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Final Report

PANEL FLUTTER OF CYLINDRICAL SHELLS

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PART A

AEROELASTIC STABILITY OF PLATES AND CYLINDERS

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ABSTRACT

Linear stability criteria are presented for the panel flutter of thin plates and thin-walled cylinders. These structures are exposed to fluid flow passing parallel to an outer surface. The expression for fluid pressure is simplified in order to emphasize the dynamic properties of the systems. The pressures are derived from steady flow relations (frequency effects are ignored). An arbitrary spatial phase angle is included in the pressure expression. As this phase angle is varied in a continuous manner, the fluid flow passes from "subsonic" character to supersonic character. The results are useful in classifying several types of instability and discussing several pathological cases which are usually treated separately.

The analysis is intended to serve as an aid to understanding the mechanism of panel flutter; however, it can be applied directly to several problems. It is accurate for the static divergence and "coupled mode" flutter of flat panels in supersonic flow, and also for divergence problems wherever experimental measurements can supply the values for the necessary aerodynamic parameters. One result is to point out the

importance of static instability for flat panels in a transonic viscous flow. A second result is to illustrate that the asymmetric divergence of cylindrical shells is very sensitive to small changes in the pressure distribution.

A, $\tilde{A}$	Aerodynamic pressure parameter; $\dfrac{\rho U^2 L^3}{\sqrt{\lvert M^2 - 1 \rvert} D}$, $\dfrac{\rho U^2 R^3}{\sqrt{\lvert M^2 - 1 \rvert} D}$
D	$Eh^3/12(1 - \nu^2)$
F	Airy stress function
h	Panel thickness
K_m	Aerodynamic pressure constant, Eq. (5)
L	Length of panel
M	Mach number
m	Axial wave number
N	Number of modes
N_x	Axial stress resultant due to initial load
N_θ	Circumferential stress resultant due to initial load
$p(x,t)$	Aerodynamic load
q	Integer
R	Radius of cylinder
t	Time
U	Flow velocity
w	Panel displacement in transverse direction
x	Spatial coordinate, flow direction
z	Spatial coordinate
δ_{qm}	Kronecker delta
ϵ	Amplitude constant
θ	Angular coordinate
λ	Eigenvalue
$\tilde{\lambda}$	Eigenvalue

ρ	Fluid density
ρ_s	Panel density
σ_x	Axial stress in cylindrical shell due to shell motion
ψ_m	Spatial phase shift
ω	Frequency, rad/sec

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1. INTRODUCTION

The elastic instability of thin panels exposed to fluid flow is under intensive study at the present time. Typical problems involve thin-walled structural elements with one surface exposed to fluid flow essentially parallel to the surface. Figure 1 illustrates the flow situation for a flat plate and a cylinder. The usual question of interest is whether the elastic panels incur divergence (static instability) or flutter (dynamic instability) at some value of flow velocity.

The fluid pressures exerted on oscillating panels are difficult to derive in many cases. The role of fluid viscosity, frequency of oscillation, and panel geometry have complicated panel flutter studies to the point where the results are often difficult to understand.

The present study is based on an intuitive simplification of the pressure distribution on the panel. It illustrates the effect of the spatial distribution of pressures. The pressures are taken from steady flow results and are hence independent of the frequency of oscillation. The results are valid only for instabilities occurring at relatively low frequencies.

An approximate solution is required because of the nature of the assumptions on the pressures. These assumptions are equivalent to a specification of the generalized forces on a discrete system. Galerkin's method is used to pose the eigenvalue problem in matrix form.

2. FLUID PRESSURES

The pressure expression used in this study is motivated by the solution for flow over an infinitely long, two-dimensional stationary wavy wall (Fig. 2). For the case of inviscid, isentropic flow, one finds that a deflection

$$w(x) = \epsilon \sin \frac{2\pi x}{\ell} \qquad (1)$$

yields a pressure of the form

$$p(x) = \epsilon \frac{\rho U^2}{\sqrt{|M^2 - 1|}} \frac{2\pi}{\ell} \cos\left(\frac{2\pi x}{\ell} + \psi\right) \qquad (2)$$

where ψ takes the value 0 for a supersonic flow and $\pi/2$ for subsonic flow. The solution is not valid near Mach 1.

The pressure expression given in Eq. (2) is "exact" within the framework of linearized potential flow for the stationary wall under consideration. We will view this expression, however, as an approximation which has been provided to describe a given physical situation: a panel of finite length with viscous flow effects, real gas effects, etc. As an example, for transonic flow, McClure[1] measured pressures of the form

$$p(x) = \frac{\epsilon \rho U^2}{\sqrt{|M^2 - 1|}} \frac{2\pi}{\ell} K \cos\left(\frac{2\pi x}{\ell} + \psi\right) \qquad (3)$$

for a stationary wavy wall. The constants K and ψ are functions of Mach number, fluid properties and wavelength. McClure found the amplitude constant K to be near unity. His measured values of ψ ranged from

20 to 45°. We hence see that values of ψ lying between 0 and 90° have physical significance in practical cases.

Let us consider the pressure expression, Eq. (3) as sufficient for our purposes. We will generalize this expression slightly by using subscripts to show the dependence of the constants K and ψ upon the wavelength. For a given deflection of a wall

$$w(x,t) = e^{i\omega t} \sum_{m=1}^{N} a_m \sin \frac{m\pi x}{L} \qquad (4)$$

one then has a pressure expression of the form

$$p(x,t) = e^{i\omega t} \frac{\rho U^2}{\sqrt{|M^2 - 1|}} \sum_{m=1}^{N} a_m \frac{m\pi}{L} K_m \cos\left(\frac{m\pi x}{L} + \psi_m\right) \qquad (5)$$

Note that each term in Eq. (4) represents a wave with length $\frac{2L}{m}$.

In the following examples, it will be assumed that the constants K_m and ψ_m are known. (This is equivalent to assuming that the generalized forces are known for the discrete system.) For example, if slender wing (Ackeret) theory were used for supersonic flow over a finite panel, Eq. (5) would result with $K_m = 1$ and $\psi_m = 0$ for all m.

3. FLAT PANEL OF FINITE LENGTH

Consider the case of a two-dimensional flat panel exposed to fluid flow over one surface Fig. 3. The plate is of uniform thickness, length L and simply supported at both ends. The aerodynamic expression of Eq. (5) will be used to provide fluid pressures above the panel.

The fluid below the panel is at rest and at the same static pressure as the upper flow.

The equation of motion for small deflections of the plate is

$$D \frac{\partial^4 w}{\partial x^4} - N_x \frac{\partial^2 w}{\partial x^2} + \rho_s h \frac{\partial^2 w}{\partial t^2} + p(x,t) = 0 \qquad (6)$$

and the boundary conditions are

$$w(0,t) = w(L,t) = \frac{\partial^2 w}{\partial x^2}(0,t) = \frac{\partial^2 w}{\partial x^2}(L,t) = 0$$

The solution is assumed to be of the form

$$w(x,t) = e^{i\omega t} \sum_{m=1}^{N} a_m \sin \frac{m\pi x}{L}$$

Galerkin's method yields a set of coupled, linear algebraic equations of motion

$$\sum_{m=1}^{N} \left\{ \left[(m\pi)^4 + \frac{N_x \ell^2}{D} (m\pi)^2 - m\pi A K_m \sin\psi_m - \lambda \right] \delta_{mq} + A K_m \cos\psi_m \, \eta_{mq} \right\} a_m = 0 \qquad (q = 1,2,\ldots N) \qquad (7)$$

where

$$\lambda = \frac{\rho_s h \omega^2 L^4}{D} \qquad \eta_{mq} = \begin{cases} \dfrac{4mq}{m^2 - q^2} & \text{if } m + q \text{ is odd} \\ 0 & \text{if } m + q \text{ is even} \end{cases}$$

$$A = \frac{\rho U^2 L^3}{\sqrt{|M^2-1|}\, D}$$

and δ_{mq} is the Kronecker delta.

This is a linear eigenvalue problem in the eigenvalue λ. It is non-Hermitian and hence in general we may have complex eigenvalues.

The characteristic polynomial is solved for the eigenvalue as a function of A and $\frac{N_x L^2}{D}$.

To interpret the stability of the system, we must remember that the frequency of oscillation varies as the square root of the eigenvalue:

$$\omega \propto \lambda^{1/2}$$

and hence

$$w(x,t) \propto e^{i\lambda^{1/2}t}$$

The square root must be considered a multivalued function of the complex variable λ. If all eigenvalues λ are real and positive, then neutral stability results. If λ is real and negative, static divergence occurs. If λ is complex, then flutter occurs.

Results have been calculated for the stability of a panel with no membrane tension ($N_x = 0$). Extensive experience with Galerkin's method as applied to fourth order differential equations has shown excellent convergence when four modes are used. Two-mode, four-mode, and eight-mode calculations were used here; the results were found to converge adequately.

The stability boundaries shown in Fig. 4 are from a four-mode analysis. For this special case, the amplitude constants and the spatial phase shift have been set equal for all modes:

$$K_1 = K_2 = K_3 = K_4 = K$$

$$\psi_1 = \psi_2 = \psi_3 = \psi_4 = \psi$$

As a result, the amplitude constant is easily incorporated into the ordinate. The figure hence emphasizes the role played by ψ.

The panel is stable for sufficiently low values of A, regardless of the value for ψ. As A increases, however, the panel becomes unstable at some critical value of A. This can be either divergence or flutter, depending on the value of ψ.

It is interesting that for $\psi = 0$ ("supersonic" flow) only flutter is possible. (Experimental evidence indicates that this theoretical solution is correct for $\psi = 0$.) Also, for $\psi = 90°$ ("subsonic" flow) only divergence is possible. These limiting cases are well known. On the other hand, for phase angles ψ between 25° and 90°, one encounters divergence first and then flutter.

The results for small values of ψ, say from 0° to 40° are important. In transonic flow, for instance, ψ depends upon boundary layer thickness, fluid viscosity, etc. If a given test were carried out for varying boundary layer properties, the type of instability might well change from a dynamic type to a static type because of this spatial phase shift. (It must be remembered that the present analysis cannot predict the single-degree-of-freedom type of flutter which often typifies transonic flow. On the other hand, this analysis is "exact" for simply supported plates which diverge and hence is sufficient to predict static instability.)

For phase angles ψ near 90°, one finds that increasing dynamic pressure causes first a static divergence, followed by dynamic instability and finally a static divergence. This might be a confusing factor in some subsonic experimental work, where spatial phase angles might be near, but not exactly, 90°.

4. ASYMMETRIC FLUTTER OF A CYLINDER OF FINITE LENGTH

The stability of a finite elastic cylinder Fig. 1 will be investigated in the same spirit as the flat panel. The shell is of uniform thickness and unstiffened. Conventional cylindrical coordinates x, r, Θ will be used. Donnell's cylinder equations are adequate to describe the deflections of interest here:

$$D\nabla^4 w - N_x \frac{\partial^2 w}{\partial x^2} - \frac{N_\Theta}{R^2}\frac{\partial^2 w}{\partial \Theta^2} + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \rho_s h \frac{\partial^2 w}{\partial t^2} + p(x,t) = 0 \quad (8)$$

$$\nabla^4 F - \frac{Eh}{R}\frac{\partial^2 w}{\partial x^2} = 0 \quad (9)$$

The boundary conditions are taken to be the freely-supported case:

$$v = w = \frac{\partial^2 w}{\partial x^2} = \sigma_x = 0 \qquad (\text{at } x = 0, L)$$

Again, for a deflection of the form

$$w(x,\Theta,t) = e^{i\omega t} \cos n\Theta \sin \frac{m\pi x}{L}$$

the fluid forces will be taken as

$$p(x,\Theta,t) = \frac{\rho U^2}{\sqrt{|M^2 - 1|}} e^{i\omega t} (\cos n\Theta) K_m \frac{m\pi}{L} \cos\left(\frac{m\pi x}{L} + \psi_m\right)$$

If one again applies Galerkin's method to the equations of motion (8) and (9), one obtains a system of linear algebraic equations

$$\sum_{m=1}^{N} a_m \left[\left\{ \left[\left(\frac{m\pi R}{L}\right)^2 + n^2 \right]^2 + 12(1-\nu^2)\left(\frac{R}{h}\right)^2 \left(\frac{m\pi R}{L}\right)^2 \left[\left(\frac{m\pi R}{L}\right)^2 + n^2 \right]^{-2} + \frac{N_x R^2}{D} \left(\frac{m\pi R}{L}\right)^2 \right.\right.$$

$$\left. + \frac{N_\theta R^2}{D} n^2 - \tilde{\lambda} - \tilde{A} K_m \frac{m\pi R}{L} \sin \psi_m \right\} \delta_{qm}$$

$$\left. + \left\{ \tilde{A} K_m \frac{m\pi R}{L} \cos \psi_m \right\} \eta_{qm} \right] = 0 \qquad (q = 1,2,\ldots N)$$

where

$$\tilde{\lambda} = \frac{\rho_s h \omega^2 R^4}{D}$$

$$\tilde{A} = \frac{\rho U^2}{\sqrt{|M^2 - 1|}\, D}$$

and η_{qm} is defined as for the plate.

These equations can be solved for the eigenvalues $\tilde{\lambda}$ as a function of the fluid dynamic pressure ratio $\tilde{A}$ and the phase shift ψ. We will consider numerical results for a case corresponding to wind tunnel tests carried out by Olson [2].

$N_x = 0$

$N_\theta = 0$

$R = 8.00$ inch

$h = 0.004$ inch

$\ell = 15.4$ inch

$\nu = 0.35$

$n = 28$

We will again choose

$\psi_1 = \psi_2 = \ldots \psi_n = \psi$

$K_1 = K_2 = \ldots K_n = K$

The results for a four mode solution are given in Fig. 5. Here it is seen, as for a flat plate, that for $\psi = 0$ only flutter can occur. For values of ψ between 60° and 120°, there is an unexpected result. The case of static divergence does indeed occur, but at relatively large values of $\tilde{A}$. In this case, if ψ is not exactly 90°, then flutter can occur at much lower values of $\tilde{A}$.

This analysis shows the danger inherent in using an aerodynamic theory which predicts that $\psi = 90°$ exactly. Resulting calculations might not reveal a flutter situation which occur at a much lower dynamic pressure ratio.

Note that the flutter boundary is very insensitive to changes in ψ from -30° to 60°. This means that the details of the pressure distribution on the cylinder are not of much importance in the stability analysis. This explains why one of the simplest aerodynamic theories, Ackeret theory, can be used with success to predict cylinder flutter which occurs at low frequencies [3].

5. CONCLUSIONS

The appearance of a spatial phase shift as a free parameter in the fluid pressure expression results in some new observations. It illustrates the change, in a continuous manner, from subsonic (or slender body) flow character to supersonic character. The intermediate values of the phase angle have physical application to the cases of viscous transonic flow over flat plates and supersonic flow over cylindrical shells.

The analysis is limited to two types of elastic instability: coupled mode flutter and divergence. The study cannot predict single degree-of-freedom flutter because of the use of steady flow relations for the fluid forces.

Several examples were studied in which the pressure amplitudes K_m were identical in all modes and the phase angles ψ_m were identical in all modes. This case was chosen because of its simplicity. Conclusions for the flat plate and the cylinder will be discussed separately.

The flat plate exhibits both divergence and flutter. For one range of the spatial phase angle ψ (-90° to -60°), the plate is stable for all dynamic pressures. For a second range of ψ (-60° to 25°), only flutter is possible. Finally, for a third range of ψ (25° to 90°), divergence is the critical form of instability, occurring at a much lower dynamic pressure than flutter. The stability diagram indicates that experiments carried out for certain phase angles might be confusing in the sense that different regions of stability and instability could be observed in turn as the dynamic pressure is raised.

Divergence occurs for flat plates at a relatively low value of dynamic pressure ratio. As a result, divergence may be a distinct problem for the case of viscous transonic flow, where previous pressure measurements indicate that the necessary phase shift does occur [1].

The cylinder example studied was for a particular cylinder geometry, chosen to match the only successful experiments to date. The cylinder exhibits coupled mode flutter over the entire phase angle range of physical interest. This flutter boundary is surprisingly insensitive to the value of ψ. This is fortunate from a practical standpoint. It means that coupled mode flutter calculations can be carried out for such a shell with less attention paid to the details of the spatial pressure distribution.

The occurrence of divergence for the cylinder is not a simple phenomenon. In the past, divergence has been predicted for some types of cylinders in supersonic flow (where axial wavelengths are long compared to circumferential wavelengths). For the cylinder studied here the divergence would be of little practical interest. Very small phase shifts from $\psi = 90°$ cause flutter to occur at much lower dynamic pressures than divergence.

It is not prudent to extend the results of this simple analysis too far. On the other hand, it can serve as a qualitative aid to investigators in panel flutter. There are times when the methods of analysis are so cumbersome that one restricts his techniques (or his interest) to only divergence or to flutter. It is apparent that one must be careful to not overlook one of the possible instabilities.

6. ACKNOWLEDGEMENT

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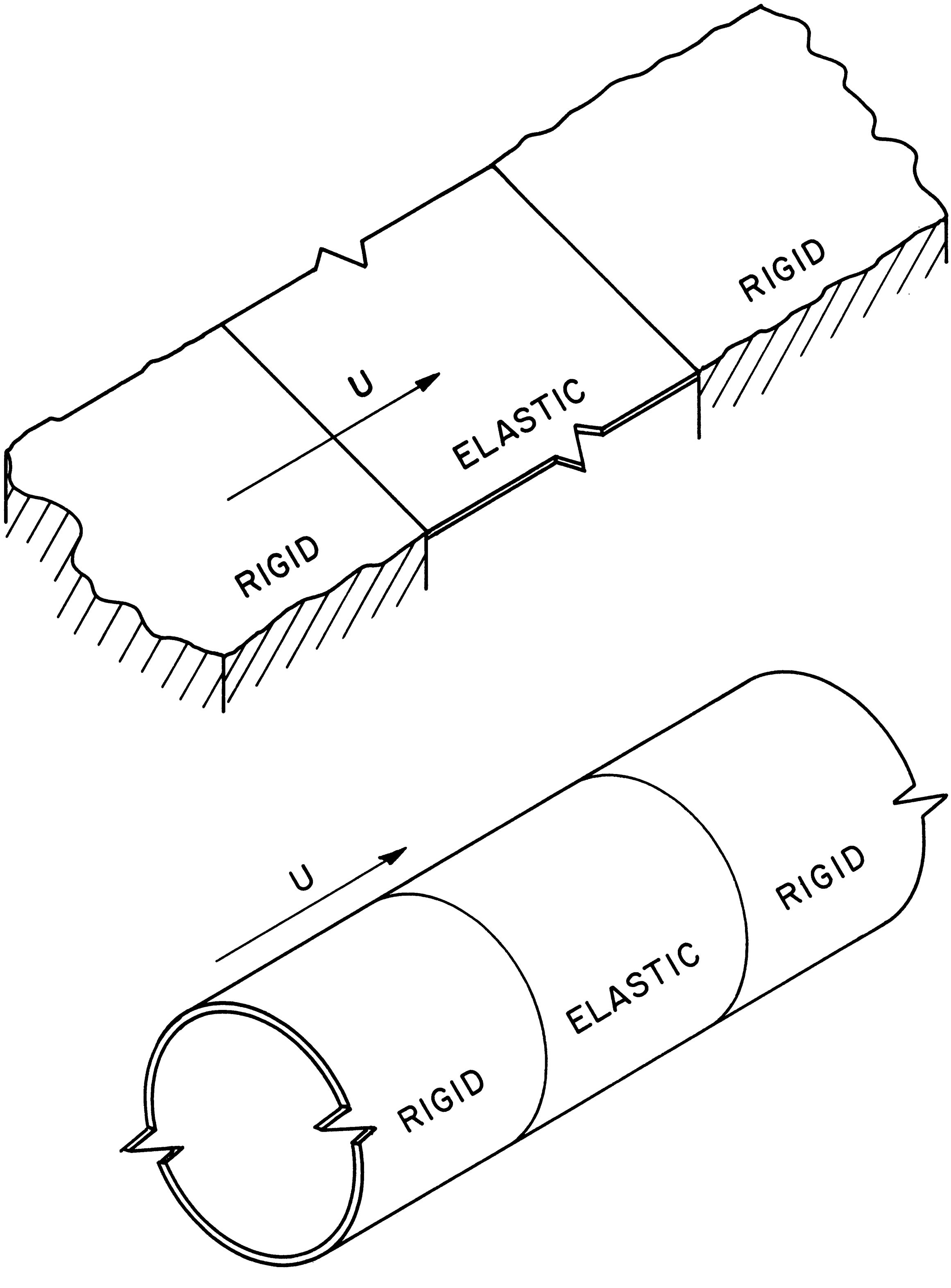

Fig. 1. Typical panel flutter problems.

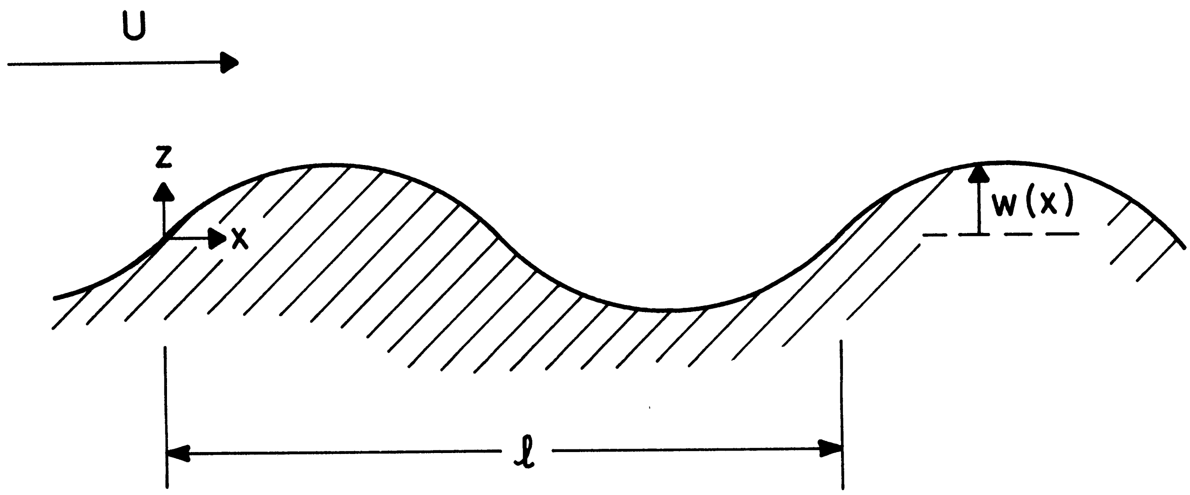

Fig. 2. Flow over an infinitely long, stationary, two-dimensional wavy wall.

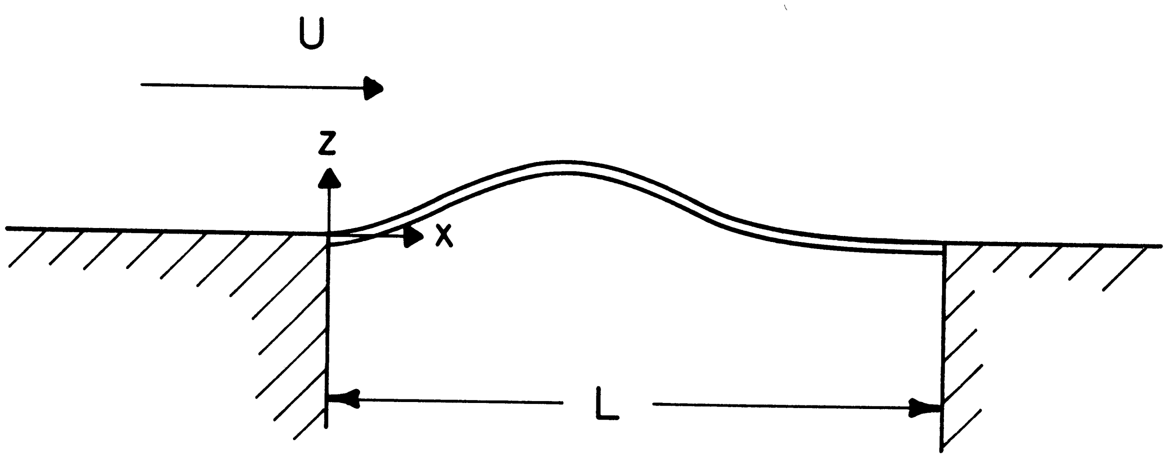

Fig. 3. Flow over a two-dimensional flat panel.

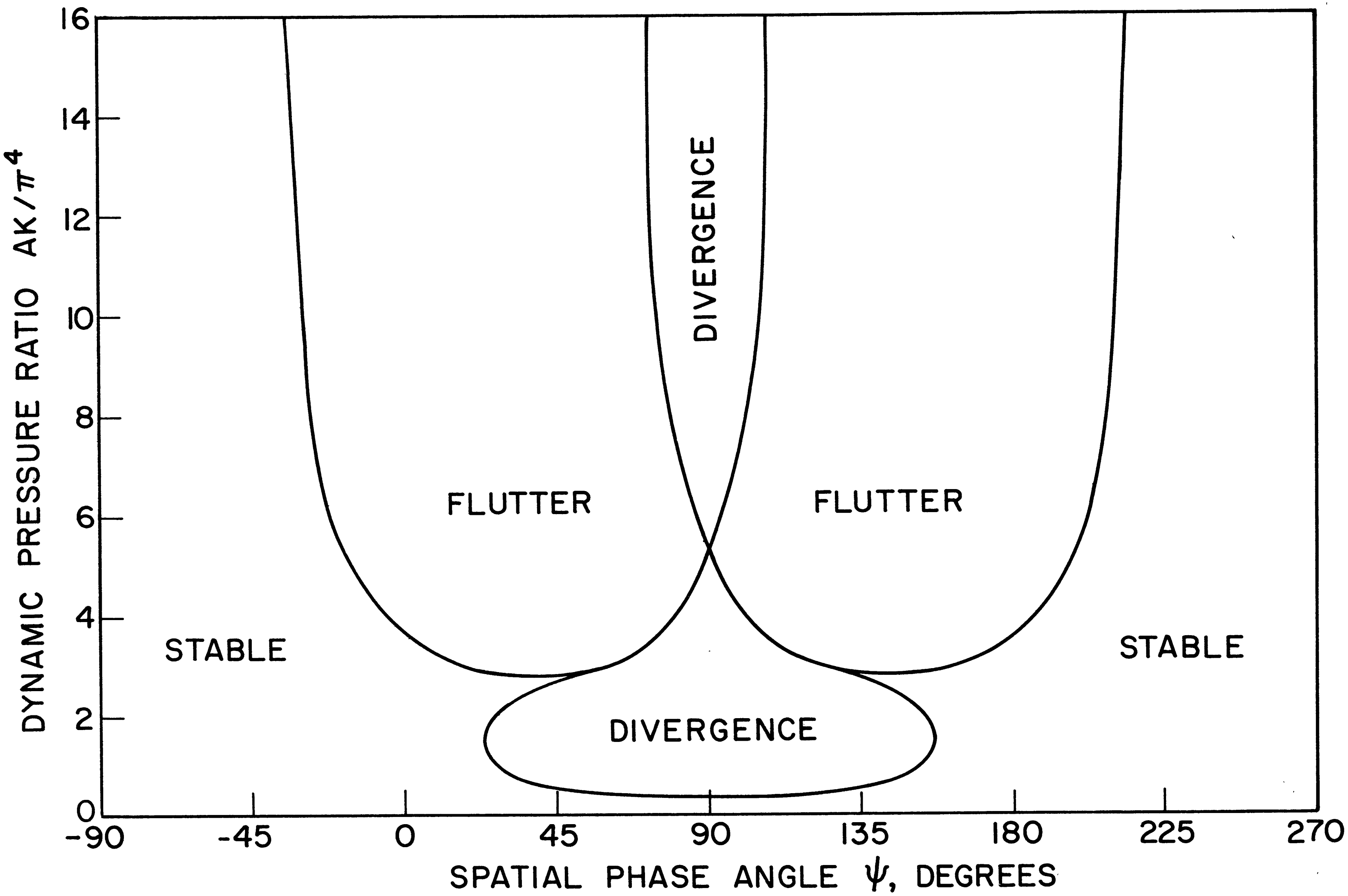

Fig. 4. Stability boundaries for a flat plate.

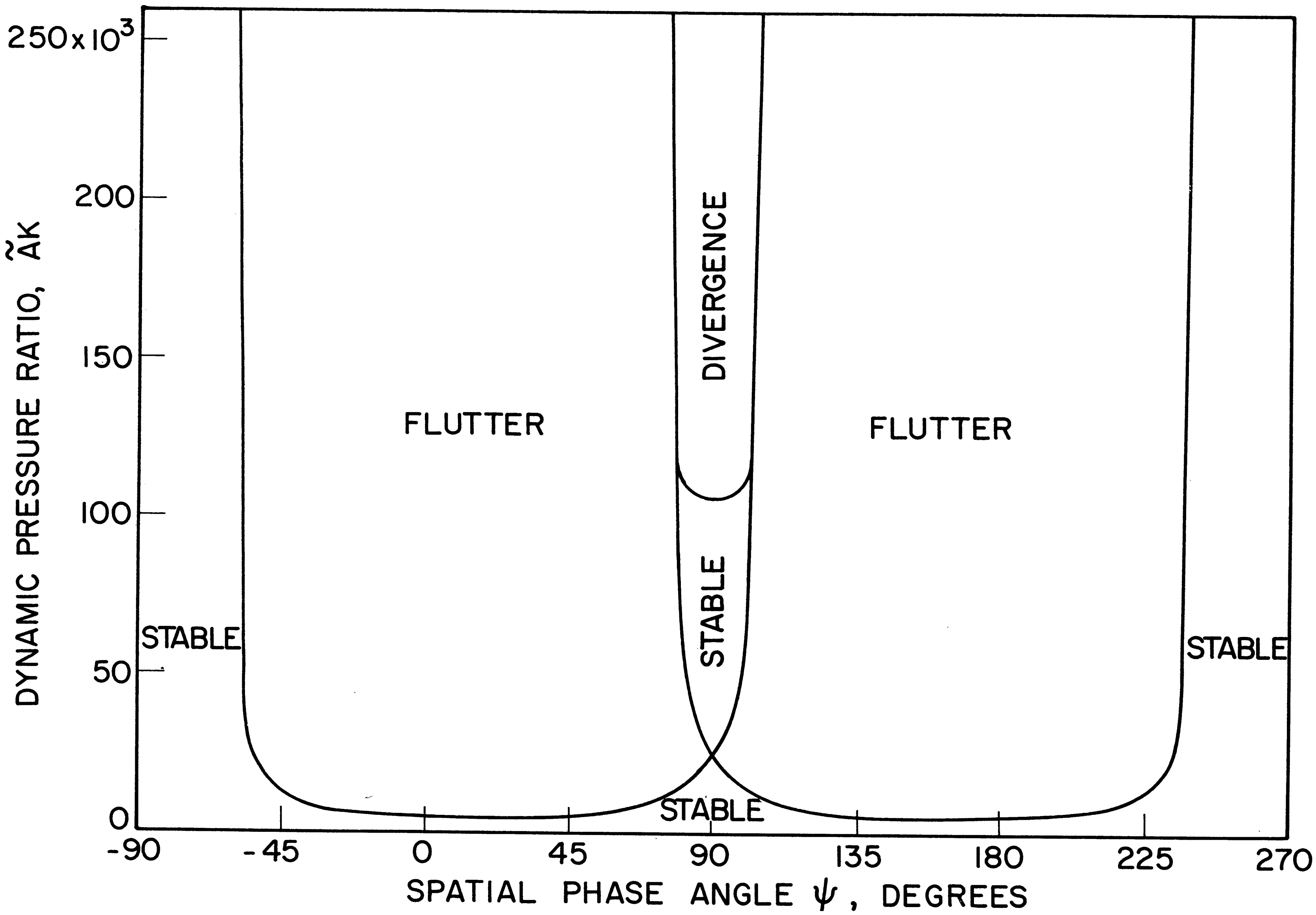

Fig. 5. Stability boundaries for a cylinder.

PART B

ENGINEERING ESTIMATES FOR SUPERSONIC FLUTTER OF CURVED SHELL SEGMENTS

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ABSTRACT

Static aerodynamic theory is used to find design curves for the flutter of curved panels. The panels are rectangular segments cut from a circular cylindrical shell. Supersonic flow is directed parallel to the generators of the shell segment. The pressure expression used is general enough to encompass a wide range of physically possible pressure distributions. Design curves are given in the form of a thickness parameter required to prevent flutter as a function of curvature and length-to-width ratio. Upper and lower bounds for the onset of coupled-mode panel flutter are given. Comparisons with other theories and experiments are made. The results are intended to aid in design of wind tunnel models for panel flutter tests.

D	$Eh^2/[12(1-\nu^2)]$
F	Airy stress function
H	Thickness parameter, $\left[\frac{\sqrt{M^2-1}}{(1-\nu^2)}\frac{E}{q}\right]^{1/3}\frac{h}{L}$
h	Panel thickness
L	Length of panel
M	Mach number
m	Axial wave number
N	Number of modes
N_x, N_Θ	Stress resultants, see equations (5) and (6)
$p(x,\Theta,t)$	Aerodynamic load
q	Integer, also dynamic pressure
R	Radius
t	Time
V	Flow velocity
W	Width of panel
W_{eff}	Effective width of panel, W/n
w	Panel displacement in radial direction
x	Spatial coordinate, flow direction
Z	Curvature parameter, $\frac{L}{R}\frac{L}{h}\sqrt{1-\nu^2}$
δ_{qm}	Kronecker Delta
Θ	Angular coordinate
Θ_o	Included angle of shell segment
$\hat{\lambda}$	Eigenvalue
ρ	Fluid density

ρ_s	Panel density
ψ	Spatial phase shift
ω	Frequency, rad/sec

LIST OF FIGURES

1. INTRODUCTION

There is a need for rough estimates of panel flutter boundaries in design work. One specific area which has not been studied extensively involves the flutter of a rectangular panel with curvature (Fig. 1). This panel would in general form a portion of a cylindrical body (Fig. 2), and would be supported at its edges by heavy bulkheads or stringers. Some portions of the outer skin of a missile would correspond to this case. For conventional aircraft, such a panel might represent a window, where the window is relatively weak compared to the surrounding structure. Such windows can be a problem in high speed flight where temperature lowers their rigidity.

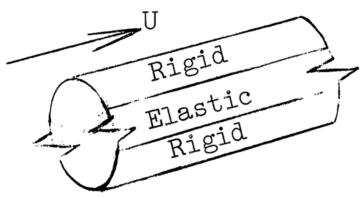
The exact mathematical solution to this problem is so difficult, and the results dependent on so many parameters, that there is serious doubt whether it is of any practical value. On the other hand, recent research [1] for cylindrical shells indicates that approximate results can be found by using a steady flow (quasi-static) theory. The approach taken here is to recommend a set of design curves developed by a simple theory with the intent that corrections to these curves are to be made as experimental data are obtained. It is felt that the design parameters used here are somewhat universal and will be the ones which will prove useful in the long run, even after more precise theories are available.

In the mathematical development of the problem, we will remain within the framework of linear shell theory and steady flow theory. A modal approach will be used, hence the aerodynamic problem resolves to finding pressures on sinusoidally deflected walls. The aerodynamic pressure distribution used in this problem will be generalized in the manner studied in Ref. [2].* A parameter ψ is introduced to typify the spatial pressure distribution. Then ψ is allowed to vary over the range of values which can be expected for such a panel under different physical conditions including boundary layer effects and length-to-width ratio effects. The result is an approximate theory which gives upper and lower limits to the panel thickness requirement to prevent flutter. The upper limit corresponds to the use of Ackeret theory, the lower limit corresponds loosely to a "slender body" type of theory. These two bounds represent extremely different flow situations, yet the dynamics of the system are so insensitive to the details of the pressure distribution as to cause a variation from upper to lower bound of only 35% for most cases.

A series of figures will be presented for design purposes. These should be especially useful in designing models for wind tunnel testing.

Previous work has been done on related problems. Dzygadlo [3] studied the elastic instability of an infinitely long elastic segment of an infinitely long cylinder. The stability boundaries were found for a traveling wave form:

$$w(x,\theta,t) = w(\theta)\, e^{i\alpha(x-V_t)}$$

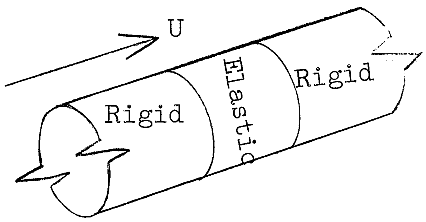


*Part A of this report.

A set of integro-differential equations of motion resulted. These were solved with the aid of a Fourier series in the Θ variable. Much effort was placed on a study of the effect of structural (Voigt) damping on the stability boundaries. For moderate amounts of damping, unexpected changes in the panel's stability resulted. The numerical results presented were not extensive. It was concluded that for small damping ratios and for fixed shell thickness and radius, the critical Mach number does not vary greatly for included angles for the segment lying between $\pi/4$ and π.

Another study of interest was by Dowell and Widnell [4]. The case considered was a finite length elastic segment in an infinitely long rigid cylindrical shell. In this case, the generalized aerodynamic forces were found for deflections of the type

$$w(x,\Theta,t) = e^{i\omega t} \cos n\Theta \sin \frac{m\pi x}{L}$$



Dowell made several comments about the stability of the shell segment merely by looking at the character of the generalized forces. First of all, in the low supersonic Mach number range, a single degree of freedom type of flutter is possible. Secondly, for shell segments with long length-to-width ratios, static divergence takes place. Flutter boundaries for the "coupled-mode" type of flutter were not presented.

Neither of these studies is easy to extend to the current problem. Neither case yields useful design curves (nor were they intended to). The approach used by Dowell would be the more easy to extend to the present case.

The present solution parallels the approach used by McElman [5] to some extent. McElman studied a curved orthotropic panel segment by using a two mode analysis with Ackeret theory. No design curves of the type shown here were presented in McElman's work. (In order to work with lower aspect ratio panels, one needs many modes rather than two.)

2. STATEMENT OF PROBLEM

Consider a cylindrical shell segment as shown in Fig. 1. Supersonic flow passes over the outer surface of the segment, with flow direction parallel to the cylinder axis. The segment is of uniform thickness and of isotropic, homogeneous elastic material. Conventional cylindrical coordinates x, r, θ are used. The shell segment is defined by

$$r = R$$

$$0 \leq x \leq L$$

$$\frac{-\theta_0}{2} \leq \theta \leq \frac{\theta_0}{2}$$

Deflection of the surface of the segment will be given by $w(x,\theta,t)$ measured from the mean radius of the shell. The edges of the shell will be "freely-supported" as defined below. The shell may be internally pressurized. No structural damping will be included.

STRUCTURAL DETAILS

The shell is thin and initially circular. Radial deflections are restricted to be small:

$$\frac{w(x,\theta,t)}{h} \ll 1$$

The in-plane motions of the shell $u(x,\theta,t)$ and $v(x,\theta,t)$ are small compared to $w(x,\theta,t)$ so that inertial effects due to in-plane motion can be neglected (Reissner's assumption). The included angle θ_o will be less than $\pi/2$ so that Donnell's shallow shell equations can be used.

$$D\nabla^4 w - \overline{N}_x \frac{\partial^2 w}{\partial x^2} - \frac{\overline{N}_\theta}{R^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \rho h \frac{\partial^2 w}{\partial t^2} + p(x,\theta,t) = 0 \quad (1)$$

$$\nabla^4 F - \frac{Eh}{R}\frac{\partial^2 w}{\partial x^2} = 0 \quad (2)$$

where D is the bending rigidity of the shell, $\overline{N}_x$ and $\overline{N}_\theta$ are constants representing the components of membrane stress due to internal pressurization and $F(x,\theta,t)$ is the stress function defined so that

$$\widetilde{N}_x(x,\theta,t) = \frac{1}{R^2}\frac{\partial^2 F}{\partial \theta^2} \quad (3)$$

$$\widetilde{N}_\theta(x,\theta,t) = \frac{\partial^2 F}{\partial x^2} \quad (4)$$

Note that $\widetilde{N}_x$ and $\widetilde{N}_\theta$ are the time dependent components of membrane stress due only to panel motion. The total membrane stresses are

$$N_x(x,\theta,t) = \overline{N}_x + \widetilde{N}_x(x,\theta,t) \quad (5)$$

$$N_\theta(x,\theta,t) = \overline{N}_\theta + \widetilde{N}_\theta(x,\theta,t) \quad (6)$$

Boundary conditions to be applied at $x = 0$, $x = L$ are

$$v = w = \frac{\partial^2 w}{\partial x^2} = \frac{\partial^2 F}{\partial \theta^2} = 0 \quad (7)$$

Boundary conditions at $\Theta = \pm \frac{\Theta_o}{2}$ are

$$w = \frac{\partial^2 w}{\partial \Theta^2} = u = \frac{\partial^2 F}{\partial x^2} = 0 \tag{8}$$

These freely supported boundary conditions have been chosen primarily because they are satisfied (term by term) by the series

$$w(x,\Theta,t) = e^{i\omega t} \sum_{m=1}^{N} a_m \cos \frac{n\pi\Theta}{\Theta_o} \sin \frac{m\pi x}{\ell} \quad \begin{pmatrix} 0 \le x \le L \\ \frac{-\Theta_o}{2} \le \Theta \le \frac{\Theta_o}{2} \end{pmatrix} \tag{9}$$

These boundary conditions are useful, however, because they result in a dynamically "weak" plate. Hence, the stability boundaries will tend to be conservative for design purposes when applied to damped plates.

At this point, the structural problem has been posed. We need to find the aerodynamic pressures $p(x,\Theta,t)$ generated at the panel surface.

AERODYNAMIC DETAILS

A strong assumption on the aerodynamic pressures will be made. The pressure on a panel deflection

$$w(x,\Theta,t) = \epsilon\, e^{i\omega t} \cos \frac{n\pi\Theta}{\Theta_o} \sin \frac{m\pi x}{L} \tag{10}$$

will be assumed to be

$$p(x,\Theta,t) = \epsilon\, e^{i\omega t} \frac{\rho U^2}{\sqrt{M^2-1}} \frac{m\pi}{L} \cos \frac{n\pi\Theta}{\Theta_o} \cos\left(\frac{m\pi x}{L} + \psi\right) \tag{11}$$

In other words, the pressure will have a magnitude equal to that given by Ackeret theory and a spatial pressure distribution that can vary as desired. (It would be possible to discuss this same assumption later in terms of generalized forces, but this is not as meaningful.)

The above assumption appears to be a strong oversimplification at first glance. One wonders how to choose a proper value of ψ for a cylindrical shell segment. In Reference 6, extensive numerical work was done for pressures on oscillating cylinders exposed to potential flow. The phase angle ψ was found to vary only between 0° and 90°. Hence, we will include values of ψ between 0° and 90° in the present study. In Reference 2, the dynamic results of such an assumption are studied in detail. It is found that the choice of ψ does not drastically affect the thickness requirement for cylindrical shells.*

STABILITY DETAILS

Galerkin's method is used to pose the problem in matrix form. The deflections of the shell segment are

$$w(x,\Theta,t) = e^{i\omega t} \cos \frac{n\pi\Theta}{\Theta_o} \sum_{m=1}^{N} a_m \sin \frac{m\pi x}{L} \tag{12}$$

Note that this expression allows n half waves in the circumferential direction of the panel. If n takes a value higher than 1, then the effective length-to-width ratio of the panel increases accordingly because there are stationary nodal lines down the length of the panel.

*Footnote: The thickness required to prevent flutter is a continuous function of ψ. For the case studied in Ref. [2], the thickness requirement has a minimum near $\psi = 30°$. This value of the thickness ratio at $\psi = 30°$ is practically identical with that at $\psi = 0°$. Because the calculation for $\psi = 0°$ has more physical meaning (Ackeret theory) it is used as a reference rather than $\psi = 30°$.

The expression for pressure, Eq. (11), is used in conjunction with Eqs. (1) and (2) to yield the set of linear algebraic equations of motion:

$$\sum_{m=1}^{N} a_m \left[\left\{ [m^2 + (\frac{L}{W}_{eff})^2]^2 + \frac{12\ Z^2\ m^4}{\pi^4 [m^2 + (\frac{L}{W}_{eff})^2]^2} + \frac{N_x L^2}{\pi^2 D} m^2 + \frac{N_\Theta L^2}{\pi^2 D} (\frac{L}{W})^2_{eff} \right. \right.$$

$$\left. \left. - \hat{\lambda} - \frac{24\ m\pi}{\pi^4 H^3} \sin \psi \right\} \delta_{qm} + \frac{24}{\pi^4 H^3} \eta_{qm} \cos \psi \right] = 0$$

$$(q = 1, 2, \ldots n)$$

where:

$$H = \left[\frac{\sqrt{M^2 - 1}\ E}{(1 - \nu^2)\ q} \right]^{1/3} \frac{h}{L}$$

$$Z = \frac{L}{R} \frac{L}{h} \sqrt{1 - \nu^2}$$

$$L/W_{eff} = \frac{Ln}{R\Theta_o}$$

$$\hat{\lambda} = \frac{\rho_s h w^2 L^4}{\pi^4 D}$$

$$\eta_{qm} = \begin{cases} 0 & m+q \text{ even} \\ \dfrac{4qm}{q^2 - m^2} & m+q \text{ odd} \end{cases}$$

Thus, a set of linear algebraic equations are obtained. The occurrence of a negative eigenvalue $\hat{\lambda}$ signifies static divergence of the panel and complex $\hat{\lambda}$ signifies flutter.

3. RESULTS

Stability boundaries have been calculated for the aerodynamic loading discussed above. All results will be given for cases with zero membrane stresses $\overline{N}_x$ and $\overline{N}_\Theta$. This theory would be more inaccurate at positive values of membrane stress which would cause higher flutter frequencies.

The results are presented using the effective length-to-width ratio L/W_{eff}, a curvature parameter Z and a thickness parameter H. The plots of H versus L/W_{eff} are given as a generalization of the work of Kordes, Tuovila, and Guy [7], and the curvature parameter Z is chosen to correspond to Batdorf's study of cylinder buckling [8].

A four mode solution for $\psi = 0$ (Ackeret theory) is given in Fig. 3. It is easily seen that curvature helps to stiffen the panel and reduce the thickness requirement. An interesting effect is obtained in the regions where H increases with increasing L/W_{eff}. This means that a panel of given physical length and width will flutter in a mode with $n > 1$, giving a higher critical value of L/W_{eff}. As an example, a panel of length 10 inches and width 2 inches has a physical length-to-width ratio of 5. If $Z = 8000$ for this panel then it must have a thickness ratio of $H = 0.065$ to prevent flutter from occurring at an effective length-to-width ratio of 15. This particular panel flutters with $n = 3$, i.e., it has two interior nodal lines extending down its length.

Results for $\psi = 90°$ are given in Fig. 4. These results are somewhat similar to the $\psi = 0$ curves except that the instabilities in the lower left corner are due to static divergence. Again, one must observe the cases where H increases with L/W_{eff} and one must choose the multiple of the geometric length-to-width ratio which gives the critical value of H.

Finally, several of the curves for $\psi = 0°$ and $\psi = 90°$ are combined in Fig. 5. These are approximate bounds for the thickness required to prevent an instability. Note that the difference between the two bounding curves is not great, particularly in certain intermediate regions of L/W_{eff}. This may be an indication as to why Ackeret theory gives relatively good results for the cylinder experiments discussed in Ref. 1. In these tests, a cylinder fluttered in a mode which yields an effective L/W_{eff} = 9.21 and with Z = 6,950. Flutter occurred at H = 0.0624.

It is felt that figures such as Fig. 5 can be very useful to designers. The curves give rough bounds for the onset of flutter or divergence for freely supported panels. As experiments are carried out, confidence can be obtained for accuracy of such curves.

It is suspected that for L/W_{eff} large, more modes are needed to ensure convergence. Gaspars and Redd [9] studied carefully the number of modes required for convergence on finite aspect ratio flat plates when Ackeret theory is used. They found that as many as 50 modes were needed for flat plates with aspect ratios of 10 or more. The present results are less sensitive to convergence problems because of the presence of curvature and because the flutter parameter H is less sensitive to error in the eigenvalue of the matrix problem.

Other theories and experiments are shown in Fig. 6. Several of the points shown correspond to work for full cylindrical shells. The problem of a shell segment is closely related to that of a full cylinder. Structurally, the major difference is that the full cylinder can flutter in modes with waves travelling in the circumferential direction whereas the segment cannot. Of particular interest in Fig. 6 are the experimental points found for full cylinders by Olson and Fung [1] and by Stearman, Lock and Fung [10]. It is now suspected that these cylinders did flutter in circumferentially travelling waves [11]. This might explain why the experimental values occurred at slightly higher values of thickness ratio than predicted by the present theory. The experiments of Tuovila and Hess [12] were carried out for a shell segment clamped all around. The tests were done at Mach 1.3, which unfortunately brings in transonic effects into the comparison. In transonic flow the unsteady aerodynamic terms are of importance and there effects are neglected in this theory.

The theories of Voss [13] and Shulman [14] both were done for a complete cylinder with the use of Ackeret theory. These should (and do) correspond with the present calculations and serve as a check.

The theory of Dyzgadlo [15] was carried out for a more exact aerodynamic theory on a finite length cylinder. These were mode calculations. 'hese appear to yield values of H slightly higher than the current work which may reflect the fact that fewer modes were used by Dzygadlo. (Gaspars and Redd [8] indicate that the thickness requirement decreases with an increase in the number of modes.)

All in all, there are no unclassified experiments known to the authors which furnish the proper comparison with the theory. Such tests would be useful.

Figures 7-9 are cross plots of the same data given in Figs. 3-5.

4. CONCLUSIONS

Design curves have been given for prevention of aeroelastic instability of curved shell segments. The calculations are approximate in the sense that unsteady aerodynamic effects are ignored and because a modal approach was used. These are the very reasons that the results are understandable, however. From a practical standpoint, these design curves, as corrected by experiment, will probably be more useful than exact theories.

The only case illustrated here was the case of freely supported edges. The results should be conservative if applied to panels with calmped edges.

One shortcoming of the current calculations is the limited number of modes used. Only four mode solutions were carried out. If more modes were used, the results presented at higher values of L/W_{eff} would become more accurate.

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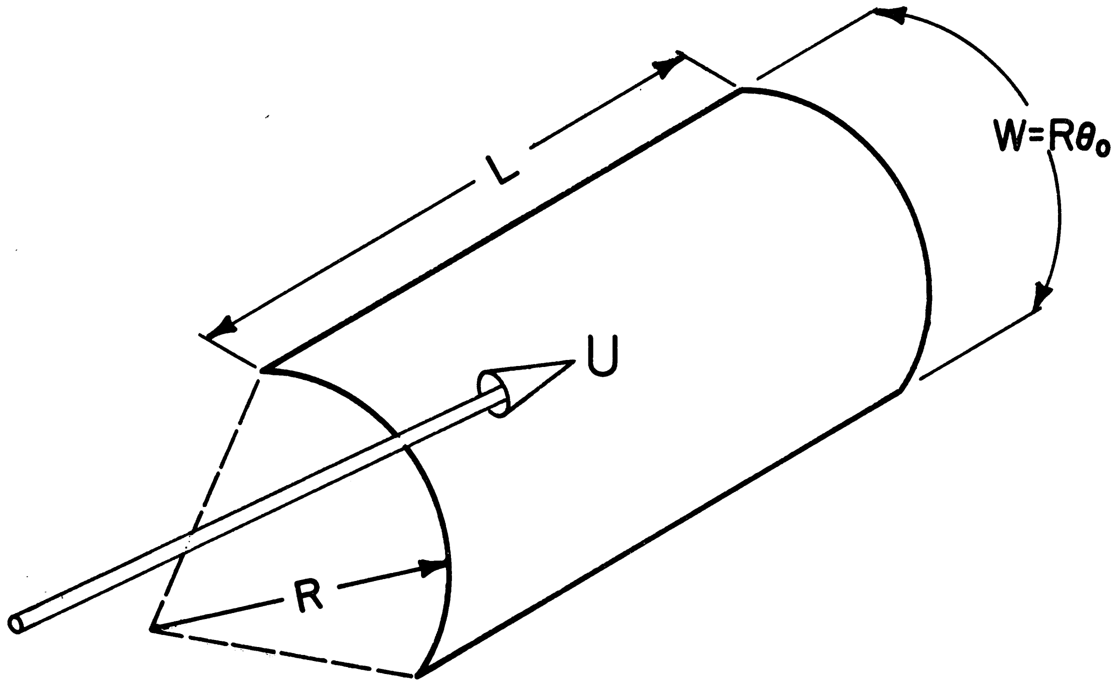

Fig. 1. Shell segment.

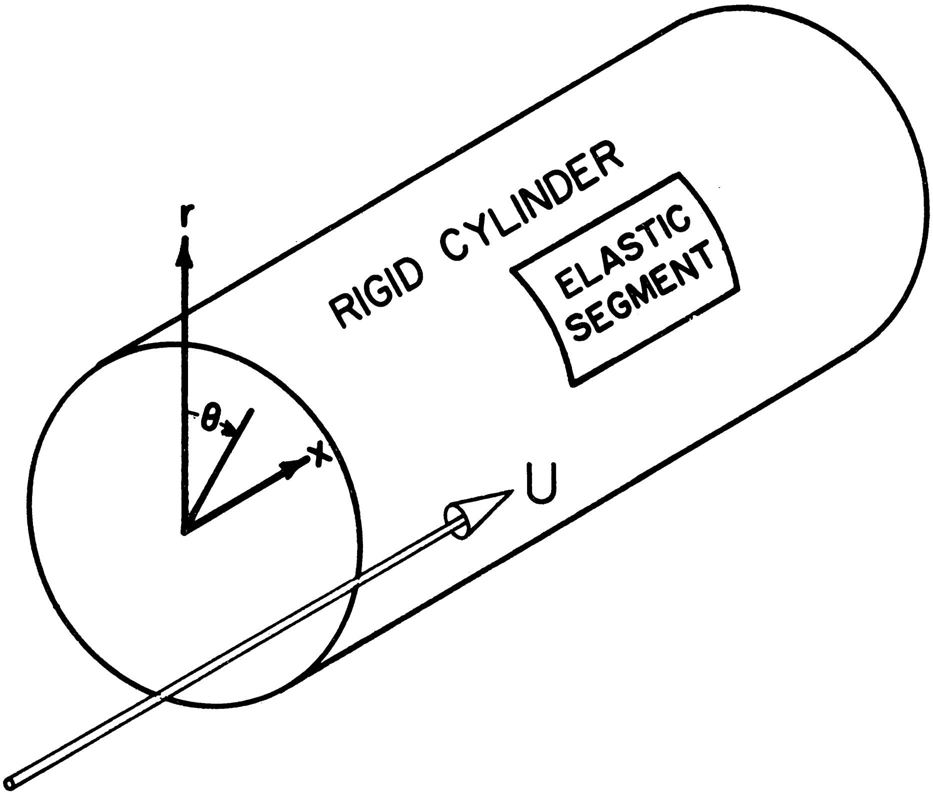

Fig. 2. Elastic shell segment imbedded in a cylinder.

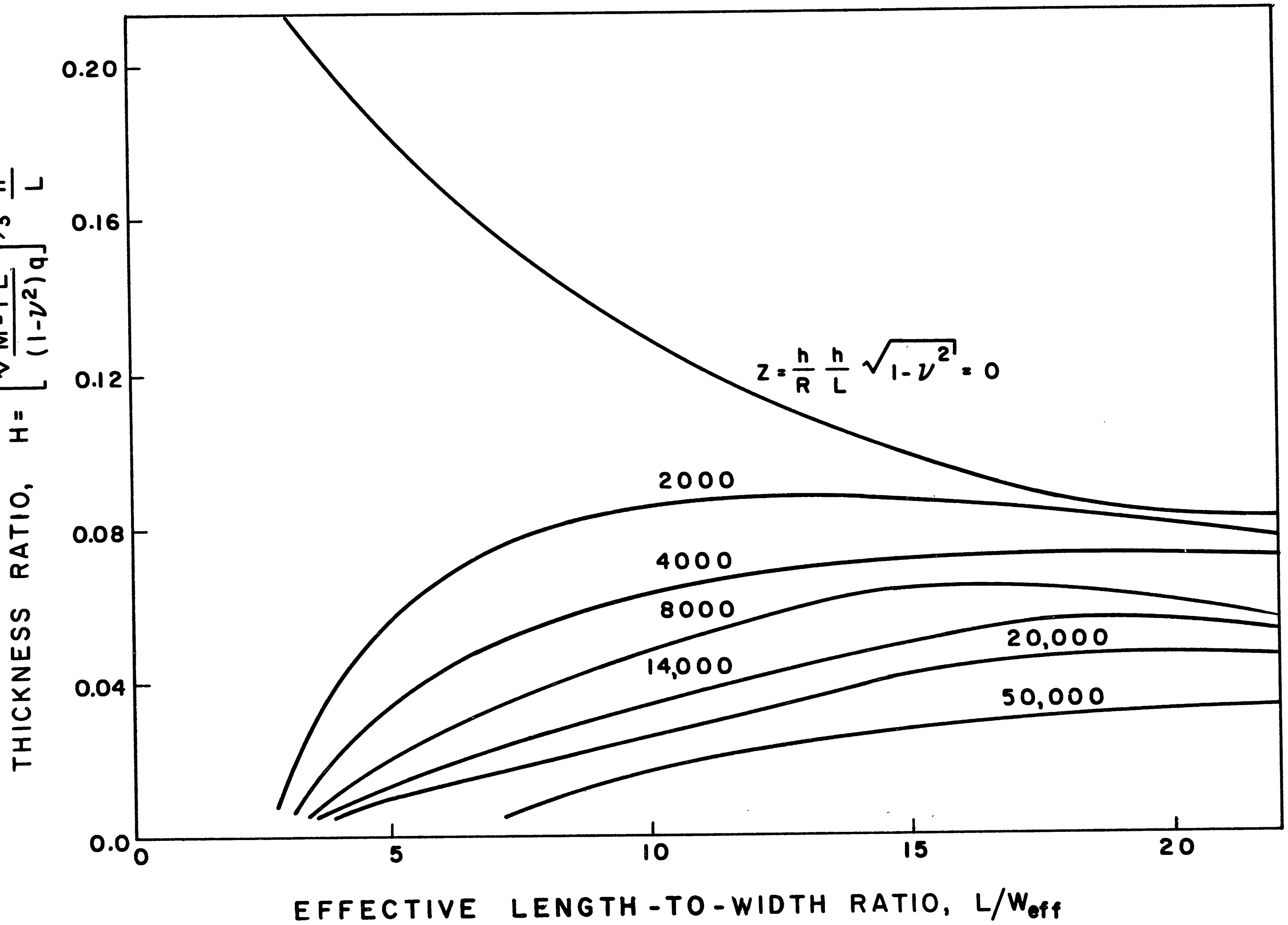

Fig. 3. Thickness requirement for a cylindrical shell segment, $\psi = 0°$ (Ackeret theory).

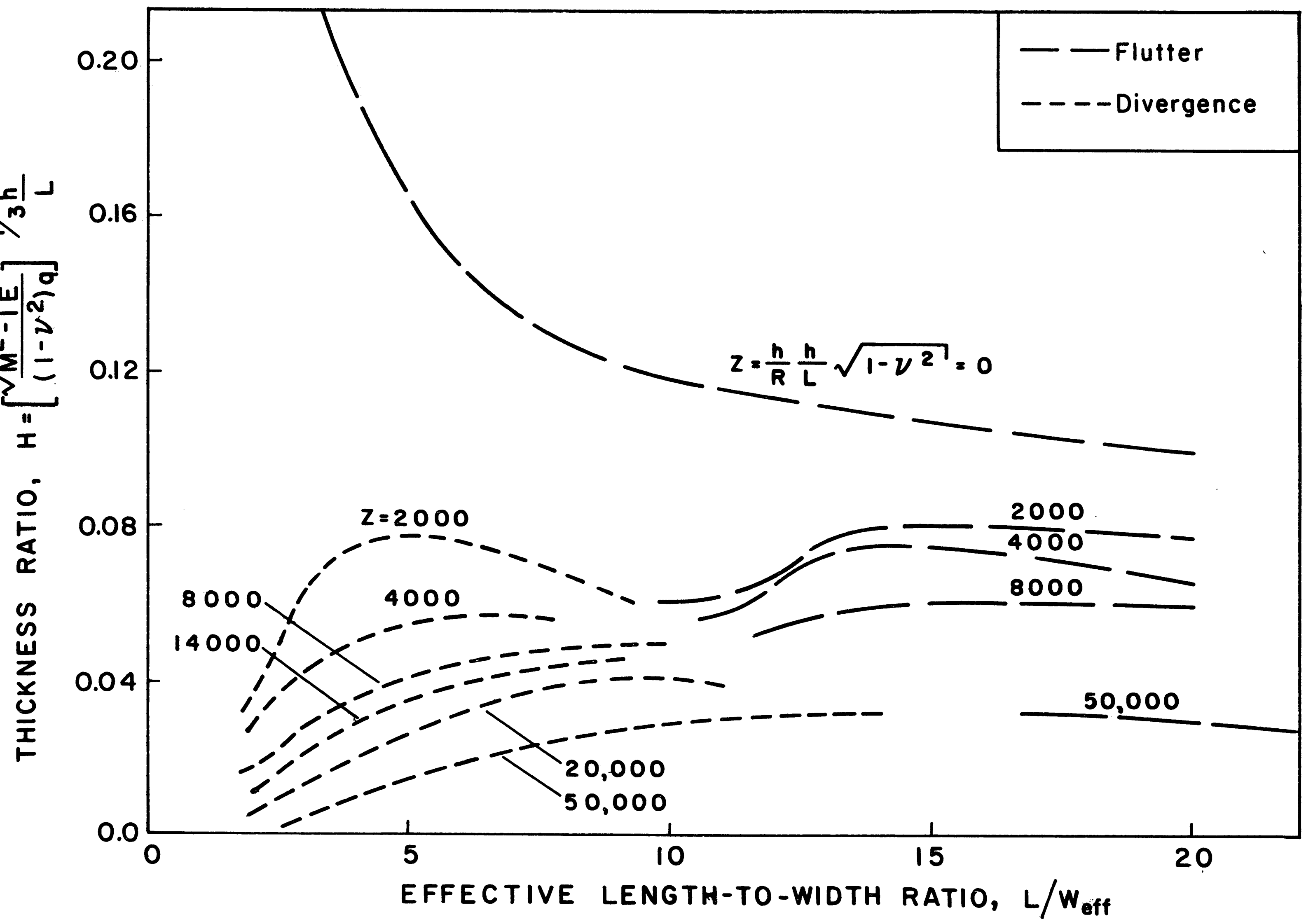

Fig. 4. Thickness requirement for a cylindrical shell segment, $\psi = 90°$ ("slender body" theory).

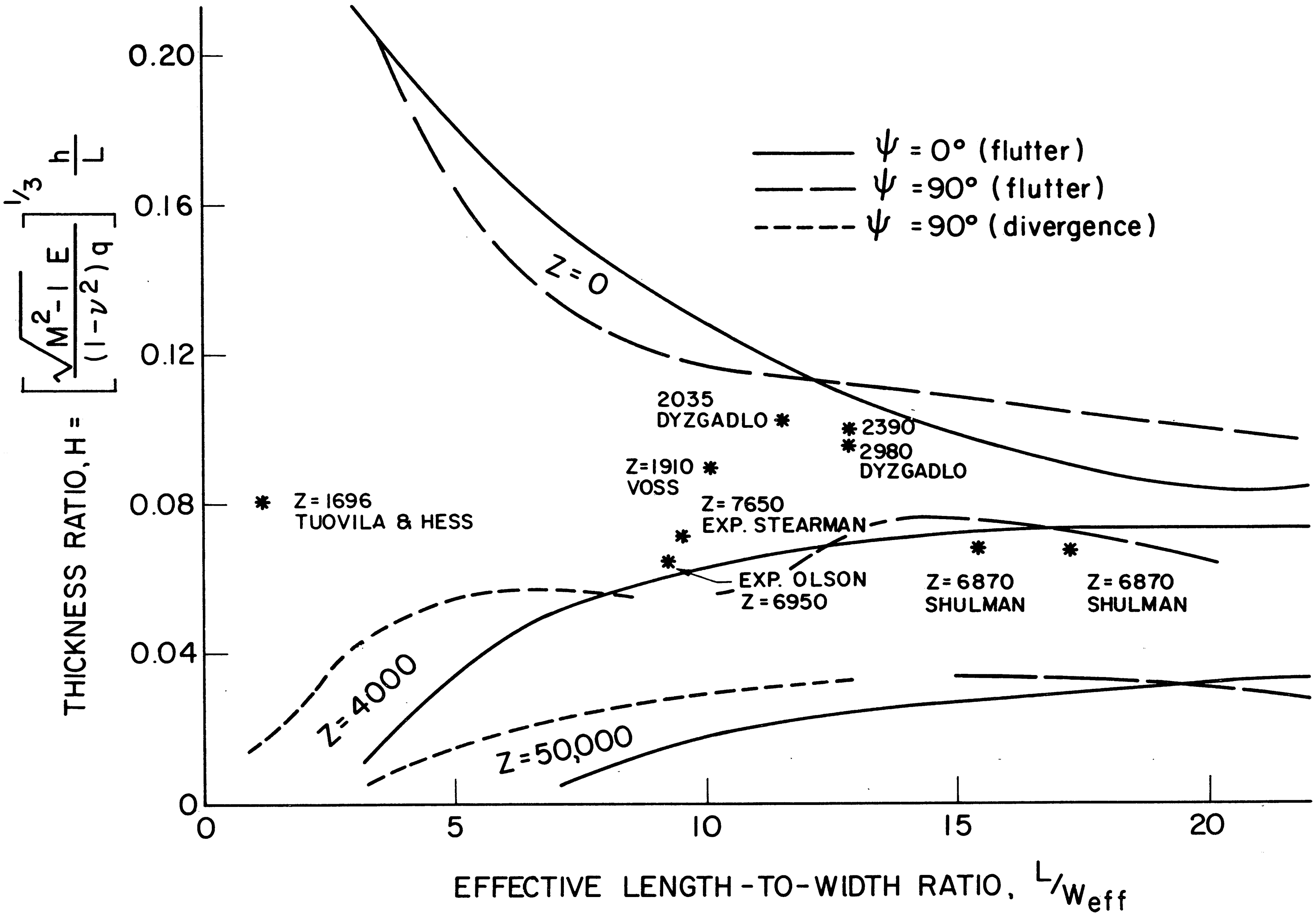

Fig. 5. Upper and lower bounds for thickness requirement. Freely supported edges.

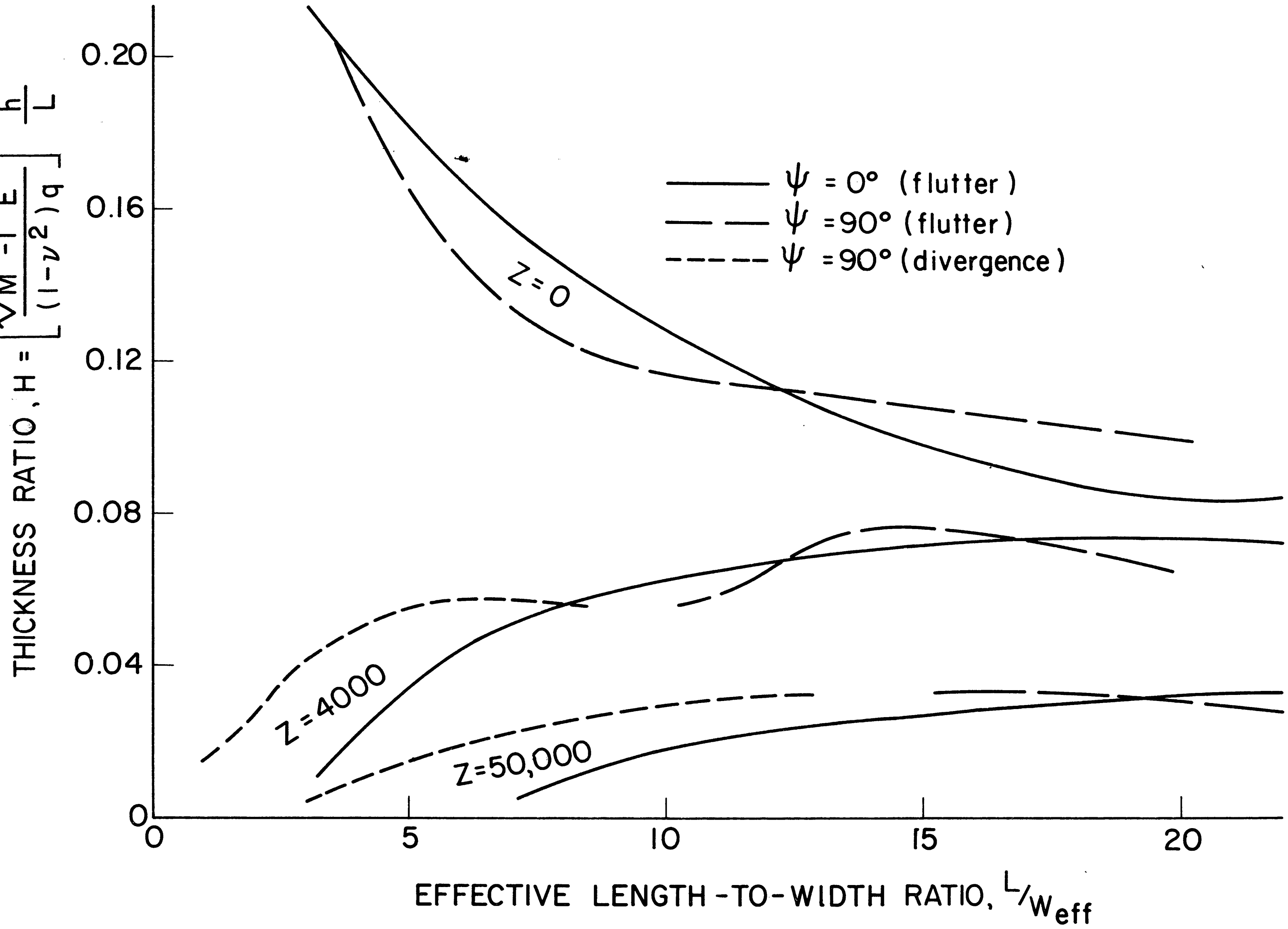

Fig. 6. Comparison with other theories and experiments.

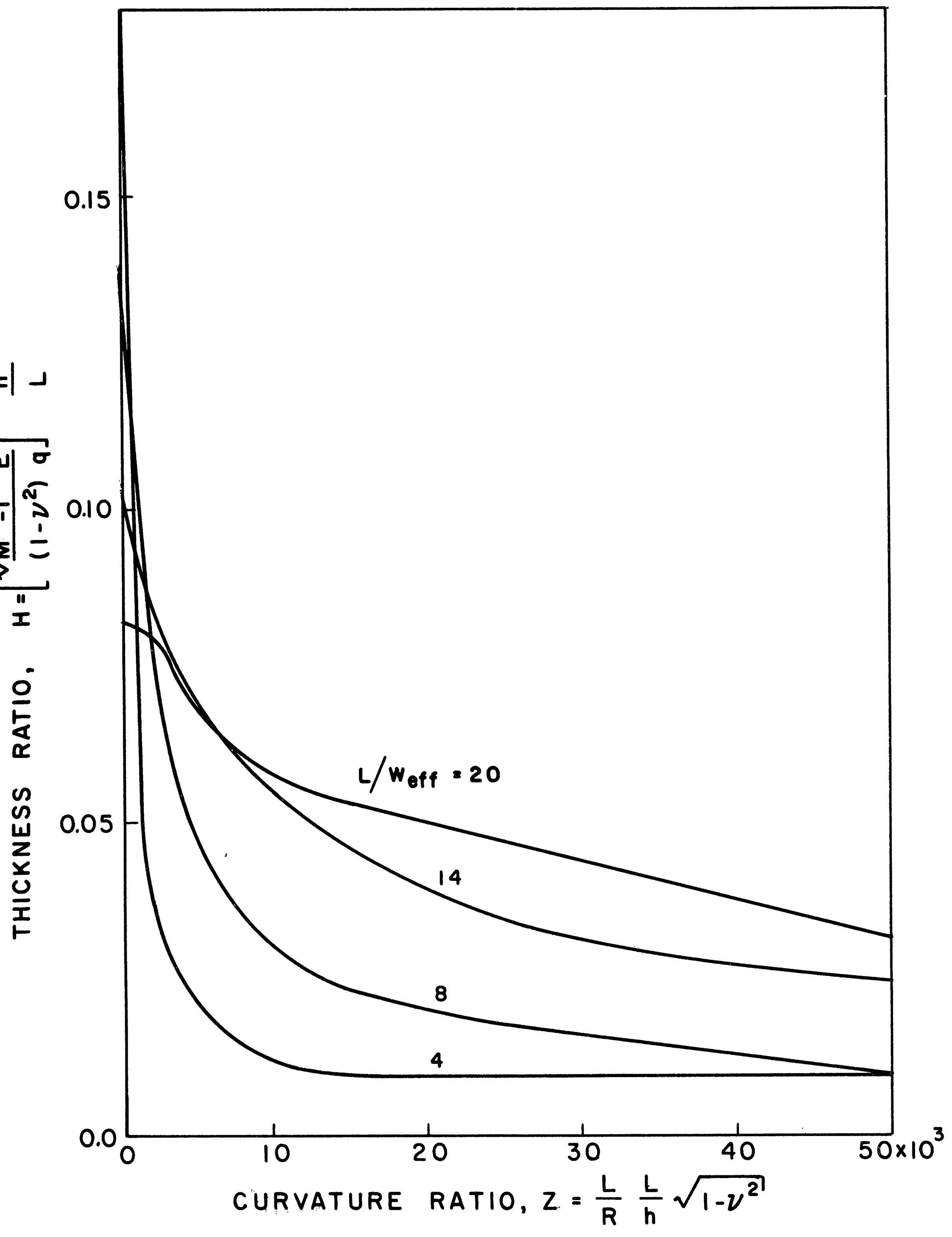

Fig. 7. Stability boundaries, $\psi = 0°$ (Ackeret theory), four mode solution.

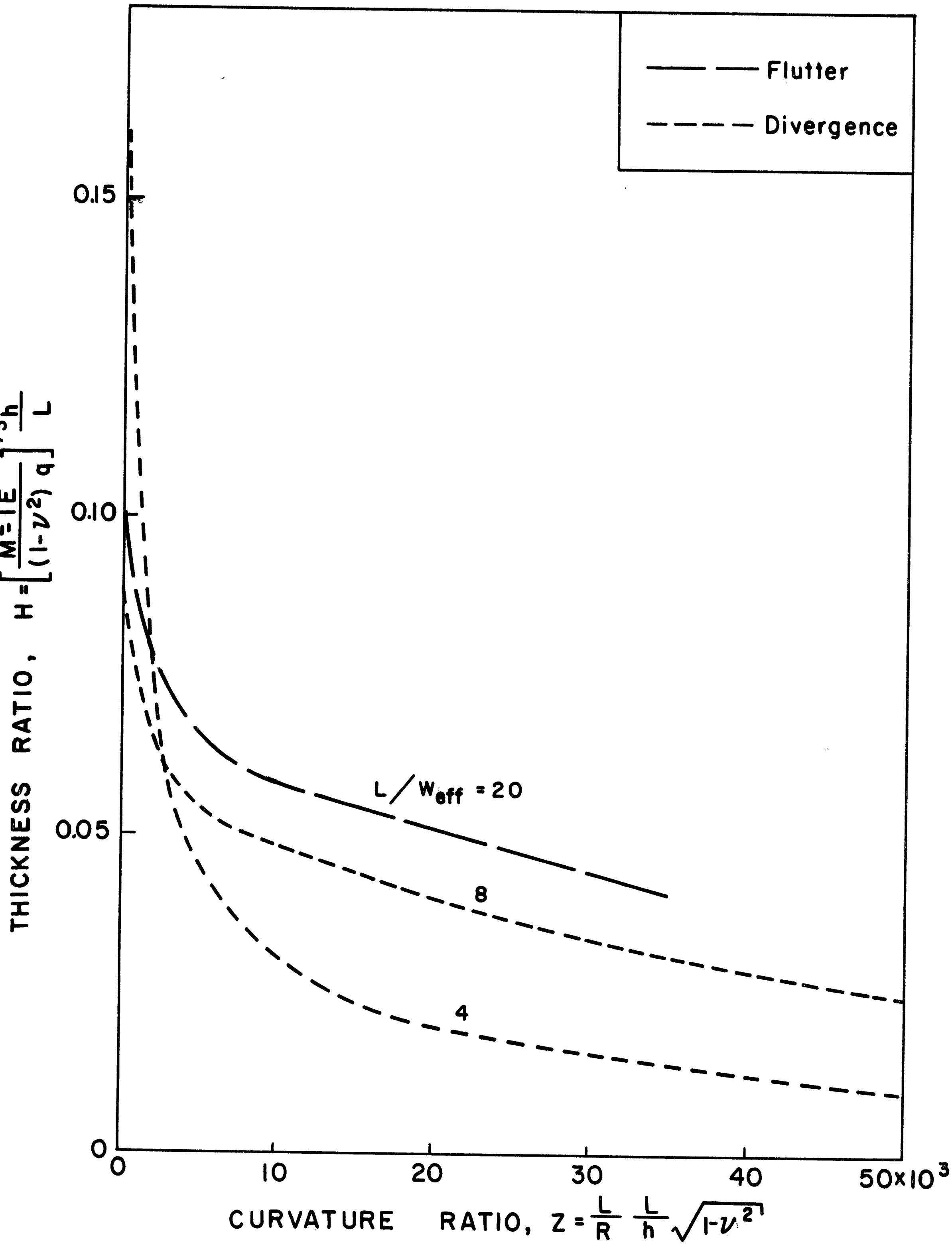

Fig. 8. Stability boundaries, ψ = 90° ("slender body" theory).

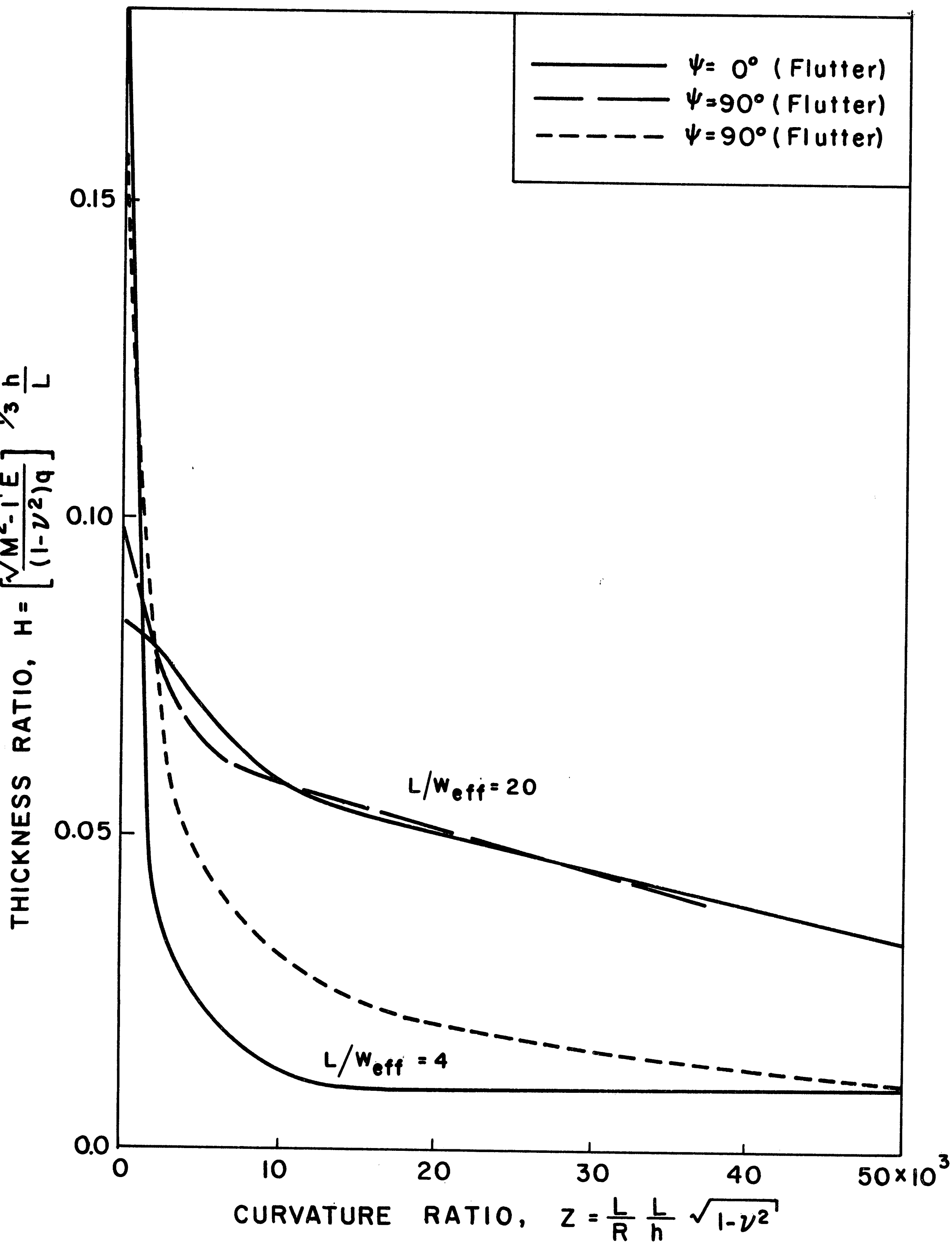

Fig. 9. Approximate upper and lower stability boundaries.