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HEAT TRANSFER THROUGH GASES CONTAINED BETWEEN TWO VERTICAL CYLINDERS AT DIFFERENT TEMPERATURES

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Abstract—Heat transfer through gases contained between vertical concentric cylinders was investigated when the radius of the inner cylinder is small compared to the radius of the outer cylinder. Experiments were performed in a modified hot wire type thermal conductivity cell in which end effects could be evaluated in addition to the overall heat transfer between the cylinders. Measurements were made with argon, helium and neon in the pressure range 0·1–620 mmHg, for the temperature differences between the cylinders of 10, 50 and 100 degC, and for the length to the outer diameter ratios between 3·3 and 6·7. The results show that below certain Rayleigh numbers and length to diameter ratios heat is transferred from the hot to cold boundary by conduction only. End effects contribute only in the corner regions. The distance to which these end effects penetrate was determined experimentally, and compared to analytical results obtained from a simple heat balance in the corner region. Relationships, based on the experimental data, were also obtained for the average heat transfer both in the corner regions and over the entire length of the cylinders. It is shown that these results may be used to estimate the error introduced in thermal conductivity and thermal accommodation coefficient measurements by neglecting end effects.

NOMENCLATURE

- a, radius of inner cylinder [cm];
- b, radius of outer cylinder [cm];
- c_p, specific heat [W/g degC];
- D, diameter of outer cylinder [cm];
- g, gravitational acceleration [cm/s^2];
- h, heat-transfer coefficient [W/cm degC];
- H, distance between potential leads [cm];
- k, thermal conductivity [W/cm degC/cm];
- q, heat flow per unit length [W/cm];
- Q, heat flow [W];
- r, radial coordinate [cm];
- S, actual length of cylinders [cm];
- T, temperature [°C];
- ΔT, temperature difference, $= T_a - T_b$ [degC];
- w, axial velocity [cm/s];
- Z, distance from corner in axial direction [cm];
- Z_p, penetration depth [cm];
- C, correction to density;
- Kn, Knudsen number, λ/D;
- Nu, Nusselt number, $= hk/D$;
- Pr, Prandtl number, $= \left.\dfrac{c_p\mu}{k}\right|_{T=\bar{T}}$;
- Ra, Rayleigh number, $= GrPr = \left.\dfrac{g\rho^2\Delta T(2b)^3}{\mu^2\bar{T}}\dfrac{c_p\mu}{k}\right|_{T=\bar{T}}$;
- w^*, axial velocity, $= \left.\dfrac{\omega\rho b c_p}{k}\right|_{\substack{T=\bar{T}\\ Z\geq Z_p}}$;
- Z_p^*, penetration depth, $= Z_p/2b$.

Greek symbols

- ρ, density [g/cm^3];
- ρ', corrected density, $= \rho(1 + C)$ [g/cm^3];
- λ, mean free path [cm];
- μ, viscosity [g/cm/s].

Subscripts and superscripts

- a, evaluated at a;
- b, evaluated at b;

c, convective;
d, departure corner;
e, due to end effects;
f, along the filament;
H, based on height, H, when $S = H$;
k, conductive;
r, radiation;
s, starting corner;
t, total;
v, in vacuum;
$\bar{\ }$, indicates average value.

INTRODUCTION

A KNOWLEDGE of heat transfer through gases contained between concentric cylinders at different temperatures is required in many problems of practical interest. It is needed, for instance, in the experimental determination of thermal conductivities and thermal accommodation coefficients of gases. Heat transfer between concentric cylinders has been measured in connection with such experiments.† To determine thermal conductivities or thermal accommodation coefficients not the total heat transfer, but only the heat conducted through the gas must be known. Thus, considerable efforts have been made in these experiments to minimize the effects of natural convection with the result that in most of the available data heat transfer due to convection is negligible. This investigation was undertaken to study heat transfer by both conduction and convection between two vertical concentric cylinders. Radiative transfer will not be considered. In most cases it is not negligible but is independent of the conductive and convective heat transfer and once the experimental conditions have been specified it can be either measured or calculated with reasonable accuracy.

In addition to studying the overall heat transfer between the cylinders, special attention is given to the evaluation of the end effects.‡ End effects become of particular importance in thermal conductivity and thermal accommodation coefficient measurements when the temperature difference between the inner and outer cylinders is large; a condition that has been encountered recently, in attempts to extend the use of the concentric cylinder geometry to higher temperatures [3–5]. This investigation was motivated by the problems arising due to end effects in such experiments. The results will be presented in general form, however, so as to be applicable to other problems of similar geometry.

† Good summaries of these experiments may be found in [1, 2].

‡ It is noted here that the end effects can be caused predominantly by convection in the gas but are not limited to convection effects only.

EXPERIMENTAL

Here we shall be concerned with the following problem: a gas is contained between two vertical concentric cylinders of radii a and b and length S. The radius of the outer cylinder b, is very large compared to a ($b/a \gg 1$) and the length S is large compared to $b(S/b > 1)$. The temperatures of the inner and outer cylinders are T_a and T_b respectively. The ratio $(T_a - T_b)/T_a$ is not necessarily small compared to one and thus the properties of the gas cannot be assumed to be constant. The two horizontal surfaces which bound the gas layer are impervious to heat and mass flow. The aim was to construct an apparatus which approximates well the above conditions and also allows the variation of the significant parameters over a wide range. The experiments were performed in a modified hot wire type apparatus [6] in which the gas, the pressure, the temperature difference between the cylinders, and the lengths of the cylinders could be varied independently. The radii a and b were fixed.

The test section (Fig. 1) consists of a thin tungsten filament supported axially in a vertical Pyrex tube. Three sections of the filament, differing in length, were isolated by potential leads [7, 9]. The effective length of the outer cylinder (distance S in Figs. 1 and 5) about each of these sections can be varied by moving teflon discs inside the tube. These moving discs and also the center filament are mounted on a

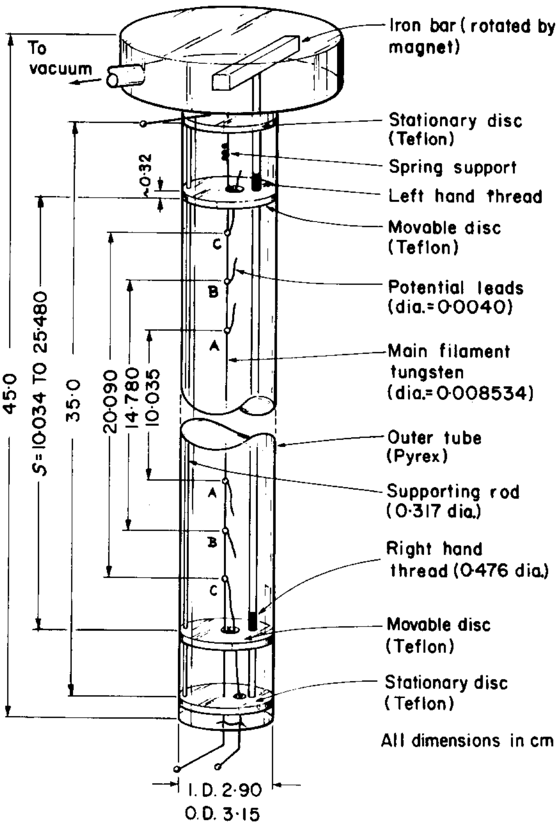

FIG. 1. Schematic of test section.

stainless steel frame. It is understood that the frame might affect the heat transfer. To test this, thermal conductivities of gases were measured with the apparatus, and it was found [6] that in the range of present experimental conditions the frame has negligible effects on the heat transfer.

The data was obtained by the following experimental procedure. At each pressure and at each temperature difference between filament and tube, the position of the movable discs was varied ranging from a distance equal to the length of the filament section ($S = H$, see Fig. 5) to the largest possible distance between the discs. For each position of the discs (i.e. for each S value) the total power input to the filament section under consideration (A–A, B–B, or C–C, Fig. 1) was measured both in the presence of the gas Q_t, and in vacuum Q_v ($\sim 1 \times 10^{-7}$ mmHg). The heat transferred from the filament to the gas due to end effects Q_e, and due to conduction Q_k, may be expressed as

$$Q_H = Q_k + Q_e = Q_t - Q_v. \qquad (1)$$

In evaluating Q_H from equation (1) it is implicitly assumed that Q_v is the sum of the heat loss from the filament by radiation and the heat conducted along the filament to the supports ($Q_v = Q_r + Q_f$), and that Q_v is independent of pressure. Data was taken after steady-state conditions were reached.

The outer cylinder was immersed in an oil bath, held at 34·50 ± 0·02°C. The inner surface of the test tube was presumed to have been at this temperature. The average temperature of the filament was determined by measuring its resistance [10]. The temperature distribution along the filament is not entirely uniform and the term filament temperature, as used here, refers to the average temperature corresponding to the measured resistance of the appropriate filement section. The temperatures of the teflon discs were not recorded.

The test tube was connected to a glass vacuum system. Pressures were measured with a McLeod gauge, a U-tube mercury manometer and an ionization gauge. The electrical measurements were made with a high precision d.c. Millivolt Standard, together with an optical galvanometer and suitable standard resistors. Test gases of highest quality ("Airco" in Pyrex) were used throughout the experiments, but no other efforts were made to obtain clean surfaces. The experimental data reported in the following were obtained with argon, neon, and helium for temperature differences of 10, 50 and 100 degC and in the pressure range 0·1–620 mmHg.

PENETRATION DEPTH

Under certain conditions, in the center part of the cylinders the heat transfer will be by conduction only; i.e. at a distance from the ends the heat transfer through the gas is described by the Fourier equation [11]. First, the conditions will be determined under which such a "conduction regime" exists.

The experiments of Eckert and Carlson [11] on natural convection in air between two vertical plates show that local heat-transfer conditions are different in opposite corners (two lower, or two upper corners) and are similar in diagonally opposite ones. The two different types of corners are denoted as starting and departure corners. The distance from the end where the end effects become negligible and the conduction regime begins is referred to as the penetration depth Z_p. Although the penetration depths in the starting and departure corners may be somewhat different [11], here no distinction will be made between them. In order to find the penetration depth we apply a heat balance in the corner region [11, 12]

$$Q_c = Q_s - Q_d = \int_0^{Z_p} q_s(Z)\,\mathrm{d}Z - \int_0^{Z_p} q_d(Z)\,\mathrm{d}Z \quad (2)$$

where Q_c is the enthalpy carried by convection through a horizontal plane at Z_p, and q_s and q_d are the heat fluxes per unit length in the starting and departure corners (Fig. 2). Equation (2) can be applied only when $Z_p < H/2$. It will be assumed now: (a) that q_s varies as $Z^{-\frac{1}{4}}$ [13];

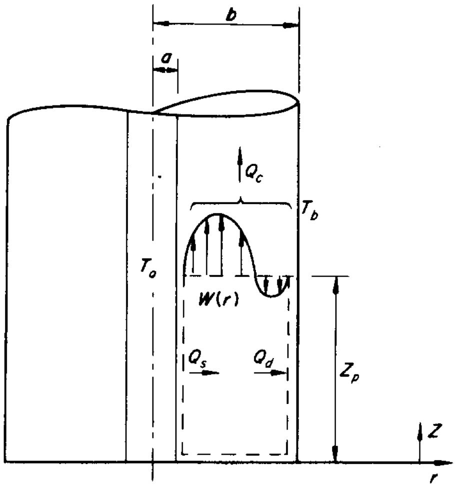

FIG. 2. Sketch of lower end of tube indicating the corner region.

(b) that at $Z = Z_p$, q_s is equal to the conductive heat transfer per unit length q_k, and (c) that q_d is independent of the position and is equal to q_k. With these assumptions equation (2) may be integrated to yield

$$Z_p = 3Q_c/q_k. \quad (3)$$

The parameters Q_c and q_k are given by the equations

$$Q_c = [\int_a^b 2\pi r \rho w(r)\,\mathrm{d}r \int_{T_a}^{T} c_p\,\mathrm{d}T]_{Z=Z_p} \quad (4a)$$

$$q_k = [2\pi a k\,\mathrm{d}T/\mathrm{d}r]_{r=a;\,Z=Z_p} \quad (4b)$$

where $w(r)$ is the axial velocity and all other symbols are defined in the nomenclature. Equation (4a) is based on the assumption that in the conduction regime ($Z = Z_p$) the flow is laminar and fully developed. In order to evaluate Q_c and q_k, the temperature and velocity distributions must be known at $Z = Z_p$. In the conduction regime heat is conducted in the radial direction only and thus, at $Z = Z_p$, the conservation equations for mass, momentum and energy are

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r} r\rho' w = 0 \quad (5a)$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\mu\frac{\mathrm{d}w}{\mathrm{d}r}\right) = g(\rho' - \bar{\rho}) \quad (5b)$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(rk\frac{\mathrm{d}T}{\mathrm{d}r}\right) = 0 \quad (5c)$$

where $\rho' = \rho(1 + C)$, and C is a small correction to the density. It was introduced so that the continuity and momentum equations could be satisfied simultaneously without knowledge of the detailed flow field in the corners [3]. In the calculations that follow, C was always less than 0·1 and generally was about 0·01. In equation (5b) the approximation has been made that density differences due to temperature differences are only of importance in producing differences in the buoyancy force. The equation of state for gases then has the form [12]

$$\rho(r)T(r) = \bar{\rho}\bar{T} = \left[\frac{2}{b^2 - q^2}\int_0^b \rho r \, \mathrm{d}r\right]\bar{T} = \text{const.} \tag{6}$$

The boundary conditions corresponding to equations (5a–5c) are

$$\left.\begin{array}{lll} r = a, & w = 0, & T = T_a, \\ r = b, & w = 0, & T = T_b. \end{array}\right\} \tag{7}$$

The temperature difference between the cylinders may be large ($T_b/T_a > 1$) and hence k and μ cannot be taken to be constants. Equations (5–7) were integrated numerically using a high speed digital computer. Solutions for equations (3–7) were obtained for a wide range of conditions, namely for three monatomic gases (A, Ne, He) and four diatomic gases (CO_2, O_2, N_2 and H_2), five temperature differences (ΔT = 10, 50, 100, 500, 1500 degC), three pressures (400, 600, 760 mm Hg) and five radius ratios (b/a = 10, 25, 50, 100, 250, using a = 0·5, 0·05, 0·005 and 0·0005 cm). The gas properties used in calculations were taken from [14, 15]. The results were expressed in terms of the dimensionless penetration depth, Z_p^*, axial velocity, w^*, and the Rayleigh number, Ra (these parameters are defined in the nomenclature). It was found that for the entire range of parameters employed in the solutions both w^* and Z_p^* can be correlated extremely well with the Rayleigh number. Calculated axial velocity profiles are shown in Fig. 3 for various Rayleigh numbers. These profiles change very little for different radius ratios as long as the radius ratio, b/a, is larger than about ten.

The calculated penetration depths are shown in Fig. 4. The results of the calculations are represented by a solid line and can be well approximated for $b/a > 10$ by the relation

$$Z_p^* = Z_p/2b = Ra/4400, \qquad (Ra \geqslant 4400). \tag{8}$$

Equation (8) is expected to be a reasonable

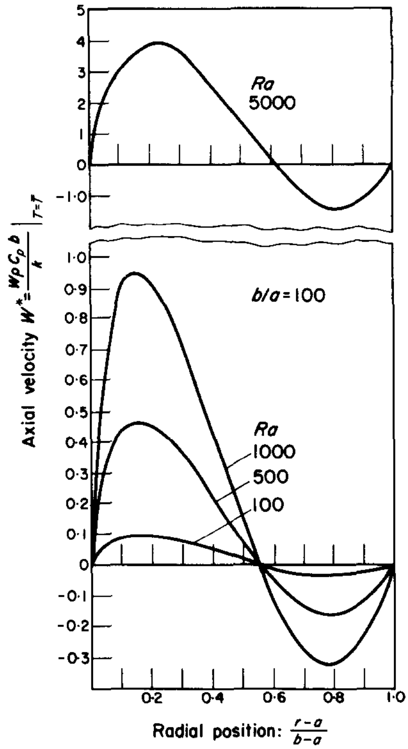

FIG. 3. Calculated axial velocity distributions (a = 0·005 cm)

approximation for Z_p^* only as long as the conductive and convective heat transfers are of the same order of magnitude in the corner region. When the convective heat transfer is small compared to the heat conduction then dimensional considerations [3] suggest that the penetration depth becomes constant, its value being of the order of the difference between the radi ($b - a$). As will be shown presently, this condition arises when $Ra < 4400$.

Penetration depths were measured as described in the previous section. A typical result indicating the variation of the heat transfer with the distance S is shown in Fig. 5. In principle, the penetration depth is reached when there is no further change in the measured heat transfer with a change in S. Here, however, the penetration depth was taken to correspond to the conditions, that for a 1 cm change in S the heat transfer changes less than 0·1 per cent compared to its maximum value. The experimentally

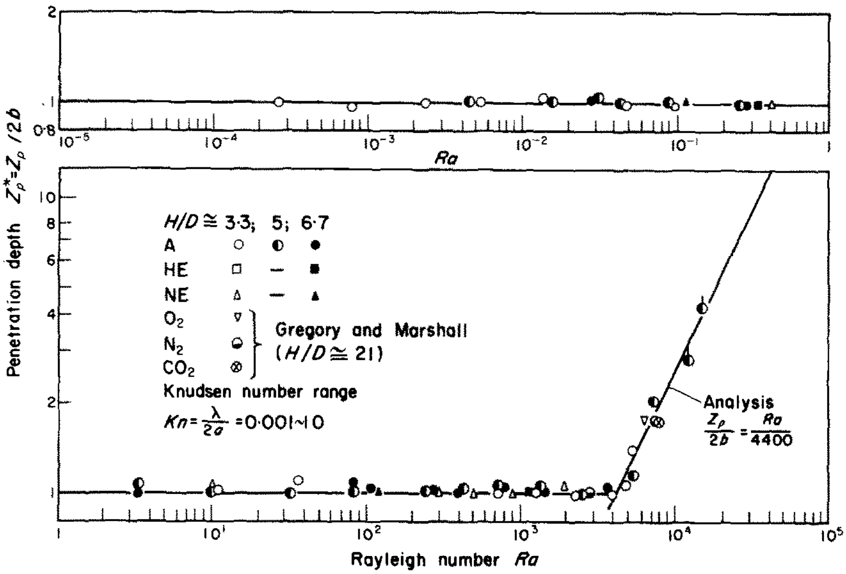

FIG. 4. Penetration depth.

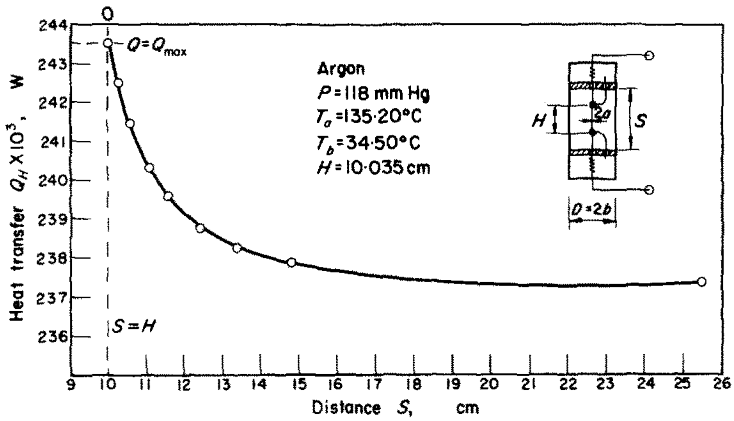

FIG. 5. Data, showing the variation of heat transfer from length H of the filament due to changes in the distance S.

determined penetration depths are shown in Fig. 4. In certain cases the condition given could not be reached and the penetration depth was obtained by extrapolation of the data. These test points are indicated by a short line attached to them. In Fig. 4, penetration depths are also shown deduced from Gregory and Marshall's thermal conductivity experiments [16, 17]. For Rayleigh numbers greater than about 4400 the data agrees fairly well with the analytical result [equation (8)], provided that $Z_p < H/2$. At lower Rayleigh numbers the penetration depth becomes a constant and equal to the diameter of the outer tube.

$$Z_p^* = 1, \qquad (Ra < 4400). \tag{9}$$

The results in Fig. 4 also indicate that, as expected, the penetration depth is independent of the cylinder length as long as $Z_p < H/2$.

In the experiments the pressure was reduced to such low levels that rarefaction effects become important. The highest Knudsen number (based on the diameter of the filament) in the experiments was about ten. At this value nearly free molecule conditions exist in the gas [18]. It is interesting to note, however, that even at such a high degree of rarefaction the penetration depths remain equal to $2b$. It is noted here also that in the present experiments the Knudsen numbers corresponding to $Ra = 4400$ were about 0·01. At these Knudsen numbers the temperature jump effects are negligible, equations (5, 7) are valid, and consequently equation (8) may be used in estimating the penetration depth.

The conditions can now be established under which a conduction regime exists, i.e. when $Z_p < 2H$. Using equations (8, 9) the limiting conditions for the conduction regime were determined and are shown in Fig. 6. In this figure the Rayleigh number is based on the radius b so that the results for the concentric cylinder geometry can be compared with the results obtained by Eckert and Carlson [11] and by Batchelor [12] for vertical parallel plates. As can be seen the Rayleigh numbers limiting the conduction regime as given by the present results are similar to the values presented by Eckert and Carlson, and are less than the values given by Batchelor. However, the slope of the line bounding the conduction regime is identical to the one given by Batchelor.

HEAT TRANSFER IN THE CONDUCTION REGIME

The conductive and convective heat transfer between the two cylinders will be now evaluated based on heat-transfer measurements performed with the moveable discs positioned at either one of the potential leads (AA, BB, or CC).

Corner region

The total heat flux between the cylinders, including the corner regions, may be written as ($Z_p < H/2$)

$$Q_H = \bar{h}_c \, \Delta T \, 2\pi b 2 Z_p + \frac{2\pi k \, \Delta T}{\log b/a}(H - 2Z_p). \quad (10)$$

The overall heat-transfer coefficient in the corner region $\bar{h}_c$, has been defined without distinguishing between the starting and departure corners. Using equation (10) one obtains the average Nusselt number in the corner region

$$\overline{Nu_c} \equiv \frac{\bar{h}_c 2b}{k} = \frac{Q_H - Q_k}{2\pi k Z_p \Delta T} + \frac{2}{\log b/a}, \quad (25 < Ra < 2 \times 10^4). \quad (11)$$

For the present data, equation (11) can be applied only in the Rayleigh number range

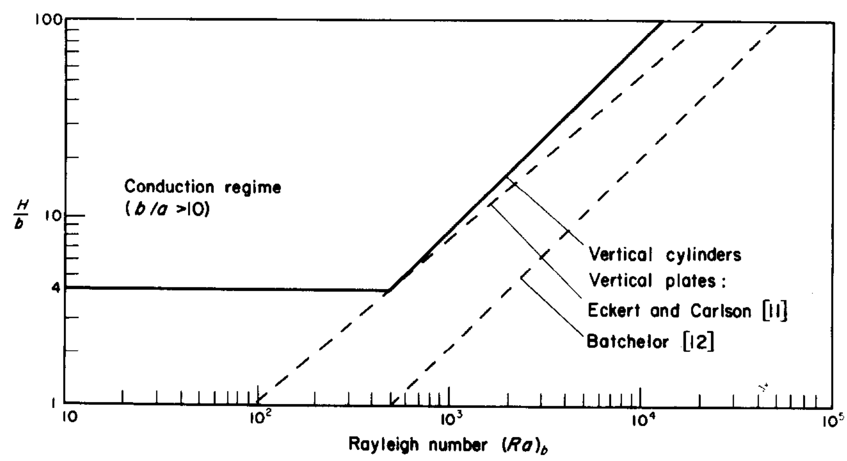

FIG. 6. Limits of the conduction regime (Rayleigh number is based on radius b).

indicated. At $Ra < 25$ rarefaction effects become significant, and the heat conducted from the filament Q_k, cannot be calculated from the Fourier equation. At $Ra > 2 \times 10^4$ the condition $Z_p < H/2$ is not valid any more.

Corner Nusselt numbers calculated from equation (11) are shown in Fig. 7. In calculating $\overline{Nu_c}$, average values for the thermal conductivity were used corresponding to the temperatures $(T_a + T_b)/2$, and Z_p was computed from equations (8) or (9). For Q_H the measured heat-transfer values were substituted. The results indicate that in the appropriate range of Rayleigh numbers $\overline{Nu_c}$ is nearly a constant, at 0·355.† In the present experiments $\log(b/a) = 5{\cdot}86$, and $\overline{Nu_c}$ can be expressed as

$$\overline{Nu_c} \cong 0{\cdot}013 + \frac{2}{\log b/a}, \quad (25 < Ra < 2 \times 10^4). \qquad (12)$$

Substituting equations (8, 9, 12) into equation (14) we get

$$\overline{Nu} \cong \frac{2}{\log b/a} + 0{\cdot}026\, D/H, \quad (25 < Ra < 4400) \qquad (15a)$$

$$\overline{Nu} \cong \frac{2}{\log b/a} + 5{\cdot}9 \times 10^{-6}(D/H)\, Ra, \quad (4400 < Ra < 2 \times 10^4). \qquad (15b)$$

The above results [equation (15)] may be used to estimate the errors arising in thermal conductivity measurements due to the neglecting of end effects. For example, in typical hot wire thermal conductivity cells $H/D = 20$ and $b/a = 200$. Then, for $Ra = 8800$, by including the end effects the Nusselt number is 0·379, while neglecting end effects it is 0·377. Thus, in this case end effects may introduce about a 1 per cent error into the measurements. It is noted that, even when end effects are minimized an error may be introduced in the thermal conductivity measurements, due to the heat transferred in the axial direction by the flowing gas. The calculations of Blais and Mann [3] for H_2 and the present calculations for CO_2, N_2, O_2 and H_2 indicate that this heat transfer is small compared to the heat conducted through the gas radially; e.g. for $\Delta T = 2000$ degC and for $2b = 2$ cm the energy associated with the gas flowing upward (or downward, Fig. 2) is ~0·5 per cent of the radial heat conduction for a 20-cm long cylinder. The percent difference is even less for smaller b and ΔT values. This

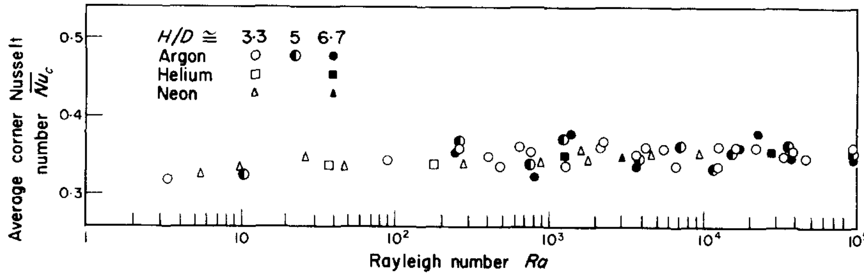

FIG. 7. Average Nusselt numbers in the corner region.

Average heat transfer

An average heat-transfer coefficient for the heat transfer between the inner and outer cylinders including the corner regions may be defined by the equation

$$Q_H = \bar{h}\, \Delta T 2\pi b\, H. \qquad (13)$$

Using equations (11, 13) one obtains for the average Nusselt number

$$\overline{Nu} \equiv \frac{\bar{h}2b}{k} = \frac{2}{\log b/a} + \frac{D}{H} \times \left(2\overline{Nu_c} - \frac{4}{\log b/a}\right)\frac{Z_p}{D}. \qquad (14)$$

† Note that the average Nusselt number remains constant at least up to $Ra = 1 \times 10^5$ (Fig. 7) even though $Z_p > H/2$.

result is borne out by the data which shows that at $\Delta T = 100$ degC the measured thermal conductivity values agree closely with previously reported values provided the test section was long enough for the end effects to be negligible [6].

Finally, it has been observed that at $Ra > 1 \times 10^5$ steady-state conditions could not be reached and significant fluctuations occurred in heat transfer. It has yet to be determined whether these fluctuations are due to turbulence or to instabilities such as observed by Elder [19] and by Vest [20] between parallel plates using fluids with high Prandtl numbers.

ACKNOWLEDGEMENT

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Résumé—Le transport de chaleur à des gaz contenus entre des cylindres concentriques verticaux a été étudié lorsque le rayon du cylindre intérieur est faible comparé au rayon du cylindre extérieur. Les expériences ont été effectuées dans une cellule de conductivité thermique du type à fil chaud modifié dans laquelle les effets d'extrémités pourraient être évalués en plus du transport de chaleur total entre les cylindres. Les mesures ont été faites avec de l'argon, de l'hélium et du néon dans la gamme de pressions de 0,1 à 620 mmHg, pour des différences de température des cylindres de 10, 50 et 100°C, et pour des rapports de la longueur au diamètre extérieur compris entre 3,3 et 6,7. Les résultats montrent qu'en dessous de certains nombres de Rayleigh et certains rapports de la longueur au diamètre, la chaleur est transportée de la paroi chaude à la paroi froide seulement par conduction. Les effets d'extrémités donnent seulement des contributions dans les régions des coins. La distance à laquelle pénètrent ces effets d'extrémités a été déterminée expérimentalement, et comparée à celle obtenue par un calcul simple du bilan de chaleur dans la région des coins. Des relations pour le transport de chaleur moyen, basées sur les données expérimentales, ont été obtenues également à la fois dans la région des coins et dans toute la longueur des cylindres. On montre que ces résultats peuvent être employés pour estimer l'erreur introduite dans les mesures de conductivité thermique et du coefficient d'accommodation thermique lorsqu'un néglige les effets d'extrémités.

Zusammenfassung—Der Wärmetransport durch Gasschichten zwischen zwei konzentrischen Zylindern wurde für den Fall untersucht, dass der Radius des Innenzylinders klein im Vergleich zum Aussenzylinder ist. Versuche wurden durchgeführt in einer umgebauten Kammer für Wärmeleitfähigkeitsmessungen nach der Hitzdrahtmethode. Hierbei konnten die Endeinflüsse neben dem gesamten Wärmestransport zwischen den Zylindern berechnet werden. Messungen wurden durchgeführt für Argon, Helium und Neon im Druckbereich von 0,1–620 mmHg, und für Temperaturdifferenzen zwischen den Zylindern von 10, 50 und 100°C. Das Verhältnis von Länge zu Durchmesser des Aussenzylinders betrug 3,3 und 6,7. Die Ergebnisse zeigen, dass unterhalb gewisser Rayleigh-Zahlen und Lählen und Längen-zu Durchmesserverhältnissen Wärme nur durch Leitung von der warmen zur kalten Berandung transportiert wird. Randeinflüsse machen sich nur in den Ecken bemerkbar. Die Länge, bis zu welcher diese Randeinflüsse vordringen, wurde experimentell bestimmt und mit Werten verglichen, die analytisch aus einer einfachen Wärmebilanz im Eckbereich erhalten wurden. Auf den Versuchswerten beruhende Beziehungen wurden erhalten für den mittleren Wärmeübergang, sowohl im Eckbereich als auch für die gesamte Länge der Zylinder. Es wird gezeigt, dass diese Ergebnisse zur Abschätzung des Fehlers herangezogen werden können, die durch Vernachlässigung der Randeinflüsse bei der Messung von Wärmeleitfähigkeiten und thermischen Akkomodationskoeffizienten entstehen.

Аннотация—Исследуется теплообмен в газах между вертикальными концентрическими цилиндрами, когда радиус внутреннего цилиндра весьма мал по сравнению с радиусом внешнего цилиндра. Эксперименты проводились в модифицированной теплопроводящей ячейке типа горячей проволочки, в которой помимо суммарного коэффициента теплообмена между цилиндрами определялись концевые эффекты. Проводились эксперименты с аргоном, гелием и неоном в диапазоне изменения давления от 0,1 до 6200 мм рт.ст. при перепадах температур на цилиндрах от 10, 50 до 100°С, причем длина составляла 3,3 и 6,7 калибров. Из результатов следует, что при более низких числах Релея и меньших калибрах тепло передается только теплопроводностью от горячей границы к холодной. Концевые эффекты имеют место только в угловых зонах. Исходя из экспериментальных данных, также получены соотношения для среднего теплообмена в угловых зонах и по всей длине цилиндров. Показано, что эти результаты можно использовать для определения погрешности при измерении коэффициентов теплопроводности и теплоусвоения, когда не учтены концевые эффекты.