SUM RULE FOR NONLEPTONIC HYPERON DECAY *

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A simple sum rule is derived which relates the p-wave amplitudes to those of the s-wave of the non-leptonic hyperon decay.

The parity violating (s-wave) amplitudes of the non-leptonic hyperon decays have been successfully explained by various methods (current algebra [1], vector dominance model [2], the gauge field model [3,4] etc.), while the situation of the parity conserving (p-wave) amplitudes is yet to be clarified [5]. Most works on the latter rely on the pole model, which is not unambiguous with regard to the type of the vertex to be used.

In this note, we point out that the current algebra technique applied in the infinite momentum reference frame enables us to derive sum rules for the p-wave amplitudes. As a matter of fact, the soft pion limit in the decay process $Y \rightarrow B + \pi$, taken in the rest frame of the initial particle Y, implies that

$$m_Y = m_B \,, \tag{1}$$

while the same approximation in the rest frame of the pion leads to a Y particle momentum of

$$\sqrt{p_Y^2 + m_Y^2} = \frac{m_Y^2 - m_B^2 + \mu_\pi^2}{2\mu_\pi} \rightarrow \infty \,, \tag{2}$$

as $\mu_\pi \rightarrow 0$,

thus avoiding the condition (1) which makes the s-wave amplitudes vanish.

Our assumptions are

(I) The weak interaction Hamiltonian H_W is an octet [6] which satisfies the condition

$$[Q_5^\alpha, H_W^{\mathrm{pv(pc)}}] = [Q^\alpha, H_W^{\mathrm{pc(pv)}}] \,, \tag{3}$$

where Q_5^α and Q^α ($\alpha = 1, 2, \dots, 8$) stand for the axial- and vector-change respectively, and $H_W^{\mathrm{pv(pc)}}$ for the parity violating (conserving) part of H_W.

(II) The PCAC (Partially conserved axial-vector current) relation between the pion fields φ^α and the axial vector current A_μ^α,

$$\varphi^\alpha = \partial A_\mu^\alpha / F_\pi \mu_\pi^2 \, \partial X_\mu \,, \qquad (\alpha = 1,2,3) \,, \tag{4}$$

$\sqrt{2}\, F_\pi$ being the $\pi^\pm \rightarrow \mu^\pm \nu$ decay constant.

The standard application of the PCAC relation to the process $Y \rightarrow B\pi^0$, in the rest frame of the pion, gives

$$\begin{aligned} \sqrt{2q_0}\langle B(\boldsymbol{p}),\ \pi^0(0)|H_W^{\mathrm{pv(pc)}}|Y(\boldsymbol{p})\rangle = -\mathrm{i}\,\frac{(\mu_\pi^2 - q_0^2)}{F_\pi \mu_\pi^2}\Big\{&\langle B|[Q^3, H_W^{\mathrm{pc(pv)}}]|Y\rangle \\ &- q_0 \sum_l \left[\frac{\langle\ |B_0^3(0)|l\rangle\langle l|H_W^{\mathrm{pv(pc)}}|Y\rangle}{q_0 + p_0^B - p_0^l + \mathrm{i}\epsilon} - \frac{\langle B|H_W^{\mathrm{pv(pc)}}|l\rangle\langle l|A_0^3(0)|Y\rangle}{q_0 + p_0^l - p_0^Y - \mathrm{i}\epsilon}\right]_{\boldsymbol{p}^l = \boldsymbol{p}}\Big\} \end{aligned} \tag{5}$$

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If we take the limit $q_0 \to 0$ under constraint (2) i.e., $q_0|\boldsymbol{p}| \to \frac{1}{2}(m_Y^2 - m_B^2)$, then we obtain a complicated sum rule, since every intermediate state can contribute in the sum of eq. (5). In order to derive a simple result, we therefore take the limits successively* ($\lim_{q_0\to 0}$ first)

$$\mathrm{Lim} \equiv \lim_{|\boldsymbol{p}|\to\infty} \lim_{q_0\to 0} . \tag{6}$$

The only intermediate state which contributes to the sum is, then, that with the mass equal to one of the external baryons. Noting that $Q^3 = I_3$, the third component of the isospin, we obtain

$$\begin{aligned}\mathrm{Lim}\ \sqrt{2q_0}\langle \mathrm{B}\pi^0|H_{\mathrm{W}}^{\mathrm{pv(pc)}}|\mathrm{Y}\rangle = & \lim_{|\boldsymbol{p}|\to\infty} \frac{\mathrm{i}}{F_\pi}\{(I_3^{\mathrm{Y}} - I_3^{\mathrm{B}})\langle \mathrm{B}|H_{\mathrm{W}}^{\mathrm{pc(pv)}}|\mathrm{Y}\rangle + \\ & + \sum_{\substack{\text{spin sum}\\ \text{of the inter-}\\ \text{mediate particle}}} [\langle \mathrm{B}|A_0^3(0)|\mathrm{B}\rangle\langle \mathrm{B}|H_{\mathrm{W}}^{\mathrm{pv(pc)}}|\mathrm{Y}\rangle - \langle \mathrm{B}|H_{\mathrm{W}}^{\mathrm{pv(pc)}}|\mathrm{Y}\rangle\langle \mathrm{Y}|A_0^3(0)|\mathrm{Y}\rangle]\} ,\end{aligned} \tag{7}$$

which gives the expression for the s- and p-wave amplitudes, A and B, upon using tables 1, 2 and 3,

$$\frac{1}{\sqrt{2}}\{A(\Sigma_+^+) - A(\Sigma_-^-)\} = A(\Sigma_0^+) = D_{\mathrm{pc}} - F_{\mathrm{pc}} + g_{\mathrm{A}}(D_{\mathrm{pv}} - F_{\mathrm{pv}})\frac{m_\Sigma - m_{\mathrm{N}}}{m_\Sigma + m_{\mathrm{N}}}$$

$$\tfrac{1}{2}\{A(\Sigma_+^+) + A(\Sigma_-^-)\} = A(\Sigma_0^0) = -\frac{1}{\sqrt{2}}\left\{D_{\mathrm{pc}} - F_{\mathrm{pc}} - g_{\mathrm{A}}(D_{\mathrm{pv}} - F_{\mathrm{pv}})\frac{m_\Sigma - m_{\mathrm{N}}}{m_\Sigma + m_{\mathrm{N}}}\right\}$$

$$A(\Xi_-^-) = -\sqrt{2}A(\Xi_0^0) = \frac{1}{\sqrt{3}}\left\{D_{\mathrm{pc}} - 3F_{\mathrm{pc}} + (D-F)(D_{\mathrm{pv}} - 3F_{\mathrm{pv}})\frac{m_\Xi - m_\Lambda}{m_\Xi + m_\Lambda}\right\}$$

$$A(\Lambda_-^0) = -\sqrt{2}A(\Lambda_0^0) = \frac{1}{\sqrt{3}}\left\{D_{\mathrm{pc}} + 3F_{\mathrm{pc}} - [g_{\mathrm{A}}(D_{\mathrm{pv}} + 3F_{\mathrm{pv}}) + 2D(D_{\mathrm{pv}} - F_{\mathrm{pv}})]\frac{m_\Lambda - m_{\mathrm{N}}}{m_\Lambda + m_{\mathrm{N}}}\right\} \tag{8A}$$

and

$$\frac{1}{\sqrt{2}}\{B(\Sigma_+^+) - B(\Sigma_-^-)\} = B(\Sigma_0^+) = -\left\{D_{\mathrm{pv}} - F_{\mathrm{pv}} + g_{\mathrm{A}}(D_{\mathrm{pc}} - F_{\mathrm{pc}})\frac{m_\Sigma + m_{\mathrm{N}}}{m_\Sigma - m_{\mathrm{N}}}\right\}$$

$$\tfrac{1}{2}\{B(\Sigma_+^+) + B(\Sigma_-^-)\} = B(\Sigma_0^0) = \frac{1}{\sqrt{2}}\left\{D_{\mathrm{pv}} - F_{\mathrm{pv}} - g_{\mathrm{A}}(D_{\mathrm{pc}} - F_{\mathrm{pc}})\frac{m_\Sigma + m_{\mathrm{N}}}{m_\Sigma - m_{\mathrm{N}}}\right\}$$

$$B(\Xi_-^-) = -\sqrt{2}B(\Xi_0^0) = -\frac{1}{\sqrt{3}}\left\{D_{\mathrm{pv}} - 3F_{\mathrm{pv}} + (D-F)(D_{\mathrm{pc}} - 3F_{\mathrm{pc}})\frac{m_\Xi + m_\Lambda}{m_\Xi - m_\Lambda}\right\}$$

$$B(\Lambda_-^0) = -\sqrt{2}B(\Lambda_0^0) = -\frac{1}{\sqrt{3}}\left\{D_{\mathrm{pv}} + 3F_{\mathrm{pv}} - [g_{\mathrm{A}}(D_{\mathrm{pc}} + 3F_{\mathrm{pc}}) + 2D(D_{\mathrm{pc}} - F_{\mathrm{pc}})]\frac{m_\Lambda + m_{\mathrm{N}}}{m_\Lambda - m_{\mathrm{N}}}\right\} . \tag{8B}$$

Here $D, F, D_{\mathrm{a}}, F_{\mathrm{a}}$ (a = pv, pc) are the d- and f- type coupling parameters defined in tables 1 and 2. In computing the Σ decay amplitudes, we omitted the contribution form the Σ^+ state since it would not be a discrete state if the limit

$$m_\Sigma - m_\Lambda > q_0 \to 0 \tag{9}$$

* Of course, there is no guarantee that these two limits will give the same result in a mathematically rigorous sense, as in most applications of the limiting procedures in physics. Only a reasonable answer obtained is an a posteriori justification. Exception is the axiomatic field theory where a rigorous proof is required, since there is no experimental data to be compared with.

Table 1

The SU_3 Clebsch-Gordan coefficients C_a (Y→B) in the relation $\langle B|H_W^a|Y\rangle = 2F_\pi i C_a(Y\to B)\bar{u}_B\Gamma_a u_Y$

(a = pv or pc, $\Gamma_{pv} = \gamma_5$, $\Gamma_{pv} = 1$)

	$C_a(Y\to B)$
$\Sigma^+\to p$	$D_a - F_a$
$\Sigma^0\to n$	$-\frac{1}{2}\sqrt{2}(D_a - F_a)$
$\Lambda\to n$	$-\frac{1}{6}\sqrt{6}(D_a + 3F_a)$
$\Xi^0\to\Lambda$	$-\frac{1}{6}\sqrt{6}(D_a - 3F_a)$
$\Xi^-\to\Sigma^-$	$D_a + F_a$
$\Xi^0\to\Sigma^0$	$-\frac{1}{2}\sqrt{2}(D_a + F_a)$

Table 2

The SU_3 Clebsch-Gordan coefficients $G(B,B')$ in the relation $\langle B|A_0^3(0)|B'\rangle = G(B,B')\bar{u}_B\gamma_4\gamma_5 u_{B'}$

B=B' except the last row	$G(B,B')$
p	$\frac{1}{2}g_A = \frac{1}{2}(D+F)$
n	$-\frac{1}{2}g_A$
Λ	0
Σ^+	F
Σ^0	0
Σ^-	$-F$
Ξ^0	$-\frac{1}{2}(D-F)$
Ξ^-	$\frac{1}{2}(D-F)$
(Λ,Σ^0)	$\frac{1}{3}\sqrt{3}D$

Table 3

The infinite momentum limit of the Dirac bilinear forms, $\bar{u}_1\Gamma u_2$ (ξ being the Pauli's two component spinor, $\boldsymbol{p}$ the unit vector)

Γ	$\lim_{\lvert\boldsymbol{p}\rvert\to\infty}\bar{u}_1\Gamma u_2\, 2\sqrt{m_1 m_2}$
1	$(m_1+m_2)\xi_1^\dagger\xi_2$
γ_4	$2\lvert\boldsymbol{p}\rvert\xi_1^\dagger\xi_2$
γ_5	$(m_2-m_1)\xi_1^\dagger\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\xi_2$
$\gamma_4\gamma_5$	$-2\lvert\boldsymbol{p}\rvert\,\xi_1^\dagger\boldsymbol{\sigma}\cdot\hat{\boldsymbol{p}}\xi_2$

Table 4

Comparison of the sum rule (14) with experimental data (in the unit $10^5\ \mathrm{sec}^{-\frac{1}{2}}$)

	A	B	
	experiment=input	experiment	theory
Σ_+^+	0(0.02 ± 0.04)	19.08 ± 0.35	19.3
Σ_-^-	1.87 ± 0.03	-0.55 ± 0.43	0(0.14±0.28)
Ξ_-^-	-2.07 ± 0.03	7.42 ± 0.65	7.6
Λ_-^0	1.52 ± 0.02	10.44 ± 0.33	10.1

had been taken. In fact, we note that*

$$m_\Sigma - m_\Lambda = O(\mu_\pi^{2/3}) \text{ as } \mu_\pi\to 0 \tag{10}$$

This can be seen by taking the limit $\mu_\pi\to 0$ in the relation

$$\langle\Sigma|\frac{\partial A_\mu^\alpha}{\partial x_\mu}|\Lambda\rangle|_{\boldsymbol{p}_\Sigma=\boldsymbol{p}_\Lambda} = i(E_\Sigma - E_\Lambda)\langle\Sigma|A_0^\alpha(x)|\Lambda\rangle = \frac{F_\pi\mu_\pi^2}{\mu_\pi^2-(E_\Sigma-E_\Lambda)^2}\langle\Sigma|J_{\varphi\alpha}(x)|\Lambda\rangle|_{\boldsymbol{p}_\Sigma=\boldsymbol{p}_\Lambda} \tag{11}$$

$$(\alpha = 1,\ 2,\ 3)$$

where $J_{\varphi\alpha}$ is the source of the π^α field. On the other hand, in deriving the formulae for the Λ decay amplitudes, we have assumed that

$$m_\Sigma - m_\Lambda = O(\mu_\pi),\quad \text{as}\quad \mu_\pi\to 0, \tag{12}$$

so that the Σ^0 intermediate can contribute. This would be the case if the relevant amplitude depends upon the ratio $\mu_\pi/(m_\Sigma - m_\Lambda)$ in an essential way, e.g. term such as $\mu_\pi^2/\{\mu_\pi^2 \pm (m_\Sigma - m_\Lambda)^2\}$ appearing as an important factor. In such a case, we should take the limit with the ratio $\mu_\pi/(m_\Sigma - m_\Lambda)$ kept constant. Lacking knowledge of the mass dependence of the amplitudes, we rely on the experimental data to guide us in the choice of limiting procedure. In fact, taking the limit (10) for the Λ-decay amplitudes leads to the result that $B(\Lambda)$ is too large by factor 2.

In eq. (8), the parameters D and F are given by [7]

* If $F_\pi = O(\mu_\pi)$ as $\mu_\pi\to 0$, then we have $m_\Sigma - m_\Sigma = O(\mu_\pi)$, however.

$$D = 0.74 \pm 0.02, \quad F = 0.49 \pm 0.02, \quad (g_A = 1.23 \pm 0.03)\,, \tag{13}$$

and the other parameters, D_a, F_a, (a = pv, pc) are unknown. Eliminating the latter, we obtain relations among the A and B amplitudes. The first two are

$$B(\Sigma^+_+) = g_A \frac{m_\Sigma + m_N}{m_\Sigma - m_N} A(\Sigma^-_-)\,, \tag{14Σ^+}$$

$$B(\Sigma^-_-) = \frac{1}{g_A} \frac{m_\Sigma + m_N}{m_\Sigma - m_N} A(\Sigma^+_+) = \sqrt{2}(D_{pv} - F_{pv})\,. \tag{14Σ^-}$$

In order to simplify the result, we assume that

$$A(\Sigma^+_+) = 0 = D_{pv} - F_{pv}. \tag{15}$$

Then the rest of the sum rule reads,

$$B(\Xi^-_-) = -(D-F)\frac{m_\Xi + m_\Lambda}{m_\Xi - m_\Lambda} A(\Xi^-_-) + \frac{2}{\sqrt{3}}(1-(D-F)^2)D_{pv}\,, \tag{14Ξ}$$

$$B(\Lambda^0_-) = [g_A A(\Lambda^0_-) - \sqrt{\tfrac{2}{3}}\, D A(\Sigma^-_-)] \frac{m_\Lambda + m_N}{m_\Lambda - m_N} + \frac{4}{\sqrt{3}}(g_A^2 - 1)D_{pv}\,, \tag{14Λ}$$

where

$$\frac{1}{\sqrt{3}} A(\Lambda^0_-) + \frac{2}{\sqrt{3}} A(\Xi^-_-) + \frac{1}{\sqrt{2}} A(\Sigma^-_-) = \frac{-4}{\sqrt{3}}\left[g_A \frac{m_\Lambda - m_N}{m_\Lambda + m_N} + (D-F)\frac{m_\Xi - m_\Lambda}{m_\Xi + m_\Lambda}\right] D_{pv}\,. \tag{16}$$

Table 4 gives the numerical result. The agreement with the experimental data is excellent. Some remarks are in order.

i) The smallness of $B(\Sigma^-_-)$ is a result of $A(\Sigma^+_+) = 0$. The experimental fact

$$\frac{P(\Sigma^+_+)}{S(\Sigma^-_-)} = \left[\frac{(m_\Sigma - m_N)^2 - \mu_\pi^2}{(m_\Sigma + m_N)^2 - \mu_\pi^2}\right]^{1/2} \frac{B(\Sigma^+_+)}{A(\Sigma^-_-)} = 1.02 \pm 0.04 \tag{17}$$

suggests an empirical relation

$$g_A = \frac{m_\Sigma - m_N}{m_\Sigma + m_N}\left[\frac{(m_\Sigma + m_N)^2 - \mu_\pi^2}{(m_\Sigma - m_N)^2 - \mu_\pi^2}\right]^{1/2} = 1.19\,. \tag{18}$$

The relations (14) and (18) explain the observed form of the Σ triangle (the isosceles right-triangle of Gell-Mann-Rosenfeld).

ii) The left hand side of eq. (16) is the Lee-Sugawara [8] combination under the constraint (15). The approximate validity of their relation for the s-wave amplitudes stems from the appearance of the mass difference on the right hand side of eq. (16). (The parameter D_{pv} calculated from the experimental data is 1.23.) On the other hand, the even better Lee-Sugawara relation for the p-wave cannot be explained in a simple manner.

iii) If $A(\Sigma^+_+)$ is small but nonvanishing, then our prediction is

$$S(\Sigma^+_+)/g_A P(\Sigma^+_+) = g_A P(\Sigma^-_-)/S(\Sigma^-_-) \tag{19}$$

i.e.

$$\alpha(\Sigma^+_+) = g_A^2 \alpha(\Sigma^-_-)\,. \tag{20}$$

We give the present experimental numbers [7], $\alpha(\Sigma^+_+) = 0.026 \pm 0.042$ and $\alpha(\Sigma^-_-) = 0.060 \pm 0.047$, although we are not sure about the validity of our approximation (soft pion limit) in computing such small quantities.

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