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SHORT PAPER

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Characteristics of ionospheric thermal radiation

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Abstract—A theoretical observation of the characteristics of ionospheric thermal radiation is made using a linear theory, based on a macroscopic concept with the aid of fluctuating electromagnetic field theory. The thermal noise power generated in the ionosphere per unit volume, per unit frequency bandwidth, and the available thermal noise power at a receiving antenna per unit frequency bandwidth are calculated. The spectral distributions of ionospheric thermal radiation are obtained and discussed in detail.

A study of the calculated power level of the thermal noise generated in the ionosphere shows that it is exceedingly low and decreases rather rapidly with an increase of frequency f. A large portion of the noise signal generated appears to be in the frequency range of $f < 10^7$ c/s, with the microwave noise signal being negligible, and appears to come mainly from the region between 60 km and 100 km of the ionosphere. Furthermore, the study reveals that the available noise power at a receiving antenna depends upon the geographical location of the antenna in general and that the power level is higher in the equatorial zone than in the polar cap zone.

For a noise signal frequency of less than 10^8 c/s, the power level increases monotonically with an increase of polar angle θ from $\theta = 0$ and reaches its maximum value at $\theta = 90°$, where it is at least a hundred times greater than at $\theta = 0$. For a noise signal with a frequency of 10^9 c/s, the available noise power has its maximum at $\theta \simeq 50°$, where the power level is comparable to that of a noise signal with a frequency of 10^6 c/s. For a noise signal of a frequency above 5×10^9 c/s, the angular dependence disappears.

1. Introduction

In a recent theoretical study made by the author on ionospheric thermal radiation, the expressions for w_0, the thermal noise power generated per unit volume, per unit frequency bandwidth, and for P_0, the available noise power at the receiving antenna per unit frequency bandwidth for the case of vertical incident measurement, have been derived, based on a macroscopic concept with the aid of the fluctuating electromagnetic field theory using the Maxwell-Langevin equations. It is the purpose of the present paper to estimate theoretically the radiation characteristics of ionospheric thermal noise by investigating the expressions for w_0 and P_0 with the aid of experimentally observed available data (e.g. Legalley and Rosen, 1964; Jackson and Kane, 1959; Jackson and Seddon, 1958) for the profile of the electron number density.

The expressions for w_0 and P_0 are given as follows (the derivation of these expressions is presented in Hsieh (1966)):

$$w_0(f, r, \theta) = kT_0\left(\frac{Z^2}{1 + Z^2}\right)(1 + 2l_{33}) \tag{1}$$

and

$$P_0(f, \theta) = \int_{h_0}^{h_0+\Delta h} \frac{\pi f}{c}\left(\frac{kT_0XZ}{1 + Z^2}\right)[l_{22} + l_{33}]\, dh, \tag{2}$$

where

$$l_{22}(f, r, \theta) = \frac{(1 + Z^2)(1 + Z^2 + Y^2) + (Y^2 + Z^2 - 3)\, G^2 \sin^2\theta}{(Y^2 + Z^2 - 1)^2 + 4Z^2},$$

$$l_{33}(f, r, \theta) = \frac{(1 + Z^2)(1 + Z^2 + Y^2)}{(Y^2 + Z^2 - 1)^2 + 4Z^2},$$

$$X = \left(\frac{\omega_p}{\omega}\right)^2,$$

$$\omega_p^2 = \frac{N_0 e^2}{m\varepsilon_0},$$

$$Y^2 = \left(\frac{\omega_b}{\omega}\right)^2 = G^2[1 + 3\cos^2\theta],$$

$$Z = \frac{\nu}{\omega},$$

$$G = -\frac{M}{3}\frac{e}{m}\left(\frac{a}{r}\right)^3\frac{1}{\omega} \text{ and}$$

$$r = a + h. \tag{3}$$

In the above expressions, r and θ are the radial and polar angular variables, respectively, in the geomagnetic polar coordinate system whose origin is located at the center of the Earth. The electronic charge e is taken as a negative value and m is the electronic mass. ω_p and ω_b are the electron plasma and gyrofrequencies, respectively, and ω is the angular frequency of the noise signal under consideration. c is the speed of light in vacua and ε_0 is the dielectric constant of vacuum. h is the height above sea level; f is the radiation frequency; a is the radius of the Earth; M is the magnetization of the Earth; and k is the Boltzmann constant. Finally N_0, T_0 and ν are electronic number density, temperature and collision frequency, respectively.

2. Asymptotic Expressions

When the radius of the Earth a is taken as 6370 km and the magnetization of the Earth M is taken as $(0{\cdot}935)/(4\pi)$ A/m² (Morgan, 1959), the factor $G(\omega, r)$ given in equation (3) can be approximated by

$$G(\omega, r) \simeq \frac{6{\cdot}95 \times 10^8}{f}\left\{1 - 3\left(\frac{h}{a}\right)\right\}, \qquad \text{for } \left(\frac{h}{a}\right) \ll 1. \tag{4}$$

Consequently G is practically invariant with respect to h for the range of height up to 200 km. On the other hand, Z, being equal to $\nu(h)/\omega$, varies considerably with h in the same region of the ionosphere. It is of interest to note that when the parameters Z and G satisfy the following conditions:

$$4Z^2 \ll 1, \qquad 3 \ll G^2 \qquad \text{(Case A)} \tag{5a}$$

or

$$4Z^2 \ll 1, \quad 4G^2 \ll 1 \qquad \text{(Case B)}, \tag{5b}$$

Equation (1) takes the following simple form:

$$w_{03}(f, h) = kT_0 Z_3^2 \tag{6a}$$

or

$$w_{04}(f, h) = 3kT_0 Z_4^2. \tag{6b}$$

On the other hand, for $|\Delta h| \ll h_0$, equation (2) takes the following form:

$$P_{03}(f, \theta) = \xi_0(\theta, h_0) k T_0 X_3 Z_3 \left(\frac{\Delta h}{\lambda_3}\right) \pi \tag{7a}$$

or

$$P_{04}(f, \theta) = 2kT_0 X_4 Z_4 \left(\frac{\Delta h}{\lambda_4}\right) \pi \tag{7b}$$

with

$$\xi_0(\theta, h_0) = \frac{2 + G^2 \sin^2 \theta}{G^2(1 + 3\cos^2 \theta)}, \tag{8}$$

where the subscripts 3 and 4 are introduced in w_0 and P_0 to indicate the fact that Case A and Case B are being considered respectively. Furthermore, for the region of the ionosphere between 85 km and 200 km, the range of frequency which satisfies the conditions (5a) and (5b) can be given as follows:

$$1{\cdot}6 \text{ mc} \leq f \leq 42 \text{ mc}, \qquad \text{for Case A} \tag{9a}$$

or

$$14 \text{ kmc} \leq f, \qquad \text{for Case B.} \tag{9b}$$

Thus equations (6a) and (7a) can be regarded as the asymptotic expressions for w_0 and P_0 for the radio-frequency range and equations (6b) and (7b) are the asymptotic expressions for the microwave-frequency range respectively.

3. Calculation

It is easily seen that once the electron temperature T_0, number density N_0 and collision frequency ν are known, calculation of the quantities w_0 and P_0 is straightforward. Unfortunately the exact knowledge of T_0, N_0 and ν as functions of position is not available at the present time. In view of the fact that many of the results of radio observation can be explained by assuming that the ionosphere is horizontally stratified, that is, that the electron density and collision frequency are functions only of the height h, this assumption is adopted for the present investigation. Furthermore, it is assumed that under the thermal equilibrium conditions the electron temperature T_0 is equal to the background gas temperature T_g. The midday mean electron number density profile $N(h)$, based on seven experimental observations—one by partial reflection technique, two by cross-modulation technique and four by rocket experiment—is used for the D-region (60 km–90 km) and the other profiles, based on two rocket observations, are used for the height range between 90 km and 180 km.

The value of the average electron collision frequency ν with the neutral particles and ions can be given approximately as follows:

$$\nu = \langle \nu_m \rangle + \langle \nu_{ei} \rangle, \tag{10}$$

where $\langle \nu_m \rangle$ and $\langle \nu_{ei} \rangle$ denote the average electron collision frequency with neutral particles and with ions respectively (e.g. SHKAROFSKY, 1961). They are given in terms of atmospheric parameters as follows:

$$\langle \nu_m(h) \rangle = 1{\cdot}9944 n_M T_0 \times 10^{-7}, \qquad \text{for } h \leq 180 \text{ km}, \tag{11}$$

where n_M is the number density of air, and

$$\langle \nu_{ei} \rangle = 8{\cdot}375 N \log \Lambda \times 10^{-6}/T_0^{3/2} \tag{12}$$

with

$$\Lambda = 1{\cdot}24 \times 10^7 \, (T_0^3/N)^{1/2}, \tag{13}$$

where N is the ion density and T_0 is the electron temperature. In the present calculation it is assumed that $N_0 = N$ and $T_0 = T_g$. ν, given in equation (10), is calculated with the aid of tabulated data for n_M and T_g (e.g. KALLMANN-BIJI, 1961). The mks system of units is employed in the present study.

4. DISCUSSION OF RESULTS

The results of calculation of the noise power spectrum show that for a given value of θ and h, w_0 decreases with an increase in f as shown in Fig. 1 except in the vicinity of $f = 10^9$ c/s. The rate of decrease is higher in the h.f. range than in the l.f. range. There is a hump, i.e. a relative minimum and a relative maximum, in the range

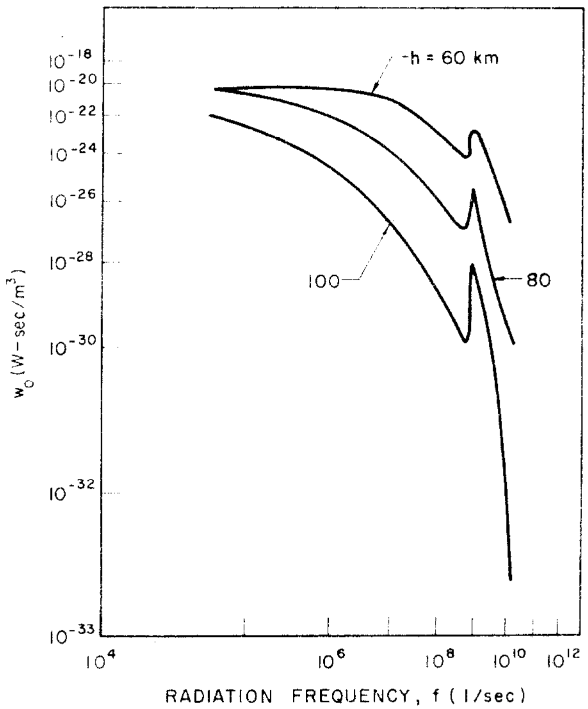

Fig. 1. The frequency spectrum of the thermal energy volume density with height as parameter for $\theta = 45°$.

$10^8 < f < 5 \times 10^9$ c/s and this hump appears to be largest for $\theta = 45°$ and it tends to disappear for $\theta = 90°$.

It is of interest to note that for the frequency range specified by the inequalities (9a) and (9b), equations (6a) and (6b) both suggest that w_0 is inversely proportional to the square of the frequency f and directly proportional to the square of the collision frequency ν for the region between 85 km and 200 km of the ionosphere. In addition, a comparison between equations (6a) and (6b) shows that at a given height h, since $dw_{04}/df = 3dw_{03}/df$, the rate of decrease of w_0 in the h.f. range is three times greater than in the l.f. range (e.g. see the case of $h = 100$ km in Fig. 1). Thus the observation made in Fig. 1 on the behaviour of w_0 can be predicted quite well by the asymptotic expressions for w_0 given by equations (6a) and (6b). It should be pointed out, however, that equations (6a) and (6b) are not applicable for a frequency range in the vicinity of $f = 10^9$ c/s and therefore a prediction of the behaviour of w_0 in this region has to be made by studying equation (1).

On the other hand, as shown in Fig. 2, for a given $f \leq 10^8$ c/s, P_0 increases rapidly with an increase of θ in the region $0 \leq \theta \leq 20°$ and increases gradually in the region

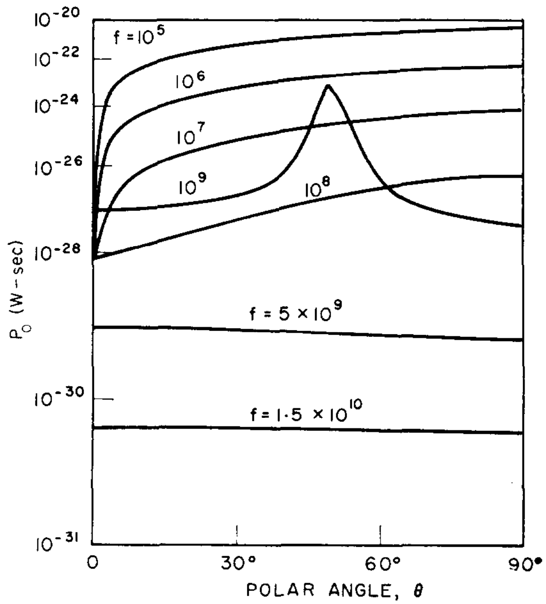

Fig. 2. The polar angular spectrum of available noise power at the receiving antenna per unit frequency bandwidth from the source lying between 60 km and 180 km.

$\theta > 20°$, then reaches its maximum value at $\theta = 90°$. For a given $f \geq 7{\cdot}5 \times 10^9$ c/s, P_0 becomes independent of θ. However, for the noise signal with $f = 10^9$ c/s, P_0 increases gradually from $\theta = 0$ to about 30°, then increases rather rapidly to reach its maximum value at about $\theta = 50°$, and finally decreases as θ increases further. The values of P_0 in the region of $\theta = 0$ and in the vicinity of $\theta = 90°$ are about the same. The maximum value of P_0 at $\theta = 50°$ for $f = 10^9$ c/s is comparable to that for $f = 10^6$ c/s. It should be noted that the asymptotic expression (7b) which predicts P_0 for the microwave range is independent of θ, while equation (7a) indicates that P_0, for the radio-frequency range, does depend upon θ in the manner specified by equation (8), which is consistent with the above observation made on the behaviour of P_0 in Fig. 2.

5. CONCLUSIONS

The spectral distribution of ionospheric thermal noise power has been obtained. At a given position (h, θ) in the ionosphere there is at least ten thousand times more thermal noise power per unit volume, per unit frequency bandwidth, generated in the frequency range $5 \times 10^4 \leq f \leq 10^6$ c/s than in the range $f \geq 10^9$ c/s. w_0, the thermal noise power generated per unit volume, per unit frequency bandwidth, decreases monotonically with an increase in the height h, which is reasonable in view of the fact that the electronic collision process plays a major role in the type of radiation under consideration, and since the collision frequency $\nu(h)$ decreases with an increase in height. w_0 appears to be independent of the polar angle θ except for the case of $f = 10^9$ c/s in which it has its maximum value at $\theta = 45°$.

The observation of the spectrum of the available noise power at the receiving antenna $P_0(f, \theta)$, per unit frequency bandwidth, with the source regions between 60 km and 180 km of the ionosphere, shows that in general it decreases with an increase of the frequency f and increases with an increase of the polar angle θ.

It should be pointed out that most of the uncertainty in the present calculation arises from a lack of complete knowledge concerning the electron temperature and the electron distribution function. Furthermore, the results shown above are based on the ideal assumption of a Maxwellian distribution, and equal electron and gas temperatures. Consequently the results of the present study should be regarded as a good qualitative analysis of the thermal radiation from the ionosphere rather than a rigorous quantitative analysis.

The calculations indicate that the power level of the thermal radiation originating in the ionosphere is indeed exceedingly low, a fact which is commonly believed. For example, at $\theta = 90°$ for $f = 10^5$ c/s, P_0 is of the order of 10^{-20} W c/s and for $f = 10^6$ c/s, it is of the order of 10^{-22} W c/s, which might not be detectable without the aid of an exceedingly sensitive detecting system. However, it is of interest to observe the dependence of P_0 on the polar angle θ, as well as on the radiation frequency f.

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