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Suspension Bridges: A New System. By STILLMAN W. ROBINSON.

(Read before the Michigan Scientific Association.)

MR. EDITOR:—The following article is an abstract of a Thesis prepared by Mr. Robinson, and read before the Faculty just before taking the degree of Civil Engineer at the last commencement. It is due to him to say, that he had numerous verifications of the principal formulas, and several processes of reduction which I have omitted; but all the formulas here given are literally copied from his paper.

Professor Rankine, in his work on "Applied Mechanics," in speaking of the linear arch which is every where normally pressed, p. 190, says, "The only arch of this kind which has hitherto been considered, is the circular arch under uniform pressure." This example is illustrated by a circular ring placed horizontally under a fluid. He then gives a second example, p. 190, called the "Hydrostatic Arch," or "The Arch of Yvon-Villarceaux," where the arch is in a vertical plane and pressed externally by a fluid whose surface is horizontal. The investigations by Mr. Robinson, given in the following article, make another, or third example. D. VOLSON WOOD,

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In the ordinary suspension bridge, the tension is greatest at the piers and least at the middle.

My object is to deduce and discuss the equations for a suspension bridge when the suspension rods are so inclined as to produce a uniform tension throughout the cable; the bridge being loaded uniformly over the span. The analysis is founded upon the following theorems:

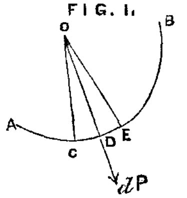
THEOREM I. *In a normally pressed arch the tension or compression is uniform throughout.*

For the tension from one point to another cannot differ unless there be a tangential component; but if all the forces are normal they cannot have any tangential component at the point where they act.

THEOREM II. *The pressure at different points varies inversely as the radius of curvature.*

Proof. Let A B, fig. 1, be any normally pressed arc, d P be any element of the normal force, and which may be considered the resultant of the tensions on each side of it.

FIG. I.



C D=D E=$\frac{1}{2}\,ds$.

O D=ρ=the radius of curvature at D.

T=the stress along the arc.

a=an infinitesimal angle C O E.

Then, by Mechanics we have

$$d\,P=\sqrt{T^2+T^2-2TT\cos.a}=2T\sin.\tfrac{1}{2}\,a,$$

which at the limit equals T a.

$$\text{But } \rho\,a=ds \quad \therefore \quad d\,P=\frac{T\,ds}{\rho} \qquad . \qquad . \qquad (1)$$

which proves the theorem. By integrating (1) the first member between O and P; the second between O and 1, we have

$$P=\frac{T}{\rho} \quad \text{or } T=P\,\rho* \qquad . \qquad . \qquad (2)$$

hence, the tension equals the pressure per unit of length multiplied by the radius of curvature at the middle of that unit.

*It will be seen that Mr. Robinson has adopted the same method for finding an expression for the tension, that many writers use for finding the relation between the power and resistance, when a rope is coiled around a cylinder and friction is considered. If we should apply external normal forces just sufficient to remove the pressure on the cylinder, there would be no friction, and hence no loss of tension, which corresponds to Theorem I.

If F be the applied force, W the resistance at the other end of a rope coiled around a cylinder, a the arc at a unit's distance, and f the co-efficient of friction; we have (see Bartlett's Analytical Mechanics, 3d Ed., p. 382)

$$F=W\,e^f$$

in which if $f=0$ we have F $=$ W, which also corresponds with Theorem I.

In the case of friction the tension varies, and if the cylinder be circular, the radius is constant; but in Theorem II., the tension is constant and the radius of curvature may or may not be constant. D.V.W.

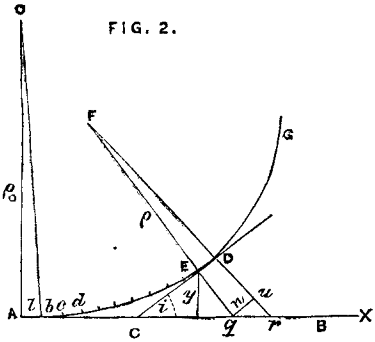
Now let A G, fig. 2, represent the arc of the cable. Let A B be the axis of X ; A O of y,

w, the weight per unit of length over the span.

AO, F E and F D radii of curvature.

By Theorem I, the suspension rods must coincide in direction with the radii of curvature; hence if E D $= d\,s$; E q and D r may be considered two consecutive suspension rods. Through E, draw E C tangent to the curve, and through q a line parallel to the tangent.

FIG. 2.



Then let D C $x = i$ O A $= \rho_0$

$q\,r = m$ Radius F E $= \rho$

$q\,u = n$

Then $w\,m$ is the weight acting vertically on $q\,r$, and which resolved normally, is the normal pressure on E D, and equals

$$w\,m \text{ sec. } i = d\,\text{P}. \quad . \quad . \quad . \qquad (3)$$

At A, $m = d\,s$ and $i = o$; hence the normal pressure at that point is

$$w\,d\,s. \quad . \quad . \quad . \quad . \qquad (4)$$

Hence, from (1), (3), and (4), we have

$$\rho\,m \text{ sec. } i = \rho_0\,d\,s \quad . \quad . \qquad (5)$$

$$\therefore \frac{d\,s}{m} = \frac{\rho}{\rho_0} \text{ sec. } i \quad . \quad . \quad . \qquad (6)$$

By similarity of triangles

$$\rho : d\,s :: \rho + y \text{ sec. } i : n \text{ or } m \cos. i$$

$$\therefore \frac{ds}{m} = \frac{\rho \cos. i}{\rho + y \text{ sec. } i} \quad . \quad . \quad . \qquad (7)$$

From (6) and (7) we find

$$\rho_0 = (\rho + y \text{ sec. } i) \text{ sec.}^2 i \quad . \quad . \qquad (8)$$

which is the equation of the curve in terms of the variables ρ, y, and i.

To find the equation when referred to rectangular co-ordinates, we substitute.

$\dfrac{\left(1 + \dfrac{dy^2}{dx^2}\right)^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ for ρ and $\sqrt{1 + \dfrac{dy^2}{dx^2}} = \sqrt{1 + \text{tang.}^2 i}$ for sec. i and

(8) becomes

$$\left(1 + \frac{dy^2}{dx^2}\right)^{\frac{3}{2}} + y\left(1 + \frac{dy^2}{dx^2}\right)^{\frac{3}{2}} \frac{d^2y}{dx^2} = \rho_0 \frac{d^2y}{dx^2} \quad . \quad . \qquad (9)$$

$$\text{Make } \frac{dy^2}{dx^2} = 2\,z \qquad \therefore \frac{d^2y}{dx^2} = \frac{d\,z}{dy}$$

and (9) becomes

$$dy+\frac{y\,dz}{1+2z}=\rho_\circ\frac{dz}{(1+2z)^{\frac{5}{2}}}$$

This is of the following well known form, usually called a linear equation.

$$dy+\mathrm{X}\,y\,dx=\mathrm{X}_1\,dx,$$

of which the integral is

$$y=e^{-\int \mathrm{X}\,dx}\int e^{\int \mathrm{X}\,dx}\,\mathrm{X}_1\,dx \quad . \quad . \quad (10)$$

Observing that $\mathrm{X}=\frac{1}{1+2z}$ and $\mathrm{X}_1=\frac{\rho_\circ}{(1+2z)^{\frac{5}{2}}}$, and using dz for dx in (10), because the expression is a function of z, not of x, and substituting in (10) and reducing gives

$$y=\frac{1}{\sqrt{1+2z}}\left(-\frac{\rho_\circ}{2(1+2z)}+\mathrm{C}\right)$$

But $y=0$ and $2z=0$ for $x=0$ $\therefore$ $\mathrm{C}=\frac{1}{2}\rho_\circ$,

$$\therefore\ y=\frac{\rho_\circ}{2}\left(\frac{1}{(1+2z)^{\frac{1}{2}}}-\frac{1}{(1+2z)^{\frac{3}{2}}}\right)$$

$$\text{or } (1+2z)^{\frac{3}{2}}=\frac{z\rho_\circ}{y}{}^{*} \quad . \quad . \quad . \quad (11)$$

$$\text{For convenience, make } 2z=u^2=\frac{dy^2}{dx^2}=\tan^2 i \quad . \quad . \quad (12)$$

$$\therefore\quad (1+u^2)^{\frac{3}{2}}=\frac{u^2\rho_\circ}{2y} \quad . \quad . \quad . \quad (13)$$

With (9), (12), and (13) eliminate y, dy, and d^2y, and we have

$$(1+u^2)^{\frac{5}{2}}+\frac{\rho_\circ u^2}{2}\frac{du}{dx}=\rho_\circ\frac{du}{dx}$$

$$\therefore\ \frac{dx}{\rho_\circ}+\frac{u^2du}{2(1+u^2)^{\frac{5}{2}}}=\frac{du}{(1+u^2)^{\frac{5}{2}}} \quad . \quad . \quad (14)$$

which by integration becomes

$$\frac{x}{\rho_\circ}+\frac{1}{2(1+u^{-2})^{\frac{3}{2}}}=\frac{1}{(1+u^{-2})^{\frac{1}{2}}}$$

$$\text{or } \frac{x}{\rho_\circ}(1+u^2)^{\frac{3}{2}}+\frac{u^3}{2}=u(1+u^2) \quad . \quad . \quad (15)$$

Substitute $(1+u^2)$ from (13) and reducing will give

$$u^2-u\frac{x}{y}=-2$$

* If in (11) we substitute $z=\frac{1}{2}\frac{dy^2}{dx^2}$ and reduce, we will obtain a differential equation between x and y, of the form of a complete cubic equation, which form is inconvenient even if it has a possible solution. Hence he obtains another equation between x and u and then eliminates u. D.V.W.

$$\therefore u = \frac{x}{2y} \pm \sqrt{\left(\frac{x}{2y}\right)^2 - 2} \quad . \quad . \quad . \quad (16)$$

which in (13) gives

$$\left[1 + \left(\frac{x}{2y} \pm \sqrt{\frac{x}{2y}}\right)^2 - 2\right]^{\frac{3}{2}} = \left[\frac{x}{2y} \pm \sqrt{\left(\frac{x}{2y}\right)^2 - 2}\right]^2 \frac{\rho_o}{2y} \quad (17)$$

which is the equation required, but it is too complex to be of *practical* use. By using the variable i, we obtain more convenient forms. Thus from (12) and (13) we obtain

$$y = \tfrac{1}{2} \rho_o \sin.^2 i \cos. i \quad . \quad . \quad . \quad (18)$$

Similarly from (12) and (15)

$$x = \rho_o (\sin. i - \tfrac{1}{2} \sin.^3 i) \quad . \quad . \quad (19)$$

$$= \tfrac{1}{2} \rho_o (1 + \cos.^2 i) \sin. i$$

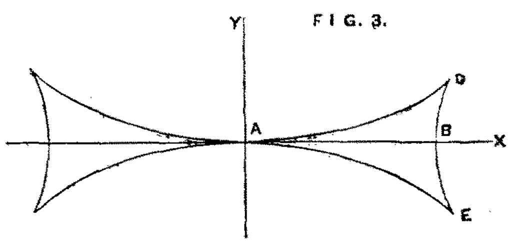
From (8) and (18)

$$\rho = \rho_o (1 - \tfrac{3}{2} \sin.^2 i) \quad . \quad . \quad (20)$$

From (18) and (19) we obtain the following table of co-ordinates for $\rho_o = 100$.

For $i = 0$	$y = 0\cdot$	$x = 0\cdot$
$i = 10°$	$y = 1\cdot485$	$x = 17\cdot096$
$i = 20°$	$y = 5\cdot495$	$x = 32\cdot200$
$i = 30°$	$y = 10\cdot825$	$x = 43\cdot350$
$i = 40°$	$y = 15\cdot825$	$x = 50\cdot998$
$i = 50°$	$y = 18\cdot860$	$x = 54\cdot132$
$i = 54° 44' 7''$	$y = 19\cdot446$	$x = 54\cdot430$
$i = 60°$	$y = 18\cdot750$	$x = 54\cdot210$
$i = 70°$	$y = 15\cdot100$	$x = 52\cdot480$
$i = 80°$	$y = 8\cdot421$	$x = 50\cdot625$
$i = 85°$	$y = 4\cdot480$	$x = 50\cdot200$
$i = 90$	$y = 0\cdot000$	$x = 50\cdot000$

FIG. 3.



with these, fig. 3 is constructed.

Discussion of the Curve.

1. From (18) and (19) we see that for the same values of i, x and y vary directly as ρ_o; hence if a table be made as above, for $\rho_o = 1$, they may be found for any other value of ρ_o, by simply multiplying the values in the table by that value.

2. To find the maximum values of x and y, differentiate (18) and (19), and place equal zero. From either we get

$$\sin.\ i=\sqrt{\tfrac{2}{3}} \qquad \cos.\ i=\sqrt{\tfrac{1}{3}} \qquad . \qquad . \quad (22)$$

or $i=54°\ 44'\ 7''$; hence both are maximum at the same point.

3. From (20) ρ is positive, for $\sin.^2 i$ less than $\frac{2}{3}$ and negative for $\sin.^2 i$ greater than $\frac{2}{3}$, and zero for $\sin.^2 i$ equal $\frac{2}{3}$. For $i=90$; $\rho=\frac{1}{2}\rho_o$.

4. From the preceding results we infer that there may be a cusp. Applying the test we first find from (18), (19), and (22) that

$$\frac{dy}{dx}=\frac{0}{0} \text{ for } i=54°\ 44'\ 7''$$

Substitute (22) in (18) and (19) and we have

$$x=\tfrac{2}{3}\rho_o\sqrt{\tfrac{2}{3}}. \qquad y=\tfrac{1}{3}\rho_o\sqrt{\tfrac{1}{3}} \qquad . \qquad . \qquad . \quad (23)$$

$\therefore\ \frac{x}{2y}=\sqrt{2}$ which in (17) gives an identical expression; also in (16) gives $u=\pm\sqrt{2}$.

Hence the first conditions are fulfilled. Now give increments to these values of x and y in (16) and it becomes imaginary; and by subtracting decrements, it remains real; hence there is a cusp at the point whose co-ordinates are given in Equation (23).

5. From (23) we have $\frac{x}{y}=2\sqrt{2}$, which being constant shows that the ratio of x and y for the maximum is independent of the loading or ρ_o.

6. Because there are two angles corresponding to every sin. and cosin.; therefore from (18) and (19) it appears that the curve is symmetrical in respect to both x and y. Hence there are four cusps, as shown in fig. 3.

7. To find where the curve cuts the axis of x, make $y=0$ in (18), which gives $\cos.\ i=0$ or $\sin.^2 i=0$. For the latter $i=0$, which is at the origin; for the former $i=90°$; which in (19) gives

$$x=\pm\tfrac{1}{2}\rho_o \qquad . \qquad . \qquad . \qquad . \quad (23a)$$

It may be asked why x and y are not continually increasing functions of each other, as in the parabola or catenary. The original premises will throw some light upon this point.

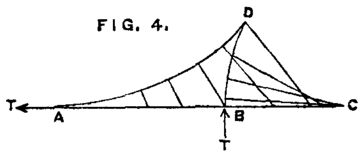
It will be remembered that the portion of w X between two radii of curvature produced, resolved normally, is the pressure. Now the further the curve be prolonged, the nearer horizontal is the radius of curvature, and if it could be prolonged so as to be horizontal, it would touch X at an infinite distance, making the load and hence the pressure infinite, also the tension infinite, which would not be consistent with Equation (2), unless the radius of curvature at the middle be infinite, which can be the case only when the curve becomes a straight line. From this popular reasoning we would conclude that the curve must return upon itself.

8. Equation (2) shows that when ρ is negative, the tension is negative, or otherwise it is compressive. This will enable us to explain the facts which pertain to the arc D B. After passing the cusp the arc is

concave to the force, (see fig. 4); hence it is compressive, but uniform and equal to the tension on the preceding part. By the curve returning as it does, the radius of curvature may become horizontal without intercepting an infinite amount of X. The exact value is hereafter shown to be ρ_0, see Equation (26a), hence the weight on this portion is $w\,\rho_0$, an amount equal the tension of the cable. See Equation (2).*

9. It will thus be seen, that if we have a chord A D, fig. 4, to resist tension, and a rod D B, to resist compression, bent into the proper curve, and forces T applied as in the figure, and a series of chords arranged normally, attached to a system of weights uniformly distributed over X, and a sufficient horizontal force applied at C to keep the chords normal; then will the combination remain in equilibrium.

FIG. 4.



10. *Length of Arc.* By calculus

$$ds = \frac{dy}{\sin. i}.$$

Differentiate (18), substitute and reduce, gives

$$ds = \frac{\rho_0}{4}(1 + 3\cos. 2\,i)\,d\,i$$

$$\therefore\ s = \tfrac{1}{4}\,\rho_0\,(i + \tfrac{3}{2}\sin. 2\,i) + c$$

But

$$s = 0 \text{ for } i = 0 \qquad \therefore c = 0$$

hence for both branches we have

$$2\,s = \tfrac{1}{2}\,\rho_0\,(i + \tfrac{3}{2}\sin. 2\,i) \qquad . \qquad . \qquad (24)$$

in which i is a linear quantity. If it be given in degrees, we use

$$\frac{\pi\,i}{180}$$

$$\therefore\ 2s = \tfrac{1}{2}\,\rho_0\left(\frac{\pi\,i}{180} + \frac{3}{2}\sin. 2\,i\right) \qquad . \qquad . \qquad (25)$$

The limits for AD are 0° and 54° 44′ 7″; for D B the limits are 54° 44′ 7″ and 90°.

For $i = 45°$, we have

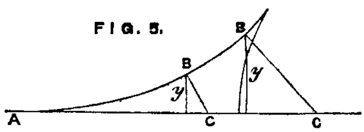
$$2s = \rho_0 \times 1{\cdot}14+$$

From (25) we see that a table whose argument is i, might be made, and calling $\rho_0 = 1$, we could find s for any other value of ρ_0 by simply multiplying by the assumed value.

* As the tension at A and C are horizontal, equal, and opposite, and the load acts vertically downward, and the tension at B vertically upward; we would infer from the principle of parallel forces that $T = w$ A C, or from eq. (26a) $T = w\,\rho_0$ as above. We see from (23a) and (26a) that B is midway between A and C; hence the whole system will balance upon B as a fulcrum, the same as if the roadway were perfectly rigid. D.V.W.

10. *To get an expression for* A C, *or one-half the span of a bridge.* See fig. 5.

FIG. 5.



Let $AC = B$

$\therefore B = x + y \text{ tang. } i,$

or combining with (18) and (19), we obtain after reduction,

$$B = \rho_0 \sin. i \quad . \quad . \quad . \quad . \quad (26)$$

At the cusp $\sin. i = \sqrt{\frac{2}{3}}$ $\quad \therefore B = \rho_0 \sqrt{\frac{2}{3}}$

For $i = 90°$ $\quad B = \rho_0 \quad . \quad . \quad (26a)$

If it be desired to attach the ties at equidistant points on the roadway, make $B = b', 2b', 3b'$, &c. in (26), and substitute the value of i thus formed in (25), and the points of attachment on the cable will thus become known.

11. *Length of the Ties.* Let $l = BC$, = the length of any one. Then, Fig. 5, and Eq. (18) give $l = y \sec. i = \frac{1}{2} \rho_0 \sin.^2 i \quad . \quad (27)$

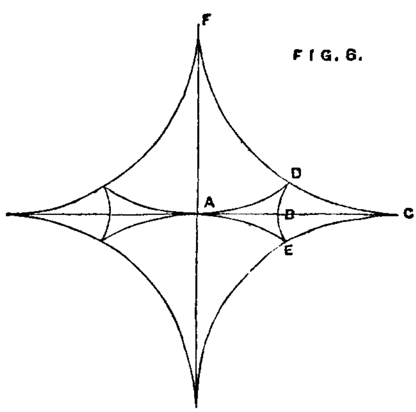
which with (26) becomes $= \dfrac{B^2}{2\rho_0}$ for the last tie.

12. *The Evolute of the Curve*, fig. 6.

Let x_1, and y_1, be the co-odinates of any point of the evolute, then we readily have; $\quad x_1 = x - \rho \sin. i$

$$y_1 = y + \rho \cos. i$$

FIG. 6.



These with (8), (19), and (20) give by eliminating $x, y, \rho. \sin. i$ and $\cos. i$;

$$y_1^{\frac{2}{3}} + x_1^{\frac{2}{3}} = \rho_0^{\frac{2}{3}}$$

which is the equation of a hypocycloid, when the radius of the generating circle is one-fourth that of the directing one*. It is represented in fig. 6. The curve is symmetrical with X and Y, and with axes inclined 45° to X and Y.

13. *Tension of the ties.* Suppose the ties are so distributed that the tension on each shall be equal, then it is required to find the points of attachment.

* This equation is deduced from a variety of problems, see Mathematical Monthly, vol. 1, p. 133.
Equation (17), or (18) and (19), are the equations of the involute of the hypocycloid, but the involute touches the evolute at the points given by eq. (23). The radius of the directing circle in this case would be ρ_0; of the generating circle $\frac{1}{4} \rho_0$.
We here have an easy mode of determining the length of the evolute FC, fig. 6. For FD evidently equals $FA = \rho_0$ and $DC = BC = \frac{1}{2} \rho_0$ (see 23a); hence the total length is $\frac{3}{2} \rho_0$, which agrees with other modes of solution.
By means of eqs. (18) and (19), we find the area ADBA to be

Let $d\,\text{P}$ = the amount of normal pressure on an element ds.

Then from (3) we have, considering B as variable, and using $d\,\text{B}$ for $m\,d\,\text{P} = w\,d\,\text{B}\sec.\,i$.

But from (26) $d\text{B} = \rho_0 \cos.\,idi$

$$\therefore d\,\text{P} = w\rho_0\,di \text{ or } \text{P} = w\,\rho_0 i \quad . \quad . \quad (28)$$

for the total normal pressure on one side. We see P varies as i.

Now assume the number of ties on half of one cable; say N; the tension on each will be $\text{P} \div \text{N}$; so for the first tie we have $\dfrac{\text{P}}{\text{N}\,w\,\rho_0} = i$, and this value of i in (24) will give s.

Substitute $2i$, $3i$, $4i$, &c. in (24), and we may find s for the 2d, 3d, 4th, &c., ties. The same in (26) gives the points of attachment to the roadway.

From the origin to the cusp, $\text{P} = w\,\rho_0 \times 0{\cdot}955+$

" " " intersection with X, $\text{P} = \frac{1}{2}\pi w\rho_0$

Hence the total normal pressure to the cusp is nearly equal the tension of the cable; and to the intersection with X, it is more than $1\frac{1}{2}$ times the tension.

14. *Horizontal Stress along the Roadway.*

An element of the horizontal force is $d\,\text{H} = wd\,\text{B}$ tang. i.

Find tang. i from (26) and substitute gives

$$d\,\text{H} = \frac{w\,\text{B}\,d\,\text{B}}{\sqrt{\rho_0{}^2 - \text{B}^2}}$$

$$\therefore \quad \text{H} = w(\rho_0 - \sqrt{\rho_0 - \text{B}^2}) \quad . \quad . \quad (29)$$

$$\text{Area} = \int y\,dx = \tfrac{1}{2}\rho_0{}^2 \int \left(\sin.^2 i \cos.^2 i - \frac{3}{2}\sin.^4 i \cos.^2 i\right) di$$

$$= \tfrac{1}{2}\rho_0{}^2 \left\{ \cos.i \left[\frac{1}{32}\sin.i + \cos.^2 i \sin.i \left(\frac{1}{4}\sin.i - \frac{1}{16}\right)\right] + \frac{1}{32}\sin.^{-1} x \right\}$$

For the area to a vertical ordinate through D, the limits are $i=0$ and $\sin.i = \sqrt{\frac{2}{3}}$

$$\therefore \text{the area} = \rho_0{}^2 \times 0{\cdot}03047+$$

For the total area ADB, the limits are $i=0$ and $i=90$

$$\therefore \quad \text{area} = \frac{1}{128}\pi\,\rho_0{}^2$$

For the area DAFD+DBCD, we may use the polar equation of the area;

$\int \frac{1}{2}\rho^2 di$ which with (20) gives

$$\tfrac{1}{2}\rho_0{}^2 \int \left(1 - 3\sin.^2 i + \frac{9}{4}\sin.^4 i \right) di$$

$$= \tfrac{1}{2}\rho_0{}^2 \left(\frac{11}{32} i + \frac{3}{16}\sin.2i + \frac{9}{128}\sin.^4 i \right)$$

which between the limits $i=0$ and $i = \frac{1}{2}\pi$ gives area DAFDBCD $= \dfrac{11}{128}\pi\,\rho_0{}^2$

Hence, the total area AFCA is

$$\left(\frac{1}{128} + \frac{11}{128}\right)\pi\,\rho_0{}^2 = \frac{3}{32}\pi\,\rho_0{}^2$$

a result which also agrees with the direct solution. D.V.W.

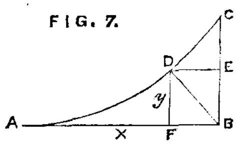
Observing that $H=0$ for $B=0$ $\quad\therefore\quad$ $C=w\rho_0$

Between the origin and cusp $H=w\rho_0 \times 0{\cdot}423+$.

This force is resisted either by compression in a rigid roadway, or by tension by fastenings at the ends. In the former the stress is greatest at the middle; in the latter, at the ends.

15. *To find ρ_0 when the span and deflection are given.*

FIG. 7.



Let $A=BC=$ height of the pier, fig. 7.

$B=AB$; $y=DF$; $x=AF$.

We have

$$B-x=y \text{ tang. } i.$$

which with (26) gives

$$B-x=\frac{By}{\sqrt{\rho_0^2-B^2}}$$

From the fig. $(B-x)^2=y\,(A-y)$; eliminating $B-x$ and reducing, gives

$$\rho_0^2-B^2=\frac{\rho_0^2 y}{A}$$

From (18) and (26) $\quad y=\frac{B^2}{2\rho_0}\sqrt{\rho_0^2-B^2}$

By eliminating y, and reducing, we have

$$\rho_0=\frac{B}{2A}\sqrt{B^2+4A^2} \qquad . \qquad . \qquad (30)$$

16. *Tension of the Cable.*

From (1) and (30)

$$T=w\rho_0=\frac{wB}{2A}\sqrt{B^2+4A^2}$$

which is identical with the expression for the tension at the pier heads of the ordinary suspension bridge, when the cable is the arc of a parabola. In the expression $T=w\rho_0$., T is independent of the span; hence the tension is the same for all spans, if ρ_0 and the load per unit of length remain the same.

17. *Total length of the cable between the pier heads.*

In fig. 7, $CD=(B-x)\sec. i$

From (19) and (26) $x=B-\frac{B^3}{2\rho_0^2}$

$$\therefore 2CD=\frac{B^3}{\rho_0^2}\sec. i.$$

which with (24) gives

$$2AC=L=\tfrac{1}{2}\rho_0\left(i+\frac{3}{2}\sin.2i\right)+\frac{B^3}{\rho_0^2}\sec. i \qquad . \qquad . \qquad (31)$$

which, with (26) and (30), will give L in terms of A and B.

The balance of the paper consists of remarks upon the relative merits of the system.