# ATMOSPHERIC PROPERTIES FROM THE INVERSION OF PLANETARY OCCULTATION DATA 

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When short wavelength radiation passes through a planetary atmosphere the various atmospheric constituents distort and spectrally filter the transmitted radiation. If we consider the case of radiation passing tangentially through a spherically stratified planetary atmosphere (i.e. during occultation of the source) the general effect of any single atmospheric distortion is of the form

$$
\begin{equation*}
\text { effect }\left(\mu_{0} r_{0}\right)=2 \int_{\mu_{0} r_{0}}^{\infty} \frac{\text { cause }(\mu r) \mu r \frac{\mathrm{~d} r}{\mathrm{~d}(\mu r)} \mathrm{d}(\mu r)}{\sqrt{ }\left[(\mu r)^{2}-\left(\mu_{0} r_{0}\right)^{2}\right]} \tag{1}
\end{equation*}
$$

where $r$ is the radius from the center of the planet to a point on the ray path, $\mu$ is the index of refraction at that point and the subscript ( $)_{0}$ refers to the condition at the tangent point of the ray that is being considered. The relation is valid for attenuation, angular refraction, and the phase shift caused by the atmosphere. This equation is an Abel integral equation and consequently has an exact mathematical inversion. The inversion relation is

$$
\begin{equation*}
\operatorname{cause}(\mu r)=\frac{1}{\pi} \frac{\mathrm{~d}(\mu r)}{\mathrm{d} r} \int_{\mu r}^{\infty} \frac{\frac{\mathrm{d}\left[\mathrm{effect}\left(\mu_{0} r_{0}\right)\right]}{\mathrm{d}\left(\mu_{0} r_{0}\right)} \mathrm{d}\left(\mu_{0} r_{0}\right)}{\sqrt{ }\left[\left(\mu_{0} r_{0}\right)^{2}-(\mu r)^{2}\right]} \tag{2}
\end{equation*}
$$

The importance of this inverse relation is obvious and it has been discussed for attenuation and angular refraction by Hays and Roble (1968).

However, we would like to summarize some of the properties of planetary atmospheres which may be obtained through the inversion of planetary occultation data in Table 1.

Table 1

| Phenomena | Cause | Effect |
| :--- | :--- | :---: |
| Refraction | $\frac{1}{\mu} \frac{\mathrm{~d} \mu}{\mathrm{~d} r} \sin Z$ | $R\left(\mu_{0} r_{0}\right)$ |
| Microwave phase shift <br> Attenuation <br> (scattering particle <br> absorption) | $\sigma_{i}(\lambda) n_{i}(r)$ | $-\ln \left(\frac{I(\lambda)}{I_{0}(\lambda)}\right)$ |
| Airglow emission | $E_{\lambda}(r)$ | $\frac{4 \pi F_{\lambda}\left(r_{0}\right)}{\Omega A}$ |

Where $\boldsymbol{Z}$ is the angle between the planet radius and ray path, the zenith angle, $R\left(\mu_{0} r_{0}\right)$ is the refraction angle, $\phi\left(\mu_{0} r_{0}\right)$ is the microwave phase shift, $\lambda$ the wavelength, $n_{i}$ is the number density of the $i^{i \text { h }}$ attenuating species, $\sigma_{i}$ is the total photo-absorption cross section, $I_{0}$ the intensity of radiation above the atmosphere, $\bar{I}$ is the intensity of radiation after traversing the atmosphere at a tangent height $r_{0} . E_{\lambda}(r)$ the volume emission rate, and $F_{\lambda}\left(r_{0}\right)$ the flux of energy into a telescope of aperture $A$ and field of view $\Omega$.

The measurement of airglow or scattered light is useful only for an instrument close to the planet with a narrow field of view in order to prevent broadening of the height resolution at the tangent point. Such measurements have been carried out in several cases. Reed and Blamont (1967) have measured the $O\left({ }^{1} D\right)$ $O\left({ }^{3} P\right) 6300 \AA$ red line emission from a satellite using a photometer with a $\frac{1_{2}}{}{ }^{\circ}$ field of view and Barth (1967) has measured the intensity of singly scattered $L y \alpha$ from the Mariner 5 spacecraft during its exit from the

Earth and during its fly-by at Venus in order to obtain hydrogen density. Microwave occultation data is also available from the Mariner fly bys to Mars and Venus and its reduction has been discussed by Phinney and Anderson (1968).

For the case of refraction and attenuation, stars offer sharp resolution for obtaining geophysical data at the tangent point and since occultation from a tangent height of 100 km occurs in 30 sec many observations per orbit may be made (Jones et al. 1962; Fischbach, 1965).

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