only that, but the last two analytical results were restricted to the zone inside of the impeller.

The above mentioned concordant results may form the basis for the assumption of a dip in the velocity profile at the outlet of the impeller in the radial direction in a centrifugal pump. But before going further in theoretical
considerations, it seemed desirable to obtain the velocity profile at the outlet of an impeller under normal operating conditions by means of an experimental investigation. This procedure would eliminate doubts concerning the unknown influence of "idealized" conditions to some extent.

A. Instrumentation and Procedure

The investigation of the velocity profile between two vanes required the development of an instrumentation set which allowed the measurement to instantaneous pressures in order to deduce velocities. For this purpose the suggestion of G. V. Edmonson has been followed and a set-up composed essentially of an oscilloscope and different pressure pick-ups was selected.

The mercury manometer used in the preceding investigation was disconnected from the Pitot tube and an appropriate pressure pick-up was connected with the nipples (a), (b) or (c) of the Pitot tube in different phases of the investigation (Figure II-7)(Photographs 1 and 2). The signals taken by the pressure pick-ups were transmitted through an amplifier to the oscilloscope, and the variations of pressure were observed. In many cases the oscilloscope was triggered by a magnetic pick-up, which transmitted the signals corresponding to a reference point on the impeller. A small triangle of steel plate with a sharp edge was located on the shaft in order to trigger the magnetic pick-up. The triggering allowed investigation of the distribution of pressure on the same point in the pump. Different runs have been made without triggering and the obtained variations were not essentially different (Figure III-1).
Figure III-1. General Arrangement of Instruments.
The procedure of flow measurement was based on the assumption that small variations in the angle of discharge at outlet, does not affect substantially the total and the static pressure \(^{(8)(29)}\). Since the average angle of outlet for a given performance characteristic had been measured in the preceding investigation, the Pitot tube hole was adjusted perpendicular to this direction, and the variations of the instantaneous total pressure were determined by the shape of the oscilloscope sweep. In the second step the Pitot tube hole was aligned in the direction of flow, turning it 90° from its initial position so that the variation of instantaneous static pressure could be determined (Figure III-2). Since the entire set-up was calibrated previously by a dead weight tester, the pressure variations read on the oscilloscope sweep could be numerically evaluated. The difference between the readings of total and static pressure gave the dynamic pressure head, from which the velocity variations were computed.

In the course of this investigation the effect of mechanical vibrations on the magnitude of the signals given by the pick-up could not be ignored\(^{(70)}\). In order to evaluate the effect due to mechanical vibrations, a special nipple between the pick-up and the Pitot tube was used. In the middle of this nipple a thin wall of metal was left undrilled and no transmission of pressure was possible to the pick-up. The signals thus transmitted to the oscilloscope were necessarily due only to the effect of mechanical vibrations.

A Tektronix oscilloscope with a dual beam was employed. In the first attempt a strain gage pick-up with a maximum pressure range up to 100 psi was used and the amplification of the signals were obtained by a normal
Figure III-3. Tektronix Balance Unit.
bridge. This set-up gave a blurred sweep so that no information could be taken from it. A consultation with different specialists of electronic instruments led to the abandonment of the use of the bridge which was considered too "noisy" for the present investigation. The Tektronix firm produced a balance unit especially for this research (Figure III-3) with which the average total and static pressures could be determined with the 100 psi strain gage pick-up. However, the instantaneous pressure changes were measured essentially by a Kristal SIM pick-up, and alternatively SIM EZ14 3177 combined with a Piezo-Calibrator Model No. 708 and Model 2, Series 99. The calibration of the SIM pick-up confirmed strictly the linearity of scale predicted by the manufacturer. Even two Kristal pick-ups and two Piezo-Calibrators were used in order to have a simultaneous picture of the measured pressure on the dual beam; in this case one was located in connection (a) and the second in connection (c) (Figure II-1). Unfortunately, however, the design of the Pitot tube did not allow simultaneous measurement of total and static pressure.

The total and static pressure measurements were made basically for the performance at 1750 RPM and 800 GPM in the center line of the impeller. A systematical investigation concerning the effect in change of speed, capacity and the axial location of the pressure hole showed that the phenomenon remained the same and only the magnitude of the measured pressure profiles differed, so that it seemed more desirable to concentrate the whole investigation on the basic case in order to obtain significant results for an analysis. The exploring of a wide range of speed was restricted
automatically, because the phenomenon was almost indiscernable at speeds under 1000 RPM. At speeds up to 2100 RPM the magnitude of the dip in the pressure profile increased, but this was not an advantage for observation since in order to obtain the entire profile on the screen the sensitivity of the oscilloscope would have to be reduced. It was already possible to obtain a pressure profile covering the whole screen at the basic speed of 1750 RPM. A reduction in capacity increased the fluctuations in the magnitude of the phenomenon, without showing any particular changes; even an increase of capacity up to 1045 GPM seemed to have the same effect with reduced intensity. A change in the axial position of the pressure hole of the Pitot tube resulted in no appreciable variation in the observed pressure profiles.

B. Observations and Measurements

The average velocity at the outlet of the impeller in a radial plane was calculated from the difference between total and static pressure measured with the Tektronix Balance Unit and the 100 psi strain gage pick-up (Photographs 3 and 4) for the basic performance 1750 RPM and 800 GPM. By calibration, this average was found to be 63.2 ft/sec. In order to obtain the instantaneous velocity profile, the SLM Piezo pick-ups were used. At the speed 1750 RPM the time corresponding to the passage of two consecutive vanes was 4.9 milliseconds and so the time on the oscilloscope was chosen almost in every case to be five milliseconds/cm. Therefore when the sweep was not magnified one cm distance in scale corresponds approximately to the passage of one vane. By magnifying the sweep, two, five or ten times, the profile could be spread over the entire screen of the oscilloscope. During this investigation 258 photographs were taken and a significant number of them is reproduced.
Taking the utmost possible care to leave no air in the pump and in the Pitot tube, the obtained total and static pressure profiles are shown in the Photographs 5 to 16. From the different sweeps for total pressure the following facts can be deduced:

1) Between two consecutive vanes a main dip exists (Photographs 5, 6, 7, 8, and 11).

2) In this region a secondary phenomenon is taking place which affects the pressure fluctuation in the formation of the main dip so that six different smaller dips, superimposed to the main profile, are observed (Photographs 5, 6, 7, 8 and 11).

3) The total average pressure in the pump is subject to a periodical variation, and the period of this change is much higher than the time required for the passage of one vane (Photographs 6 and 8).

4) Certain periodical changes are taking place in the magnitude of the main dip (Photographs 5 and 7).

The corresponding photographs for the static pressure profile (Photographs 13 to 16) confirms in a certain degree these results. However, the magnitude of the main dip is smaller than in the case of total pressure, and approaches the magnitude of the secondary dips so that the case of static pressure offers a less clear picture than the total. Nonetheless, it is to be noticed that the number of the secondary dips remains equal to six between two consecutive vanes in all cases.

The principal interest of this investigation was concentrated on the velocity distribution and it was thought that by taking an adequate number of photographs for total and static pressure, a mean value profile might be evaluated. But the shape of these profiles allows no doubt that with this procedure the determination of the mean profile from the secondary dips would remain arbitrary in a large sense. Also in the formation of a
mean value profile, no information is available for the choice of the procedure which would best represent the physical effect; so that for example in Photograph 10, an arithmetical mean value profile would reduce the magnitude of the effective main dip without any justification. It should be noticed that with experimental equipment allowing a simultaneous measurement of total and static pressure this uncertainty would be avoided. During different runs it was observed that if a small air bubble was left in the connection of the pressure pick-up, the formation of secondary dips was appreciably attenuated without any noticeable change in the main dip magnitude. This observation led to the intentional introduction of a small air bubble in the Pitot tube in order to obtain a more uniform pressure profile. The small air bubble was employed like an "elastic pillow", that absorbs the sudden and small pressure changes. It is obvious that, in principle, a small part of energy was lost for compression of the air bubble, but taking into account the very small volume of the air bubble the change of enthalpy cannot be of an appreciable amount. The transmission of pressure cannot be affected by the presence of the air bubble other than by this small amount.

1. Observations With Air Bubble in Pitot Tube

The volume of the introduced air bubble has not been controlled but in many cases it was adequately efficient to absorb almost all the secondary dips; in some other cases the secondary dips were still to be observed but the general tendency of the pressure profile was not too much affected by their presence.
The typical variation of total pressure has been given in Photographs 17 to 21, where the sweep was magnified, one, two, five and ten times. The corresponding static pressure profiles can be seen from the Photographs 22 to 26. Another series of photographs has been taken showing the successive waves on the same sweep when it was magnified ten times both for total (Photographs 27 and 31) and static pressure (Photographs 32 and 35). A study of these curves leads to the following results:

1) The existence of a main dip in all total pressure profiles are to be observed. The same dip is to be observed in the static pressure profile, although in appreciably reduced magnitude when the volume of the introduced air bubble was sufficient to annulate the formation of the secondary dips. In a number of other cases, the introduction of the small air bubble did not substantially affect the formation of the secondary dips. It is to be noticed that the number of the secondary dips remains constant by six in all photographs when they are distinctly discernable.

2) The average total and static pressure in the pump seems to fluctuate following a periodical function of the form $\Delta p = p_1 \sin (\omega_1 t + \delta_1) + p_2 \sin (\omega_2 t + \delta_2)$ (Figure III-4), where the maximum and minimum amplitude for $(p_1 + p_2)$ and $(p_1 - p_2)$ are reached. This general tendency is to be observed in different oscilloscope photographs mentioned above, and is demonstrated clearly in the Photographs 36 and 37) where four and three sweeps were taken on the same picture with the highest available sensitivity. A very roughly estimated average measurement of $T_2 \approx 28$ milliseconds and $T_3 \approx 170$ milliseconds with $p_1 - p_2 = 0.35$ psia and $p_1 + p_2 = 0.7$ psia led to an equation of $\Delta p_m = 0.525 \sin 2\pi t + 0.175 \sin 242t$. The problem of mean pressure fluctuation is beyond the scope of this investigation and no further attempt has been made to observe or to substantiate its formation. The above mentioned periodical function could be considered as the most simple relation giving roughly the effect of these changes. In the following, these periodical variations of mean pressure would be considered as a moving coordinate system in order to investigate the pressure changes between two consecutive vanes.
Figure III-4. Mean Pressure Fluctuation in the Pump.
Figure III-5. Fluctuations in the Magnitude of the Main Dip.
Figure III-6. Comparison for Results Obtained with and without Air Bubble for Total Pressure.
3) The magnitude of the main dip in total and static pressure profiles are subject to appreciable periodical changes as it can be seen on all photographs (Figure III-5 and Photograph 38), and these changes may vary by about 50 percent. The fluctuation of the magnitude in the main dip, seems to follow a periodical function also. This too has a maximum and minimum in its amplitude variation, so that it will constitute essentially a periodical function of at least two terms. The periodical change in the magnitude of the dip has not been followed in this investigation since these results were not expected and the equipment and instrumentation were not appropriate and adequate to explore this field. However it must be noticed that the fluctuation of about 50 percent noticed above is relative to the dip magnitude. If these were compared to the absolute value of effective pressure the dip magnitude will be about ten percent and the fluctuation will not exceed five percent. The periodical changes in the magnitude of the main dip might be the reason for the ± 50 percent scattering of points in the investigation of Osborne and Morelli(62) with a method where the experimental errors could not be more than ± two percent.

The presence of fluctuations in the main dip in total and static pressure made it obligatory to operate with the average of a certain number of values. This average has been calculated from 61 oscilloscope photographs as well as from 18 separate oscilloscope photographs for total and static pressures. In both cases slight numerical deviations exists, but both have given the same character for the pressure profiles. Before proceeding to the calculation of the velocity profile it was found out that the pressure signals due to mechanical vibrations were to be considered. The mechanical vibrations were registered using the method described in a foregoing paragraph. The obtained oscilloscope (Photographs 33 to 43) clearly show a tendency of the creation of six different impulses during the passage of one vane, and the existence of one main dip is more or less discernable. The introduction of an air bubble could not materially affect the existence
of these six different impulses so that in order to have a simple basis for a comparison calculation, the effects of the mechanical vibrations were considered as a simple sine function with an amplitude equal to the magnitude of the main dip from the photographs. By subtracting the average values of static pressure and pressure signals due to mechanical vibration from the average values of the total pressure, the velocity head and the velocity profile were formed (Figure III-2) for each average of the 61 and the 18 Photographs.

2. Observation Without Air Bubble in Pitot Tube

As mentioned in the beginning of this chapter, operating without an air bubble, the total pressure profiles show the presence of six different secondary dips (Photographs 5 to 12); the same secondary dips are to be observed in all static pressure profiles (Photographs 13 to 16). It has been pointed out earlier that the building of an average value profile could be appreciably influenced by arbitrary choice of origin and the procedure of taking the average in this case. It was decided therefore to use the results obtained with an entrained air bubble as a basis for a comparison calculation and then to follow qualitatively the changes in the different profiles for the cases where no air bubble exists. Figure III-6 shows the changes in total pressure profiles when no air bubble is present in the Pitot tube; a similar graphical representation could be made for the static pressure, where generally the magnitude of the secondary dips were smaller. It is evident that in the derivation of the velocity profile, six superimposed secondary dips will be present. Should the magnitude
of the secondary dips in the static pressure profile be in the range of the corresponding dips in the total pressure profile, it would be qualitatively reduced to the case of velocity profile with air bubble. Even if the magnitude of the secondary dips in the static pressure profile exceeded the magnitude of the corresponding secondary dips in the total pressure profile, the same characteristic dips in the velocity profile could remain, but they would change their direction (Figure III-2).

It might be concluded that for a comparison calculation for the velocity profile the measurements made with an air bubble can be admitted as a basic case and that the differences due to the absence of air bubble can be qualitatively followed on the results obtained in this basic case. If simultaneous measurement of total and static pressure had been recorded, it should have been possible to calculate the velocity profile and its instantaneous shape. However it would have been necessary to obtain a certain average even in this case since unsteady flow existed.

C. Comparison Between the Experimental and Derived Velocity Profiles From the Proposed Theory.

The shock effect, described in the first part of this research between the discharge from the impeller and the double vortex flow in the volute casing resulting in a transient building of a potential flow pattern can be followed further in the radial direction. The first streamline leaving the impeller in the separation zone of flows discharged between two consecutive vanes can be assumed to be a logarithmic spiral with
Figure III-7. Velocity Distribution in Potential Flow Pattern.
the equation (Figure III-7)

\[ \varphi = 132 \frac{\log r/r_2}{\tan \alpha_{2m}} \]

As can be calculated for the case of flow leaving an unobstructed impeller\(^{(66)}\), it should be pointed out that the flow is an unsteady flow and that the formation of the potential flow pattern is instantaneous and is periodically renewed with every new pressure wave. The space in which the shock effect occurs is strictly the domain limited by two consecutive logarithmic spiral pathlines and the so-called "shock circle". An attempt can be made to determine the different streamlines in this domain using elaborate methods of conform mapping. The object of this calculation was to obtain a comparative basis for the experimental average velocity profile and as the dimensions of the considered domain was small it seemed expedient to represent the shock circle and the separation spiral with their correspondent tangents on their point of intersection. This simplification will reduce the case to a known elementary conform mapping function (Appendix IV), and it will be valid insofar as the curve can be represented by its tangent. The results for the middle zone between two consecutive vanes would remain accurate, and theoretically deviation should be expected in the vicinity of vanes. The logarithmic spiral path line determines the angle between itself and any considered shock circle and is constant with an average value of \(180-15 = 165^\circ\). In the case of the axial velocity distribution the shock circle location has been chosen arbitrarily, here five different possible shock circles
a-a with a radius of 5.802 inches
b-b with a radius of 5.927 inches
c-c with a radius of 6.052 inches
d-d with a radius of 6.177 inches
e-e with a radius of 6.302 inches

have been considered and five different velocity profiles have been theoretically calculated (Appendix IV) so that the mean velocity of 63.2 ft/sec remained constant for all of them. The theoretical velocity profiles for all corresponding shock circles are reproduced in Figure III-8 and it should be noticed that comparatively small changes in location of the shock circle produce a consistent but substantial change in the shape of the theoretical velocity profile. On the same diagram the experimental velocity profiles for the averages of 61 and 18 oscilloscope pictures were introduced. The comparison of both kinds of profiles shows that a dip exists in both of them, but the magnitude is much more attenuated in the case of experimental profiles than in the different theoretical curves. It should be noticed that in the vicinity of vanes the approximation in the procedure of calculation might also have affected the difference in magnitude to some extent. The shape of the experimental curves were similar in tendency for the average of the 61 and 18 pictures. However, they do not quite correspond to the shape of the theoretical profiles obtained assuming that the shock was produced on the surface of a cylinder. Since the entier theoretical considerations were based upon the existence of a shock circle or cylinder this fact must be substantiated. The cylindrical assumption was the simplest one but
Figure III-8. Theoretical Velocity and Experimental Velocity Profile.
still was arbitrary, so that from the difference in the shape of the experimental and theoretical velocity profiles it must be concluded that the shock effect can not be produced on a simple cylindrical surface but rather must be on a surface which is more or less undulated. If it could be assumed that the intersection points of experimental and theoretical velocity profiles represent the locations of one point on the undulated surface, a two dimensional shock flow surface can be traced point by point, as has been done in Figures III-9 and III-10 in full lines for the cases of the 61 and 18 averages. This assumption could be supported by analysis if after having obtained the shock surface in full lines in Figures III-9 and III-10, the Laplace equation could be solved for the new graphical boundary conditions. This solution should reproduce the original velocity distribution. The problem might lead to a boundary value problem of the first or second kind and the carrying out of the solution of the Laplace equation in this particular case might be the subject of another investigation. The point by point construction of the shock surface for both averages (Av. 61 and Av. 18) are represented in Figures III-9 and III-10 by full lines, and are of similar shape with the same tendency in both cases. The Av. 61 and Av. 18 curves have been taken as previously mentioned, with the introduction of an air bubble in the Pitot tube.

The case without an air bubble can be now easily followed in a qualitative sense. The difference between the two cases consists in the existence of six superimposed secondary waves on the velocity profile. If, in Figure III-8 the velocity profile with six secondary dips were introduced,
Figure III-9. Shock Surface for Av. 61.
Figure III-10. Shock Surface for Av. 18.

SHOCK SURFACE AV. 18

WITH AIR BUBBLE, CONSTRUCTED

WITHOUT AIR BUBBLE, QUALITATIVE

RADIUS OF
IMPELLER  5.437 inches
PITOT TUBE  5.625
a-a  5.802
b-b  5.927
c-c  6.052
d-d  6.177
e-e  6.302
it would undoubtedly lead to the determination of six different undulations on the previously constructed shock surface profile. These are shown qualitatively with broken lines on Figures III-9 and III-10. The real shock effect occurs only on this undulated surface with six different dips. It should be noticed that this consideration is only qualitative, and does not give any information about the magnitude of these undulations. However, the establishment of its existence alone may open a field for explanations concerning the functioning of a centrifugal pump.

Any attempt to evaluate the magnitude of secondary dips should be made with instrumentation providing simultaneous readings for total and static pressure in order to be able to evaluate an instantaneous velocity profile.

D. Consequences of the Proposed Theory

The investigation of the velocity profile between two consecutive vanes in a radial plane, confirms the previously developed theory of paragraph II-E, and introduces additionally the determination of the shape of the assumed shock surface. The unsteady flow leaving the impeller seems to form some sort of an instantaneous deformable, movable fluid gear with a certain number of axial directed teeth. The surface of this gear constitutes the shock flow surface between the double vortex flow in the volute and the flow discharged from the impeller. Along the undulated, and deformable fluid surface of these teeth there occurs an exchange of mass and energy simultaneously. Every new pressure wave renews the system of the gear. This new conception of unsteady flow at the outlet of an impeller is capable of
IV CONCLUSION

This investigation led to the experimental observation of the presence of a main dip in velocity profile in the axial and in the radial directions between two consecutive vanes at the outlet of an impeller in an industrial centrifugal pump. There are then no restrictions involved in the investigation by idealization in the machinery.

Observations within the frame of this research work, as well as some observation outside of it made by different investigators are in complete disagreement with conclusions made from established theory of turbo-machinery. An attempt has been made to propose a new theory which relates the flow in the volute and the flow discharged from the impeller with different flow identities before mixing. The new theory relates the shock effect between the two flow identities.

These considerations led to a theoretical evaluation of the velocity profile, the description of the shape of the shock surface and to the conception of the mechanism about transmission of energy and mass between the impeller and volute casing.

The shock surface seems to constitute some sort of instantaneous, deformable, movable fluid gear system, which is transient but which is renewed by every new pressure wave. The number of teeth on this fluid gear is constant, and the teeth are axially directed for the performance of maximum efficiency. They change their inclination in one or the other directions following a decrease or increase in capacity for a constant speed. The unsteady exchange of mass and energy is occurring simultaneously along the teeth of the fluid gear.

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Following the principle of shock flow, the analytical results seem to be in concordance with the measurements and different predictions based on the new theory seem to agree with observations.

During this research work a conviction grew that different refinements, enlargements, and derivations were highly desirable in order to obtain a more acute and irrefutable representation of the phenomena, as it has been occasionally pointed out in the text.

Primarily the following investigations should be carried out:

1) The repetition of the same investigation with an appropriate instrumentation given simultaneous readings for total and static pressure which could be read instantaneously on the dual beam of an oscilloscope.

2) Further study of the periodical mean pressure changes in the volute (70).

3) The study of the periodical changes in the magnitude of the main dip in total and static pressure profile.

4) The solution of Laplace equation for the graphically obtained shock flow surface and derivation of velocity distribution between two vanes. If the first attempt should not give concordant results with the experimental curve appropriate changes can be made along the shock surface so that successive attempts will presumably lead to the superposition of the two curves for the velocity profile, and so determine exactly the instantaneous shape of the shock flow surface.

5) The changes of velocity and angle of outlet as a function of capacity given in Figure II-13, II-14, II-15, and II-16, show a consistent character of resonance diagram of the second degree, and thus a remarkable similarity with the resonance curves obtained for ships oscillation with a Frahm tank. An attempt to study their nature seems to be promising.
explaining different observed phenomena which remained incompatible with existing theories:

1) The existence of a main dip in the axial and the radial direction is established as a kinematic property of the shock effect between two flows.

2) The secondary dips in the radial direction are due to the undulations on the shock surface.

3) The mechanical vibrations are related to the number of teeth formed between two consecutive vanes. The number of them remains constant under the same performance conditions and this fact has been observed clearly in the mechanical vibration photographs. (See note.)

4) The existence of two or more dips in the axial direction can be substantiated by a change in inclination of the teeth on the fluid gear following an increase or decrease in capacity (Figure III-ll).

5) Following the instantaneous potential flow at the outlet, it can be predicted that the angle of outlet would decrease from the pressure side to the suction side between two vanes in a radial plane. This result seems to be in concordance with the analytical solution worked out for an idealized case by Rashed(70).

Note: It can be concluded from the shape of the velocity profile in the axial direction (Figure II-7) that in the basic case of 1750 RPM, 800 GPM the dips of the secondary waves, which formed the space between the teeth of the unsteady "fluid gear" are axially directed and perpendicular to the peripheral plane of the impeller (Figure III-lla). The only dip observed in the velocity profile is due to the shock effect between the two mixing flows and in this case the efficiency is maximum. Reducing the capacity of the pump causes the teeth on the fluid gear system to be inclined to the left (Figure III-llb) which explains the shape of the
Figure IV-1. Fluid Gear System.
velocity profile at 600 GPM, and the formation of double dips. The decrease
of efficiency may be considered in part, as a consequence of inclination of
teeth. With an increase of capacity the teeth on the fluid gear system
inclines to the opposite direction and this inclination can go so far
(Figure III-11c) that an observation made in the axial direction must
necessarily give different dips in the velocity profile. As was observed
in the case of 1045 GPM (Figure II-21) the decrease in efficiency is also
partly due to the inclination of the teeth. The calculation methods of
mechanical vibrations and critical speed can be revised considering the
formation of the fluid gear system. The number of impulses involved in
the calculation should be increased following the number of teeth in the
fluid gear between two consecutive vanes.
APPENDIX I

Nomenclature

\( a_n, a'_n \)  acceleration of the mass \( dm \)
\( r \)  radius of the mass \( dm \)
\( V_m, V'_m \)  relative velocity of the mass \( (dm) \)
\( R, R' \)  radius of curvature on the relative path
\( u \)  peripheral velocity of impeller
\( p, p' \)  pressure
\( \gamma \)  specific weight
\( C \)  constant
\( n, n' \)  distance of the mass \( (dm) \) on the normal to the relative path
\( V_{mo}, V'_{mo} \)  mean relative velocity
\( n_0, n'_0 \)  total distance between the guide walls of the curved channel
\( b, b' \)  total distance between the guide walls of the curved channel in perpendicular direction to the plan considered
\( \beta, \epsilon \)  angle between radius of impeller and normal to the relative path
\( \Omega \)  potential of external force field
\( \omega \)  angular velocity
APPENDIX I

DISTRIBUTION OF VELOCITY AT THE OUTLET OF AN IMPELLER
CONSIDERING THE MASS FORCES

A. Velocity Distribution in a Radial Plane

The acceleration of a mass particle in the normal direction can be written (APP. I, Figure 1)

\[ a_n = r \cdot \omega^2 \cdot \cos \beta + \frac{V_m^2}{R} - 2uV_m \quad (1) \]

\[ (-r) \quad (n) \quad (n) \quad \text{Absolute Coriolis} \quad \frac{dr}{dn} = -dn \cdot \cos \beta \]

\[ a_n = \frac{V_m^2}{R} - 2uV_m - \omega^2 \frac{dr}{dn} \quad (2) \]

\[ \frac{V_m^2}{R} - 2uV_m - \omega^2 \frac{dr}{dn} \quad \text{dm} = (p - p')df \quad (3) \]

The energy equation for the relative velocity in a incompressible fluid is

\[ \frac{V_m^2}{2g} + \frac{p}{\gamma g} + \frac{\Omega}{2g} - \frac{u^2}{2g} = \text{constant} \quad (4) \]

This equation can be differentiated in (n) direction

\[ \frac{V_m}{g} \frac{dV_m}{dn} + \frac{dp}{\gamma dn} - \frac{\omega^2 r}{g} \frac{dr}{dn} = 0 \quad (5) \]

Appropriate simplification reduces the equation to

\[ \frac{dV_m}{dn} \cdot \frac{V_m}{R} + 2\omega = 0 \quad (6) \]

A well known equation in Stodola\(^{(88)}\) in Flügel\(^{(26)}\) and Rashed\(^{(70)}\), which can be also written in the form

\[ \frac{dV_m}{dn} = \frac{V_m - 2uR}{R} \quad (7) \]

This equation has been solved by graphical methods or by means of certain assumption in the work of the above mentioned authors. In order to obtain
Figure A-I-1. Mass Forces in Radial Plane.
Figure A-I-2. Velocity Distribution in Radial Plane.
an analytical solution, a special assumption has been preferred in the case
of the outlet point at the impeller, for the integration of Equation (6).
The radius of curvature $R$ would be assumed to be constant at one point of
the curve on the normal to it. With every considered point the constant
value of $R$ is changing so that the assumption is non-restrictive. Besides
at the outlet of the impeller the radius of curvature being very large
even infinite and the distance between the guide walls very small, the
assumption of a constant radius of curvature along the normal on a point
cannot be very far from reality. With this assumption the separation of
variable leads to the exact solution,
\[ V_m - 2\omega R = Ce^\frac{n}{R} \]  \hspace{1cm} (8)

An approximate solution could be obtained by developing $e^{n/R}$ in series:
\[ V_m - 2\omega R = Ce^\frac{n}{R} = C(1 + \frac{n}{R} + \frac{n^2}{2R^2} + \cdots ) \]  \hspace{1cm} (9)

The integration constant $C$ can be determined in terms of mean velocity
$V_{m0}$ defined as follows:
\[ V_{m0} = \frac{\int_0^{n_0} V_m b \cdot dn}{bn_0} = \frac{Q}{bn_0} \]  \hspace{1cm} (10)

using only the three first terms of the series the computation leads to:
\[ C = \frac{V_{m0} - 2\omega R}{1 + \frac{n_0}{2R} + \frac{n_0^2}{6R^2}} \]  \hspace{1cm} (11)

With this value of the constant the velocity function is
\[ V_m = \frac{V_{m0} - 2\omega R}{1 + \frac{n_0}{2R} + \frac{n_0^2}{6R^2}} \left(1 + \frac{n}{R} + \frac{n^2}{2R^2}\right) + 2\omega R \]  \hspace{1cm} (12)
which can be remodeled and written in the form

$$V_m = \frac{V_{mo}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} + \omega n_o \frac{1 - 2 \frac{n}{n_o} + \frac{n_o}{3R} - \frac{n}{n_o R}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}}$$  \hspace{1cm} (13)$$

For outside where \( n = 0 \)

$$V_m = \frac{V_{mo}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} + \omega n_o \frac{1 + \frac{n_o}{3R}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} = V_{max}$$  \hspace{1cm} (14)$$

For inside where \( n = n_o \)

$$V_m = \frac{V_{mo}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} - \omega n_o \frac{1 + \frac{2}{3} \frac{n_o}{R}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} = V_{min}$$  \hspace{1cm} (15)$$

Equation (13) can be written

$$V_m = \frac{V_{mo}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} + \omega n_o \frac{1 + \frac{n_o}{3R}}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} - \frac{\omega}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} n(2 + \frac{n}{R})$$

$$V_m = V_{max} - \frac{\omega}{1 + \frac{n_o}{2R} + \frac{n_o^2}{6R^2}} n\left(2 + \frac{n}{R}\right)$$  \hspace{1cm} (16)$$

The velocity distribution tends to be linear if \( n_o/R \) is negligible compared with (2); at the inlet this can have some effect, but at the outlet the linearity is a good approximation and it might be expressed in the form:

$$V_{max} = V_{mo} + \omega n_o$$

$$V_{min} = V_{mo} - \omega n_o$$

It can be concluded taking in account only the mass forces, the relative and absolute velocity of the flow between 2 consecutive vanes in the radial plane is increasing from the pressure side up, following the
exponential relation (8); at the outlet the increase of velocity can be considered as linear, with a good approximation.

**B-Velocity Distribution in an Axial Plane**

The acceleration will be

$$a_n' = -rw^2\cos\varepsilon + \frac{V_m'^2}{R'} + 0$$  (17)

$$\cos\varepsilon = -\frac{dr}{dn'}, \quad a_n' = rw^2\frac{dr}{dn'} + \frac{V_m'^2}{R'}$$  (18)

The force of a mass (dm)

$$a_n' \cdot dm = (p' - p) \cdot df$$  (19)

$$\frac{df}{dn'} = \frac{1}{g} \left( rw^2\frac{dr}{dn'} + \frac{V_m'^2}{R'} \right) = dp \cdot df$$  (20)

$$rw^2\frac{dr}{dn'} + \frac{V_m'^2}{R'} = \frac{1}{p} \frac{dp}{dn'}$$  (21)

From the energy Equation (4), it can be calculated, as it has been done in the foregoing paragraph

$$\frac{V_m'^2}{R'} = -V_m' \frac{dV_m'}{dn'}$$  (22)

Separation of variables and integration leads to:

$$V_m'^2 = e^{-\int\frac{dn'}{R'}} + \ln C = Ce^{\frac{n'}{R'}} = C \left( 1 - \frac{n'}{R'} + \frac{n'^2}{2R'^2} - \cdots \right)$$  (23)

The constant C will be determined introducing the mean velocity

$$V_m' = \frac{\int_{Q_0} V_m' b'dn'}{b'n_0} = C \left( 1 - \frac{n_0}{2R'} + \frac{n_0'^2}{6R'^2} \cdots \right)$$  (24)

$$C = \frac{V_m'}{1 - \frac{n_0}{2R'} + \frac{n_0'^2}{6R'^2}}$$
Figure A-I-4. Velocity Distribution in Axial Plane.
The velocity function is obtained as

\[
V'_m = V'_{m0} \frac{1 - \frac{n'_1}{R'_1} + \frac{n'_2}{2R'_1^2}}{1 - \frac{n'_1}{2R'_1} + \frac{n'_2}{6R'_1^2}}
\]  

(25)

\(V'_{m0}\) will be reached for \(n' = 0\)

\[
V'_{\text{max}} = \frac{V'_{m0}}{1 - \frac{n'_1}{2R'_1} + \frac{n'_2}{6R'_1^2}}
\]  

(26)

It can be concluded that taking into account only the mass forces the relative and absolute velocity of the flow in an axial direction is increasing from the inlet side up following the relation (23); this increase can be considered linear as an approximation.
APPENDIX II

Nomenclature

r \quad \text{radius in a radial plane}

\xi \quad \text{generatrix of the control surface cones}

b \quad \text{normal distance between two streamlines}

t \quad \text{time}

\theta \quad \text{angle of rotation in a radial plan}

\nu \quad \text{Volume}

M \quad \text{moment}

m \quad \text{mass}

V \quad \text{absolute velocity}

\rho \quad \text{density}

s \quad \text{path vector}

z \quad \text{distance in z direction}

\epsilon \quad \text{angle between the generatrix of the control surface cone and the axis of the pump}

\omega \quad \text{angular velocity}

p \quad \text{pressure}

\Omega \quad \text{potential of external force field}

\gamma \quad \text{specific weight}

u \quad \text{peripheral velocity}

F \quad \text{Force}

\beta \quad \text{angle between peripheral and relative velocity}

v \quad \text{relative velocity}

\alpha \quad \text{angle between peripheral and absolute velocity}
\( \lambda \) ratio between velocity and mean velocity
\( \varphi, \psi \) function defined through Equation (27)
\( V_0 \) average velocity
\( \varphi_0, \psi_0 \) constant values of \( \varphi \) and \( \psi \)
\( Z \) number of vanes
\( C \) constant
\( \psi_1, \psi_2 \) constant values defined in text
\( A, B \) constant values defined in text
\( V_{ro} \) average radial velocity
\( r_1 \) inlet radius in radial plan

Subscripts
\( z \) in \( z \) direction
\( u \) in peripheral direction
\( r \) in radial direction
\( m \) in the direction of the generatrix
APPENDIX II

THE DISTRIBUTION OF VELOCITY AT THE OUTLET BETWEEN TWO VANES IN THE RADIAL DIRECTION, CONSIDERING THE MASS FORCES AND FOLLOWING THE THEORY OF SPANNAKE

A. Dynamic Considerations

From Appendix II Figures 1, 2, 3, the following geometric values and their infinitesimal changes are considered:

\[ r \quad \text{---} \quad dr \quad \xi \quad \text{---} \quad d\xi \quad b \quad \text{---} \quad b + \frac{\partial b}{\partial \xi} \quad d\xi \]

In \( (dt) \) time the control surface is turning about \( (d\theta) \) angle, the angular momentum law can be written for the differential volume \( d\mathbf{U} = r \cdot d\theta \cdot d\xi \cdot b \)

\[
dM_z = dF_U \cdot r = dm \cdot \frac{d(V_U \cdot r)}{dt} = \rho \cdot r \cdot d\theta \cdot d\xi \cdot b \cdot \frac{d(V_U \cdot r)}{dt} \tag{1}
\]

The substantial rate or change in \( d(V_U \cdot r) \) is the sum of local and connective part of it, \( V_U \cdot r = f(s, t) \) in an unsteady flow

\[
d(V_U \cdot r) = \frac{\partial (V_U \cdot r)}{\partial t} dt + \frac{\partial (V_U \cdot r)}{\partial s} ds
\]

\[
\frac{d(V_U \cdot r)}{dt} = \frac{\partial (V_U \cdot r)}{\partial t} + \frac{\partial (V_U \cdot r)}{\partial s} \frac{ds}{dt} = \frac{\partial (V_U \cdot r)}{\partial t} + v \frac{\partial (V_U \cdot r)}{\partial s} \tag{2}
\]

In order to evaluate the rate of \( \frac{\partial (V_U \cdot r)}{r \cdot d\theta} \) it may be written \( s = f(r, \theta, z) \) and

\[
\frac{\partial (V_U \cdot r)}{\partial s} = \frac{\partial (V_U \cdot r)}{\partial s} \frac{dz}{ds} + \frac{\partial (V_U \cdot r)}{r \cdot d\theta} \frac{dr}{ds} + \frac{\partial (V_U \cdot r)}{\partial r} \frac{dr}{ds} ...	ag{3}
\]

Some relations are evident:

\[
\begin{align*}
\text{dr} &= d\xi \cdot \sin \epsilon = ds \cdot \sin \alpha \cdot \sin \epsilon \\
\text{dz} &= d\xi \cdot \cos \epsilon = ds \cdot \sin \alpha \cdot \cos \epsilon = \frac{dr}{\tan \epsilon} \\
\text{r} \cdot d\theta &= ds \cdot \cos \alpha \\
&\quad \cdots \quad V_m = V \sin \alpha \\
V_z &= V_m \cdot \cos \epsilon = V \sin \alpha \cdot \cos \epsilon \\
V_r &= V_m \cdot \sin \epsilon = V \sin \alpha \sin \epsilon
\end{align*}
\]

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\[
\frac{\partial (V_u \cdot r)}{\partial s} = \frac{\partial (V_u \cdot r)}{\partial z} \sin \alpha \cos \varepsilon + \frac{\partial (V_u \cdot r)}{r \cdot \partial \theta} \cos \alpha + \frac{\partial (V_u \cdot r)}{\partial r} \sin \alpha \sin \varepsilon \quad \ldots \quad (4)
\]

Velocity \(V\) is the resultant of \(V_z, V_u, V_r\):

\[
V \frac{\partial (V_u \cdot r)}{\partial s} = V_z \frac{\partial (V_u \cdot r)}{\partial z} + V_u \frac{\partial (V_u \cdot r)}{r \cdot \partial \theta} + V_r \frac{\partial (V_u \cdot r)}{\partial r} \quad \ldots \quad (5)
\]

The local part of it is

\[
\frac{\partial (V_u \cdot r)}{\partial t} = - \omega \cdot \frac{\partial (V_u \cdot r)}{\partial \theta} \quad \ldots \quad (6)
\]

Combining (5) and (6) in (2) the substantial part is obtained

\[
\frac{d (V_u \cdot r)}{dt} = V_z \frac{\partial (V_u \cdot r)}{\partial z} + V_u \frac{\partial (V_u \cdot r)}{r \cdot \partial \theta} + V_r \frac{\partial (V_u \cdot r)}{\partial r} - \omega \frac{\partial (V_u \cdot r)}{\partial \theta} \quad \ldots \quad (7)
\]

The change of tangential force \(dF_u\) (Figure 4) can be computed. Considering the mass force due to pressure, the left part of Equation (1) can be written:

\[
dF_u = b \cdot d^2 \left( p - \frac{\partial p}{r \cdot \partial \theta}\right) = - r \cdot \partial \theta \cdot \partial \theta \cdot b \cdot \frac{\partial p}{r \cdot \partial \theta} \quad \ldots \quad (8)
\]

Introducing the values of Equations (5), (6) and (8) in (1):

\[
g \frac{\partial (p / \gamma)}{\partial \theta} + V_z \frac{\partial (V_u \cdot r)}{\partial z} + V_u \frac{\partial (V_u \cdot r)}{r \cdot \partial \theta} + V_r \frac{\partial (V_u \cdot r)}{\partial r} - \omega \frac{\partial (V_u \cdot r)}{\partial \theta} \quad \ldots \quad (9)
\]

The Bernoulli equation states

\[
g \left( \frac{p / \gamma + \Omega}{2} \right) - \omega \cdot V_u \cdot r = \text{constant} \quad \ldots \quad (10)
\]

Taking into account \(V^2 = V_z^2 + V_u^2 + V_r^2\)
Equation (1) can be differentiated:

\[ g \frac{\partial (p/\rho)}{\partial \theta} + v_z \frac{\partial V_z}{\partial \theta} + u \frac{\partial U}{\partial \theta} + v_r \frac{\partial V_r}{\partial \theta} - \omega \frac{\partial (V_u \cdot r)}{\partial \theta} = 0 \]  \hspace{1cm} (12)

In Equation (9) \( \frac{\partial (V_u \cdot r)}{r \partial \theta} = \frac{\partial (V_u)}{\partial \theta} \) because \( r \) is not a function of \( \theta \),

Equation (9) and (12) combined are reduced to

\[ V_r \frac{\partial (V_u \cdot r)}{\partial r} + V_z \frac{\partial (V_u \cdot r)}{\partial z} = V_r \frac{\partial (V_r)}{\partial \theta} + V_z \frac{\partial (V_z)}{\partial \theta} \]  \hspace{1cm} (13)

The operator \( \frac{\partial}{\partial \xi} = V_z \frac{\partial}{\partial \theta} + V_r \frac{\partial}{\partial r} \)

leads to \[ \frac{\partial (V_u \cdot r)}{\partial \xi} = \frac{\partial (V_m)}{\partial \theta} \]  \hspace{1cm} (14)

The following relations

\[ d\xi = \frac{dr}{\sin \xi} \hspace{1cm} V_m = \frac{V_r}{\sin \xi} \hspace{1cm} d\xi = \frac{dz}{\cos \xi} \hspace{1cm} V_m = \frac{V_z}{\cos \xi} \]

may lead to \[ \frac{\partial (V_u)}{\partial \theta} = \sin^2 \xi \cdot \frac{\partial (V_u \cdot r)}{\partial \theta} \]  \hspace{1cm} (15)

\[ \frac{\partial (V_z)}{\partial \theta} = \cos^2 \xi \cdot \frac{\partial (V_u \cdot r)}{\partial \theta} \]  \hspace{1cm} (16)

B. Kinematic Considerations

From Appendix II Figure 7 it can be deduced:

\[ V_u = u + V_m \cdot \cot \beta \]  \hspace{1cm} (17)

\[ V_u \cdot r = u \cdot r + V_m \cdot r \cdot \cot \beta = w \cdot r^2 + V_m \cdot r \cdot \cot \beta \]  \hspace{1cm} (18)
\[
\frac{\partial (V_m \cdot r)}{\partial \xi} = 2r \cdot \omega \frac{dr}{\partial \xi} + \frac{\partial (V_m \cdot r)}{\partial \xi} \cot \beta + (V_m \cdot r) \frac{\partial (\cot \beta)}{\partial \xi}
\]
\[
= 2r \cdot \omega \sin \epsilon + \left( \frac{\partial V_m \cdot r}{\partial \xi} + V_m \frac{\partial r}{\partial \xi} \right) \cot \beta + V_m \cdot r \frac{\partial (\cot \beta)}{\partial \xi}
\]
\[
\frac{\partial (V_u \cdot r)}{\partial \xi} = 2r \omega \sin \epsilon + \left( \frac{\partial V_m \cdot r}{\partial \xi} + V_m \cdot \sin \epsilon \right) \cot \beta + V_m \cdot r \frac{\partial (\cot \beta)}{\partial \xi} \tag{19}
\]

C. **Continuity**

May be written
\[
V_m \cdot r \cdot \partial \omega \cdot b + V_u \cdot d \cdot b = \left( V_m + \frac{\partial V_m}{\partial \xi} \right) (r + dr) \partial \omega \left( b + \frac{\partial b}{\partial \xi} d \xi \right) + \left( V_u + \frac{\partial V_u}{r \cdot \partial \theta} \cdot r \cdot \partial \theta \right) b d \xi \tag{20}
\]

As no material transport between streamlines is occurring, disregarding terms of second order:
\[
\frac{\partial (V_m \cdot r)}{\partial \xi} = \frac{\partial V_m \cdot r}{\partial \xi} + V_m \frac{\partial r}{\partial \xi} - \frac{\partial b}{\partial \xi} - \frac{\partial V_u}{\partial \theta} \tag{21}
\]

Taking \( \frac{\partial V_m}{\partial \xi} r + V_m \frac{\partial r}{\partial \xi} \) from (21) and inserting in (19)
\[
\frac{\partial (V_u \cdot r)}{\partial \xi} = 2r \omega \sin \epsilon - \left( \frac{V_m \cdot r}{b} \frac{\partial b}{\partial \xi} + \frac{\partial V_u}{\partial \theta} \right) \cot \beta + V_m \cdot r \frac{\partial (\cot \beta)}{\partial \xi} \tag{22}
\]

The value from (14) \( \frac{\partial (V_u \cdot r)}{\partial \xi} = \frac{\partial V_m}{\partial \theta} \) inserted in (22) gives
\[
\frac{\partial V_m}{\partial \theta} = 2r \omega \sin \epsilon - \left( \frac{V_m \cdot r}{b} \frac{\partial b}{\partial \theta} \sin \epsilon + \frac{\partial V_u}{\partial \theta} \right) + V_m \cdot r \frac{\partial (\cot \beta)}{\partial r} \sin \epsilon \tag{23}
\]

The differentiation of (17), \( u \) being constant, leads to
\[
\frac{\partial V_u}{\partial \theta} = \frac{\partial V_m}{\partial \theta} \cot \beta + V_m \frac{\partial (\cot \beta)}{\partial \theta} \tag{24}
\]
and inserting the value of \( \frac{\partial V_m}{\partial \theta} \) from (24) into (23):

\[
\frac{\partial V_m}{\partial \theta} = 2\omega \sin \epsilon - \cot \beta \left( \frac{V_m \cdot r}{b} \frac{\partial \beta}{\partial \theta} \right) \cos \beta + V_m \sin \beta \frac{\partial (\cos \beta)}{\partial \theta} + V_m \cdot r \frac{\partial (\cot \beta)}{\partial \theta} \sin \epsilon
\]

\[
\frac{\partial V_m}{\partial \theta} (1 + \cot^2 \beta) = \sin \epsilon \left( 2\omega r + V_m \cdot r \left[ \frac{\partial (\cos \beta)}{\partial r} - \cot \beta \left( \frac{1}{b} \frac{\partial \beta}{\partial r} + \frac{1}{r \sin \epsilon} \frac{\partial (\cot \beta)}{\partial \theta} \right) \right] \right)
\]

\[
\frac{\partial V_m}{\partial \theta} = \sin \epsilon \sin^2 \beta \left( 2\omega r + V_m \cdot r \left[ \frac{\partial (\cot \beta)}{\partial r} - \cot \beta \left( \frac{1}{b} \frac{\partial \beta}{\partial r} + \frac{1}{r \sin \epsilon} \frac{\partial (\cot \beta)}{\partial \theta} \right) \right] \right)
\]

\[\text{(25)}\]

The definition of mean velocity is \( V_{mo} = \frac{\Delta \Omega}{2\pi rb} \), a velocity ratio \( \lambda = \frac{V_m}{V_{mo}} \) can be defined so we have

\[
\frac{\partial V_m}{\partial \theta} = V_{mo} \frac{\partial \lambda}{\partial \theta}
\]

\[\text{(26)}\]

and the equation (25) can be written in dimensionless form

\[
\frac{\partial \lambda}{\partial \theta} = \sin \epsilon \sin^2 \beta \left( 2\omega r + V_m \cdot r \left[ \frac{\partial (\cot \beta)}{\partial r} - \cot \beta \left( \frac{1}{b} \frac{\partial \beta}{\partial r} + \frac{1}{r \sin \epsilon} \frac{\partial (\cot \beta)}{\partial \theta} \right) \right] \right)
\]

\[
\frac{\partial \lambda}{\partial \theta} = \sin \epsilon \sin^2 \beta \frac{2\omega r}{V_{mo}} + \sin \epsilon \sin \beta \lambda \left( \frac{r \partial (\cot \beta)}{\partial r} - \cot \beta \left( \frac{r}{b} \frac{\partial \beta}{\partial r} + \frac{1}{\sin \epsilon} \frac{\partial (\cot \beta)}{\partial \theta} \right) \right)
\]

\[\text{(27)}\]

(\( \phi \)) and (\( \psi \)) can be introduced as follows:

\[
\frac{\partial \lambda}{\partial \theta} = \phi(r, \beta, b, \epsilon, \theta) + \lambda \cdot \psi(r, \beta, b, \epsilon, \theta)
\]

\[\text{(28)}\]

In (28) for a given point \((r)\), \((b)\), and \((\epsilon)\) are constant and assuming \( \frac{\partial \beta}{\partial \theta} = 0\), because \( \beta \) is constant at streamlines

\[
\frac{\partial \lambda}{\partial \theta} = \phi_0 + \psi_0 \cdot \lambda
\]

\[\text{(29)}\]

\[
\frac{\partial \lambda}{\partial \theta} = \psi_0 (\frac{\phi_0}{\psi_0} + \lambda) \quad \text{------} \quad \frac{\partial \lambda}{\partial \theta} = \psi_0 \cdot \phi_0 \]

\[
\frac{\partial \lambda}{\partial \theta} = \psi_0 \frac{\phi_0 + \lambda}{\psi_0} = \psi_0 \cdot \phi_0
\]
and integrating

\[ \ln(\lambda + \phi_0/\psi_0) = \psi_0 \cdot \theta + \text{constant} \text{ or } \lambda + \phi_0/\psi_0 = C \cdot e^{\psi_0 \theta} \]

or

\[ \lambda = C \cdot e^{\psi_0 \theta} - \frac{\phi_0}{\psi_0} \]  \hspace{1cm} (30)

Determination of the constant \( C \) is obtained by the introduction of mean velocity \( V_{mo} \):

\[ V_{mo} = \frac{\Delta \Omega}{2 \pi r b} = \frac{Z}{2 \pi r b} \int_{\theta=0}^{\theta=2\pi/Z} V_{m \cdot b \cdot r} \, d\theta \]

\[ \frac{Z}{2 \pi} \int_{0}^{2\pi/Z} d\theta = 1 \quad \text{--------} \quad \int_{0}^{2\pi/Z} (Ce^{\psi_0 \theta} - \frac{\phi_0}{\psi_0}) \, d\theta = \frac{2\pi}{Z} \]

\[ \frac{C}{\psi_0} \frac{e^{2\pi/Z} \phi_0}{Z} - \frac{2\pi}{Z} = 2\pi \quad \text{--------} \quad \frac{C}{\psi_0} (e^{2\pi/Z} \psi_0 - 1) = \frac{2\pi}{Z} (1 + \frac{\phi_0}{\psi_0}) \]

\[ C = \frac{2\pi \psi_0}{Z} \frac{1 + \phi_0/\psi_0}{e^{2\pi/Z} - 1} \]  \hspace{1cm} (31)

with introduction of a new constant \( \psi_1 = \frac{2\pi \psi_0}{Z} \)

\[ C = \psi_1 \frac{1 + \phi_0/\psi_0}{e^{\psi_1} - 1} \]  \hspace{1cm} (32)

\[ \lambda = \psi_1 \frac{1 + \phi_0/\psi_0}{e^{\psi_1} - 1} e^{\psi_0 \theta} - \frac{\phi_0}{\psi_0} \]  \hspace{1cm} (33)

which can be written in the form

\[ \lambda = \frac{\psi_1}{e^{\psi_1} - 1} e^{\psi_0 \theta} - \frac{\phi_0}{\psi_0} (1 - \frac{\psi_1}{e^{\psi_1} - 1} e^{\psi_0 \theta}) \]

a change in constant again leads \( \psi_2 = \frac{\psi_1}{e^{\psi_1} - 1} \)

\[ \lambda = \psi_2 e^{\psi_0 \theta} - \frac{\phi_0}{\psi_0} (1 - \psi_2 e^{\psi_0 \theta}) \]  \hspace{1cm} (34)
Figure A-II-5. Velocity Distribution in Radial Plane.

Figure A-II-6. Case of Logarithmic Spiral Blades.
since \( \psi_0 = \sin \epsilon \cdot \sin^2 \beta \cdot \frac{2u}{V_{mo}} \) from (27)

\[
\psi_0 = \sin \epsilon \cdot \sin^2 \beta \cdot r \left( \frac{\partial (\cot \beta)}{\partial r} - \cot \beta \cdot \frac{\partial b}{\partial r} \right) \text{ from (27)}
\]

\[
\frac{\psi_0}{\psi_0} = 2 \frac{u}{V_{mo}} \frac{1}{r} \left( \frac{\partial (\cot \beta)}{\partial r} - \cot \beta \frac{\partial b}{\partial r} \right)
\]

\[
\lambda = \psi_2 e^{\psi_0 \theta} - 2 \frac{u}{V_{mo}} \frac{1 - \psi_2 e^{\psi_0 \theta}}{r} \left( \frac{\partial (\cot \beta)}{\partial r} - \cot \beta \frac{\partial b}{\partial r} \right)
\]

This equation can be written in a simple form:

\[
\lambda = A e^{\psi_0 \theta} + B
\]

in general, where \( A \) and \( B \) are constants for a given point and

\[
V_m = \cdot V_{mo} = AV_{mo} e^{\psi_0 \theta} + B \cdot V_{mo}
\]

which means an exponential variation of \( V_m \) between two consecutive vanes.

**Particular Case of Logarithmic Spiral Blades with \( b = \text{const.} \)**

Equation of log. spiral is \( r = r_1 e^{\text{const.} \cdot \theta} \)

\[
\frac{dr}{d\theta} = \text{const.} \cdot r_1 e^{\text{const.} \cdot \theta} \quad \text{---} \quad \tan \delta = \frac{r}{\partial r/\partial \theta} = \frac{1}{\text{const.}}
\]

\[
\cot \delta = \text{const} = \tan \left( \frac{\pi}{2} - \delta \right) = \tan \beta
\]

\[
r = r_1 e^{\theta \cdot \tan \beta} \quad \text{between} \quad \theta = 0, \theta = \frac{2\pi}{Z}
\]

\( \beta = \text{const.} \). Radius of Curvature being \( \rho = r \sqrt{\lambda + \tan^2 \beta} \).

Equation (27) is reduced to:

\[
\frac{\partial b}{\partial r} = 0 \quad \frac{\partial (\cot \beta)}{\partial r} = 0
\]

and with the same assumption \( \frac{\partial b}{\partial \theta} = 0 \)

\[
\frac{\partial \lambda}{\partial \theta} = \sin^2 \beta \cdot \frac{2u}{V_{ro}} \quad \text{---} \quad \lambda = \frac{V_r}{V_{ro}}
\]
\begin{equation}
\frac{d\lambda}{d\theta} = \text{and integrating: } \lambda = \theta + C \tag{37}
\end{equation}

For the constant (C) introducing mean velocity:

\begin{equation}
V_{ro} = \frac{\Delta Q}{2\pi r b} = \frac{Z}{2\pi b} \int_{0}^{2\pi} r b \cdot d\theta \quad \text{------- } \int_{0}^{2\pi} \lambda \cdot d\theta = \frac{2\pi}{Z}
\end{equation}

\begin{equation}
\int_{0}^{2\pi} \frac{Z}{(\phi_0 r b \cdot d\theta + c \cdot d\theta)} = \frac{2\pi}{Z} \quad \frac{\phi_0}{2} \theta^2 + C \theta \bigg|_{0}^{\frac{2\pi}{Z}} = \frac{2\pi}{Z} \quad C = 1 - \frac{\phi_0}{Z}
\end{equation}

and Equation (37) becomes

\begin{equation}
\lambda = 1 + \phi_0 (\theta - \frac{\pi}{Z}) \tag{38}
\end{equation}

\begin{equation}
\lambda = 1 + 2 \sin^2 \beta \frac{u}{V_{ro}} (\theta - \frac{\pi}{Z})
\end{equation}

\begin{equation}
V_r = V_{ro} + 2 \sin^2 \beta \cdot u (\theta - \frac{\pi}{Z}) \tag{39}
\end{equation}

which means linear change with \(\theta\).

Considering only the effect of mass forces the change of velocity between two consecutive vanes in a radial plane is given through the exponential relation\(^{(36)}\).
APPENDIX III

Nomenclature

C  velocity vector

\( t \)  time

\( \Omega \)  potential of external force field

\( p \)  pressure

\( \rho \)  density, specific mass

\( \varphi \)  potential function of the velocity vector

\( u \)  velocity component in x direction

\( v \)  velocity component in y direction

\( w \)  velocity component in z direction

\( E_t \)  total energy
APPENDIX III

CORRELATION BETWEEN ENERGY INPUT AND FORMATION
OF UNSTEADY FLOW PATTERN

A. Vector Form

Energy input into the flow at the outlet of an impeller in a
centrifugal pump is a continuous process governed by successive pressure
waves that introduce new accelerated fluid particles. Following Euler,
the acceleration is the substantial differential of velocity:

$$\frac{DC}{dt} = \frac{\partial C}{\partial t} + C \times \text{grad } C = \text{grad } (-p/\rho + \Omega)$$

where \( \Omega \) is the potential of external force field so \( \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z} \) represent
mass force per unit of mass.

$$C \times \text{grad } C = C \times (C \cdot \nabla) - C \cdot \text{rot } C = \text{grad } \frac{C^2}{2} - C \cdot \text{rot } C$$

$$\frac{DC}{dt} = \frac{\partial C}{\partial t} + \text{grad } \frac{C^2}{2} - C \times \text{rot } C = \text{grad } (-p/\rho + \Omega)$$

$$\text{grad } \left( \frac{C^2}{2} + \frac{p}{\rho} - \Omega \right) = -\frac{\partial C}{\partial t} + C \times \text{rot } C$$

The left side of this equation contains the change of total energy with the
position, so a change of total energy in ONE CHOSEN DIRECTION can be obtained
through:

- \( \frac{\partial C}{\partial t} = 0 \) and \( C \times \text{rot } C \neq 0 \) a turbulent flow
- \( \frac{\partial C}{\partial t} \neq 0 \) and \( C \times \text{rot } C = 0 \) unsteady potential flow
- \( \frac{\partial C}{\partial t} \neq 0 \) and \( C \times \text{rot } C \neq 0 \) unsteady turbulent flow

But if the change of total energy is taking place in the DIRECTION OF FLOW,
neither \( C \) nor \( (\text{rot } C) \) being zero, the vector product \( C \times \text{rot } C = 0 \), and the
flow has only an unsteady pattern.
If potential flow can be assumed so that the vector $\vec{c}$ field itself is a gradient of a scalar field $\varphi$; $c = \nabla \varphi$ his rotation is:

$$\text{rot } \vec{C} = \text{rot } \nabla \varphi = \nabla \times \nabla \varphi = 0.$$ 

In the same time, space and time integration being independent:

$$\frac{\partial \vec{C}}{\partial t} = \frac{\partial}{\partial t} \nabla \varphi = \nabla \frac{\partial \varphi}{\partial t}.$$ 

The equation is reduced to

$$\nabla \left( \frac{C^2}{2} + \frac{p}{\rho} - \Omega + \frac{\partial \varphi}{\partial t} \right) = 0$$

and integrating it along a streamline

$$\frac{C^2}{2} + \frac{p}{\rho} - \Omega + \frac{\partial \varphi}{\partial t} = f(t).$$

$f(t)$ is an arbitrary function of time because the previous equation states only that the space differential of the expression in the brackets, is zero; however, the integration constant may change with time so that the sum of total energy in the flow can still be changed arbitrarily by a running impeller.

B. Differential Form

The Euler equation can be written for the substantial derivative:

$$\frac{Du}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\partial \Omega}{\partial x}$$

With the previous definition of $\Omega$, this equation leads to:

$$\frac{\partial (-p/\rho + \Omega)}{\partial x} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial z} + v \left( \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right) + w \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right).$$
and noting: \( u \frac{\partial u}{\partial x} = \frac{\partial (u^2/2)}{\partial x} \)

it can be expressed

\[
\frac{\partial}{\partial x} \left( \frac{u^2 + v^2 + w^2}{2} + \frac{p}{\rho} - \Omega \right) = - \frac{\partial u}{\partial t} + v \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) + 2 \left( \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} \right).
\]

The assumption of potential flow leads:

\[
\begin{align*}
    u &= \frac{\partial \phi}{\partial x} & v &= \frac{\partial \phi}{\partial y} & w &= \frac{\partial \phi}{\partial z} \\
    \frac{\partial v}{\partial x} &= \frac{\partial^2 \phi}{\partial x \partial y} & \frac{\partial u}{\partial y} &= \frac{\partial^2 \phi}{\partial x \partial y} & \text{to } u^2 + v^2 + w^2 = c^2
\end{align*}
\]

and

\[
\frac{\partial}{\partial x} \left( \frac{c^2}{2} + \frac{p}{\rho} - \Omega \right) = - \frac{\partial u}{\partial t}
\]

with cyclical permutation of coordinates:

\[
\begin{align*}
    \frac{\partial}{\partial y} \left( \frac{c^2}{2} + \frac{p}{\rho} - \Omega \right) &= - \frac{\partial v}{\partial t} \\
    \frac{\partial}{\partial z} \left( \frac{c^2}{2} + \frac{p}{\rho} - \Omega \right) &= - \frac{\partial w}{\partial t}
\end{align*}
\]

The left side of the last three equations contains the change of the total energy with respect to position; setting

\[
E_t = \frac{c^2}{2} + \frac{p}{\rho} - \Omega
\]

\[
\frac{\partial E_t}{\partial x} + \frac{\partial E_t}{\partial y} + \frac{\partial E_t}{\partial z} = - \left( \frac{\partial u}{\partial t} + \frac{\partial v}{\partial t} + \frac{\partial w}{\partial t} \right)
\]

it means that the change of total energy is bound to the existence of an unsteady flow.

The potential flow assumption is not indispensable if the change of energy in the DIRECTION OF FLOW is considered; in this case the (x) and (c) directions may be chosen the same and so the rotation terms of the equation are reduced to zero. From these equations, it can be easily shown
again that the change of energy for any direction may result from a turbulent flow, from an unsteady potential flow or from both of the combined.

It might be concluded that the change of total energy in a flow is always accompanied by an unsteady flow pattern, if this change is occurring in the direction of the flow.

This appendix demonstrates also that in order to increase the energy in the flow, it is sufficient and necessary to develop an unsteady flow pattern by any conceivable mechanical means. The development of this unsteady flow can be obtained by introduction of a "centrifugal" or "centripetal" forces arbitrarily. For example, a typhoon or a whirlwind is a natural "pump" which develops a potential energy in its "eye," but the necessary centripetal circulation flow pattern is provided by the surrounding changing winds. In all languages the commonly established term of "centrifugal pump" is a misleading one if it is meant to represent the characteristic effect of the pump. Besides "centripetal pump" also has been manufactured using the same flow principles. The German term "Kreiselpumpe" translated as "Impeller-pump" may give a more accurate idea of the characteristic effect for this machinery.
APPENDIX IV

NOMENCLATURE

z complex variable
w conformal mapping function of z
a constant
α angle between two asymptotes
φ velocity potential function
ψ stream function
r distance from the point of intersection between two asymptotes
n defined in text
ψ angle between the origin axis and the vector of the considered point
v velocity vector
v₀ mean velocity
ℓ distance between two vanes
APPENDIX IV

CONFORMAL MAPPING FUNCTION FOR SHOCK FLOW PATTERN

A two dimensional flow parallel to the x axis given through the function \( az = a(x + iy) \) is the simplest function which may be used to derive the complex function of the potential shock flow pattern between impeller outlet and the shock boundary of the two vortices of the volute through the elementary transformation function

\[
W = \frac{a}{n} Z^n \quad \text{and} \quad n = \frac{\Pi}{\alpha}
\]

where \( \alpha \) is the angle between the two asymptotes.

A. In the case of the axial direction (Figure 1) the outlet flow is directed nearly normal to the "wall" built by the two vortices flow of the volute, so \( \alpha = \frac{\Pi}{2} \) or \( n = 2 \) and the complete function is:

\[
W = \frac{a}{2} Z^2 = \frac{a}{2} (x + iy)^2
\]

\[
= \frac{a}{2} (x^2 - y^2) + iaxy
\]

\[
= \Phi + i\ Psi
\]

where the velocity potential is represented by \( \Phi \) and the stream function by \( \Psi \). The streamlines \( \Psi = \text{const.} \) are equilateral hyperbolae. The velocity components are to be found through differentiation of the velocity potential,
Figure A-IV-1. Potential Flow in Axial Direction.
Figure A-IV-2. Potential Flow in Radial Direction.
giving:

\[ v = \sqrt{\left( \frac{\partial \psi}{\partial x} \right)^2 + \left( \frac{\partial \psi}{\partial y} \right)^2} \]

\[ = a \sqrt{x^2 + y^2} = ar \]

The above derived relations serve to calculate the streamlines \( \psi = \text{const.} \), and to trace them. Also the velocities may be obtained similarly for every point of the flow.

B. In the case of a radial section the outlet flow and the "wall" built by the two vortices flow are forming an angle of \( \alpha = 180 - 15 = 165^\circ \) (Figure 2).

\[ n = \frac{\pi}{\alpha} = \frac{180}{165} = 1.09 \]

In this case the use of polar coordinates gives \( z = re^{i\varphi} \)

\[ w = \frac{a}{n} r^n e^{in\varphi} = \frac{a}{n} r^n (\cos n\varphi + i \sin n\varphi) \]

\[ \varphi = \frac{a}{n} r^n \cos n\varphi = \text{const.} \]

\[ \psi = \frac{a}{n} r^n \sin n\varphi = \text{const.} \]

In the particular case of 800 GPM 1750 RPM with the average \( n = 1.09 \)

\[ w = \frac{a}{1.09} r^{1.09} (\cos (7.09\varphi) + i \sin (7.09\varphi)) \]

and the velocity vector

\[ \nabla = \frac{\partial \psi}{\partial z} = az^{-1} = ar^{-1} e^{i(n-1)\varphi} \]
has the absolute value

$$|\vec{v}| = ar^{n-1} = ar^{0.09}$$

For every assumed shock circle there is one value of (a) and the mean
velocity between two consecutive vanes is defined through the equation

$$v_0 = \frac{\int ar^{0.09} \, dx}{\ell}$$

where (\ell) is the distance between two vanes. This integration has been
effectuated graphically so that for every supposed shock circle the mean
velocity \( v_0 \) should have the experimental value of \( v_0 = 63.2 \) ft/sec.
General View of the Pump
Directional Pitot Tube
Static Pressure Average with Tektronix Balance Unit

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 1.3.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope SENSITIVITY  5, 2, l, 0.5 Millivolt/cm
   Tektronix TIME  0.2 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION  1 cm = 5 psi at sensitivity 500μ
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure α₀ = 195° Static pressure α₀ = 285°
7. Without air bubble, with air bubble, unknown
Difference Between Total and Static Pressure with the Tektronix Balance Unit

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 0.2 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 5 psi at sensitivity 500μ volt/cm
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure α₀ = 195° Static pressure α₀ = 285°
7. Without air bubble, with air bubble, unknown
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 92 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 2 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube, a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube  a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.1  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope  SENSITIVITY  5, 2, 1, 0.5 Millivolt/cm
   Tektronix  TIME  5, Milliseconds/cm
   Dual beam  SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION  1 cm = 10 psi at sensitivity 50
5. Camera exposure time  1/10  1/25  1/50  1/100
6. Total pressure  \( \alpha_0 = 195^\circ \)  Static pressure  \( \alpha_0 = 285^\circ \)
7. Without air bubble, with air bubble, unknown
Total Pressure without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
   0.01 0.1 1 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 2, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope SENSITIVITY  5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME  5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 205^\circ$
7. Without air bubble, with air bubble, unknown

Total Pressure Without Air Bubble
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708    0.01    0.1    1
   Piezo Calibrator Mod. 2 Serie 99    0.5    5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$, Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM P2 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5
4. Oscilloscope
   Tektronix SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube: \( a, b, c \)
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope
   SENSITIVITY: \( \frac{1}{5}, 2, \frac{1}{2}, 0.5 \) Millivolt/cm
   Tektronix
   TIME: \( \frac{5}{2} \) Milliseconde/cm
   Dual beam
   SWEEP MAGNIFIED \( \frac{1}{1}, 2, 5, 10 \)
   TRIGGERING, from outside \('ON, OFF\')
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time: \( 1/10, 1/25, 1/50, 1/100 \)
6. Total pressure \( \alpha_0 = 195^\circ \)
   Static pressure \( \alpha_0 = 285^\circ \)
7. Without air bubble, with air bubble, unknown
Static Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5
4. Oscilloscope
   SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.1 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 2 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure Without Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 143177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope SENSITIVITY  5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME  5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope
   SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope  SENSITIVITY  5, 2, 1, 0.5 Millivolt/cm
   Tektronix  TIME  2  Millisecond/cm
   Dual beam  SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50, 1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Ann Arbor, Michigan
December 12, 1959
Photograph No. 20

Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope
   SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope
   SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 2 Milliseconds/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_a = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit

<table>
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<tr>
<th>Piezo Calibrator Mod. No. 708</th>
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<th>0.1</th>
<th>1</th>
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4. Oscilloscope

<table>
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<th>Tektronix</th>
<th>TIME</th>
</tr>
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<tbody>
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<table>
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<tr>
<th>Dual beam</th>
<th>SWEEP MAGNIFIED X 1, 2, 5, 10</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>TRIGGERING, from outside ON, OFF</td>
</tr>
<tr>
<td></td>
<td>CALIBRATION 1 cm = 10 psi at sensitivity 50</td>
</tr>
</tbody>
</table>

5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier Bridge, Tektronix Balance unit

<table>
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<th>Piezo Calibrator</th>
<th>Mod. No. 708</th>
<th>0.01</th>
<th>0.1</th>
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</tr>
</thead>
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<tr>
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<td>Mod. 2 Serie 99</td>
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</table>

4. Oscilloscope

<table>
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<tr>
<th>Sensitivity</th>
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<th>Millivolt/cm</th>
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<tr>
<td>Tektronix TIME</td>
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<td>Milliseconde/cm</td>
</tr>
<tr>
<td>Dual beam SWEEP MAGNIFIED X</td>
<td>1, 2, 5, 10</td>
<td></td>
</tr>
<tr>
<td>TRIGGERING, from outside</td>
<td>ON, OFF</td>
<td></td>
</tr>
<tr>
<td>CALIBRATION</td>
<td>1 cm = 10 psi at sensitivity 50</td>
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</tr>
</tbody>
</table>

5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure \( \alpha_0 = 195^\circ \) Static pressure \( \alpha_0 = 285^\circ \)
7. Without air bubble, with air bubble, unknown
1. Location of pressure pick up on the Pitot tube  a, b, c
2. Pick up: Strain gage 100 psi, SLM c, SLM PZ 14.3177 a
3. Amplifier: Bridge, Tektronix Balance unit
4. Oscilloscope  SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm (for both)
   Tektronix  TIME 5 Milliseconds/cm
   Dual beam  SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = a 10, c 13.62, psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Static Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
   Scales: 0.01 0.1 1
   0.5 5
4. Oscilloscope SENSITIVITY: 5, 2, 1, 0.5 Millivolt/cm
   TIME: 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PII 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
   0.01 0.1 1
   0.5 5
4. Oscilloscope: SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 2 Milliseconde/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube  a, b, c
2. Pick Up: Strain gage 100 psi SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope  SENSITIVITY  5,  2,  1,  0.5 Millivolt/cm
   Tektronix  TIME  5  Millisecond/cm
   Dual beam  SWEEP MAGNIFIED X 1,  2,  5,  10
   TRIGGERING, from outside ON, OFF
   CALIBRATION  1 cm = 10 psi at sensitivity 50
5. Camera exposure time  1/10  1/25  1/50  1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

first wave
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b c
2. Pick up: Strain gage 100 psi SIM, SIM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope
   Tektronix SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Dual Beam TIME 5 Milliseconde/cm
   SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 125^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

second wave
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5 2 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1 2, 5 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION L cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

third wave
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SIM, SIM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

fourth wave
Total Pressure with Air Bubble

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick up: Strain gage 100 psi SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
   0.01 0.1 1
   0.5 5
4. Oscilloscope
   Tektronix
   TIME: 5
   Dual beam
   SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside
   ON, OFF
   CALIBRATION: 1 cm = 10 psi at sensitivity 50
5. Camera exposure time: 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

fourth wave
Static Pressure, with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown

first wave
Static Pressure, with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain Gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON OFF
5. Camera exposure time 1/1- 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
   second wave
Static Pressure, with Air Bubble

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5
4. Oscilloscope
   Tektronix SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Dual Beam TIME 5 Millisecond/cm
   SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure \( \alpha_0 = 195^\circ \) Static pressure \( \alpha_0 = 285^\circ \)
7. Without air bubble, with air bubble, unknown

third wave
Static Pressure, with Air Bubble

1. Location of pressure pick up on the Pitot tube  a,  b,  c
2. Pick up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope  SENSITIVITY  5,  2,  1,  0.5 Millivolt/cm
   Tektronix  TIME  5
   Dual beam  SWEEP MAGNIFIED X 1,  2,  5,  10
   TRIGGERING, from outside  ON, OFF
   CALIBRATION  1 cm = 10 psi at sensitivity 50
5. Camera exposure time  1/10  1/25  1/50  1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

fourth wave
Pressure Fluctuation in Pump

1. Location of pressure pick up on the Pitot tube  a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscopes
   SENSlIVITY  5, 2, 1, 0.5  Millivolt/cm
   Tetronix TIME  5  Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION  1 cm = 10 psi at sensitivity 50
5. Camera exposure time  1/10  1/25  1/50  1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Pressure Fluctuation in Pump

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SIM, SIM PZ 14.3177
3. Amplifier Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown
Fluctuation in the Magnitude of the Main Dip

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick up: Strain gage 100 psi, SIM, SIM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance Unit
   Piezo Calibrator Mod. No. 708
   Piezo Calibrator Mod. 2 Serie 99
   0.01 0.1 1
   0.5 5
4. Oscilloscope: SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Milliseconds/cm
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 10 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Mechanical Vibrations

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SLM, SLM PZ 14.3177
3. Amplifier Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5
   Dual beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown
Mechanical Vibrations

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick up: Strain gage 100 psi, SIM, SIM PZ 143177
3. Amplifier Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708 0.01 0.1 1
   Piezo Calibrator Mod. 2 Serie 99 0.5 5
4. Oscilloscope SENSITIVITY 5, 2, 1, 0.5 Millivolt/cm
   Tektronix TIME 5 Millisecond/cm
   Dual Beam SWEEP MAGNIFIED X 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION lcm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10 1/25 1/50 1/100
6. Total pressure $\alpha_0 = 195^\circ$ Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

Mechanical Vibration
### Mechanical Vibrations

1. Location of pressure pick up on the Pitot tube: a, b, c
2. Pick Up: Strain gage 100 psi, SLM, SLM FZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   - Piezo Calibrator Mod. No. 708: 0.01 0.1 1
   - Piezo Calibrator Mod. 2 Serie 99: 0.5 5
4. Oscilloscope:
   - Sensitivity: 5, 2, 1, 0.5 Millivolt/cm
   - Tektronix TIME: 5 Milliseconds/cm
   - Dual beam SWEEP MAGNIFIED: x 1, 2, 5, 10
   - TRIGGERING, from outside: ON, OFF
   - CALIBRATION: 1 cm = 13.63 psi at sensitivity 50
5. Camera exposure time: 1/10, 1/25, 1/50, 1/100
6. Total pressure $\alpha_o = 195^\circ$ Static pressure $\alpha_o = 285^\circ$
7. Without air bubble, with air bubble, unknown

**Mechanical Vibration**
Mechanical Vibrations

1. Location of pressure pick up on the Pitot tube a, b, c
2. Pick-up: Strain gage 100 psi, SIM, SIM PZ 14.3177
3. Amplifier: Bridge, Tektronix Balance unit
   Piezo Calibrator Mod. No. 708  0.01  0.1  1
   Piezo Calibrator Mod. 2 Serie 99  0.5  5
4. Oscilloscope
   Tektronix
   Dual beam
   TIME
   SWEEP MAGNIFIED  × 1, 2, 5, 10
   TRIGGERING, from outside ON, OFF
   CALIBRATION  lcm = 13.63 psi at sensitivity 50
5. Camera exposure time 1/10  1/25  1/100
6. Total pressure $\alpha_0 = 195^\circ$  Static pressure $\alpha_0 = 285^\circ$
7. Without air bubble, with air bubble, unknown

mechanical vibration


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