INTERFEROMETRIC RECONSTRUCTION OF PHASE OBJECTS USING DIFFUSE 'CODING' AND TWO HOLOGRAMS

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In contrast with previously described multiply-exposed single holograms, used for interferometry according to a principle first described by Gabor, Stroke, Restrick, Funkhouser and Brumm [1], interferometric image 'coding' and 'decoding' may be accomplished in diffuse light with two separately recorded holograms.

The possibility of adding the complex amplitudes of the electric field vectors in two wavefronts by successively double-exposing a single hologram was first suggested by Gabor, Stroke, Restrick, Funkhouser and Brumm [1], and demonstrated by them for an 'image-synthesis' application. Several interferometric applications (e.g. of the two-beam type) suggested by that work have also been demonstrated in a number of cases [2-4]. Another early example of a multiple-exposed hologram was described by Stroke and Falconer [5] in connection with 'multi-directional illumination and moving scatterers'. In these and other comparable experiments, single holograms were used for interferometric and other purposes, with or without 'double' or multiple exposure.

It is of a particular interest to show that complex amplitudes in two wavefronts may be interferometrically added by means of two separate holograms, recorded independently in two separate exposures. The interferometric addition may be performed, for example, by placing the two holograms next to each other, with the emulsions in close 'contact' and suitably aligned during the reconstruction process. It is also of interest to demonstrate that an interferometrically 'decoded' image may be obtained, with the aid of a suitable 'decoding' hologram \( H_2 \) in a case where the first hologram \( H_1 \) of a phase object, of interest, has been recorded by illuminating the object with a randomly diffused wavefront, or indeed by letting the hologram 'look' at the phase object through a diffusing glass (or other diffusor) placed next to it.

The mathematical theory of the image-reconstruction obtained with the two 'adjacent' holograms (in physical or equivalent optical contact) is comparable to the theory given in our ref. 1, for the general case, and in our ref. 2 for a special case, for single multiply-exposed holograms, provided that it is understood that the intensities in the two 'partial holograms' added in the single holograms of our ref. 1 and 2 are

Fig. 1. Holographic interferometry arrangement used both in the recording and in the reconstruction. \( P = \text{phase object (see reconstructed image in fig. 2)}, D = \text{diffuser}, L = \text{collimating lens}, H_1, H_2 = \text{holograms}. \) The recording of \( H_1 \) is obtained by interference of the field scattered by \( P+D \) with the coherent reference background produced by \( L \). The recording of \( H_2 \) is obtained by interference of the reference background with the field scattered by \( D \) alone (with \( P \) removed); a clear glass plate of thickness comparable to \( H_1 \) was placed in front of \( H_2 \) during the recording, for compensation (\( H_1 \) was recorded through its glass support, a Kodak 649 F microflat plate). The white arrows show the light path used to illuminate \( H_1+H_2 \) in the reconstruction (\( P \) and \( D \) are not used in the reconstruction; \( P+d \) and \( d \), in \( H_1 \) and \( H_2 \) are symbolic). The 75 mm length of \( D \) gives the scale of the arrangement.
now simply added by means of the two separately recorded holograms, placed in close contact with each other. Let \( a_R \) be the complex amplitude distribution produced on the hologram \( H_1 \) by a reference source \( R \) (situated in the focal plane of the lens \( L \) in fig. 1), and let \( a_{p+D} \) be the complex amplitude distribution produced on \( H_1 \) by the arrangement object \( P + \text{diffusor} \ D \), where the field scattered by the phase object \( P \) may actually be recorded through the diffusor, as in fig. 1. (For the theory of holography with diffusely illuminated phase objects, see e.g. refs. 5 and 6). Let \( a_{D} \) be the complex amplitude produced in the plane of the holograms by the diffusor \( D \) alone (with the object \( P \) removed). The intensity recorded in the first hologram \( H_1 \) is
\[
I_1 = a_R^* a_{p+D} + a_{p+D}^* a_R + a_R^* a_{D} + a_{D}^* a_R,
\]
and the intensity recorded in the second hologram \( H_2 \) is
\[
I_2 = a_R^* a_{p+D} + a_{D}^* a_R + a_{p+D}^* a_{D} + a_{D}^* a_{p+D}.
\]
In the reconstruction process, the two holograms are placed carefully aligned, with the emulsions (i.e. \( I_1 \) and \( I_2 \)) in contact. The complex amplitude of the wave transmitted through \( H_1 \) and \( H_2 \) in the two separated image-forming side-band terms [6] is
\[
(H_1 + 2)_+ = a_R (a_{p+D}^* + a_{D}^*) \quad \text{and} \quad (H_1 + 2)_- = a_R (a_{p+D} + a_{D}),
\]
which will give by holographic (Fourier or Fresnel [6]) transformation complex-amplitude distributions in the side-band images proportional to \( (A_{p+D} + A_{D}) \) and \( (A_{p+D} - A_{D}) \), where \( A_{p+D} = a_{p+D}^* a_{p+D} \) and \( A_{D} = a_{D}^* a_{D} \), respectively. The complex amplitudes in the waves transmitted through the phase object \( P \), respectively the diffusor \( D \), when they are illuminated by a plane spatially coherent monochromatic wave (here 6328 Å laser light). It immediately follows that the complex amplitudes in the reconstructed images will be proportional to \( (A_{p+D} + 1) \), respectively \( (A_{P} + 1) \), showing that an interferometrically decoded image has indeed been reconstructed. Fig. 2 shows an interferometrically decoded image of a phase object letter "φ" and word "phase") obtained according to this principle, in the arrangement of fig. 1.

Physically, the complex addition obtained with two holograms may be understood, also, by noting that the complex addition results from a superposition of the image-forming wavefront, reconstructed by the first hologram \( H_1 \), on the image-forming wavefront reconstructed by the second hologram \( H_2 \). In a sense, our two-hologram holography arrangement may be considered as an interferometric 'image decoding' arrangement, in which a 'coded' hologram \( H_1 \) of a phase object \( P \) may be made to produce a 'decoded' image with the aid of a separately recorded 'decoding' hologram. Clearly, with a phase object having no steep phase gradients (compared to the gradients in the diffusor), the coded hologram \( H_1 \) will not be capable of revealing its 'key' (produced by the diffusor) by any holographic or interferometric process. The second hologram \( H_2 \), with the recording of the 'key' is required for the reconstruction of the image of the phase object.

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