ployment of the steamer, might be taken at £32,079 including all contingencies. Supposing the reef to have been within a boating distance, say $1\frac{1}{2}$ mile and $\frac{1}{2}$ mile from Suez, if the steamer had been equipped at Suez, and had been continuously employed, then, on this supposition, the cost might have been £42,082. The remaining expenditure, £13,029 was entirely exceptional, arising mainly from the steamer being equipped at Bombay. Proc. Inst. Civil Eng., Nov. 10, 1863.

For the Journal of the Franklin Institute.

Trussed Arch.

By De Volson Wood, Prof. of C. E. University of Michigan.

The problem which I propose to discuss may be stated as follows:

Let the arch of the Truss be a parabola, or if it be polygonal, let the vertices of the polygon be in a parabola; the tie which joins the ends of the arch be horizontal; all the parts of the truss be reduced to mathematical lines, and the joints perfectly flexible. Let the load over any part of the truss be uniform—or, what is better, let the weights upon the joints equal each other.

It is well known that for an uniform load over the whole length of a parabolic arc, there are no strains in it but compression (or tension), and hence, if the load be above the arch there will be no strains upon the ties and braces; and if the load be below, the ties will simply sustain the total load; hence the strains upon the several parts are easily computed. I will, therefore, proceed to the case of a partially loaded truss. Let the horizontal tie be divided into equal parts by the trussing, and let each part be called a bay.

Let N = total number of bays in the horizontal tie,

n = the number of a bay which corresponds to the number of a brace or pair of braces,

F = the force of compression in the arch at any point,

$F_1 = H$ = horizontal force in the tie,

F_2 = the strain on a brace or tie in the truss,

i = the inclination of the arch to the horizontal,

θ = the inclination of a brace, or tie, to the vertical,

D = greatest depth of the truss = distance from the vertex of the parabola to the horizontal tie,

h = depth of the truss at any point,

l = length of bay,

p = one of the equal weights which constitutes the load,

V and V_1 = vertical re-actions of the supports, and let n be counted from the V support,

x be horizontal and positive towards V,

y be vertical and positive downwards, then if θ is on the right of y it will be positive; if on the left, negative.

x', y' be the co-ordinates of c,

x'', y'' be the co-ordinates of b,

$2p_1$ the parameter of the parabola.

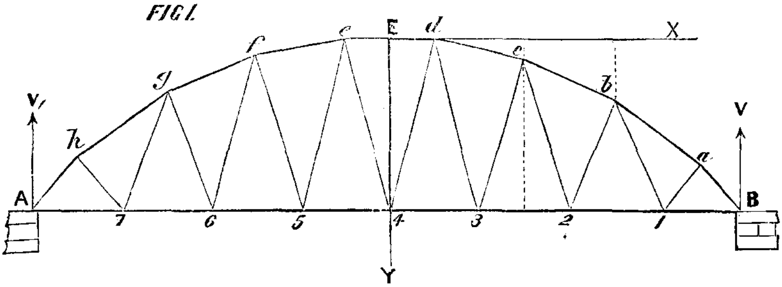
For the equation of the arch we have,

$$x^2 = 2p_1 y \text{ or } \tfrac{1}{4}N^2 l^2 = 2p_1 D \quad \therefore \; 2p_1 = \frac{N^2 l^2}{4D} \text{ which gives}$$

$$x^2 = \frac{N^2 l^2}{4D} y \text{ or } y = \frac{4D}{N^2 l^2} x^2 \quad . \quad . \quad . \quad (1)$$

It can easily be shown that if the truss be uniformly loaded from any brace to the remote end, the strain upon the brace will be greater than if there be an additional uniform load between it and the near end. I shall therefore consider the case of such an uniform load.

FIG. 1.



First, take the case of triangular trussing as shown in Fig. 1. Let equal weights rest upon the joints c, d, e, f, g, and h, and none upon a and b. This will insure, as stated above, a greater strain on $c2$ than if b or a, or both were loaded with the same weights. Suppose now that a vertical section is made just at the right of c; said section will intersect cb, $c2$, and 3 2, and the strains in these bars must be in equilibrium with the forces between the section and B; in other words they must be in equilibrium with V_1, and since V acts vertically, we have the vertical components in cb and $c2$ equal V, and the horizontal components in the same bars equal the strain on 3 2, or equal H. Hence using the notation given above, we readily have

$$F \sin i + F_2 \cos \theta = V \quad . \quad . \quad . \quad (2)$$

$$F \cos i + F_2 \sin \theta = H \quad . \quad . \quad . \quad (3)$$

Multiply (2) by cos i, (3) by sin i, subtract and reduce gives

$$F_2 \cos \theta = \frac{V - H \operatorname{tang} i}{1 - \operatorname{tang} \theta \operatorname{tang} i} \quad . \quad . \quad . \quad (4)$$

This formula is general, whatever be the curve of the arch.

Calling $a\,1\,b$ the first pair of braces (or ties) counting from B; let $b\,2\,c$ be the n^{th} pair; then will

2 1 be the n^{th} bay; 3 2 the $(n+1)^{th}$ bay, and

$N - n$ = the number of loaded joints,

$\therefore (N-n)p$ = the total load on the truss,

$Nl = AB$ = total length of truss,

$\frac{1}{2}(N-n)l$ = distance from the centre of the load to A, Then by the principle of the lever we have

$$\text{V}\text{N}l = (\text{N}-n)p \cdot \tfrac{1}{2}(\text{N}-n)l$$

$$\therefore \quad \text{V} = \frac{(\text{N}-n)^2}{2\text{N}}p. \quad . \quad . \quad . \quad . \quad (5)$$

We also have from the figure,

$$\text{B}m = (n+\tfrac{1}{2})l$$

$$x' = \text{EK} = (\tfrac{1}{2}\text{N}-n-\tfrac{1}{2})l \ \therefore \text{ By (1) we have } y' = \frac{\text{D}}{\text{N}^2}(\text{N}-2n-1)^2$$

$$x'' = \text{EL} = (\tfrac{1}{2}\text{N}-n+\tfrac{1}{2})l \quad . \quad . \quad y'' = \frac{\text{D}}{\text{N}^2}(\text{N}-2n+1)^2$$

$$\text{tang } i = \frac{y''-y'}{l} = \frac{4\text{D}}{\text{N}^2 l}(\text{N}-2n) \quad . \quad . \quad . \quad (6)$$

$$\text{tang } \theta = \frac{\frac{1}{2}l}{\text{D}-y'} = \frac{l\text{N}^2}{2\text{D}[\text{N}^2-(\text{N}-2n-1)^2]} \quad . \quad . \quad (7)$$

To find the tension in the bar 3 2, we take the origin of moments at c, and we readily have

$$\text{H}(\text{D}-y') = \text{V}(n+\tfrac{1}{2})l$$

$$\therefore \text{H} = \frac{\text{N}^2 l\text{V}}{2\text{D}(2\text{N}-2n-1)} \quad . \quad . \quad (8)$$

Substitute (5), (6), (7), and (8) in (4), gives

$$\text{F}_2 \cos\theta = \frac{(\text{N}-n)^2 p}{2\text{N}}\left[\frac{4n^2-1}{4n\text{N}-4n^2-1}\right]$$

$$= \frac{(\text{N}-n)}{2\text{N}}p\left[n - \frac{\text{N}-2n}{4n(\text{N}-n)-1}\right] \quad . \quad (9)$$

We see in (9) that when n is less than $\frac{1}{2}$ N the fraction in the parenthesis is negative; but when it exceeds $\frac{1}{2}$ N, it becomes positive, and observing that when it is greater than $\frac{1}{2}$ N, less than one-half the bridge is loaded, we have this peculiar result: the strain upon *a* tie or brace, is greatest when the truss is loaded between it and the nearer end. We may also observe, that in practical cases the omission of the fraction in the parenthesis, will not give an error of more than one-fifth the true value, and generally the error will be much less. Hence we have approximately

$$\text{F}_2 \cos\theta = \frac{(\text{N}-n)np}{2\text{N}} * \quad . \quad . \quad . \quad (10)$$

If (10) were true we observe that the strains on any brace will be the same if the load extend from it to either end.

It is easy to show, geometrically, that the vertical components of the strains on each of the braces which constitute a pair, as has been designated, are equal. To give a further application of the analysis, I will prove it by equation (4). Now let a vertical section be made, just at the left, but infinitely near b. The section will cut b 2, bc, and

*Bow in his excellent Treatise on Bracing, gives this as the *exact* formula.

12. Tang i and V, will remain the same, but tang θ is negative and equal $\frac{\frac{1}{2}l}{D-y''}$; . . . - (10′)

and to find H we have $H\,(D-y'')=V\,(n-\frac{1}{2})\,l$.

$$\therefore \quad H=\frac{\frac{1}{2}(2n-1)\,l}{D-y''}V \qquad (11)$$

These in (4) give

$$F_2\cos\theta=V\,\frac{1-\dfrac{2n-1}{D-y''}\cdot\dfrac{2D}{N^2}(N-2n)}{1+\dfrac{2D}{(D-y'')}\cdot\dfrac{(N-2n)}{N2}} \text{ which reduced gives}$$

$$F_2\cos\theta=\frac{(N-n)^2}{2N}\,p\left[\frac{4n^2-1}{4nN-4n^2-1}\right] \text{ the same as before.}$$

But the strains on the braces of a pair will not be equal, for they are unequally inclined. To find F_2, we find θ from (7) and use it in (9).

Example.—Let $N=8$ and $D=l$.

To show more clearly the value of the fraction in the parenthesis, Eq. (9), I will keep it separate in the following table:

No. of the pair of braces, or $n=$	Vertical component of the strains on the n^{th} pair; or $F_2\cos\theta$, Eq. (9.)	Inclination of the braces; or θ, Eq. (7.)	Values of $\cos\theta$.	Numbers in the second column divided by those in the fourth: or, F_2.
0	$0+\frac{8}{2}\quad p=4{\cdot}000\,p.$	*Ba*, 64°53′	0·4245	9·422 p
1	$(\frac{7}{16}-\frac{7}{72})\quad p=0{\cdot}3403\,p.$	*a*1, 64°54′ *b*1, 39°22′	0·4245 0·7731	0 802 p 0·440 p
2	$(\frac{3}{4}-\frac{3}{94})\quad p=0{\cdot}7189\,p.$	*b*2, 39°22′ *c*2, 30°11′	0·7731 0 8644	0 928 p 0·831 p
3	$(\frac{15}{16}-\frac{5}{472})\quad p=0\,9270\,p.$	*c*3, 30°11′ *d*3, 26°55′	0·8644 0·8916	1·072 p 1·038 p
4	$p=1{\cdot}0000\,p.$	*d*4, 26°55′ *e*4, 26°55′	0·8916 0·8916	1·121 p 1·121 p
5	$(\frac{15}{16}+\frac{3}{472})\,p=0{\cdot}9438\,p.$	*e*5, 26°55′ *f*5, 30°11′	0·8916 0·8644	1·057 p 1·090 p
6	$(\frac{3}{4}+\frac{1}{94})\quad p=0{\cdot}7606\,p.$	*f*6, 30°11′ *g*6, 39°22′	0·8644 0·7731	0·879 p 0 998 p
7	$(\frac{7}{16}+\frac{1}{72})\quad p=0{\cdot}4514\,p.$	*g*7, 39°22′ *h*7, 64°54′	0·7731 0·4245	0·583 p 1·060 p
8	$p=0{\cdot}000\,p.$			

We may readily conceive these strains to be produced by an uniform load moving without shock over the truss from B to A; and the same strains in a reverse order may be produced by a movement in the opposite direction.

When N is even we may have $n=\frac{1}{2}N$, which in (9) gives $F_2\cos\theta=\frac{1}{8}Np$.

Next suppose the load on the horizontal tie. This is the more natural place for the surcharge. In this case the vertical force sustained by each pair when they are all equally loaded, is p. But if only a portion of the truss be loaded, equation (9) will not apply, as may be seen from the following statements. To produce the greatest strain on $c3$, we unload joints 1 and 2, and load 3, 4, 5, 6, and 7; and calling the bay $2-1$, the n^{th}, the load will be $(\text{N}-n)p$; and the centre of the loading will be $\frac{1}{2}(\text{N}-n+1)\,l$ from A; hence, to find V we have

$$\text{V}\,.\,\text{N}l=\tfrac{1}{2}(\text{N}-n)(\text{N}-n+1)l.$$

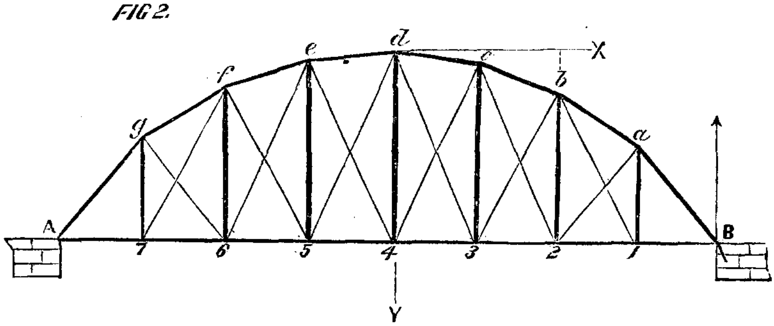
$$\therefore\ \text{V}=\frac{(\text{N}-n)(\text{N}-n+1)l}{2\text{N}}. \qquad (12)$$

If joint 2 be loaded, the vertical force on $c2$ will not be the same as on $b2$, but we may find it on $b2$ by making a section just to the left of b; and substitute in (4) the values of (6), (10′), and (12). But we observe that these values are all the same as those before used, except V; hence we have at once, for the strain on the first of the n^{th} pair,

$$\text{F}_2\cos\theta=\frac{(\text{N}-n)(\text{N}-n+1)}{2\,\text{N}}\,p\left[\frac{4n^2-1}{4n\text{N}-4n^2-1}\right] \qquad (13)$$

Next consider panel trussing as shown in Fig. 2, and let the load be upon the lower chord, and let the bays in the lower chord be of equal lengths. It will make some difference in the strains whether they be resisted by ties or braces.

FIG 2.



First consider braces.

Let x' and y' be the co-ordinates of c.

x'' y'' be the co-ordinates of b.

3 2 be the n^{th} bay, and call $c2$ the n^{th} brace, and

$e3$ the n^{th} tie. Suppose the load is on from A to 3, and off from B to 2, including 2; then the load is $(\text{N}-n)p$; and V the same as (12). The equation of the curve is given by (1), hence we have

$$x'=(\tfrac{1}{2}\,\text{N}-n)l\ ,\ y'=\frac{\text{D}}{\text{N}^2}(\text{N}-2n)^2$$

$$x''=(\tfrac{1}{2}\text{N}-n+1)l\ ,\ y''=\frac{\text{D}}{\text{N}^2}(\text{N}-2n+2)^2$$

$$\text{tang } i = \frac{y'' - y'}{l} = \frac{4\text{D}}{l\text{N}^2}(\text{N} - 2n + 1) \quad . \quad . \quad . \quad (13a)$$

$$\text{tang } \theta = \frac{l}{\text{D} - y'} = \frac{l\text{N}^2}{4\text{D}n(\text{N} - n)} \quad . \quad . \quad . \quad (14)$$

Now conceive a section made just at the right, but infinitely near c, so as to cut cb, $c2$, and 3 2; then will equation (4) be applicable. It cuts $b3$, but it is not in action when $c2$ is. To find H, we have by an equation of moments,

$$\text{H}(\text{D} - y') = \text{V}nl$$

$$\therefore \text{H} = \text{V}\frac{l\text{N}^2}{4\text{D}(\text{N} - n)}$$

These in (4) give by reduction

$$\text{F}_2 \cos\theta = \frac{(\text{N} - n)(\text{N} - n + 1)(n - 1)n}{n\text{N} - \text{N} - n^2 + 2n - 1} \cdot \frac{p}{2\text{N}} = \frac{(\text{N} - n)n}{2\text{N}}p \quad . \quad (15)$$

This is a maximum for $n = \frac{1}{2}\text{N}$, for which it equals $\frac{1}{8}\text{N}p$; hence, if there are less than N bays the central brace will not be strained as much as for half the truss loaded as for the whole truss loaded; if $\text{N} = 8$, it will be strained the same; if greater it will be strained more for half the truss loaded than for the whole loaded. We also see that for the partial load the central ones are strained more than the end ones. The formula does not give the vertical strain on Ba; for it is really $\text{V} = \frac{1}{2}(\text{N} - 1)p$; but for $n = 1$, (15) gives $\frac{\text{N} - 1}{2\text{N}}p$. The reason of this failure will be found by observing that in making the reduction of (15), a factor $(n - 1)$ is cancelled in both terms of the fraction, which factor is 0 for $n = 1$, which would make the fraction $\frac{0}{0}$.

Example.—Let $\text{N} = 8$. D has disappeared in the reduction, but it must be known to get θ. Let $\text{D} = 2l$. We have

No. of the brace, or $n =$	Vertical component of the strain on the n^{th} brace, or $\text{F}_2 \cos\theta$; eq. (15)	Inclination of the braces, or θ, eq. (14)	Values of $\cos\theta$.	Strains on the n^{th} brace, or F_2 eq. (15)
1 or 7; or g 6	$\frac{7}{16}p$	48°48′	0·6587	0·6641 p
2 or 6; or $b1$ or $f5$	$\frac{12}{16}p$	33°41′	0·8321	0·9013 p
3 or 5; or $c2$ or $e4$	$\frac{15}{16}p$	28° 4′	0·8824	1·0624 p
4 ; or $d3$	1 p	26°34′	0·8944	1·1182 p

Although the braces which we have considered, at the rear end of he load incline towards the load, yet they must incline both ways as n the figure to completely brace the truss. For this partial load, the vertical strain on $a1$, is the same as on $b1$; and if the load extends from 3 to A, the vertical forces on $c2$ and $b2$ are equal. Similarly for the other pairs; hence, observing the numbers in the second column of the preceding table, and we have for the actual strains on the vertical ties taken in their order from either end, when the load extends from the tie to the other end: $\frac{12}{16}p$, $\frac{15}{16}p$, p, $\frac{15}{16}p$, $\frac{12}{16}p$, $\frac{7}{16}p$.

Now suppose there are ties instead of braces, in the panels. Making the section at b and it will cut the acting bars cb, $b3$, and $3\ 2$. When it is loaded from 3 to A, $c2$ will not act.

$$\text{tang}\ \theta = -\frac{l}{\text{D} - y''} \qquad . \qquad . \qquad . \qquad (16)$$

$$\text{and H} \ . \ (\text{D} - y'') = \text{V}\,(n-1)\,l$$

$$\therefore \ \text{H} = \frac{\text{V}\,(n-1)\,l}{\text{D} - y''} = \frac{\text{V}l\text{N}^2}{4\text{D}\,(\text{N} - n + 1)}$$

$$y'' = \frac{\text{D}}{\text{N}^2}(\text{N} - 2\,n + 2)$$

V is given in (12) and tang i in (13a); these substituted in (4) give

$$\text{F}_2 \cos\theta = \text{V}\frac{1 - \dfrac{(n-1)l}{\text{D} - y''} \ . \ \dfrac{4\,\text{D}}{\text{N}^2 l}(\text{N} - 2n + 1)}{1 + \dfrac{l}{\text{D} - y''} \ . \ \dfrac{4\,\text{D}}{\text{N}^2 l}(\text{N} - 2n + 1)}$$

$$= \text{V}\frac{n-1}{\text{N} - n} = \frac{(\text{N} - n + 1)\,(n-1)}{2\,\text{N}} \qquad . \qquad . \qquad (17)$$

This formula also fails, for $n = \text{N}$, because, in making the final reduction, we dropped a factor, $\frac{\text{N} - n}{\text{N} - n}$, which for $n = \text{N}$ becomes $\frac{0}{0}$, but it should $= 0$. It is true for all the other ties.

If in (17) we write $n + 1$ for n, we will have $(\text{N} - n)n$, which is the same as (15); hence, the vertical component of the strain on a brace, when braces are used, is the same as on the tie in the next panel, when ties are used; and as the inclinations, $c2$ and $c4$, for instance, are the same, the *actual* strains will be the same. Hence, referring to the preceding table, we have $\frac{7}{16}p$ for the vertical strain on the *second* tie (or $a2$); $\frac{12}{16}p$ on the *third* tie (or $b3$), &c.

The general principles of the methods here used are applicable to those cases in which the bays of the lower chord are not equal; but in such cases we cannot obtain as symmetrical expressions as those here found.

Railroad Cuttings and Embankments.—Side Depths and Side Stakes. By Oliver Byrne, C.E.

From the Lond. Civ. Eng. and Arch. Jour., Feb., 1864.

(Continued from page 152.)

In an embankment (Fig. 7), given the breadth of the roadway AB $= 32$ feet; the height CF $= 18$ feet; the side slopes as 1 to $\frac{2}{3}$ (BI : ID : : 1 : $\frac{2}{3}$); the fall of the surface F to M $= 1$ in $26\frac{1}{2}$ (FN : NM : $26\frac{1}{2}$: 1); the rise of the surface from F to K to be