slight deviations at energies larger than 5 keV. The present results for Cs are in good agreement with the one reported in ref. 10 as far as the same incident energy is concerned. Considering the measurements of ref. 10 which reveal a F_{-1} maximum of 0.21 ± 0.04 for Cs at 0.75 keV, the F_{-1} maxima shift from Cs to Li towards higher energies. This observation is to be expected according to the theory of such processes [13]. The value of the maxima decreases from Cs to Li monotonically from approximately 0.21 to 0.06. It seems therefore at first that Cs should be the most powerful charge exchanger. However, the production of a 0.75 keV high intensity hydrogen beam and the transport to the charge exchanger is a difficult task. The choice of sodium as the charge exchanger has the great advantage that the incident beam can have a much higher energy without too great a loss in the charge exchange yield. Consequently, the intensity of the primary beam is greatly increased, which in turn results in a larger negative beam intensity coupled with a superior beam quality. The charge exchange yield of sodium decreases only slowly with increasing energy and suggests its use as a charge exchanger yielding a reasonable result for protons up to about 10 keV. Sodium has been used successfully as a charge exchanger in the ETH polarized ion source since more than six months. This period of experience shows also that the handling of Na is much easier than that of Cs and K.

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CRITICAL ORDER-PARAMETER RELAXATION AND HYPERSONIC ATTENUATION IN HELIUM *

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The asymmetry of the temperature dependence, the broad symmetric maximum, and the frequency dependence of hypersonic attenuation in helium near the λ-point can be understood qualitatively in terms of the critical relaxation of fluctuations in the order parameter.

Measurements of the first-sound attenuation in helium near the λ-point have revealed three unexpected features: a) an asymmetric temperature dependence [1, 2], $|\tau|^{-1}$ below and $\tau^{-\frac{1}{2}}$ above T_λ ($\tau \equiv T/T_\lambda - 1$); b) a broad symmetric maximum [2] centered at the λ-point; c) an ω^2 frequency dependence [2, 3] below *and* above the λ-point. These features cannot be easily recon-

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ciled with the present scaling ideas. Instead, we show that these features can be understood within a simple phenomenological theory of the critical relaxation of fluctuations in the order parameter.

The static phenomenological theory of helium [4], which includes the correct static scaling laws, has been recently extended [5] to yield the singular behavior of dynamic properties in the hydrodynamic limit ($k\xi \ll 1$) where k is the (second-sound) wave vector and $\xi = \xi_0 \tau^{-\frac{2}{3}}$ is the coherence length. We now extend the theory to the critical limit ($k\xi \gg 1$) and include the relaxation of the order parameter $\Psi(r,t) = \sum_k \Psi_k(t)\exp(ikr)$,

$$(\gamma\,\partial/\partial t + \alpha + E_k)\Psi_k + \beta\sum_{pq}\Psi^*_{p+q}\Psi_p\Psi_{k+q} = 0 \qquad (1)$$

$$E_k = D|\tau_c/\tau|^{\frac{1}{3}} k^2 Q_k\,. \qquad (2)$$

Here $|\Psi(r,t)|^2$ is the condensate density; $\alpha = \alpha_0\tau$; α_0, β, γ are the phenomenological parameters; $D = \hbar^2/2m$; τ_c is the Ginzburg critical temperature; (for helium, $\tau_c \approx 0.3$); and Q_k is a homogeneous function of $k\xi$ with limits: $Q_k = 1$ ($k\xi \ll 1$) and $Q_k = (k\xi)^{-\frac{1}{2}}$ ($k\xi \gg 1$). Thus in the hydrodynamic limit, $E_k = Ak^2$, $A = D|\tau_c/\tau|^{\frac{1}{3}}$; whereas in the cricital limit, $E_k = Bk^{\frac{3}{2}}$, $B^4 = D^3\tau_c\alpha_0$ [6]. [For a superconductor, eq. (2) should be replaced by $E_k = Dk^2(1+|\tau_c/\tau|^{\frac{1}{3}}Q_k)$.]

Below T_λ, the equilibrium solution of eq. (1) is the uniform solution $|\Psi_0|^2 = -\alpha/\beta$. Spontaneous fluctuations of the order parameter about this equilibrium solution will relax in accordance to eq. (1). In particular, the relaxation time for the *linear* fluctuating Fourier component $\delta\Psi_k(t)$ is

$$\theta_k = \gamma(E_k - 2\alpha)^{-1}\,, \qquad (3)$$

and the mean square fluctuation is

$$\langle\,|\delta\Psi_k|^2\rangle = k_B T(E_k - 2\alpha)^{-1}\,. \qquad (4)$$

Assuming statistical independence of the various Fourier components $\delta\Psi_k$, we calculate the mean relaxation time by

$$\theta = \sum_k\langle|\delta\Psi_k|^2\rangle\theta_k \Big/ \sum_k\langle|\delta\Psi_k|^2\rangle\,. \qquad (5)$$

In the hydrodynamic limit, $\theta = \theta_0 - (\pi-2)/(4-\pi) \times (\gamma/4\alpha)$, $\theta_0 = -\gamma/2\alpha$; whereas in the critical limit

$$\theta = -(\gamma\xi_0^{\frac{3}{2}}/B)\,\ln|\alpha\xi_0^{\frac{3}{2}}/B + \tau|\,.$$

Above T_λ, the effects of nonlinear fluctuations [7] are important. The simplest approximation to include the nonlinear fluctuations above T_λ is a self-consistent Hartree approximation [8]; the net effect of which is to make the substitution in (3), (4); $-2\alpha \to \alpha + \Sigma$, where

$$\Sigma = \beta(4\pi A/k_B T)^{-\frac{3}{2}} F_{\frac{3}{2}}(X) = \Sigma_0\tau^{\frac{1}{2}}\,,$$

F_n is the usual nth Bose-Einstein integral, and $X = (\Sigma+\alpha)/k_B T$. In this approximation, the relaxation time is still given by (5).

The classical expression for the first-sound attenuation, α_1, is given by

$$\alpha_1 = S\omega^2\theta(1+\omega^2\theta^2)^{-1} \qquad (6)$$

where S is the strength of the attenuation, ω is the first-sound (angular) frequency and θ is the relaxation time for restoration to equilibrium. Following Landau and Khalatnikov [9], we identify θ with the *order-parameter* relaxation time, eq. (5). (In ref. 9, only θ_0 was used.) Note that the dividing line between the hydrodynamic and critical regions, $k\xi \approx 1$ or $\nu \approx 5\times 10^{10}\tau$ Hz, concurs [2] with the condition $\omega\theta \approx 1$. Thus from the classical expression (6), the sound attenuation is: $\alpha_1 \sim \omega^2\tau^{-1}$ in the hydrodynamic region below T_λ; $\alpha_1 \sim (\ln|1/\tau|)^{-1}$ in the critical region; and $\alpha_1 \sim \omega^2\tau^{-\frac{1}{2}}$ in the hydrodynamic region above T_λ.

An implicit assumption in the above analysis is that $k < \xi_c^{-1}$, $\xi_c \equiv \xi(\tau_c)$. In the event that $k > \xi_c^{-1}$, we note that the energy spectrum changes to $\sim Dk^2$ over all temperature regions [6]. Furthermore the uniform equilibrium solution is not a good unperturbed solution, and nonlinear fluctuations are important on both sides of the λ-point. Hence for high enough frequencies, the sound attenuation should be symmetrical. An estimate of the threshold frequency to this supercritical region ($k\xi_c > 1$) is $\nu_c \approx 10^{10}$ Hz. It is not clear whether the recent measurements of α_1 [10] at 10^9 Hz is reflecting the supercritical behavior or the bending over of α_1 in between the hydrodynamic and critical limits.

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