LOGARITHMIC CORRECTIONS TO THE THEORY OF THE λ-TRANSITION IN ^{4}He

V. K. WONG
Department of Physics, University of Michigan, Ann Arbor, Michigan 48104 USA

Received 3 October 1968

The logarithmic singularity in the specific heat is used to investigate the logarithmic corrections to the phenomenological theory of the λ-transition in liquid helium-4. A justification of the theory of Mamaladze is also given.

In a previous communication [1], a new phenomenological theory for the λ-transition of liquid helium, based on a scaling law analysis and the identification of the order parameter as the condensate density n_0, was presented. The critical exponents of various physical quantities predicted by this theory were in good agreement with experimental data, up to a point. The reservation expressed here arises from the inability of the scaling-law analysis to distinguish between a (τ^x) dependence and for example a $(\tau^x \ln \tau^{-1})$ dependence, where x is some critical exponent and $\tau = 1 - T/T_\lambda$. Because of this ambiguity in the scaling-law analysis, we use, in this note, the observed logarithmic singularity in the specific heat as a basis to investigate the logarithmic corrections to the theory of ref. 1. We also point out a possible justification of the somewhat *ad hoc* theory of Mamaladze [2,3].

The phenomenological theory begins with the Gibbs free-energy density written in the form

$$f\{\Psi\} = f_{\mathrm{I}}(\tau) + A(\tau)|\nabla\Psi|^2 - B(\tau)|\Psi|^2 + \tfrac{1}{2}C(\tau)|\Psi|^4 , \tag{1}$$

where the notation of ref. 1 is followed. In the bulk sample at equilibrium, eq. (1) yields

$$n_0 \sim |\Psi|^2 = B/C \quad \text{and} \quad f = f_{\mathrm{I}} - \tfrac{1}{2}B^2/C .$$

In the presence of low-speed superflow, eq. (1) gives the following expression for the superfluid density [4]: $\rho_s = (2m^2/\hbar^2)AB/C$. Finally for the theory to be valid, we need the fluctuations in the order-parameter to be small, which can be expressed by condition:

$$(\xi_\perp/e\xi_\parallel) \approx (\sqrt{2}/8\pi e)(C/B^{\frac{1}{2}}A^{\frac{3}{2}})\, k_B T_\lambda \ll 1.$$

The theory of ref. 1 can be summarized by $A \sim \tau^{-1/3}$, $B \sim \tau$, $C \sim$ const., with $f - f_{\mathrm{I}} \sim -\tau^2$, $\rho_s \sim \tau^{2/3}$ and $(\xi_\perp/e\xi_\parallel) \sim$ const.

From the observed logarithmic singularity in the specific heat c_p over 4 decades [5], we have that $f - f_{\mathrm{I}} = -f_0\, \tau^2 \ln|\Lambda/\tau|$, where $f_0 \approx$ $\approx (5/16)nk_B T_\lambda$, n is the number density at T_λ and $\Lambda = e^{11/2} \approx 245$. From recent measurements of ρ_s over 3 decades [6,7], we see that $\rho_s \sim \tau^{2/3}$ Therefore it is desirable to construct a theory such that (I) $B^2/C \sim \tau^2 \ln|\Lambda/\tau|$; (II) $AB/C \sim \tau^{2/3}$; and (III) $A^{3/2}B^{1/2}/C \sim$ const. Condition III expresses the expectation that the fluctuations are small right up to T_λ. Unfortunately, these three conditions on the three coefficients are *not* compatible; and one of these conditions must be changed. From this point of view, the theory of ref. 1 represents the replacing of condition (I) by (I') $B^2/C \sim \tau^2$ in the spirit of the scaling-law analysis. However, a) as c_p is measured to a precision one order of magnitude greater than ρ_s; b) as there exists an exact logarithmic singularity in the specific heat for the two-dimensional Ising model; and c) as ρ_s is *not* (rather n_0 is) analogous to quantities which can be measured in other second-order transitions; we retain condition (I) in preference to (II). In place of condition (II), we use (II') $B/C \sim \tau$, which is derived from the microscopic result [8,9] $n_0 \sim \tau$. The conditions (I), (II'), (III) determine unambiguously the following temperature dependences: $A(\tau) = A_0(\tau^{-1}\ln|\Lambda/\tau|)^{1/3}$; $B(\tau) = B_0\tau\ln|\Lambda/\tau|$; $C(\tau) = C_0\ln|\Lambda/\tau|$. Note that ρ_s is now proportional to $(\tau^2\ln|\Lambda/\tau|)^{1/3}$.

To evaluate the coefficients, we observe that there are yet no experimental data on $n_0(\tau)$. Therefore, we introduce the change in variable

$\tilde{\Psi} = (2mA)^{\frac{1}{2}} \hbar^{-1}\Psi$ (such that $\rho_s = m\,|\tilde{\Psi}|^2$) into eq. (1) to obtain

$$f\{\Psi\} = f_I(\tau) + (\hbar^2/2m)|\nabla\tilde{\Psi}|^2 - \alpha(\tau)|\tilde{\Psi}|^2 + \tfrac{1}{2}\beta(\tau)|\tilde{\Psi}|^4\,, \tag{2}$$

where

$$\alpha(\tau) = \alpha_0\,(\tau^2 \ln|\Lambda/\tau|)^{\frac{2}{3}}\,,$$

$$\beta(\tau) = \beta_0\,(\tau^2 \ln|\Lambda/\tau|)^{\frac{1}{3}}\,, \tag{3}$$

with $\alpha_0 = (\hbar^2/2m)\,B_0/A_0$ and $\beta_0 = (\hbar^2/2m)^2 \times C_0/A_0^2$. If the logarithmic corrections are omitted in eq. (3), then eqs. (2,3) reproduce the somewhat ad hoc theory of Mamaladze [2,3]. In particular, no $|\tilde{\Psi}|^6$ term is needed, contrary to the suggestion of Amit [10] and in agreement with experimental data [3]. Note that f is a functional of Ψ, not $\tilde{\Psi}$, so that the instability of eq. (2) with respect to $\tilde{\Psi}$ is irrelevant. Finally, we remark that the data of ρ_s [7] can be fitted well with $\rho_s = 1.05\,mn\,(\tau^2 \ln|\Lambda/\tau|)^{\frac{1}{3}}$. Using this result for ρ_s and the expression for f derived from c_p, we obtain $\alpha_0 = 1.8 \times 10^{-16}$ erg and $\beta_0 = 7.7 \times 10^{-39}$ erg cm^3.

Similar logarithmic corrections apply to the phenomenological theory of critical points [11,12]. For example, the corrected coefficients of the two-dimensional Ising model [11] are: $A(\tau) \sim \tau^{-1/4}$; $B(\tau) \sim \tau^{7/4} \ln|\Lambda_I/\tau|$; $C_n(\tau) \sim \ln|\Lambda_I/\tau|$; where $\Lambda_I = e^{3/2}$.

I would like to thank Dietrich Stauffer for directing my attention to the problem of logarithmic corrections.

References

1. V.K.Wong, Phys. Letters 27A (1968) 269.
2. Yu.G.Mamaladze, Zh. Eksp. i Teor. Fiz. 52 (1967) 729; Soviet Phys. JETP 25 (1967) 479.
3. Yu.G.Mamaladze, Phys. Letters 27A (1968) 322.
4. B.D.Josephson, Phys. Letters 21 (1966) 608.
5. M.J.Buckingham and W.M.Fairbank, Progr. Low Temp. Physics, Vol. III, ed. C.J.Gorter (North-Holland, Amsterdam, 1961) p. 85.
6. J.R.Clow and J.D.Reppy, Phys. Rev. Letters 16 (1966) 887.
7. J.A.Tyson and D.H.Douglass Jr., Phys. Rev. Letters 17 (1966) 472; 622 (E); J.A.Tyson, Phys. Rev. 166 (1968) 166.
8. A.Z.Patashinskii and V.L.Pokrovskii, Zh. Eksp. i Teor. Fiz. 46 (1964) 994; Soviet Phys. JETP 19 (1964) 677.
9. M.A.Moore and R.B.Stinchcombe, Phys. Letters 24A (1967) 619.
10. D.J.Amit, Phys. Letters 26A (1968) 466.
11. V.K.Wong, Phys. Letters 27A (1968) 358.
12. D.J.Amit and M.Luban, Phys. Letters 27A (1968) 487.

* * * * *

HOLOGRAPHIE UND INTERFERENZVERSUCHE MIT INHOMOGENEN OBERFLÄCHENWELLEN

H. NASSENSTEIN
Abteilung für Angewandte Physik der Farbenfabriken Bayer AG, Leverkusen

Received 7 October 1968

Holography with the inhomogeneous surface wave of total reflection, reconstruction of this surface wave, and interference of two surface waves are discussed.

Kürzlich wurde über eine holographische Technik berichtet, bei der das Referenzbündel in der photographischen Schicht an der Grenzfläche Gelatine - Luft totalreflektiert wird [1,2]. Aufnahme und Rekonstruktion des Hologramms erfolgen bei diesem Verfahren jedoch in konventioneller Weise. Wie bekannt, tritt aber bei der Totalreflexion die Strahlung auch ins dünnere Medium ein und breitet sich dort in Form einer inhomogenen Oberflächenwelle in sehr dünner Schicht längs der Grenzfläche aus. Es erschien nun interessant festzustellen, ob auch solche inhomogenen Wellen bei der Holographie benutzt werden können.

Die Versuche wurden mit Photoplatten durchgeführt, deren Glasträger einen grösseren