A MODEL OF *CP* VIOLATION *

Y. TOMOZAWA
Randall Laboratory of Physics, The University of Michigan, Ann Arbor, Michigan 48104, *USA*

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A model of the weak interactions mediated by a nonet or an octet of W-mesons is presented. The model contains the *CP* violating phase intrinsically.

We construct a weak interaction Lagrangian L_W out of nonets of hadronic currents, $(J_\lambda)_\beta$, $(\alpha, \beta = 1,2,3)$, W-mesons [1], $(W_\lambda)_\beta$, and a charged leptonic current, l_λ,

$$\begin{aligned} -L_W = & g_1 \, \mathrm{Tr}\,(JW) \\ & + g_2\{(JW)^2_3 \exp i\chi + (WJ)^3_2 \exp - i\chi\} \\ & + g_3\{(JW)^3_2 \exp - i\varphi + (WJ)^2_3 \exp i\varphi\} \\ & + g_l(lW^1_2 + l^+W^2_1) \end{aligned} \tag{1}$$

Here the 3×3 hermitian matrix $(J_\lambda)^\alpha_\beta$ or $(W_l)^\alpha_\beta$ is a sum of the corresponding SU_3 octet and the singlet matrix (in a nonet symmetric form), all the coupling constants g_i are real and the complex phases are inserted for a purpose to be discussed later. All the currents consist of V-A form and the leptonic current is μ-e symmetric. The space-time indices will be supressed whenever no confusion arises.

The free Lagrangian of the massive W-mesons is assumed to be SU_3 symmetric, and the effective Lagrangian which is responsible for low energy reactions is constructed by contraction of the W-mesons. In doing so, we require the following two experimental facts to be satisfied:

(I) No $|\Delta S| = 2$ transition in the lowest order, and

(II) Universality of the leptonic interactions in the sense of Gell-Mann-Levy-Cabibbo [2].

It is easy to see that while the first requirement demands that

$$g_2^2 = g_3^2 \tag{2}$$

and

$$\chi - \varphi = \pm \tfrac{1}{2}\pi, \ (\mathrm{mod.}\ 2\pi), \tag{3}$$

the second one leads to the condition

$$g_l = g \tag{4}$$

where we have introduced the definitions

$$g_1 = g \cos\theta \tag{5}$$

and

$$g_2 = g_3 = g \sin\theta \tag{6}$$

The effective Lagrangian is then

$$\begin{aligned} -L^{(2)}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} [& \tfrac{1}{2}\mathrm{Tr}(JJ)\cos^2\theta + \\ & + \sqrt{2}\{(JJ)^2_3 \exp i(\varphi \pm \tfrac{1}{4}\pi) + \mathrm{h.c.}\} \sin\theta\cos\theta + \\ & + \{(JJ)^2_2 + (JJ)^3_3\} \sin^2\theta + l^+l^+ + \\ & + l(J^1_2 \cos\theta + J^1_3 \sin\theta \exp i\varphi) + \mathrm{h.c.}] \end{aligned} \tag{7}$$

where

$$\tfrac{1}{2}\sqrt{2}G_F = g^2/M_W^2 \tag{8}$$

is the Fermi coupling constant, M_W being the mass of the W-meson. If we used octets instead of nonets for $(J)^\alpha_\beta$ and $(W)^\alpha_\beta$, the condition for the phase angles and the form of the resulting effective Lagrangian would be slightly different †, al-

† For the octet case, eq. (3) should be replaced by

$$\chi - \varphi = \pm \tfrac{1}{3}\pi \tag{3'}$$

and the first three terms in the parentheses of eq. (7) by

$$\begin{aligned} & \tfrac{1}{2}\{\mathrm{Tr}(JJ) - \tfrac{1}{3}(\mathrm{Tr}J)^2\} \cos^2\theta + \\ & + \sqrt{3}\{(JJ)^2_3 \exp i(\varphi \pm \tfrac{1}{6}\pi) + \mathrm{h.c.}\} \sin\theta \cos\theta + \\ & + \tfrac{1}{2}\{\mathrm{Tr}(JJ) + (\mathrm{Tr}J)^2 - 2J^1_1 \mathrm{Tr}J\} \sin^2\theta . \end{aligned}$$

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though the essential features of the model still remain the same.

The following remarks concerning expressions (1) and (7) are in order:

(1) All the isospin selection rules are consistent with the observed ones, ($\Delta I = \frac{1}{2}$ rule for the $|\Delta S| = 1$ transition and $\Delta I = 1$ rule for the $\Delta S = 0$ semileptonic transition), as well as the $\Delta S = \Delta Q$ rule for the semileptonic transitions.

(2) The Cabibbo angle is naturally introducted as the ratio of the coupling constants of the SU_3 symmetric and the SU_3 breaking hadronic terms.

(3) All CP violating effects will have amplitudes of order G_F^2 or higher except in processes with the real production of W_2^3 and W_3^2 mesons. In this case the CP violating amplitude will be of order, $g = (\sqrt{5\sqrt{2}}\, M_W/m_N) \times 10^{-3}$, ($m_N$ being the nucleon mass). In fact, the $W_3^2 = W_{\overline{K}^0}$ meson is coupled to the current $g\{J_2^3 \cos\theta + (J_2^2 \exp(\pm\frac{1}{2}i\pi) + J_3^3) \times \sin\theta \exp i\varphi\}$, so that we expect a maximum CP violation in the $W_{\overline{K}^0}$ production or its decay process through the $\Delta S = 0$ current. To order g, the CP violation will be observed also in the production of the other types of the neutral W mesons through mixing, while no CP violation will be found in the charged W meson production in this order.

(4) The electromagnetic interactions induce the $\Delta I = \frac{3}{2}$, $\Delta S = 1$ component in $L_{\mathrm{eff}}^{(2)}$. However this term is in phase with the $\Delta I = \frac{1}{2}$, $\Delta S = 1$ term, so that it soes not lead to any observable CP violation to first order in G_F.

(5) There is no CP violation to second order in G_F for the K^0-$\overline{K}^0$ mixture, according to the Lagrangians (1) and (7). This is because the complex phase for the $\Delta S = 1$ nonleptonic decay interaction appears as a common factor which is unobservable.

In order to explain the observed CP violation of the K^0-$\overline{K}^0$ system, we consider two possibilities:

[A] Contraction of two W-mesons in eq. (1) gives a term of order G_F^2,

$$G_F^2 (J_\lambda)_3^2 \{\langle T((J_\lambda)_2^2 (J_\nu)_2^2)\rangle_0 - \langle T((J_\lambda)_3^3 (J_\nu)_3^3)\rangle_0\} \times (J_\nu)_3^2 \sin^2\theta \cos^2\theta \exp(2i\varphi) , \tag{9}$$

where T stands for a Wick ordered product and $\langle\ \rangle_0$ for the vacuum expectation value. The corresponding diagram is given in fig. 1. This term vanishes in the SU_3 symmetric limit of the strong interactions, otherwise it would contribute to the $\Delta S = 2$ transition. If this term is indeed non-zero, the $\Delta S = 2$ terms in $(L_{\mathrm{eff}}^{(2)})^2$ and eq. (9) lead to CP violation through the mass matrix of the K^0-$\overline{K}^0$ mixture because they have an observalbe relative phase $\pm\frac{1}{2}\pi$.

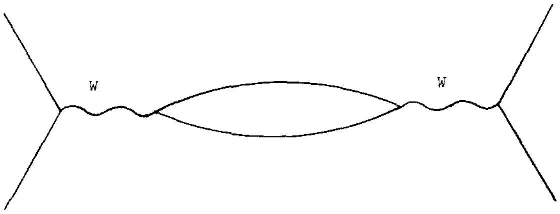

Fig. 1. Diagram corresponds to the expression (9). [The solid lines stand for hadrons.]

There are two ways of achieving a non-vanishing result for the term (9).

A-1 The SU_3 symmetric breaking medium strong interaction is a candidate, since this interaction discriminates the SU_3 index 3 from the rest, and therefore makes the term (9) finite. If the medium strong interactions coupling strength relative to the SU_3 symmetric strong interaction is r_1, then the K^0-$\overline{K}^0$ mixing parameter [3] ϵ which is a measure of the CP violation, is also of the order of r_1. Since $|\epsilon| = 2 \times 10^{-3}$, we have to require that

$$r_1 \lesssim 10^{-3} , \tag{10}$$

otherwise we would meet too large CP violation. Dispite significant SU_3 breaking phenomena in the low energy region (the mass splitting of low lying multiplets or violation of SU_3 sum rules for low energy reactions, etc.) the condition (10) is not unreasonable in a quark model [4]. In fact, the succes of the Gell-Mann-Okubo mass formula, the fact that the SU_3 breaking effect seems to decrease in reactions at higher energies or the observation that the Regge trajectories of various multiplets have parallel slopes, might be considered as evidence to favor this point of view. We also note that the effect of mass splitting of low mass multiplets on the expression (9) (diagram of fig. 1) would be small since it is dominated by the cut off of the integral over intermediate state momenta.

A-2 The electromagnetic interaction ‡ com-

‡ The electromagnetic interactions themselves do not make the term (9) non-vanishing, since they transform as a SU_3 tensor T_1^1, so that they do not give any splitting between the SU_3 indices 2 and 3.

bined with the medium strong interaction is another possibility. These interactions are responsible for the SU_3 violation in the magnetic moments of baryons or in the electromagnetic form factors of any multiplets. In like manner, they would give a contribution to ϵ, which is crudely estimated to be

$$r_2 \approx \alpha \times (\text{say } \tfrac{1}{5}) \approx 10^{-3} \; (\text{or} \approx \alpha/\pi) \,. \tag{11}$$

[B] The neutral leptonic current l^0_λ may exist in L_W as

$$-L_W^{NL} = \sqrt{r_3}\, g l^0 W_2^2 \,, \tag{12}$$

where $\sqrt{r_3}$ is the coupling strength relative to that of the charged leptonic term. The resulting additional term in L_{eff} is then

$$- L_{eff}^{NL} = \sqrt{r_3}\, G_F l^0 \times \times \{ J_2^2 \cos\theta + (J_3^2 \exp i\varphi + J_2^3 \exp - i\varphi) \sin\theta \} \,. \tag{13}$$

It is easy to see that $(L_{eff}^{(2)})^2$ and $(L_{eff}^{NL})^2$ have $\Delta S = 2$ transition terms with different phases, and hence they produce $|\epsilon| = r_3$. From the present experimental data for various neutral leptonic currents [5], we obtain upper limits for the parameter r_3,

$$\begin{aligned} (r_3)_{\nu\bar{\nu}} &< 2 \times 10^{-3} \\ (r_3)_{e\bar{e}} &< 2.2 \times 10^{-5} \\ (r_3)_{\mu\bar{\mu}} &< 4.9 \times 10^{-5} \,. \end{aligned} \tag{14}$$

From these numbers, we may conclude that within this model the possibility of the neutral currents being responsible for the observed *CP* violation is excluded for the ($\bar{e}$e) or ($\bar{\mu}\mu$) currents.

With these provisions, [A] and/or [B], we continue our remarks:

(6) The prediction of this model for *CP* violation in the K^0-$\bar{K}^0$ system is equivalent to that of the superweak theory [6], except for case B, where the phase of the *CP* violation parameter ϵ is undetermined. This is because, in case A, the effective Lagrangian (9) contributes to the mass matrix * of the K^0-$\bar{K}^0$ system and therefore [3]

$$\epsilon = - \text{Im}\, M_{12}/(\tfrac{1}{2}\gamma S - i\Delta m) \,, \tag{15}$$

while in case B, the Lagrangian (13) can contribute both to the decay and mass matrices, giving

$$\epsilon = (i\, \text{Im}\, \Gamma_{12} - \text{Im} M_{12})/(\tfrac{1}{2}\gamma_S - i\Delta m) \tag{16}$$

Here M_{12}, γ_S and Δm stand for the off diagonal element of the mass matrix, the decay rate of the short lived K^0 and the mass difference of the two neutral kaons. ($\Delta m = m_L - m_S$.)

(7) In $(L_{eff}^{(2)})^2$, we have a term with $\Delta S = 1$ transition

$$\tfrac{1}{2} G_F^2 (l^+ J_1^2)(l J_3^1) \sin\theta \cos\theta \exp i\varphi \,, \tag{17}$$

which has a phase factor different from that of the hadronic $\Delta S = 1$ term, the latter being $\exp i(\varphi \pm \frac{1}{2}\pi)$. This difference in phase leads to *CP* violation of order G_F^2 in the $|\Delta S| = 1$ transition. As a result, the *CP* violation in the $\Delta S = 0$ transition, such as in the electric dipole moment of neutron, occurs in order G_F^3.

All the experimental data at the present time are consistent with this model.

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* We define the K^0-$\bar{K}^0$ mixing matrix by $\mathcal{M} = \Gamma + iM$, and call the hermitian 2×2 matrices Γ and M the decay matrix and the mass matrix respectively.

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