EFFECT OF FLUCTUATIONS ON THE THERMAL CONDUCTIVITY OF He I NEAR T_λ *

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A microscopic calculation of the contribution of order-parameter fluctuations to the thermal conductivity of liquid helium I near T_λ is shown to give results in agreement with experiments and the dynamic scaling hypothesis.

The anomalous increase in the thermal conductivity [1,2] in helium I near T_λ has been successfully analyzed in terms of the dynamic scaling hypothesis [3,4], mode-mode coupling [5] and the generalized Landau theory [6]. The above theories are phenomenological in nature.

Aslamazov and Larkin [7] have calculated microscopically the contribution of the order-parameter fluctuations to the electric conductivity of a normal metal above the superconducting transition in the classical range. Tsuzuki [8] has extended the calculation to the critical range.

The purpose of this letter, which is the analog of refs. 7 and 8 for the superfluid transition, is to report a microscopic calculation of the contribution of order-parameter fluctuations to the thermal conductivity of helium I near T_λ.

Recently the authors [9] have been able to reformulate the theory of Patashinskii and Pokrovskii [10] to describe consistently the order-parameter fluctuations in the experimentally accessible temperature range near T_λ. The order parameter appropriate to the superfluid transition is the Bose field operator $\psi(r,t)$, and hence order-parameter fluctuations are described by the fluctuation propagator $G(ll') = -i\langle T\{\psi(l)\,\psi^+(l')\}\rangle$, $l = \eta, t_l$. The Fourier transform can be written as $G^{-l}(k,\ \omega \pm i0^+) = \omega - W(k,\omega) \mp i\frac{1}{2}\Gamma(k,\omega)$. The microscopic theory gives the following results for small k and ω near T_λ:

$$W(k,\omega) = W(k,0) \approx \begin{cases} A_1 k^2 + \hat{\eta} & 0 < k < k_1 \\ A_0 k^{\frac{3}{2}} + \hat{\eta} & k_1 < k < k_0 \\ k^2/2m + \hat{\eta} & k_0 < k \end{cases} \quad (1)$$

and $\Gamma(k,\omega) = \gamma_0 \omega$, where $A_0^4 = (2/15\pi)(T_\lambda V_0)^2$, $A_1^3 = (T_\lambda V_0)^2/432\pi^2\,\hat{\eta}$, $\hat{\eta} = \frac{3}{2}nV_0\tau$, $\tau = (T-T_\lambda)/T_\lambda$, $V_0 = \int dr V(r)$, $V(r)$ is the interparticle potential, the momenta k_0, k_1 satisfy $A_0 k_0^{\frac{3}{2}} = k_0^2/2m$ and $A_1 k_1^2 = A_0 k_1^{\frac{3}{2}}$ respectively, and γ_0 is a constant. An essential feature of the present theory is the damping Γ of the critical fluctuations, which was ignored in ref. 10. Since the damping is of the same order of magnitude as the real part, it is misleading to refer to helium near T_λ as behaving like an ideal gas of quasi-particles.

The thermal conductivity κ can be found from eq. (88b) in ref. 11 in terms of the auto correlation function of $j' = j^q + \lambda g$. We use the notation of ref. 11 and choose $\lambda = (p + Ts)/nm$, which gives

$$j' = [Ts(1) - \frac{Ts}{n}n(1) + p(1) - p + Ts]v(1) \quad (2)$$

If we ignore the fluctuations of entropy and density, (2) reduces to $j' = Ts v$, and the contribution of order parameter fluctuations to κ is

$$\kappa' = \frac{1}{2} s^2 \lim_{\omega \to 0} [\ \lim_{\to 0} \langle v_x(k,\omega)\, v_x(-k,-\omega)\rangle]. \quad (3)$$

The correlation function $\langle v\,v\rangle$, which has the structure of a two-particle correlation function, is factorized into a product of two fluctuation propagators in analogy to refs. 7 and 8. The corresponding integral for κ' is

$$\kappa' = \frac{1}{6}\frac{s^2}{m^2}\frac{1}{\beta}\sum_n \int \frac{dp}{(2\pi)^3}p^2\,Q^2\,G(p,\omega_n+i0^+)\,G(p,\omega_n). \quad (4)$$

The vertex function Q is determined by the Ward identity

$$kQ = -m\,\nabla_k G^{-1}(k,0) = 2mA_1 k. \quad (5)$$

Integrals similar to (4) are performed in refs. 7 and 8, and we find

$$\kappa' = (32\pi)^{-1}(T_\lambda s/n)^2 \gamma_0(\hat{\eta}A_1)^{-\frac{1}{2}} \propto \tau^{-\frac{1}{3}}. \quad (6)$$

It is seen that the contribution of order-parameter fluctuations to κ diverges as $\tau^{-\frac{1}{3}}$ consistent with the phenomenological theories [3-6] except for the absence of logarithmic factors. The extended dynamical scaling [4] and mode-mode coupling [5] theories give a critical temperature dependence for κ' proportional to $\tau^{-\frac{1}{3}}C_p^+/C_p^{-\frac{1}{2}}$ where C_p^+ and C_p^- is the specific heat at constant pressure above and below T_λ respectively*. The factors of C_p arise in ref. 5 from the normalization of the entropy fluctuations which have been neglected here. The origin of C_p factors in the other theories can also be considered as arising from the entropy fluctuations. The pro-

* Refs. 3 and 6 obtain $\tau^{-\frac{1}{3}}C_p^{-\frac{1}{2}}$.

blem of including the contribution of entropy fluctuations to the thermal conductivity into a consistent, microscopic theory is discussed in ref. 9.

References
1. G. Ahlers, Phys. Rev. Letters 21 (1968) 1159.
2. M. Archibald, J. M. Mochel and L. Weaver, Phys. Rev. Letters 21 (1968) 1156.
3. R. A. Ferrell et al., Ann. Phys. (NY) 47 (1968) 565.
4. B. I. Halperin and P. C. Hohenberg, Phys. Rev. 177 (1969) 952.
5. J. Swift and L. P. Kadanoff, Ann. Phys. 50 (1968) 312.
6. D. Stauffer and V. K. Wong, to be published.
7. L. G. Aslamazov and A. I. Larkin, Soviet Phys. - Solid State 10 (1968) 875.
8. T. Tsuzuki, Prog. Theoret. Phys. (Kyoto) 41 (1969) 41.
9. H. Gould and V. K. Wong, to be published.
10. A. Z. Patashinskii and V. L. Pokrovskii, Soviet Phys. - JETP 19 (1964) 677.
11. L. P. Kadanoff and P. C. Martin, Ann. Phys. (NY) 24 (1963) 419.

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IONICITIES, EFFECTIVE CHARGES, DIELECTRIC CONSTANTS, AND ATOMIC POLARIZABILITIES OF III-V SEMICONDUCTORS

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Relative sharing times of a valence electron are used to calculate particular dielectric constant and polarizabilities of the constituent atoms from optical effective charges and dielectric constants of the compound.

The real chemical bond in III-V compounds is usually described as a superposition of an ionic structure $A_{III}^{3+} B_V^{3-}$ (probability w) and a covalent structure $A_{III}^{1-} B_V^{1+}$ (probability $1-w$). For both the electrons in one of the four tetraedrical bonds follows:

$$\Psi^2(1,2) = w \cdot \psi_{ion}^2(1,2) + (1-w) \cdot \psi_{cov}^2(1,2). \quad (1)$$

Disregarding overlap (for the values of overlap integrals of III-V compounds see ref. 1 we put $\psi_{cov}(1,2) = \frac{1}{2}\sqrt{2}(\Phi_{III}(1) \cdot \Phi_V(2) + \Phi_{III}(2)\Phi_V(1))$ for the covalent structure and $\psi_{ion}(1,2) = \Phi_V(1)\Phi_V(2)$ for the ionic structure, resp., (Φ_{III}, Φ_V –normalized electronic wave functions of the III- and the V-atom, resp., in the crystal.).

Integrating (1) over variables of one electron we see that a valence electron spends $\frac{1}{2}(1\mp w)$ of its time in the wave function of the III-atom and the V-atom, resp. (These sharing times can be

used for lending weight to atomic relativistic effects of constituents to predict the same effects of the compounds [1].) This means, the net number of valence electrons is $8(1\mp w)/2$ at the III-atomic site and the V-atomic site, resp., in the crystal. For $w = 1/4$ the atoms posses their natural number of valence electrons; the bond is neutral (configuration $A_{III}^0 B_V^0$).

Introducing different optical dielectric constants ϵ_{III} and ϵ_V for the III-atoms and V-atoms, resp., in the crystal [2], one can deduce "macroscopic" charges $e_{III/V}^* = 8(1\mp w)/2 \, \epsilon_{III/V}$ from the "microscopic" net charges mentioned above. Their difference produces the effective charge e_c^* defined by Callen [3], which splits the longitudinal and transverse optical frequencies.

$$e_c^* = (e_V^* - e_{III}^*) \cdot 1/2 \quad (2)$$

The sum of the "macroscopic" charges per bond