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THEORETICAL AND EXPERIMENTAL MODELS OF THE DIFFUSE RADAR BACKSCATTER FROM MARS

For work through March 31, 1992

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Objectives

Our general objective for this work has been to develop a theoretically and experimentally consistent explanation for the diffuse component of radar backscatter from Mars. The strength, variability, and wavelength independence of Mars' diffuse backscatter are unique among our Moon and the terrestrial planets. This diffuse backscatter is generally attributed to wavelength-scale surface roughness and rock clasts within the Martian regolith. Through the combination of theory and experiment, we would bound the range of surface characteristics that could produce the observed diffuse backscatter. Through these bounds we gain a limited capability for data inversion. Within this umbrella, specific objectives are:

a) To better define the statistical roughness parameters of Mars' surface so that they are consistent with observed radar backscatter data, and with the physical and chemical characteristics of Mars' surface as inferred from Mariner 9, the Viking probes, and Earth-based spectroscopy.

b) To better understand the partitioning between surface and volume scattering in the Mars regolith.

c) To develop computational models of Mars' radio emission that incorporate frequency dependent, surface and volume scattering.

Progress through December 31, 1990

Earth-based, time-domain radar surveys of Mars exhibit a specular return from a sub-Earth region of Mars followed by diffuse returns from concentric rings that are centered on the sub-Earth point. A characteristic of the diffuse return is that the radar incidence angle increases from zero degrees at the sub-Earth point to nearly 90° at ring diameters that correspond to Mars' limb. An interpretation of the diffuse backscatter beyond inferences based upon the Rayleigh roughness criterion requires some assumption about the roughness statistics of the Mars surface, and requires a scattering theory that permits roughness at all scales.

Several investigators have argued that natural surfaces are scaling [1, 2]. We explore that possibility through an Earth analog of a Mars volcanic region. The topographies of several debris flow units near the Mount St. Helens Volcano were measured at lateral scales of millimeters to meters in September 1990. Mount St. Helens was chosen because of its ease of access and its extremely young terrains. Our objective was to measure the surface roughness of the debris flows at scales smaller than, on the order of, and larger than the radar wavelength of common remote sensing radars. We used a laser profiling system and surveying instruments to obtain elevation data for square areas that varied in size from 10 cm to 32 m. The elevation data were converted to estimates of the power spectrum of surface roughness. The conversions were based upon standard periodogram techniques, and upon a modified spectral estimation technique that we developed.

The surfaces that we examined were located in the debris avalanche west-northwest of the volcano, along the North Fork Toutle River Valley. Most of the debris was deposited during and immediately after the eruption of Mount St. Helens on 18 May 1980. Since then, the deposits have undergone significant erosion by wind and water. A geologic description of the debris avalanche is given by Glicken [3].

Laser Profiler. A 2D laser profilometer was developed specifically for this experiment. Its main component is a surveying electronic distancemeter (EDM) which uses an infrared laser to measure distance. The EDM is mounted on an XY table that is supported 1.5 m above the ground (the minimum range of the EDM) by 4 tripods. Stepper motors are used to move the mounting platform across the table in both directions, allowing the EDM to scan a 1 m^2 surface area. The stepper motors and the EDM are controlled by a laptop computer, allowing the system to run unattended after startup. DC power is provided by two 12 V marine batteries.

Debris surfaces were spray-painted to increase reflectivity. The EDM laser has a spot diameter of ~1.5 mm. In its most precise mode, the standard deviation of the measured surface height is 3 mm. Because each measurement requires 2–3 seconds, a typical scan of 10 cm x 10 cm with sample interval Δ = 2 mm (2601 points) takes ~1.8 hours. Since time was a limiting factor, an increase in surface area required a corresponding increase in Δ. Scans were performed at each site using at least two sampling intervals. A typical surface height grid is shown in Figure 1.

Surveying. Larger-scale topography was surveyed with a self-leveling level and stadia rod. Square grids with sides of 16 or 32 m were delineated by cables with markers at 1 m intervals. A typical survey grid is shown in Figure 2.

Surface Spectra. Estimates of roughness power spectra were developed from the elevation data, but the processing steps differed for profilometer and survey data. Profilometer data suffered from errors which prevented use of standard spectral estimation techniques. These errors were of two types: (1) incorrect height impulses due to overheating of the EDM, and (2) intermittent level shifts due to instability in the EDM. Removal of errors due to overheating involved a combination of median and quartile difference filtering, and is fully described in [4].

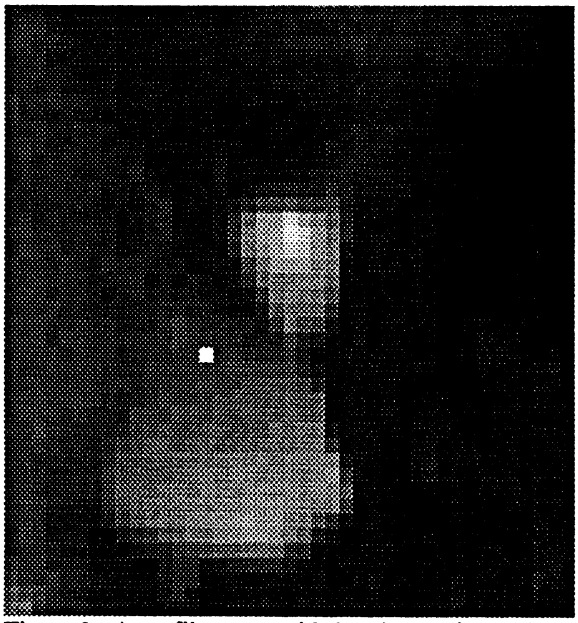

Figure 1. A profilometer grid showing surface height (dark = low). The grid measures 40 cm x 40 cm, with data points spaced 1 cm apart. Total height variation is 10.76 cm.

Figure 2. A typical survey grid showing surface height (dark = low). The grid measures 32 x 32 m, with data points 1 m apart. Total height variation is 8.28 m.

Level shifts of 2-7 mm often occurred after 20–30 minute periods of normal EDM operation. This caused subtle horizontal bands in the profilometer scans. Although these level shifts were small compared to variations in topography, they might corrupt spectral estimates enough to make their elimination worthwhile. We developed a procedure of spectral estimation by linear sampling in which we assume that the surface statistics are isotropic. Linear profiles (rows of the profilometer scan) then become samples of the surface. By using individual scan rows (and removing the mean surface height or dc level from each), we avoid the level shifts because the reference level is stable within most rows. Rows in which level shifts occur are discarded. Furthermore, averaging the autocorrelation estimates obtained in different rows reduces the effect of any remaining impulse errors as well as the variance inherent to a surface random process. The assumption that the surface statistics of the debris flows are isotropic is justified by observation. The debris flows show no directional structure.

The algorithm for obtaining a spectral estimate from linear profiles is developed rigorously in [4]. We show that if the surface height Z(x,y) is a wide-sense stationary random process that is ergodic in correlation, and if the x direction is along the row of the profilometer scan, then the autocorrelation function $R_Z(r)$ ($r^2 = \delta x^2 + \delta y^2$) for an isotropic surface is

$$R_Z(r) = R_Z(\delta_x, 0) = \lim_{T_y \to \infty} \left[\frac{1}{2T_y} \int_{-T_y}^{T_y} dy \lim_{T_x \to \infty} \left\{ \frac{1}{2T_x} \int_{-T_x}^{T_x} dx\, Z(x + \delta_x, y)\, Z^*(x, y) \right\} \right] \quad (1)$$

We use a lag window in our spectral estimate to reduce side lobes and to assure a positive spectral estimator. The window, W(r), which is analogous to the Parzen window used in 1D spectral estimation, is:

$$W(r) = \left(\frac{2}{\pi}\right) \frac{1}{2a^2} \Pi\left(\frac{r}{2a}\right)^{**2} \quad (2)$$

where $a = (N_x-1)\Delta$, N_x is the number of samples in direction x, Δ is the sample interval, Π is the 2D rect function which is centro-symmetric, and **2 indicates that the function is convolved with itself in two dimensions. The spectral estimation then becomes:

$$\hat{S}_Z(k) = 2\pi \int_0^{(N_x-1)\Delta} dr\, r\, R_Z(r)\, W(r)\, J_0(kr) \quad (3)$$

where J_0 is the zero-order Bessel function. $S_Z(k)$ is therefore the Hankel transform of $R_Z(r)$. A typical plot of $S_Z(k)$ is shown in Figure 3.

SURVEY SPECTRA. Spectral estimates from the survey data were more straightforward. in that a standard spectral estimator, the periodogram, was used in the estimation of the survey spectra. A typical periodogram estimate resulting from the survey data is shown in Figure 4.

These spectra and the many others that we have obtained indicate that, over spatial wavelengths of millimeters to decameters, the debris flows at Mt. St. Helens are scaling. These results will be reported at the International Geoscience and Remote Sensing Symposium (IGARSS'91) at Espoo, Finland, June 3-6.

SCATTERING MODEL. We are modifying a vector scattering theory to incorporate multi-scale roughness. We believe that the Phased Wiener-Hermite model shows greatest promise for this application, and are coding it for initial tests. We will report progress with the model at the Progress in Electromagnetics Research Symposium (PIERS) at M.I.T., July 1-5, 1991.

FURTHER WORK DURING THIS PERIOD. Our highest priorities are to complete development of the multi-scale, theoretical scattering model and to test it against airborne radar data for Mt. St. Helens. Further, we will obtain existing compilations of radar data from Mars radar investigators. Finally, we will begin the process of manufacture scattering surfaces (by a computer driven milling machine) having scaling roughness statistics that are consistent with Mars roughness at scales observed by Viking.

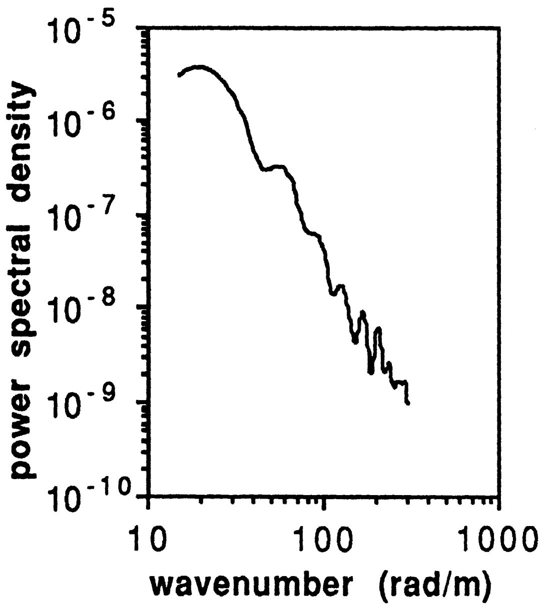

Figure 3: Surface spectral estimate $S_Z(k)$ for a typical profilometer scan.

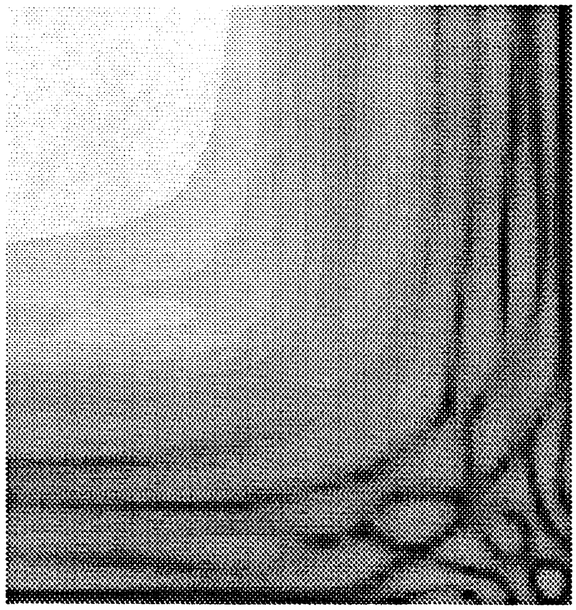

Figure 4: Periodogram spectral estimate for a typical surface survey.

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Progress January 1, 1991, through March 31, 1992

Our efforts during the past year have focused on obtaining correct estimates of the surface roughness spectra of measured debris flow units at Mount St. Helens and on developing the capability to manufacture analog surfaces with prescribed statistics for laboratory scattering studies.

Surface Spectra. We believe that we have developed an algorithm which estimates the surface spectrum with reduced variance while minimizing the effects of data errors which were present in the surface profile measurements. In the process of developing this algorithm, we determined that Fourier-based spectral estimators, which are commonly used in published studies of surface spectra, are not sensitive to the slope of a power-law spectrum. We will explain this problem and our choice of an alternative spectral estimator in the section Estimator Selection.

Surface Profile Measurements. As reported previously [1], the surface roughness of the debris flow units at Mount St. Helens were measured at scales larger than, on the order of, and smaller than the radar wavelength of common remote sensing radars. Small-scale roughness was measured using a 2D laser profilometer, which scanned a surface area of up to 1 m^2 with a laser-based surveying electronic distancemeter (EDM) to measure the surface height profile. Large-scale topography was surveyed, using a surveying level and stadia rod to measure surface heights over square grids with sides of 32 m.

Data Errors. The profilometer data suffered from data errors which made the spectral estimation process less straightforward. The errors were of two types: (1) incorrect height impulses due to overheating of the EDM, and (2) intermittent level shifts due to an instability in the EDM. Removal of the incorrect surface heights due to overheating involved a combination of median and quartile difference filtering, and is fully described in [2]. Median interpolation was then used to replace the incorrect values which had been filtered out. Tests in which holes were introduced and then replaced (via median interpolation) in artificial data sets showed that median interpolation has minimal effect on the resultant roughness statistics, primarily due to the row averaging described below.

The level shift errors resulted in horizontal bands in the profilometer scans between which the reference level shifted by 2–7 mm. To minimize the effects of the level shifts, we developed a procedure of spectral estimation by linear sampling in which the rows of the profilometer scan serve as linear samples of the surface. This procedure is described in the section Modified Estimation Algorithm.

Estimator Selection. As reported previously, several investigators have argued that natural surfaces are scaling [3–7]. Linear profiles of such surfaces have (over the range of measurable spatial frequencies) a surface roughness spectrum (power spectral density of surface height) which has a the form of a power law:

$$S_Z(f) = cf^{-\beta}$$

where $1<\beta<3$. Such a spectrum is a straight line on a log-log plot with a slope of $-\beta$.

Initial attempts to convert the measured surface profiles to estimates of the roughness spectrum involved the use of Fourier-based spectral estimators. The most common of these is the *periodogram,* which is described in numerous texts on signal processing and spectral estimation. The formula given by Kay [8] is

$$\hat{P}_{PER}(f) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x[n]\ \exp(-j2\pi fn)\right|^2$$

Another common Fourier-based estimator is the *Blackman-Tukey spectral estimator,* given by Kay as

$$\hat{P}_{BT}(f) = \sum_{k=-(N-1)}^{N-1} w[k]\hat{r}_{xx}[k]\ \exp(-j2\pi fk)$$

where w[k] is a lag window and the estimate of the autocorrelation function is

$$\hat{r}_{xx}[k] = \frac{1}{N}\sum_{n=0}^{N-1-k} x[n]x[n+k]$$

The results obtained using either of these estimators on our measured profiles suffered from high enough variance that we were not sure that functions fit to the resultant spectra were accurate representations of the actual spectra. To answer this question, we calculated the expected value for the Blackman-Tukey estimator, which can be written as follows:

$$\left\langle\hat{P}_{BT}(f)\right\rangle = \int_{-1/2\Delta}^{1/2\Delta} \widetilde{W}(f-\theta)\left\langle\hat{P}_{PER}(\theta)\right\rangle d\theta$$

where

$$\left\langle\hat{P}_{PER}(f)\right\rangle = \Delta\int_{-1/2\Delta}^{1/2\Delta} \widetilde{W}_B(f-\xi)\tilde{P}_{xx}(\xi)\ d\xi$$

and where $\widetilde{W}$ is the Fourier transform of the lag window, $\widetilde{W}_B$ is the transform of a triangular window, P_{xx} is the true power spectral density, and the sampling interval Δ is included to preserve the scaling so that spectra produced at different scales may be compared.

The expected values of the Blackman-Tukey estimates are shown in Figure 1 along with the true spectra for power-law spectra with $\beta = 2.0$ and $\beta = 2.4$. Ignoring the differences in the absolute level, we see that the slope ($-\beta$) of the two estimates is almost the same, although the true spectra have visibly different slopes. Similar results are obtained by examining the expected value of the periodogram estimator for the power-law spectrum case.

To avoid the insensitivity to slope exhibited by the Fourier-based estimators, we next tried a Capon or Minimum Variance spectral estimator, described in chapter 11 of Kay [8]. The Capon estimate is obtained by first calculating an estimate of the covariance matrix $\mathbf{R}_{ZZ}$, whose elements are defined as

$$[\mathbf{R}_{ZZ}]_{ij} \equiv \langle Z^*[n]Z[n+i-j]\rangle$$

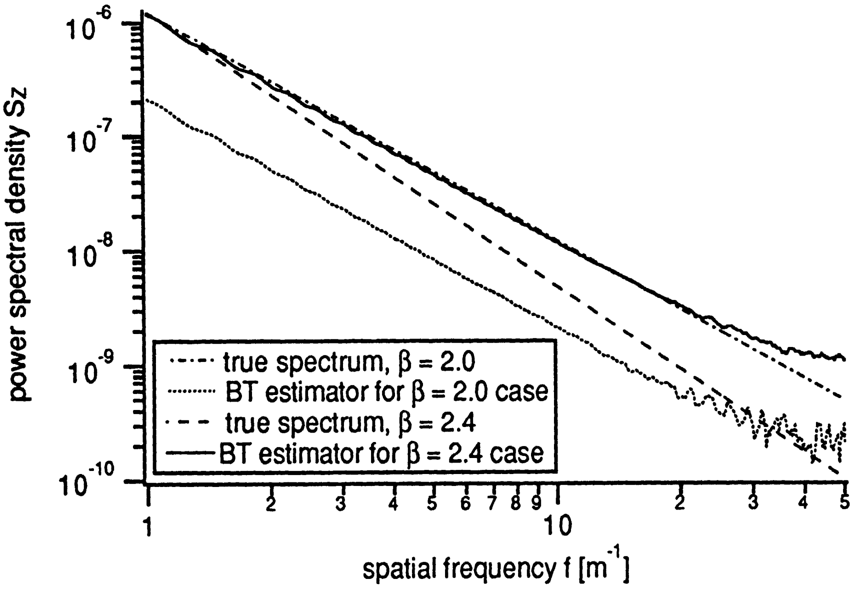

Figure 1. Expected values of the Blackman-Tukey estimate of two power-law spectra (β = 2.0 and 2.4) along with the true spectra.

The elements of the covariance matrix are estimated as follows:

$$\left[\hat{R}_{ZZ}\right]_{ij} = \frac{1}{2(N-p)}\left[\sum_{n=p}^{N-1} x[n-i]\,x[n-j] + \sum_{n=0}^{N-1-p} x[n+i]\,x[n+j]\right]$$

The Capon estimator is then given by

$$\hat{P}_{MV}(f) = \frac{p\Delta}{e^H \hat{R}_{ZZ}^{-1} e}$$

where p is the dimension of the covariance matrix and

$$e = [\quad 1 \quad e^{j2\pi f} \quad e^{j4\pi f} \quad \ldots \quad e^{j2\pi(p-1)f}\ ]^T .$$

To determine the performance of the Capon estimator in the optimal case, we generated artificial 8192-point surface profiles with spectra $S_Z(f) = cf^{-\beta}$ where $\beta = 2.0$ and $\beta = 2.4$ and calculated Capon spectral estimates for 512 frequencies with $p = 70$. The results are shown in Figure 2. The Capon estimate is clearly more sensitive to the slope of the power-law spectrum.

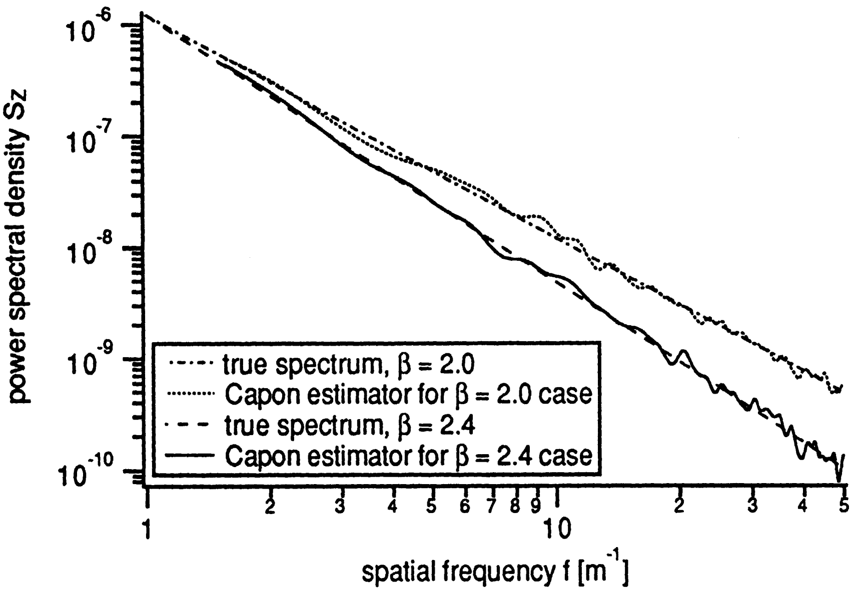

Figure 2. True spectra and Capon spectral estimates for artificial 8192-point surface profiles with $\beta = 2.0$ and 2.4.

Modified Estimation Algorithm. Having selected the Capon spectral estimator, we needed to develop an algorithm for calculating an estimate of the 2D surface roughness spectrum using linear samples, a constraint imposed by the level shift errors described earlier. Recall that a linear profile of a scaling surface has a surface roughness spectrum which has the form of a power law. If the surface is known or can be assumed to have isotropic surface statistics, the two-dimensional roughness spectrum of the surface also has the form of a power law:

$$S_Z(f_r) = a f_r^{-\gamma}$$

where the radial spatial frequency $f_r = (f_x^2 + f_y^2)^{1/2}$ and $2 < \gamma < 4$. If we assume that (1) the surface roughness spectrum is power-law down to some minimum frequency f_c which is below the lowest frequency that we can measure and (2) the roughness spectrum is finite below f_c, we can relate the power-law spectra of the surface and profile:

$$a = \frac{\Gamma[(\beta + 1)/2]}{\sqrt{\pi}\,\Gamma(\beta/2)} c$$

$$\gamma = \beta + 1$$

Therefore, our estimation algorithm is as follows: (1) assume isotropic surface statistics; (2) for each row, calculate a Capon spectral estimate; (3) average the estimates over the rows; (4) fit the single or multiple-scale averaged estimate(s) with a power-law function $S_Z(f) = cf^{-\beta}$ (if appropriate); and (5) convert the profile (1D) spectra to surface (2D) spectra, using the above two relations.

Although this method of determining the surface spectrum from linear profiles was made necessary by the presence of the level shift errors, it is not without benefit. One grid of measured surface heights yields many linear profiles. While the profile rows are not completely independent, the row spectral estimates can still be averaged to obtain a significant decrease in the variance. This averaging would not be possible with a two-dimensional estimator, given the same measured data.

Spectral Results. One of the roughest debris flows which was still in a relatively pristine state (i.e., not smoothed by sedimentary deposits) was measured at three different spatial scales. The three averaged Capon spectral estimates are shown in Figure 3, along with a power-law spectrum fit to the spectra. The linear character of the spectral estimates suggests that a power-law spectrum is appropriate for a primary debris flow surface. This implies that the surfaces are scaling for spatial frequencies between 0.08 and 100 m^{-1}. The equation of the power law fit for linear profiles of this surface is

$$S_Z(f) = (2.80 \times 10^{-4})\, f^{-2.29}$$

Converting to a surface (2D) spectrum, we obtain the following power law:

$$S_Z(f) = (1.52 \times 10^{-4})\, f^{-3.29}$$

The above example shows the importance of using a spectral estimator appropriate to a surface with a power-law spectrum. Had we used a Fourier-based estimator instead, we would have obtained a spectrum with β close to 2.0, indicating that the surface was rougher than it is.

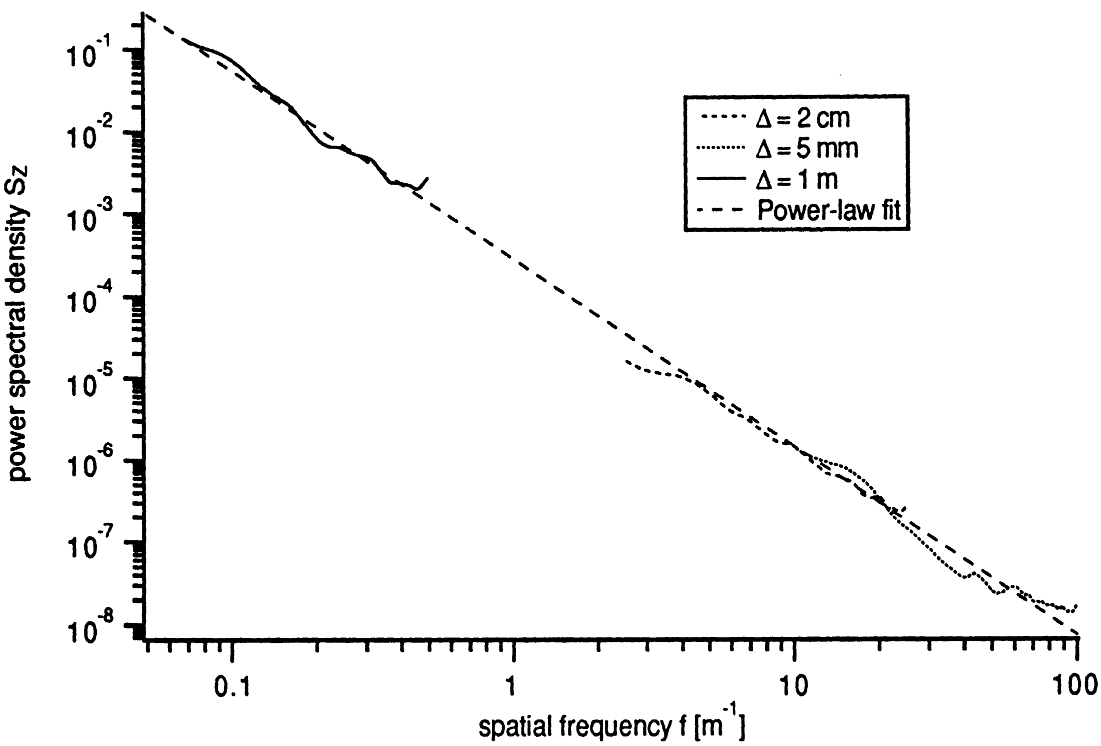

Figure 3. Capon spectral estimates of a debris flow surface at three spatial scales. A power-law fit to the spectra is also shown.

These results were presented at the 1992 Lunar and Planetary Science Conference [9] in Houston.

Volcanic Surface Analogs. The estimates of the surface roughness spectra of the measured volcanic debris flows will be used to design artificial surface analogs with similar surface characteristics. These surface analogs will then be used in scattering studies in which the scattering properties of the surfaces will be measured in the laboratory. During the last few months, significant effort has been devoted to developing a manufacturing process for these surfaces.

The initial plan was to generate a surface height profile by computer and use this profile as input to a computer-driven milling machine at an area machining company. Our requirements for the milled surface were as follows: (1) area: 1 m x 1 m; (2) total surface height variation: 15 cm; (3) material: wax, polymer, plastic, or similar material; and (4) accuracy: within 1 mm of surface model. When we contacted numerous machining shops in the area, we found that most could not handle such a large surface due to either physical machine limitations or insufficient memory capacity. The single company which did have the capability to handle such a surface was unable to give firm estimates of the cost because our specifications were subject to change, being dependent on the spectral analysis which was in progress. However, we determined that the cost of hiring out would very likely be more than the cost of purchasing our own equipment and manufacturing the surfaces ourselves.

After seeking proposals from several manufacturers, we have purchased a milling system which has the capability of producing a surface which meets the requirements stated above. We have also purchased associated CAD/CAM software which will translate our surface model into machine instructions and control the milling system. We plan to make the system operational and begin production of test surfaces in the next few weeks.

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