The toughness of the Krupp steel affords an easy solution of this traction problem, which has caused much speculation in the minds of many railway managers.

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General Problem of Trussed Girders. By DE VOLSON WOOD,
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I wish in the following article to show how to investigate truss bridges from the fundamental equations of statics.

THE PROBLEM. *It is proposed to find the strains upon the several pieces which form a truss when acted upon by any system of forces.*

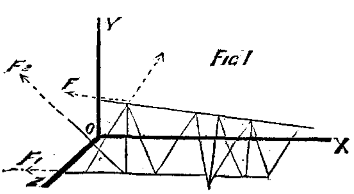
The demonstration is founded upon the following proposition, which follows directly from statics.

PROPOSITION. *If a frame of any form or combination be acted upon by any system of external forces, and the frame be completely divided into two parts by an ideal plane, the strains which act along the bars which are divided by the plane, are in equilibrium with the external forces which act upon the part of the frame on either side of the intersecting surface.*

We may readily conceive that the forces on one side of the ideal plane, hold the frame for the others to act, and the pieces transmit the forces (strains) from one part to the other; hence we may conceive that the forces on *either* side of the plane cause the strains.

We have then only to substitute forces for the strains and treat of them according to the principles of statics.

Let fig. 1 represent the combination of bars (or pieces). Take the origin of co-ordinates at O anywhere in the plane of section.



Let Y, be vertical and positive upwards.

X, horizontal and positive on the side of the section that the forces are considered.

Z, also horizontal and positive to the right of X as we look from Y towards O.

The precautions in regard to the signs are only necessary to enable us to use the same sign throughout the discussion and the applications.

Let P, P_1, P_2, &c., be the applied forces,

a, b, and c, the angles which P make with X, Y, and Z, respectively; and the same letters with subscripts, the corresponding angles of P_1, P_2, &c.

F, F_1, F_2, &c., the strains on the bars which are intersected by the plane.

α, β, and, γ the angles of F with X, Y, and Z, respectively, and the same letters with subscripts for the corresponding angles of F_1, F_2, &c.,

ΣX the sum of all the X components of the forces; similarly for ΣY and ΣZ.

Now conceive that the frame is divided and one part removed, and that in the plane section forces are substituted which will cause the same strains on the intersected bars that existed before the section was made; it is evident that the forces are in equilibrium the same as before one part was removed.

As the forces are in equilibrium we have

$$\begin{array}{ll} \text{the forces:} & \Sigma X = 0 \\ & \Sigma Y = 0 \\ & \Sigma Z = 0 \\ \text{and the couples:} & \Sigma Zy - \Sigma Yz = 0 \\ & \Sigma Xz - \Sigma Zx = 0 \\ & \Sigma Yx - \Sigma Xy = 0 \end{array}$$

which by development gives—

$$\text{the forces: } \left.\begin{array}{l} P\cos a + P_1 \cos a_1 + \&c. + F\cos\alpha + F_1\cos\alpha_1 + \&c. = 0 \\ P\cos b + P_1\cos b_1 + \&c. + F\cos\beta + F_1\cos\beta_1 + \&c. = 0 \\ P\cos c + P_1\cos c_1 + \&c. + F\cos\gamma + F_1\cos\gamma_1 + \&c. = 0 \end{array}\right\} \quad (1)$$

and the couples

$$\left.\begin{array}{ll} \text{around } X, & \Sigma P(y\cos c - z\cos b) + \Sigma F(y\cos\gamma - z\cos\beta) = 0 \\ \text{"} \quad Y, & \Sigma P(z\cos a - x\cos c) + \Sigma F(z\cos\alpha - x\cos\gamma) = 0 \\ \text{"} \quad Z, & \Sigma P(x\cos b - y\cos a) + \Sigma F(x\cos\beta - y\cos\alpha) = 0 \end{array}\right\} \quad . \quad (2)$$

Equation (1) may be written more concisely, thus:

$$\left.\begin{array}{l} \Sigma P\cos a + \Sigma F\cos\alpha = 0 \\ \Sigma P\cos b + \Sigma F\cos\beta = 0 \\ \Sigma P\cos c + \Sigma F\cos\gamma = 0 \end{array}\right\} \quad . \quad . \quad . \quad (3)$$

Discussion of equations (1) *and* (2).

1°. In the earlier part of the discussion we will suppose that the applied forces, P, P_1, &c., are given, and hence their components may be readily computed. Considering them as known quantities and placing them in the second member we have—

$$\text{the forces: } \left.\begin{array}{l} \Sigma F\cos\alpha = -\Sigma P\cos a \\ \Sigma F\cos\beta = -\Sigma P\cos b \\ \Sigma F\cos\gamma = -\Sigma P\cos c \end{array}\right\} \quad . \quad . \quad . \quad (4)$$

and the couples;

$$\left.\begin{array}{ll} \text{around } X, & \Sigma F(y\cos\gamma - z\cos\beta) = -\Sigma P(y\cos c - z\cos b) \\ \text{"} \quad Y, & \Sigma F(z\cos\alpha - x\cos\gamma) = -\Sigma P(z\cos a - x\cos c) \\ \text{"} \quad Z, & \Sigma F(x\cos\beta - y\cos\alpha) = -\Sigma P(x\cos b - y\cos a) \end{array}\right\} \quad . \quad (5)$$

2°. In (4) and (5) there are six independent equations; hence the problem is determinate, and may be solved by elimination, if the plane intersects six bars or less. If it intersects more than six it is indeterminate, unless conditions are established among the strains.

3°. If the frame has an axis, let X coincide with or be parallel to it, then we call

ΣP cos a the DIRECT STRESS, ΣP cos b and ΣP cos c the total SHEARING STRESS, ΣP (cos $b - y$ cos a) the BENDING COUPLE, and the other two the TWISTING COUPLES.

Practical examples of such a general character very rarely occur.

4°. The strains upon the bars depend upon their position, and not upon the trussing between them and either end.

5°. Let all the forces be in the plane XY. Then will $c = 90°$, $\gamma = 90°$, $(a+b) = 90°$, and $(\alpha + \beta) = 90°$, and equations (4) and (5) become

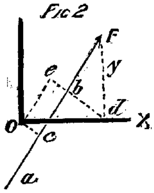
$$\left.\begin{aligned} \Sigma F \cos \alpha &= -\Sigma P \cos a \\ \Sigma F \cos \beta &= -\Sigma P \cos b \text{ also} = \Sigma F \sin \beta \\ \Sigma F (x \cos \beta - y \cos \alpha) &= -\Sigma P (x \cos b - y \cos a) \end{aligned}\right\} \quad (6)$$

in which the first is the *direct stress*, the second the *shearing stress*, and the third the *bending couple*.

The quantity $(x \cos \beta - y \cos \alpha)$ is the length of the perpendicular let fall from the origin on F. For, let ab, fig. 2, be a bar along which F acts; O the origin.

The point of application of the force may be anywhere along ab; let it be at F, and draw Fd perpendicular to X; db and Oc perpendicular to ab, and Oe parallel to $a b$.

Fig 2



Then $Od = x$; $dF = y$, $bdF = \alpha$, and $bFd = \beta$

$bd = y \cos \alpha$, $ed = x \cos \beta$

$\therefore\ Oc = eb = ed - bd = x \cos \beta - y \cos \alpha$.

Call this perpendicular q, and the second member Mz, and the equation becomes $\Sigma Fq = -Mz$.

6°. From (6) we see that the problem is determinate when the plane section intersects three bars. If more, conditions must be established among the strains.

Suppose the plane intersects three pieces; viz: the upper and lower chords and a brace (or tie), and

let F $=$ the strain on the upper chord;

$F_1 =$ the strain on the lower chord;

$F_2 =$ the strain on the brace;

α, α_1 and α_2, the corresponding angles with X,

q, q_1 and q_2, the lever arms of the strains.

Then (6) will become

$$\left.\begin{aligned} F \cos \alpha + F_1 \cos \alpha_1 + F_2 \cos \alpha_2 &= -\Sigma P \cos a \\ F \sin \alpha + F_1 \sin \alpha_1 + F_2 \sin \alpha_2 &= -\Sigma P \cos b \\ Fq \quad + F_1 q_1 \quad + F_2 q_2 \quad &= -\Sigma P (x \cos b - y \cos a) \end{aligned}\right\} \quad (7)$$

EXAMPLE. Suppose that while the frame ABC, fig. 3, is being put in position, it is acted upon by the three forces P, P_1, P_2.

Let the frame be 50 feet long and weigh 100 lbs. per foot.

Also $P_1 = 200$ lbs.; $\beta_1 = 150°$ $\therefore$ $\alpha_1 = -60°$

$P_2 = 300$ lbs.; $\beta_2 = 170°$ $\therefore$ $\alpha_2 = -80°$

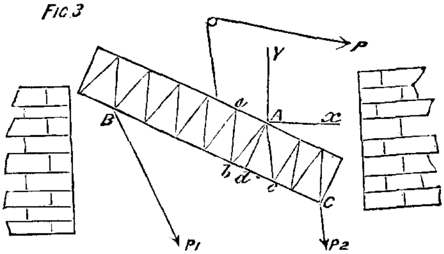
from which we may find P and α; for we have

$$\Sigma X = P \cos \alpha + 200 \cos 60° + 300 \cos 80° = 0$$

$$\Sigma Y = P \sin \alpha + 200 \sin(-60°) + 300 \sin(-80°) - 50 \times 100 = 0$$

$$\therefore \quad P = 5470 \text{ lbs}; \ \alpha = -88° \, 24\tfrac{1}{2}.$$

FIG. 3



Now suppose we wish the strains on Aa, Ab, and bc. Intersect these bars by a vertical line or plane, and to simplify the case take the intersection infinitely near A. Let Aa and bc be parallel, and make an angle of $-30°$ with X, and Ab an angle of $+20°$; A$d = q = 6$ feet, q and q_2 are zero.

According to the proposition, we may consider the forces on the right or left of A, and as there is but one external force on the right we will consider the forces on that side. Suppose there is 20 feet of frame on the right of A, and the center of gravity is 10 feet from Y; then will the moment be $20 \times 100 \times 10 = 20000$. Call the lever arm of P_2 15 feet. The strain on Aa is tensive, hence $a = 150°$; on bc it is compression, hence $a_1 = -30°$; on Ab it is compression, hence $a_2 = 20°$. Hence equation (7) become

$$F \cos 150° + F_1 \cos(-30°) + F_2 \cos 20° = -\ 152{\cdot}08$$

$$F \sin 150° - F_1 \sin\ 30° + F_2 \sin 20° = \quad 5468{\cdot}64$$

$$-F_1 \times 6 \quad = -(20000 + 300 \times 15)$$

The signs of 6 F_1 is minus because it tends to turn the system from right to left—the signs of the other moments are positive because they tend to turn it in the opposite direction. From these we find

$F = 10862$ lbs, $F_1 = 4083\tfrac{1}{3}$ lbs. and $F_2 = 6082$ lbs.

7°. Let all the forces be vertical: or, $b = 0°$ or $180,°$ $a = \pm 90°$ These in (7) give:

$$\left.\begin{aligned} F \cos \alpha + F_1 \cos \alpha_1 + F_2 \cos \alpha_2 &= 0 \\ F \sin \alpha + F_1 \sin \alpha_1 + F_2 \sin \alpha_2 &= -\ \Sigma \pm P \\ F q \quad + F_1 q_1 \quad + F_2 q_2 &= -\ \Sigma \pm P x \end{aligned}\right\} \qquad (8)$$

The plus sign belongs to forces which act vertically upward. It is well here to observe that if we consider tension as positive we *may* consider compression as negative; or we may consider both as positive and attribute the sign to the trigonometrical function—the same that we do with the applied forces—in which case a compression may be considered as a thrust whose direction is changed 180.° The hypotheses are essentially the same, although the latter is the one commonly used in establishing equations.

The first of equations (8) shows that the total compressive strain equals the total tensile ones.

The *shearing* stress, as found by the second of equations (7), *is greater on any vertical section when the longer part of the frame into which it is divided by the plane, is loaded and the shorter part unloaded.*

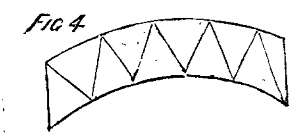
For if we conceive the longer part loaded—say from V_1 to the section, in fig. 5, and the remaining part unloaded—then will the total shearing stress on all vertical sections between the plane section and the other end be the same, and equal the reaction of the support, V; but if any load be now added to the shorter part, V will sustain a part of it while all of it must be subtracted from V to find the *shearing* stress on the section; hence it is less than before any load was placed on the shorter part.

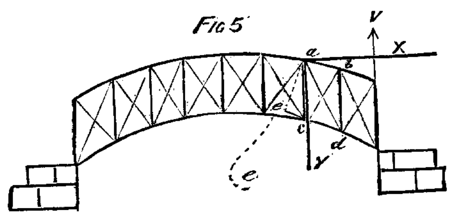
It is easy to show that the sum of the moments, ΣPx, as given by the third of (7) is greatest when the frame is loaded throughout.

Equations (8) are applicable to trusses of all forms, but they may be reduced so as to be more convenient for particular cases.

8°. Let both chords be convex upwards.

If the space occupied by the truss be divided into trapezoids by vertical bars, as *ac* and *bd*, fig. 5, and the diagonal ones so arranged that only one shall act at a time—or what is equivalent, the diagonal bars shall be so arranged as to be subject to only one kind of strain, I shall call it the *panel* system. But if it be divided into triangles, as in fig. 4, I shall call it the *triangular* system.





A bar makes the same angle with the axes that a line does which is drawn through the origin and parallel to the bar. We may assume any position of the bar to make positive angles with the axes, but after assuming the position we observe that a bar which inclines the opposite way makes a negative angle—but to be more specific we will assume, that when a line, or its prolongation, which is drawn through the origin and parallel to the bar, falls between +X and +Y the angle is positive —otherwise, it is negative. Still further, to avoid numerous negative angles which would otherwise occur in the position which we shall assume for the origin, I will make Y positive downwards. This will be equivalent to changing the signs of the vertical forces, and equations (8) become

$$\left.\begin{aligned} F\cos\alpha + F_1\cos\alpha_1 + F_2\cos\alpha_2 &= 0 \\ F\sin\alpha + F_1\sin\alpha_1 + F_2\sin\alpha_2 &= \Sigma \pm P \\ Fq + F_1 q_1 + F_2 q_2 &= -\Sigma \pm Px \end{aligned}\right\} \quad . \quad (9)$$

in which plus P acts vertically upwards.

To determine the angle of the strains we conceive that forces are applied to the bars in the section which shall produce the same strains as before the section was made; then draw a line from the origin parallel to, and in, the direction of the action of the force, and it will make the same angle with the axes that the force does. We also observe that $+\alpha$ is reckoned from $+$X towards $+$Y.

To prepare equations (9) for use, we will make a section at the right of, but indefinitely near to a, in fig. 5.

Let i = the angle which the upper chord makes with X.
i_1 = the angle which the lower chord makes with X.
θ = the angle which the brace $a\,d$ makes with Y.
V = the vertical reaction of the support.
x_1 = the lever arm of V.
h = ac = the vertical depth of the frame at the origin.
Σ_0^x P = the sum of all the loads between the origin and the end.
$\Sigma_0^x \text{P}x$ = the moments of all the loads between the origin and V.

Take the origin at a. The strain on ab is compressive, to produce which requires that F act from a towards b; hence $\alpha = i$. F_1 must act at c, and towards e; hence $\alpha_1 = 180° + i_1$; and F_2 acts from a towards d; hence $\alpha_2 = 90 - \theta$. $q = 0$; $q_1 = ae = h \cos i_1$; $q_2 = 0$. These values reduce (9) to

$$\left.\begin{array}{l} \text{F} \cos i - \text{F}_1 \cos i_1 + \text{F}_2 \sin \theta = 0 \\ \text{F} \sin i - \text{F}_1 \sin i_1 + \text{F}_2 \cos \theta = \text{V} - \Sigma_0^{x_1} \text{P} \\ \qquad \text{F}_1 h \cos i_1 \qquad\quad = \text{V}x_1 - \Sigma_0 \text{P}x \end{array}\right\} \quad . \quad . \quad (10)$$

For maximum *shearing* stress $\Sigma_0^x \text{P} = 0$; and we have

$$\left.\begin{array}{l} \text{F} \cos i - \text{F}_1 \cos i_1 + \text{F}_2 \sin \theta = 0 \\ \text{F} \sin i - \text{F}_1 \sin i_1 + \text{F}_2 \cos \theta = \text{V} \\ \qquad \text{F}_1 h \cos i_1 \qquad\quad = \text{V}x_1 \end{array}\right\} \quad . \quad . \quad . \quad (11)$$

It must be observed that V in this case is not the same as in the preceding, but it must be computed from the given data.

EXAMPLE. Suppose that in a bridge which is 120 feet long it is observed that at 30 feet from the end the upper chord is inclined 15° to the horizon; the lower chord 20,° the brace 45,° and the depth is 10 feet. It is required to find the strains upon these bars when there is an uniform load of one ton per foot over the whole length of the bridge: also the strains at the instant of maximum shearing, when it moves off without shock.

For the former part of the problem we use (10) and readily have

$$\left.\begin{array}{l} \text{F} \cos 15° - \text{F}_1 \cos 20° + \text{F}_2 \sin 45° = 0 \\ \text{F} \sin 15° - \text{F}_1 \sin 20° + \text{F}_2 \cos 45° = 60 - 30 = 30 \\ \qquad 10\ \text{F}_1 \cos 20° = 60.30 - 30.15 = 1350. \end{array}\right\}$$

These solved give

$$\text{F} = 77,\ \text{F}_1 = 143, \text{ and } \text{F}_2 = 85 \text{ tons.}$$

The results are given to the nearest whole number.

For the maximum shearing, the load, according to the principles before stated, must extend over 90 feet. For this we find $\text{V} = \frac{90 \cdot 45}{120} = 33\frac{3}{4}$, which in (11) gives

$$\left.\begin{array}{ll} F\cos 15^\circ - F_1\cos 20^\circ - F_2\sin 45^\circ & = 0 \\ F\sin 15^\circ - F_1\sin 20^\circ + F_2\cos 45^\circ & = 33\tfrac{3}{4} \\ 10F_1\cos 20^\circ & = 1012\tfrac{1}{2} \end{array}\right\}$$

which being solved will give

$$F = 43\cdot 2,\ F_1 = 107\cdot 6,\ F_2 = 84\cdot 4 \text{ tons},$$

which compared with the preceding shows that the strain on the brace is nearly as great as when the whole bridge is loaded. We shall hereafter see that there are cases in which the strain on the braces is *greater* for maximum shearing than when the whole frame is loaded.

If bc is brought into action as a brace, it must be done by the loading between b and v.

(To be Continued.)

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The Cinematics of the Slide Valve. By ALBERT ASTON, U.S. Corps of Naval Engineers.

As experience has proved the slide valve, cutting off by lap, to be, of all devices, the best adapted to locomotive and screw-propeller engines, a set of reliable formulas, embracing all the practical conditions, for determining the proportions necessary for its proper action, will doubtless be acceptable to those engaged in its construction.

In the following discussion, strict mathematical accuracy is not aimed at, as that would only uselessly complicate the problem. The results given by the formulas, however, will, within the limits of practice, differ from the absolute truth only by insensible quantities.

The principal dimensions to be determined are the throw of the eccentric and the lap of the valve. For these we have given the length of stroke of the piston, or the length of the crank; the point of cutting off, and the width of opening of the steam-port.

The action of an eccentric is to be regarded as precisely the same as that of a crank, the centre of the eccentric corresponding to the centre of the pin of the crank. The distance from the centre of the shaft to the centre of the eccentric will be called, for convenience, the radius of the eccentric.

If a valve be without lead or lap, and the connecting-rod and eccentric-rod are supposed to be infinitely long, it will be in mid-position, or at half its stroke, when the piston is at the beginning of its stroke, its edges will coincide with the edges of the steam-ports, and its stroke will be exactly double the width of the port; but if lap on the steam side be given to the valve, its length will be correspondingly increased beyond the edges of the ports, consequently its stroke must be increased by exactly the increased length of the valve, and the centre of the eccentric must be revolved about the shaft from its mid-position until the extreme edge of the valve again coincides with the edge of the port, as the valve must be on the point of opening when the piston is at the commencement of its stroke.

It will be seen by inspecting the diagram, Plate III, that the valve will open while the centre of the eccentric passes through the arc pd, and that