It would be a natural remark to make, that the greater number of the engines and machines which have been so long at work have been greatly altered from the original construction by the exigencies and accidents of practical wear and its attendant repairs. Like John's old knife, which still remained "John's old knife," though it had received about half-a-dozen new blades in succession, and about as many new handles, it might be doubted whether these engines really represent their original construction. In one sense this may be true, and it is probable that only a portion of the original raw metal is still in combination. But then it must be remembered that in piecemeal repairs any part repair is in a great measure tied to the original form. The new blade of the old knife has to conform to the existing handle, and the new handle has to be made to the shape of the blade. The same must be, more or less, the case with repairs to an engine or machine. At the same time, such a consideration points to the absolute necessity for a correct history—or rather engine biography—being appended to each engine, pointing out where and what alterations and repairs have been made.

The Patent Museum at South Kensington, only yet needs the "Novelty" to complete the trio of the engines which figured at the great Rainhill contest. It is not now known where the "Novelty" at present is, or indeed, whether it is even still in existence. We believe that Mr. F. P. Smith is anxiously looking about for it, and we hope that he may be successful in his search. What lessons of the way in which "Fortune turns her wheel," may be read by a look at these old engines,—the progenitors of the apparatus,—the use of which is stamping its impress on present life in so many direct and indirect ways. The two Stephensons are now dead, but their engine, which killed Mr. Huskisson, raised their fortunes above those of perhaps any other engineers of their time. Timothy Hackworth is now dead, but his works at Newcastle, like those of the Stephensons, are still flourishing. The Messrs. Braithwaite are now living, but their factory has been given up, and their partner of 1829, the Swedish engineer, Ericsson, is now building "Novelties," in the way of gunboats for the Federals. In centuries hence, time will have thrown its halo of distance round these engineers, and their doings will be scanned with a kind of romantic interest. Let us hope that, in the course of the next century, the Government will have provided a decent building for the objects which represent such important interests, but which are now sheltered in the crowded and dark shed at South Kensington.

For the Journal of the Franklin Institute.

General Problem of Trussed Girders. By De Volson Wood, Prof. C.E., University of Michigan.

Continued from page 107.

20°. It is found by subjecting equation (65) to experiment, that R is not constant for beams of different forms, when made of the same material. It is also found that the value of R which is found from rectangular beams does not equal the ultimate resistance of the material to tension or compression.

For instance, if a rectangular wooden beam be suspended at its ends and broken by a weight P placed at the middle, we have, from the formula which was deduced for this case in the preceding article,

$$R = \frac{3 P l}{2 b d^2}$$

in which all the quantities in the second member may be measured, and hence R may be found. Now, bearing in mind, that, in this case R is the ultimate resistance of a unit of fibres most remote from the neutral axis, and we would expect to find that it equals the force necessary either to pull asunder, or to crush a bar of the same material whose section is unity, when the force is applied in the direction of the length.

But an examination of tables, in which are entered the ultimate resistances to tension, compression, and values of R, shows that the disagreement is too great and too uniform to be attributed to errors of experiment. There is a very good table of this kind in the appendix of "Mosley's Mechanics and Engineering." A careful examination shows that the value of R is generally, indeed almost always, between the others. For instance, if the resistance to tension—which we will hereafter call S,—is greater than the resistance to compression—which we will call C;—then we find that R is less than S and greater than C. This is the case with wrought iron and most kinds of wood. If C be greater than S, R will be less than C and greater than S, as in the case of cast iron. We find the same result by comparing tables in Weisbach's Mechanics and Engineering.

Hence we infer *that the theory is not perfect; in other words, it does not represent the true law of resistance of beams.*

This fact induced Professor Barlow, a few years since, to investigate the law of resistance more carefully and more thoroughly than ever before; and as a result he announced a new law, called "*The Resistance to Flexure.*" I consider the term unfortunate; for all the resistances in a beam which resist bending may properly be called "*resistances to flexure,*" but he intended to include only a certain class of resistances which he thought was developed by bending, and the value of which he tried to determine at the instant of rupture. I think it may more properly be called "*a resistance to longitudinal shearing.*"

He reported the results of his investigations to the Royal Society (England) in 1855, and afterwards published the data, experimental and analytical results upon which the theory is founded, in the *Civil Engineer and Architect's Journal,* vol. xix, page 9, and vol. xxi, page 111. They are also published in the *Journal of the Franklin Institute,* vol. xxxii, pp. 4 and 73. These articles from a scientific point of view, are looked upon as among the most valuable that have ever been written upon this subject.

His complete theory involves *two laws* of resistance; the *first* of which is essentially the same as that given in the preceding number, 19° ; and the second of which he calls "*the resistance to flexure.*"

The first *law* includes, and is founded upon, the following principles:

1. The fibres on the convex side are extended, and those on the concave side are compressed.

2. There is a neutral surface, along which the fibres are neither extended nor compressed.

3. The resistance to extension and compression varies directly as the distance of the fibre from the neutral axis.

4. The ultimate resistance which acts along the most remote fibre is S on the side of tension and C on the compressed side: S being the tenacity of the material for a unit of section, and C the resistance to crushing per unit of section.

The second *law* is founded upon the following principles:

1. The *longitudinal shearing* is a resistance acting in addition to the direct extension or compression; and is really the cohesive resistance which is developed between two adjacent planes of fibres which are unequally elongated or compressed.

2. This resistance is evenly distributed over the transverse section; and, consequently, (within the limits of its operation) its centre of action on the compressed part will be at the centre of gravity of the compressed section; and, similarly, in respect to the extended part. This resistance per unit of section Barlow calls φ.

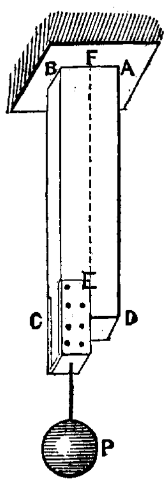
3. It is proportional to and varies with the *inequality* of strain between the fibres nearest the neutral axis and those most remote, and hence

4. The "resistances to longitudinal shearing" in open built beams will be to that in a solid beam of the same material at the instant of rupture, as the depth of the solid part, is to the distance of the outside of the solid part from the neutral axis.

5. Sections which were normal to the axis of the beam before flexure, will remain normal during flexure.

6. Rupture of solid beams will take place when the strain on a unit of section of the fibres most remote from the neutral axis, is $S + \varphi$ or $C + \varphi$, according as one or the other is first reached.

FIG 30



To show more clearly the nature of the longitudinal shearing, I will refer to an example given by Barlow. Suppose that P, in Fig. 30, is just sufficient to pull asunder the bar F B C E, in which case it is supposed that all the fibres are equally strained, and each unit resists a force equal to S. Now if another bar, A B C D, were substituted for the former, and P applied to the same section, C E, it is evident that P would not break the bar; for the fibres just at the left of F E, will restrain those just at the right, so that they will not be elongated as much as they would if the part A F E D were removed. The restraining of the fibres just at the right of F E will have its effect upon the next plane of fibres, and so on to B C. Now this restraining influence, this cohesion between the fibres is the "resistance to longitudinal shearing;" and it is evident that a portion of the force P, must be absorbed in overcoming it, so that it cannot produce a strain of S on the fibre B C.

A phenomenon similar to this takes place in a bent beam. The fibres unequally distant from the neutral axis are unequally strained, and hence develop the "longitudinal shearing;" and it seems evident that this strain absorbs some of the bending force. Hence, it takes a greater force to produce a strain S on the outer fibres, than it would if this resistance did not exist.

Although this theory seems rational, yet it remains to be seen whether it is the *true* one, or even whether it is better for practical purposes than the theory commonly used.

In analyzing it, we observe that all the resisting forces are parallel to the axis of X; and taking the applied forces parallel to the axis of Y, and we will immediately obtain equations (61) and (62); and hence the remarks which follow them apply to this theory.

I will now show how to develop the third of equations (62). Suppose that the beam ruptures on the side of tension. Let the origin be on the neutral axis; d_1 the distance of the most remote fibre from the neutral axis; and the other notation as in number 19°. Then proceeding as we did to obtain equation (65), and we find for the resistance according to the first law above named, $\frac{SI}{d_1}$; and according to the second law $\varphi\iint y\,dy\,dx$. Hence, the moment of resistance is,

$$\Sigma Fy = \frac{SI}{d_1} + \varphi\iint y\,dy\,dx \quad . \quad . \quad (84)$$

If the beam be rectangular, and the neutral axis at the centre of the section, b the breadth, d the depth, then $I = \frac{1}{12} b d^3$; $d_1 = \frac{1}{2} d$

$$\int_0^b \int_{-\frac{1}{2}d}^{+\frac{1}{2}d} y\,dy\,dx = \tfrac{1}{4} b d^2;$$ and (84) becomes

$$\Sigma Fy = \frac{bd^2}{12}(2S + 3\varphi) \quad . \quad . \quad . \quad (85)$$

If the beam be supported at its ends, and P be applied at the middle; we have $\Sigma Fy = \frac{1}{4} Pl$, and (85) becomes,

$$2S + 3\varphi = \frac{3Pl}{bd^2} \quad . \quad . \quad . \quad . \quad (86)$$

If $\varphi = 0$, equation (86) reduces to the *form* which is commonly used. It is evident that two experiments are necessary in order to determine both S and φ; but they can be more satisfactorily determined by taking a large number of experiments, and reducing the equations by the method of least squares.

If the theory be correct, the value of S thus found should be the same as that found by pulling a bar asunder by a force applied in the direction of its length. Barlow found for cast iron that $\varphi = 0{\cdot}85\,S$ for a mean value; and for wrought iron $\phi = \frac{1}{2}$ s nearly. (*See Civ. Eng. and Arch. Jour.*, vol. xxi, pages 114 and 116.)

This "longitudinal shearing resistance" not only has a value at the instant of rupture, but must act during all the stages of flexure up to the instant of rupture. The actual deflection therefore should be less than that found by the theory in common use; but as the observed deflections are generally small, no marked discrepancy between computed and observed results has, so far as I know, been detected.

21° *To find the position of the neutral axis.*

This may be found experimentally or analytically. The former has great advantage over the latter for any particular case, but the latter is essential for general purposes. The former must be resorted to, to confirm or vitiate a theory.

Position found by Experiment.—I will note a few examples where it has been determined experimentally.

Barlow made a very delicate experiment upon a solid rectangular cast iron beam, supported at its ends and loaded at the middle, in which he found the neutral axis to coincide exactly with the axis of the beam. (*See Civ. Eng. and Arch. Jour.*, volume xix, page 10.) He found the same result with a wrought iron beam under similar circumstances. (*See same Journal*, vol. xxi, page 115.) For malleable iron he found it to be $\frac{1}{4}$ or $\frac{1}{5}$ the depth of the beam from the compressed side. (*See Barlow's Strength of Materials*, page 330.) It should be observed that this experiment was made several years before the former ones, and that the means employed were not as delicate as in former examples. It is possible that a more careful experiment would show it to be nearer the centre of the sections than that given above.

A valuable set of experiments was made at the "Conservatorie des Arts et Méliers" in 1856, from which I infer, that in a wrought iron beam whose cross section is a double T, the neutral axis passes through or very near the centre of gravity of the sections. (*See Morin, Résistance des Matéraux*, page 137.) In the same work on pages 129, 130 and 131 are given the results of experiments upon rectangular wooden beams of various qualities, which show that it passes through or very near the centre of the transverse sections. So far as these examples are concerned the results are conclusive, and very strongly indicate that it generally coincides with or is very near the centre of gravity of the sections, when the deflecting force acts normal to the axis of the beam.

Position found by Calculation.—It is well known that within the elastic limits, the resistance of any fibre to extension is directly proportional to its elongation, and the latter is proportional to the modulus of elasticity. This is also true, to some extent, considerably beyond the elastic limit; but in the state bordering upon rupture it is not true, and it is difficult to measure the strains in any way so as to make them available for analytical purposes.

A. *Suppose that the deflecting forces are normal to the axis of the beam;* then we have only to develop the first of equations (61).

a. Let the strains upon the fibres be directly as their distances from the neutral axis.

I will first investigate those cases in which the elastic limits are not passed.

1. Let the modulus of elasticity for tension equal that for compression.

Let R_t = the strain upon a unit of fibres most remote from the neutral axis on the side of tension.

R_c = similar strain on the compressed side.

d_t = the distance of the most remote fibre from the neutral axis on the side subjected to tension.

d_c = similar distance on the compressed side, and other notation the same as before used.

The elongations of the fibres on one side of the neutral axis, equal the compressions of those on the other side which are equally strained; and because the strains are directly proportional to the distance of the fibres from the neutral axis, we have

$\frac{R_t}{d_t} = \frac{R_c}{d_c} = s =$ the strain at a unit's distance from the neutral axis; and at any distance y, the strain is $s\,y$;

$\therefore s\,y\,dy\,dz =$ the strain on any elementary fibre.

Hence after integrating once, we have $s\int_0^{d_t} zy\,dy$ which integrated so as to include the total extended part, gives the total tension. Similarly $\int_0^{d_c} zy\,dy$ gives the total compression. Observing that the angle between these forces is 180°, and we have by substituting in the first of (61)

$$s\int_0^{d_t} zy\,dy - s\int_0^{d_c} zy\,dy = 0$$

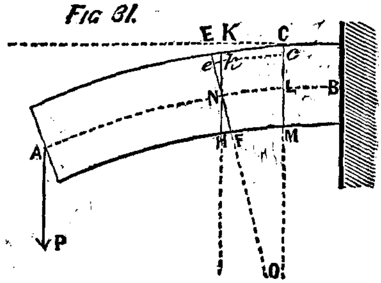
$$\text{or} \int_0^{d_t} zy\,dy + \int_{d_c}^{0} zy\,dy = \int_{d_c}^{d_t} zy\,dy = 0 \quad . \quad (87)$$

Now from the mechanics we know that the ordinate of the centre of gravity of any section is

$$\bar{Y} = \frac{\int zy\,dy}{\int ydz},$$

and if the origin be at the centre of gravity, $\bar{Y} = 0, \therefore \int zy\,dy = 0$, which is the same as (87). Equation (87) was obtained by taking the origin on the neutral axis; hence *the neutral axis coincides with the centre of gravity of the sections.*

Fig 31.



2. Let the moduli of elasticity be unequal.

Let E_t = the modulus of elasticity for tension,

E_c = the modulus of elasticity for compression,

$y = N\,k$ = any ordinate, fig. 31,

$\lambda = k\,e$ = the elongation of the fibre at k,

$dx = L\,N$ = the distance between two consecutive sections.

$dy\,dz$ = the area of a fibre,

p = the force which would produce an elongation or compression equal to λ of any fibre.

Let a beam be bent under the action of any number of parallel forces which are normal to its axis, so as to produce the neutral axis A N L B, Fig. 31.

Let C M and K H be two consecutive sections and normal to the neutral axis before flexure, then since they remain normal during flexure they will take the position of C M and E F, and will meet if produced in some point, as O.

Now, instead of the deflecting forces, we may conceive the beam to be severed along the plane K H and a force applied to each fibre acting in the direction of its length, those above L N elongating, and those below compressing the fibres, and each acting with such intensity as to produce the same elongations and compressions as the deflecting forces. Let p be one of the forces, producing an elongation or compression equal λ. We know that within the elastic limits, the force necessary to produce this elongation varies directly as the modulus of elasticity, the section and the elongation, and inversely as the length of the bar; hence,

$$p = \frac{E_t\,\lambda\,dy\,dz}{dx} \text{ or } = \frac{E_c\,\lambda\,dy\,dz}{dx} \quad . \quad . \quad (88)$$

according as it is tensive or compressive.

But because the sections remain normal to the neutral axis, we have from the similarity of triangles.

$$\frac{\lambda}{y} = c = \text{constant}, \quad . \quad . \quad . \quad (89)$$

equal the elongation or compression of a fibre at a unit's distance from the neutral axis.

$$\therefore\ \lambda = c\,y \text{ which in (88) gives}$$

$$p = \frac{c\,E_t\,y\,dy\,dz}{dx} \text{ or } \frac{c\,E_c\,y\,dy\,dz}{dx} \quad . \quad . \quad (90)$$

Hence the total tensive force is $\frac{c\,E_t}{dx}\iint_0^{d_t} y\,dy\,dz$

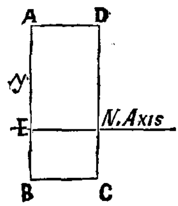
and the total compressive force is $\frac{c\,E_c}{dx}\iint_0^{d_c} y\,dy\,dz$;

hence the first of (61) becomes

$$\text{E}_t\iint_0^{d_t} y\,dy\,dz - \text{E}_c\iint_0^{d_c} y\,dy\,dz = 0 \quad . \quad (91).$$

Equation (91) can be solved when the form of the section is known.

FIG 32



EXAMPLE.—Suppose the sections are rectangular, Fig. 32.

Let A D $= b$, A B $= d$,

$y =$ A E $=$ the superior limit,

$d - y =$ E B $=$ the inferior limit,

$\frac{\text{E}_t}{\text{E}_c} = r$, for convenience.

Take the origin at E,

Then (91) becomes $r\int_0^b\int_0^y y\,dy\,dz = \int_0^b\int_0^{d-y} y\,dy\,dz$

$$\text{or } \tfrac{1}{2}\,r\,b\,y^2 = \tfrac{1}{2}\,b\,(d-y)^2$$

$$\therefore y = \frac{d}{1+\sqrt{r}} \quad . \quad . \quad . \quad (92)$$

If $r = 1$; $y = \frac{1}{2}d$,

$r = 0$; $y = d$,

$r = \infty$; $y = 0$.

If y is known, the ratio of the moduli are easily found; for by (92) we have

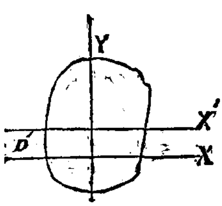
$$r = \left(\frac{d-y}{y}\right)^2 = \frac{\text{E}_t}{\text{E}_c}$$

Equation (91) is not so easily solved for the more complex forms.

3. To find the position of the neutral axis so as to give a minimum strength.

To solve this we have to make the first member of (65) a minimum.

FIG 33.



We assume that R is the same for all positions of the neutral axis. We have called I the moment of inertia about the line of intersection of the neutral plane and the section.

Let I$'$ $=$ the moment of inertia about an axis parallel to the former and which passes through the centre of gravity,

D $=$ the distance between the axis,

A $=$ the area of the section,

$a =$ the distance of the most remote fibre from the axis which passes through the centre,

$d_1 = a +$ D $=$ the distance of the most remote fibre from the neutral axis.

Then from the well known formula of reduction, we have

$$\text{I} = \text{I}' + \text{D}^2\,\text{A}.$$

Hence, the first member of (65) becomes

$$\text{R}\,\frac{\text{I}}{d_1} = \text{R}\frac{\text{I}^1 + \text{D}^2\,\text{A}}{a + \text{D}} \quad . \quad . \quad . \qquad (93)$$

which is to be a minimum. Observing that I^1, a, D, and A are constants; and differentiating, placing equal zero and solving, gives,

$$\text{D} = -a \pm \sqrt{a^2 + \frac{\text{I}^1}{\text{A}}} \quad . \quad . \qquad (94)$$

The positive value of D makes the second differential co-efficient positive; hence, it is the minimum; and the negative value is the maximum. The maximum value is really the minimum of the negative values.

EXAMPLES.—1. Suppose the sections are rectangular. Let d be the depth, and b the breadth.

Then $a = \frac{1}{2}d$; $\text{I}^1 = \frac{1}{12}b\,d^3$; $\text{A} = b\,d$,
which in (94) gives

$$\text{D} = \begin{cases} +\,0{\cdot}07732d \\ -\,1{\cdot}07732d \end{cases}$$

The positive value in (93) gives $\text{R}\,\frac{\text{I}}{d_1} = 0{\cdot}1547\,\text{R}\,b\,d^2$. . (95)

If the neutral axis passes through the centre of the sections, $\text{D} = 0$ and (93) gives $\frac{1}{6}\,\text{R}\,b\,d^2 = 0{\cdot}1666\,\text{R}\,b\,d^2$. . . . (96)

2. Suppose the sections are circular.

Then, $a = r$; $\text{I}^1 = \frac{1}{4}\pi r^4$; $\text{A} = \pi r^2$

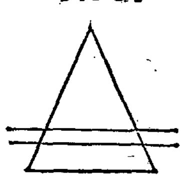
$$\therefore \text{D} = \begin{cases} +\,0{\cdot}11807\,r \\ -1{\cdot}11807\,r \end{cases}$$

$$\text{and } \text{R}\frac{\text{I}}{d_1} = 0{\cdot}7415\,r^3\,\text{R}.$$

If the neutral axis passes through the centre of gravity of the sections $\text{D} = 0$ and (93) becomes

$$\text{R}\,\frac{\text{I}}{d_1} = 0{\cdot}7854\,r^3\,\text{R}$$

FIG 34



3. Let the sections be isosceles triangles, Fig. 34. Let d = the altitude, b = the base.

Then, $a = \frac{2}{3}d$; $\text{I}^1 = \frac{1}{36}\,b\,d^3$; $\text{A} = \frac{1}{2}\,b\,d$

$$\therefore \text{D} = 0{\cdot}0403\,d$$

$$\therefore \text{R}\,\frac{\text{I}}{d_1} = 0{\cdot}04043\,d$$

If $\text{D} = 0$ we have $\frac{\text{RI}}{d_1} = 0{\cdot}04166\,d$

This analysis leads to some interesting considerations. According to the second hypothesis it appears that the neutral axis will be nearer the side which has the greater modulus of elasticity. That is, if the modulus for compression is greater than for tension, the neutral axis will be nearer the compressed side. But the modulus for compression is nearly the same as for tension in all the materials with which the engineer has to deal; hence, so long as the elasticity is unimpaired, the neutral axis passes *very near* the centre of the sections. But as the strains are increased, and the elasticity becomes impaired the third hypothesis shows that the neutral axis moves from its original position towards the side which offers the greatest ultimate resistance to the strain, until, at the instant of rupture, the distance between the centre and neutral axis is D, equation (94).

The latter reasoning furnishes an amusing paradox; for if, in a rectangular beam, we suppose that the ultimate resistances to compression and tension, as well as the moduli of elasticity, are exactly equal, it follows that both sides of the beam would be equally impaired by the strains; and hence, the neutral axis could not move, but must remain at the centre, in which case, the beam would be stronger than if one of the resistances were slightly increased.

The relative strength of the beam in the two cases is shown by equations (95) and (96). The paradox is purely theoretical, for no material is so perfectly homogeneous as to realize the conditions upon which it is founded.

If a rectangular beam be supported at its ends and is broken by a weight P, applied at the middle, we know that the moment of the weight is $\frac{1}{4}Pl$; and this must equal the moment of resistance; hence we have,

according to equation (95) $0{\cdot}1547\, Rbd^2 = \frac{1}{4}Pl$ (97)

" " (96) $0{\cdot}1666\, Rbd^2 = \frac{1}{4}Pl$ (98)

We see that the same general law holds in these equations; viz: that the resistance of a rectangular beam varies directly as the breadth and square of the depth, and inversely as the length.

We have supposed that R is constant, and hence must be determined by means which are independent of any theory of transverse resistance, unless we succeed in establishing the true theory; for each theory gives a value peculiar to itself.

Thus, in equations (97) and (98), all the quantities except R may be found by direct measurement and R found by simple computation; but the two equations give different values. It makes no difference which equation is used, provided we use the same one in practice that we do for obtaining the value of R; but unless the theory be correct R must be determined for each form of beam used in practice. When the true theory is established, it will give the same value for the constants in all forms of beams which are made of the same material.

I am not aware that the last hypothesis has been investigated before, and I have not now the time to apply it to experiments to see if it possesses much merit. A glance at it, however, inclines me to the opinion that it possesses little or no advantage either theoretically or practically over the commonly received hypothesis which fixes the neutral axis at the centre of the sections. In connexion with Barlow's theory it may be found to be of great value. In my next I will develop the formulas which will be applicable to the case.

(To be continued.)