

STATISTICAL ANALYSIS OF  
PRESSURE MEASUREMENTS

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I Introduction

Over the past several years a certain amount of non-repeatability has been noticed in the static pressure measurements made in the U. M. E. R. I. Mach number 1.90 channel. At the time of the testing conducted under project M-950, a number of test section static pressures were measured in an attempt to locate the cause of this non-repeatability. The present report contains a statistical analysis of this static pressure data. The technique used cannot possibly determine explicitly the cause of the non-repeatability. Rather, we may expect to eliminate certain causes. At the same time it is hoped that this analysis will help to answer the question of how accurately a static pressure can be measured in this tunnel using mercury manometers.

II Standard Deviations of Static Pressure Orifices Bank I.

In order to get some idea of the accuracy of the readings of static pressure orifices and of total head tubes a study of the standard deviation of these readings was undertaken. The wall orifices 167, 467, 567, and 667 were included in the same bank as the tubes that were connected to the orifices on the wedge probe. This bank only had 15 manometers in use so that the camera could be put up closer in order that the manometer print would be bigger and easier to read. For this reason and since the manometer prints were read in such a way that the interpolation was done with a dial gauge the errors introduced in reading these

tubes are somewhat less than those in the tubes in the other bank, which means that the corresponding standard deviations will be less. The wall orifice data for 167, 467, 567, 667 was taken continuously with the wedge probe data so that there was a total of 37 runs split up among four days upon which to base the average and standard deviations at Mach number 1.90.

Table I gives a list of previously obtained orifice pressure ratios listed in the M = 1.90 calibration report, the present average orifice pressure ratio and the standard deviation of the orifice pressure ratios based on the four day mean. The standard deviation would be somewhat less if the standard deviation were computed each day from the daily mean, since this mean changes with barometric pressure and is different on successive days. The first column in Table I is not complete since previous wall orifice data was not available for the same orifices as were used in the present series of tests at Mach numbers 1.45 and 2.85.

Since the standard deviations of  $p/p_b$  are the same irregardless of the value of  $p/p_b$  and hence the Mach number, it seems probable that the cause is an error in reading (e.g., an error in the manometer) the pressure  $p$  and that the error is constant and independent of the value of  $p$ . The standard deviation of the pressure itself is then  $p_b \times \sigma p/p_b = 29$  (.00047) = .0136" Hg.

Assuming that all the  $\sigma$ 's are basically the same we will try statistically to analyze them. We have 16 values of  $\sigma$  for which the mean value is  $\bar{\sigma} = .00047$ . If it can be shown that  $p/p_b$  is distributed normally, which it is roughly as will be proved later, then theory indicates that the standard deviation of a series of measurements of  $\sigma$ ,  $\sigma(\sigma)$  will be  $\sigma(\sigma) = \frac{\sigma}{\sqrt{N}}$  where  $N$  is number of observations used to compute each individual  $\sigma$ . So that in our case  $N = 37$  and  $\sigma(\sigma) = \frac{\sigma}{\sqrt{37}} = .000055$  (the value of  $\sigma(\sigma)$ )

computed from the values of  $\sigma$  itself turned out to be  $\sigma(\sigma) = .00007$ . Each value of  $\sigma$  should be within a two  $\sigma(\sigma)$  band about the mean 95% of the time. That is

$$\sigma = +.00047 \pm .00011 = \left\{ \begin{array}{l} .00036 \\ .00058 \end{array} \right\}$$

95% of the time. If we look at the values of  $\sigma$  listed we see that all the  $\sigma$ 's except one fall in this range.

If we consider the second column of Table I as being accurate (being based on 37 runs) and the points from the calibration report (column 1) as being compared to it, then the values should be within  $2\sigma$  95% of the time, taking  $\sigma$  at its average value .00047. Orifices 167, 467, and 567 are well within this band, in fact they are within the one sigma band (probability of anyone point being in this band is 68%), but orifice 667 is on the edge of the  $5\sigma$  band which is highly improbable. This suggests that there has been some major change in orifice 667 since the last test, such as the appearance of a leak.

### III Standard Deviations of Static Pressure Orifices Bank II.

Table II contains the mean values of  $p/p_b$  and the associated standard deviations for orifices 152, 157, and 161. The mean value of these standard deviations is .00062 with  $\sigma(\sigma) = .00015$ . There are four standard deviations which lie outside the mean value of  $\bar{\sigma} \pm 2\sigma(\sigma)$ , i.e.,  $.00062 \pm .00030$ . The fact that the mean value of  $\sigma$  for these orifices is larger than the mean value of  $\sigma$  for orifices 167 through 667 is probably due to the greater accuracy with which it was possible to measure the heights of the mercury columns for orifices of 167 through 667.

### IV Standard Deviations of Total Head in Boundary Layer.

Table III lists the various values of the ratio of total head

to barometric pressure which were measured with a five prong total head rake in the boundary layer of the Mach number 1.90 channel. Fig. 1 shows the location of the total head rake in the test section and the position of each of the five total head tubes in the rake. The standard deviations of these total head ratios are approximately three to four times the standard deviations measured for the static to barometric pressure ratios at orifices 167 through 667. There are three immediately obvious reasons for this larger standard deviation in the boundary layer total heads:

- 1) the total head tubes cannot possibly be as solidly mounted with respect to the test section as the static pressure orifices so that it is possible that the total head tubes oscillated slightly during a run;
- 2) the measurements were made in the boundary layer so that a large total head gradient normal to the test section floor is present and hence any slight movement of the total head tube will in effect give some average reading over a portion of the total head gradient in the boundary layer;
- 3) the total head tubes were in a separate manometer bank from orifices 167 through 667 and since this bank contained more tubes the photographs of the tubes were necessarily less readable (this probably means that the total head standard deviations are really only two to three times as large as comparable static pressure standard deviations).

For the total head tube readings  $\sigma(\sigma) = .00042$  and all of the total head readings lie in a band  $\pm 2\sigma$  from their mean value, i.e., all readings are  $.00178 \pm .00084$ .

## V Distribution Function

An attempt was made to find the distribution function for the deviations of the measured pressure ratios from their mean value. The reason for doing this was to try to ascertain whether the measuring errors were random (i.e., whether or not they had a normal probability distribution) and to see if it might be possible to infer something about the errors from their probability distribution. If the errors are normally distributed then the probability that any given measurement lies within the mean value  $\pm \lambda \sigma$  can be determined for any value of  $\lambda$ . In general it is difficult to discuss the probability of the occurrence of an error of given magnitude if the distribution function is not known. However, irregardless of the distribution function which applies to the measurements under consideration we still obtain some idea as to the accuracy of the measurements by applying the Tchybecheff inequality which may be stated as follows: the probability of a measurement lying outside the range, mean value  $\pm \lambda \sigma$  is less than  $\frac{1}{\lambda^2}$  for  $\lambda > 1$ . For example if  $\lambda = 2$  the probability of incurring an error greater than  $\pm .00094$  in the static to barometric pressure ratio is .25 if the measurements have been made with the same accuracy as those in Table I where  $\sigma = .00047$ . The results obtained using this inequality are of course extremely conservative since if we assume the data to be distributed according to the normal law the probability would be .0455.

Since there appears to be some correlation between the barometric pressure and the orifice pressure ratio (see section VII) and since the barometric pressure varied only a small amount during one day although it varied considerably from day to day, the deviations of the pressure ratios were taken with regard to their daily mean. Also the three orifices 467, 567, and 667 were taken together in the study since in the application of

the methods used to determine a distribution function it is necessary to have as many points as possible in order to make any clear cut statements. It was believed that the three orifices could be grouped together because physically they represent the same system, orifice, connecting tube and manometer tube, and statistically they have comparable standard deviations.

The Gram-Charlier method was used to find the distribution function. In this method the distribution function is approximated by a sum made up of the normal curve and a linear combination of its derivatives. Ordinarily the normal curve plus a combination of its third and fourth derivatives are used to describe the distribution function. The constants in this law are determined in such a way that the curve has the second, third and fourth moments the same as the experimental data (i.e., the variance, the skewness and the flatness are matched). The higher moments are very sensitive to small changes in the experimental data so that normally no higher moments are computed and no more terms are taken in the Gram-Charlier approximation.

The means for each day and the deviations from them were computed, then according to the Gram-Charlier method the deviations were grouped into classes and the moments and experimental probabilities were computed from this data. The class interval was taken as .0002 in the pressure ratio since this gave a fairly good compromise between the number of classes and the number of deviations in each class and also it was felt that this was just about the minimum difference in pressure ratio that has any meaning when the reading accuracy is considered.

From the value of the moments computed from the data the parameters in the Gram-Charlier law can be easily computed (see Ref. 1). The mean and standard deviation give the first term, the normal law, and the skewness and flatness determine the size of the other two terms. In the present case the curve that was obtained was not far different from the

normal law and if more data had been available it is possible that the form of the curve would be more nearly normal although there is no definite reason why it should be.

In order to check the accuracy of a distribution function it is necessary to compare the experimental and theoretical values of the probability that a point is in each of the class intervals. This comparison is the basis of the Pearson goodness of fit test. The experimental probabilities are computed by dividing the number of points in the intervals by the total number of observations. The theoretical probabilities are computed as the area under the distribution curves between the two ends of the class interval. The area can be obtained from tables of the area under the normal curve and of the derivatives of the normal curve. A graph of these probabilities for the normal distribution (first approximation to the Gram-Charlier), the Gram-Charlier distribution, and the experimental points are shown in Fig. 2.

When the experimental and theoretical probability distribution law were determined, the Pearson  $\chi^2$  (goodness of fit) test was applied to establish how well the theoretical curve represented the data. The result of the test gives the probability P that if the tests are repeated that larger deviations from the expected values of the probabilities would be found. The probability P is .4 for the Gram-Charlier distribution and .2 for the normal distribution having the same mean and standard deviation as the data. A probability P = .5 would mean that we are as likely as not to get a better set of data on repeating the tests whereas a very low value of P means that it will not be strange if when we repeat the tests we get larger deviations from the assumed law, this would either mean that the assumed distribution is wrong or that the data is not representative. In the present case P even for the normal distribution is not small enough



to invalidate the normal distribution. The difference between the normal and Gram-Charlier distributions is not very great in the present case so that we can probably use either one unless later data shows that the distribution function is actually skew in the direction found here or does possess some flatness.

## VI Correlation Coefficients

In attempting to isolate the cause of the non-repeatability it was decided to try to find some correlation between  $p/p_0$  and as many of the various physical quantities e.g.,  $p_0$ ,  $T_0$ , etc., as might effect the value of  $p/p_0$ . The correlation coefficients which indicate the importance of the trends are obtained from the usual formula:

$$r_{xy} = \frac{n \sum xy - \sum x \sum y}{\sqrt{[\sum x^2 - n(\sum x)^2][\sum y^2 - n(\sum y)^2]}}$$

In interpreting the various values which the correlation coefficient  $r_{xy}$  may assume we should notice first that

- 1)  $r_{xy} = \pm 1$  indicates that there is some definite linear relationship between  $x$  and  $y$  such that for every given value of  $x$  there is a unique value of  $y$ ;
- 2)  $r_{xy} = 0$  indicates that a knowledge of  $x$  gives no indication whatsoever as to the value of  $y$  and vice versa.

In correlation theory two least square error lines (regression lines) are fitted to the data points one which assumes  $x$  is accurate and all the error is in  $y$  (the regression line  $y$  on  $x$ ) and the other in which all the error is assumed in  $x$  and  $y$  is taken as accurate (the regression line  $x$  on  $y$ ). The theory shows that if  $x$  has a standard deviation  $\sigma_x$  when

no account of the correlation is taken and a standard deviation  $\sigma_{x,y}$  when the deviations are taken parallel to the x axis from the regression line x on y then these two standard deviations are related by the formula

$$\sigma_{x,y} = \sigma_x \sqrt{1 - r_{xy}^2}$$

If for example  $r_{xy}$  is equal to 1 then  $\sigma_{x,y} = 0$  and therefore the sum of the squares of the distances of the point from the correlation line is zero which means that all the points must be on the correlation line.

There remains two questions with respect to the value of the correlation coefficient namely:

- a) what is the significance of  $0 < |r_{xy}| < 1$ ;
- b) what is the accuracy of a value of  $r_{xy}$  which has been determined from a sample of the data.

Some idea as to the accuracy of  $r_{xy}$  when computed from a finite sample of data may be obtained from Fig. 3 which is a plot of the probable error in  $r_{xy}$  as a function of  $r_{xy}$  computed from the formula

$$P.E.(r_{xy}) = \frac{1 - r_{xy}^2}{\sqrt{N}} (.6745)$$

where N is the number of observations on which  $r_{xy}$  is based.

The question of the significance of a value of  $0 < |r_{xy}| < 1$  is not so easy to answer but it is usually approached in the following way. We consider the other correlation coefficients that we might have obtained if the data were completely scrambled (i.e., the  $x$ 's paired with different  $y$ 's), some of these coefficients might be large and some might be small. Ordinarily a correlation coefficient is considered good if a better one

could be obtained from the scrambled data less than 10% of the time. Charts based on this .10 probability (see Ref. 4) indicate that if the coefficient is based on 37 observations then a coefficient  $r_{xy} = .28$  could be obtained 10% of the time from data which basically has no correlation so that a value less than .28 is insignificant as far as drawing conclusions are concerned. Furthermore, the probability of getting an  $r_{xy} = .42$  at random from uncorrelated data is .01 and that of getting  $r_{xy} = .32$  at random is .05. In other words if we obtain a correlation coefficient of .6 or .7 from 37 observations there is an extremely small chance that it is actually obtained from data that is really unrelated.

Correlation coefficients of one are not obtained even though there may be physically some definite relation between the two quantities being correlated because there is a measuring error in determining the quantities which clouds the underlying relation.

Basically what is attempted in a correlation study is to vary one parameter and to observe the behavior of the others to try to determine their connection. A natural difficulty that is encountered in the present problem is that the basic parameters such as barometric pressure and atmospheric temperature can not be varied independently or at will. In the present series of tests at  $M = 1.90$  we were in the fortunate position, from the statistical point of view, of having the barometric pressure vary by about an inch in approximately equal steps over the period of 4 days during which the tests were run. In the case of the bag temperature the conclusions are probably not very trustworthy since the temperature did not vary over a very wide range and the accuracy of the reading is probably bad in comparison with the extent of its variation.

It should also be kept in mind that the correlation study can

only indicate when two variables fluctuate together but it can not determine which variable causes the change in the other. In fact it is quite possible that neither of the variables is the actual cause of the fluctuations but that some third variable produces the changes in both the original variables.

#### VII Correlation - Coefficients Computed from Data.

It was originally intended to correlate  $p/p_b$  with the shift in the shock pattern as observed on the schlieren photographs. This was found to be impossible since the accuracy with which the <sup>position of the</sup> shock wave pattern could be determined was of the same order of magnitude as the observed shift in the pattern.

This might be interpreted to mean that the shift in the Mach pattern is so small that it can not be the cause of the changes in the pressure ratios. At least there is no large translation of the pattern which would shift the pressures although there may be a small rotation of the lines corresponding to a change in strength of the waves.

All correlation coefficients obtained are presented in Table IV. These correlations fall into five major groups:

- 1) correlations of  $p/p_b$  vs. bag temperature
- 2) correlations of  $p/p_b$  vs. deviations in reference pressure
- 3) correlations of  $p/p_b$  at one orifice with  $p/p_b$  at another orifice.
- 4) correlations of  $p/p_b$  vs  $p_b$
- 5) correlations of total head in the boundary layer vs.  $p_b$  and  $p/p_b$

1) Bag Temperature.

The correlations of  $p/p_b$  at orifices 467 and 567 with the bag temperature,  $T_{bag}$ , are given in Table IV as  $-.38$  and  $-.29$  respectively. As has been previously pointed out a correlation of  $-.29$  is quite meaningless while a correlation of  $-.38$  is marginal, so that it would appear that there is no connection between  $p/p_b$  and  $T_{bag}$ . It is however hard to make definite statements since the range over which  $T_{bag}$  varied was very small (of the order of  $20^\circ$  over the 4 days that the test were run). In order to really get the effect of  $T_{bag}$  it would be desirable to vary the bag temperature between early morning and noon or to heat the air in the bag artificially in order to give a wide range in  $T_{bag}$ .

2) Deviations in Reference Pressure.

Since any change in barometric pressure will cause a change in the height of the mercury columns used to measure the absolute zero reference pressure as well as a change in the height of the mercury columns used to measure the static pressure at any orifice, it was felt that there might be some connection between the value of the reference pressure as measured from a manometer photograph and the value of  $p/p_b$  the pressure ratio. This might be due to some lag between the change in barometer and the change in reference pressure possibly as a result of dirt in the mercury causing the mercury to stick to the glass. An error in the reference pressure would cause an error in the read value of the orifice pressure without a change in  $p_b$  so that the ratio  $p/p_b$  would be changed correspondingly. It had been already established that there is a connection between  $p/p_b$  and  $p_b$  (see section VII, (4)) so that it

was necessary to remove the effect of  $p_b$  on the reference pressure as a possible cause. This was done by computing the daily average of the reference pressure reading and then plotting the deviations from this daily average versus the  $p/p_b$  from a representative orifice 667 (see Fig. 4). The correlation coefficient was computed as  $.27 \pm .1$  so that it is safe to say that there is no connection between changes in the height of the reference column of mercury and the static pressure ratios.

### 3) Correlations between $p/p_b$ at Two Different Orifices.

The location of all orifices is shown in Fig. 5. From Table IV it appears that orifices 467, 567, 667 are all correlated with each other and that the correlation coefficients vary in value from .62 to .76. These coefficients are well above the marginal values of correlation coefficients quoted previously. It thus appears that all the orifices in the vertical plane 12 inches upstream of the tunnel and perpendicular to the flow direction give high and low readings approximately simultaneously. On the other hand the correlation coefficients between orifice 467 and orifices 100, 121, 152 and 157 are  $-.29, 0, -.52, +.04$  so that there seems to be no systematic connection between all these orifices and the orifices in the vertical plane with 467. This may be due to the fact that some of the nozzle side orifices might have been plugged. One possible explanation for this lack of correlation is the fact that the manometer connected to orifice 467 was in a different bank from the manometers connected to orifices 100, 121, 152 and 157. The table also shows that the correlation coefficient between orifices 100 and 121 is .29, between orifices 152 and 157 is  $-.06$ , and between orifices 152 and 161 is  $-.09$ . All of these values are

below the value quoted previously as being the minimum value to which any significance may be attached. It may be significant that orifice 152 is upstream of the nozzle - test section juncture while orifices 157 and 161 are downstream of this juncture.

It is possible that the junctures in the ceiling and floor and the junctures in the side walls do not produce identical effects so that their combined effect on any one orifice may not be the same as their combined effect on some other orifice. This would then account for the correlation coefficients between orifices 152, 157, 161 and 467. The table of correlation coefficients gives a correlation of .59 between orifices 157 and 161. These two orifices are 4 inches apart in the streamwise direction.

4) Correlations of  $p/p_b$  versus Barometric Pressure.

Perhaps the most astounding result of the entire analysis was the correlation of  $p/p_b$  with  $p_b$ . Fig. 6 shows a plot of  $p/p_b$  from orifice 667 vs.  $p_b$ . The value of the correlation coefficient is +.65. The heavy line is the line of regression of  $p/p_b$  on  $p_b$  i.e., if it is assumed that all of the error is in  $p/p_b$  and a least square fit is made than this line will be obtained. The lines parallel to this heavy line are at a distance of  $\pm 1.5 \sigma_{p/p_b}$  from the heavy line so that 87% of the points should lie within this band. The two horizontal lines are at a distance of  $\pm 1.5 \sigma_{p/p_b, p_b}$  from the mean value of  $p/p_b$  so that if there were no correlation 87% of the points should fall within this band. For orifice 667  $\sigma_{p/p_b} = .00049$  and  $\sigma_{p/p_b, p_b} = .00038$ .

The first column in Table IV gives a list of correlation coefficients for several orifice pressure ratios when correlated with the barometric pressure. An interpretation of the results

given there is not very easy. If as was mentioned before a coefficient of about .32 or more is taken as non-trivial then we see that there are few trivial ones.

To determine the effect of making temperature corrections (for the expansion of the mercury and the scales) on the correlation coefficients these corrections were omitted on both the manometer and barometric pressures for orifice 467. This orifice was then correlated to the barometric pressure and the coefficient was found to be .42 instead of the value .46 which was found previously when all the temperature corrections were made. Although the corrections seemed to have a small effect in this case they were included in all other computations never the less.

All the orifices closest to the test section 167, 467, 567 and 667 have a positive correlation with the barometric pressure, that is they have an increasing pressure ratio as the barometric pressure increases. All the other orifices have either a negative or approximately zero correlation with  $p_b$ .

In order for the pressure ratio  $p/p_b$  to change with barometric pressure either 1) there should be theoretical grounds for the variation i.e.,  $p/p_b$  actually a function of  $p_b$ . 2) the pressure  $p$  being a difference in level between a reference and a manometer tube may have some correlation with  $p_b$  if either of these tubes is influenced more than the other by the barometric pressure. Reason 2) has been eliminated above. One possible explanation of this behavior is that the orifices with positive correlation are in one bank of manometer tubes and the negative ones in the other however this is tentatively ruled out since the definite zero correlation between orifices 667 and 152 and the reference pressure indicates



that probably this isn't the cause since the pressure ratios for these orifices were independent of the reference pressure as they should be.

When the deviations of the reference pressure from its mean for one bank is plotted versus the deviation for the other bank it is found that there is a strong positive correlation as would be expected since both are responding similarly to changes in barometric pressure. This means that if one bank had its pressure ratios with a positive correlation with barometric pressure then the other bank should have the same sort of positive correlation.

A few other possible explanations of the correlation of pressure ratio with barometric pressure are given here. First the loads on the tunnel and its deflections may vary with  $p_b$  causing a varied channel size or a varied shock strength from the wall junctures. The pressure ratio across the walls of the tunnel should not vary with  $p_b$  but the magnitude of the force applied to the sides of the tunnel will depend on  $p_b$ . A second possible explanation is that there may be some leakage around the seals or through the walls that varies with  $p_b$  and causes a change in the boundary layer which in turn effects the tunnel flow and of course the orifice pressure ratio. Lastly the changes in  $p_b$  may produce changes in the Reynolds number of the flow through the channel which could change the pressure ratio at an orifice either directly through a change in the displacement thickness of the boundary layer or indirectly by a change in the location transition point.

It was considered possible that the 37 runs on the four days were not 37 independent observations to determine the trend of

orifice pressure ratio with  $p_b$  but rather that all the runs on each day should be averaged together and only the days should be considered as independent. If this were true then it would be much easier to get a significant looking result that actually has no meaning. If we have four points to determine a correlation line we can get a correlation coefficient of .9 at random 10% of the time even if there is no basic relationship between the variables and we can get one of .99 random 1% of the time. In order to check to see what the correlation would be if only the days could be considered independent, all the runs on each day were averaged as to barometric pressure and orifice pressure ratio for orifices 667, 157 and 152. Then on the basis of these four average points for each orifice the correlation coefficients were computed. They were +.997 for orifice 667, -.696 for 157, and -.869 for 152 where as they were previously +.65, -.43 and -.67 respectively. The correlation for 667 is very impressive and probably indicates that it is not accidental that the four days give a consistent result. The other orifice correlations that were computed were not as high but they substantiate the findings assuming the 37 runs independent to some extent.

5) Total Head in the Boundary Layer.

As has been previously pointed out the positive value of the correlation coefficient between  $p/p_b$  at orifices 167, 467, 567, and 667 and the barometric pressure  $p_b$  may be due to changes in the boundary layer thickness. Further evidences that this may be the case are the correlation coefficient between total head in the boundary layer and  $p/p_b$  at orifice 667 which is -.39 and the correlation coefficient between total head in the boundary layer and

barometric pressure which is  $-.57$ . The total head tube in the boundary layer for which these correlations were computed was  $.13$  inches above the floor. This total head tube will give some indication of the thickness of the boundary layer, a higher total head to barometric pressure ratio corresponding to an increased velocity at the point  $.13$  inches above the floor and hence a decreased boundary layer displacement thickness. This fact is borne out by the value of the correlation coefficient between the total head tube  $.13$  inches above the floor and the total head tube  $.43$  inches above the floor which is  $.94$  (see Fig. 7) and means that one reading increases with the other total head reading so that the change in boundary layer thickness is probably the cause. This is a remarkably high correlation coefficient especially in view of the larger value of the standard deviations associated with the total head tubes. We can now say that as the total head at one point in the boundary layer increases, the velocity at that point must increase. Then the boundary layer displacement thickness,  $\delta^*$ , decreases and the effective channel area increases so that  $p/p_b$  decreases. This trend is also indicated by the correlation coefficients  $-.39$  and  $-.57$  which say that a decrease in  $p_b$  gives an increase in total head which gives a decrease in  $p/p_b$ . Furthermore, this agrees with the correlation coefficients between  $p/p_b$  for orifice 167, 467, 567 and 667 and  $p_b$  which are positive and hence indicate a decrease in  $p/p_b$  with a decrease in  $p_b$ .

#### VIII Conclusions:

The following general conclusions are apparent:

- 1) The maximum error in measuring the static to stagnation pressure

ratio is approximately  $\pm .0010$ ;

- 2) This error appears to be a random error which is inherent in the measuring technique used; if the dependence of  $p/p_0$  on  $p_0$  is eliminated.
- 3) A definite correlation exists between the static to stagnation pressure ratio and the stagnation pressure;
- 4) The underlying causes of this correlation are not understood but some possible explanations are pointed out.

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TABLE I

$p/p_b$  And Standard Deviations  
For Orifices 167 Thru 667.

Mach Number	Orifice	Calibration Report	Present Data	$\sigma$
1.45	167		.2860	.00037
1.45	267		.2861	.00040
1.45	367		.2966	.00040
1.45	467		.2934	.00044
1.45	567		.2929	.00042
1.45	667		.2962	.00046
1.90	167	.1165	.1165	.00067
1.90	467	.1390	.1394	.00048
1.90	567	.1107	.1112	.00044
1.90	667	.1110	.1135	.00049
2.85	167		.0301	.00049
2.85	267		.0303	.00041
2.85	367		.0309	.00053
2.85	467		.0313	.00048
2.85	567		.0310	.00051
2.85	667		.0324	.00053

TABLE II

$p/p_b$  And Standard Deviations  
For Orifices 152, 157 And 161.

Number Day Of Runs	Orifice 152	$\sigma_{152}$	Orifice 157	$\sigma_{157}$	Orifice 161	$\sigma_{161}$
51-12-19 8	.1310	.00074	.1355	.00065	.1429	.00069
-20 7	.1317	.00073	.1368	.00070	.1437	.00109
-21 10	.1328	.00055	.1363	.00085	.1432	.00061
-22 10	.1321	.00028	.1356	.00036	.1433	.00023

TABLE III

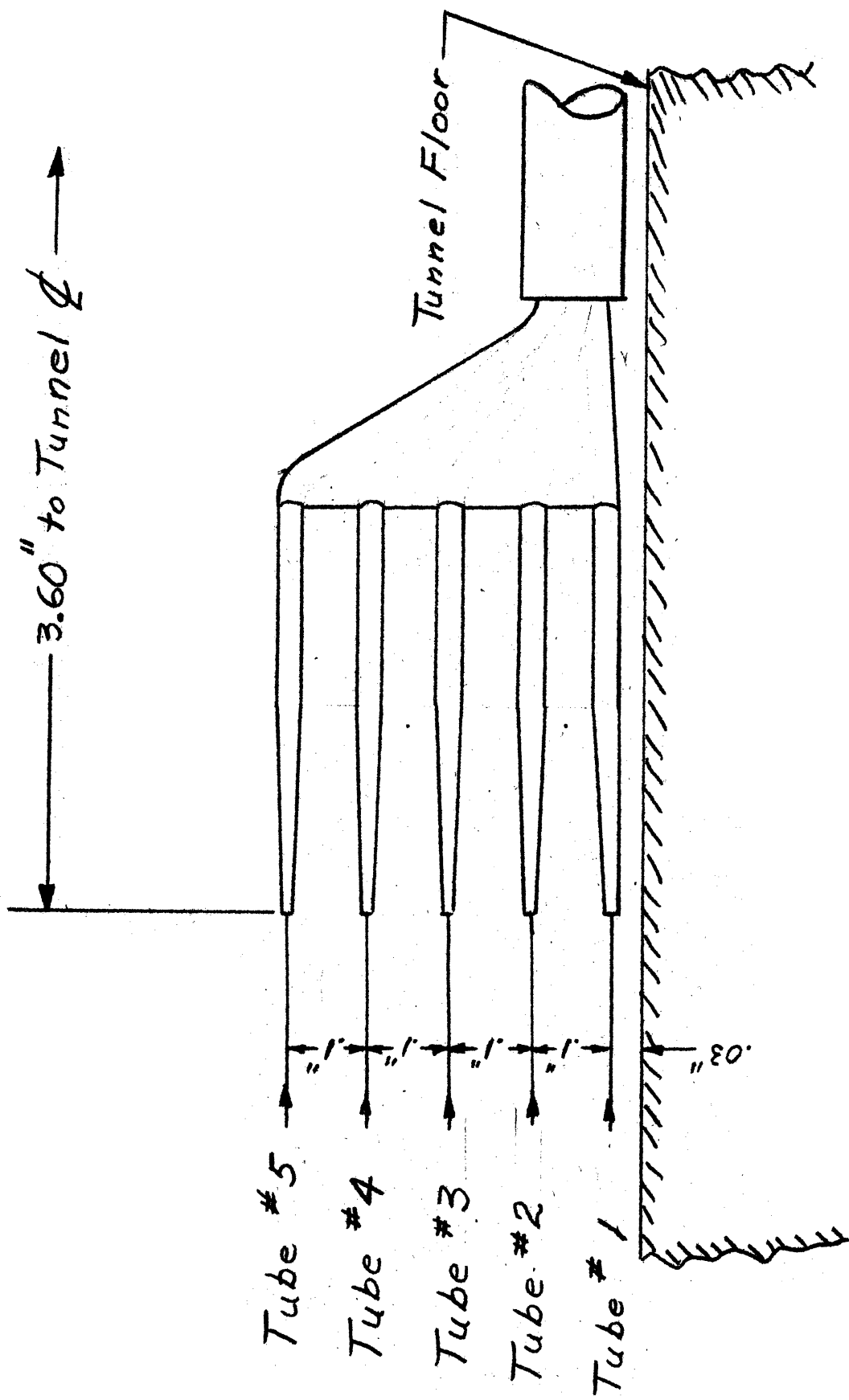
Total Head Readings In The Boundary  
Layer And Their Standard Deviations.

y	Number Of Runs	$(p\delta/p_b)_1$	$\sigma_1$	$(p\delta/p_b)_2$	$\sigma_2$	$(p\delta/p_b)_3$	$\sigma_3$	$(p\delta/p_b)_4$	$\sigma_4$	$(p\delta/p_b)_5$	$\sigma_5$
2-19	7	.2935	.0018	.3778	.0018	.4426	.0023	.4976	.0022	.5519	.0019
-20	7	.2967	.0014	.3812	.0023	.4460	.0014	.5011	.0015	.5548	.0014
-21	14	.2976	.0018	.3811	.0021	.4452	.0014	.5002	.0012	.5539	.0012
-22	15	.2950	.0016	.3781	.0019	.4437	.0020	.4986	.0022	.5527	.0021

TABLE IV

Correlation Coefficients

	P <sub>b</sub>	T <sub>bag</sub>	467	567	667	100	152	157
167.....	+0.55							
467.....	+0.46	-0.38						
567.....	+0.62	-0.29	+0.62					
667.....	+0.65		+0.76	+0.73				
100.....	-0.53		+0.29					
121.....	-0.13		0			+0.29		
152.....	-0.67		-0.52					
157.....	-0.43		+0.04					-0.06
161.....	-0.11						-0.09	+0.59
Total Head Tube No. 2 in B.L. ....	-0.57							-0.39
467 without Temp. Corrections.....	+0.42							
667 Using 4 Points.....	+0.997							
157 Using 4 Points.....	-0.70							
152 Using 4 Points.....	-0.87							

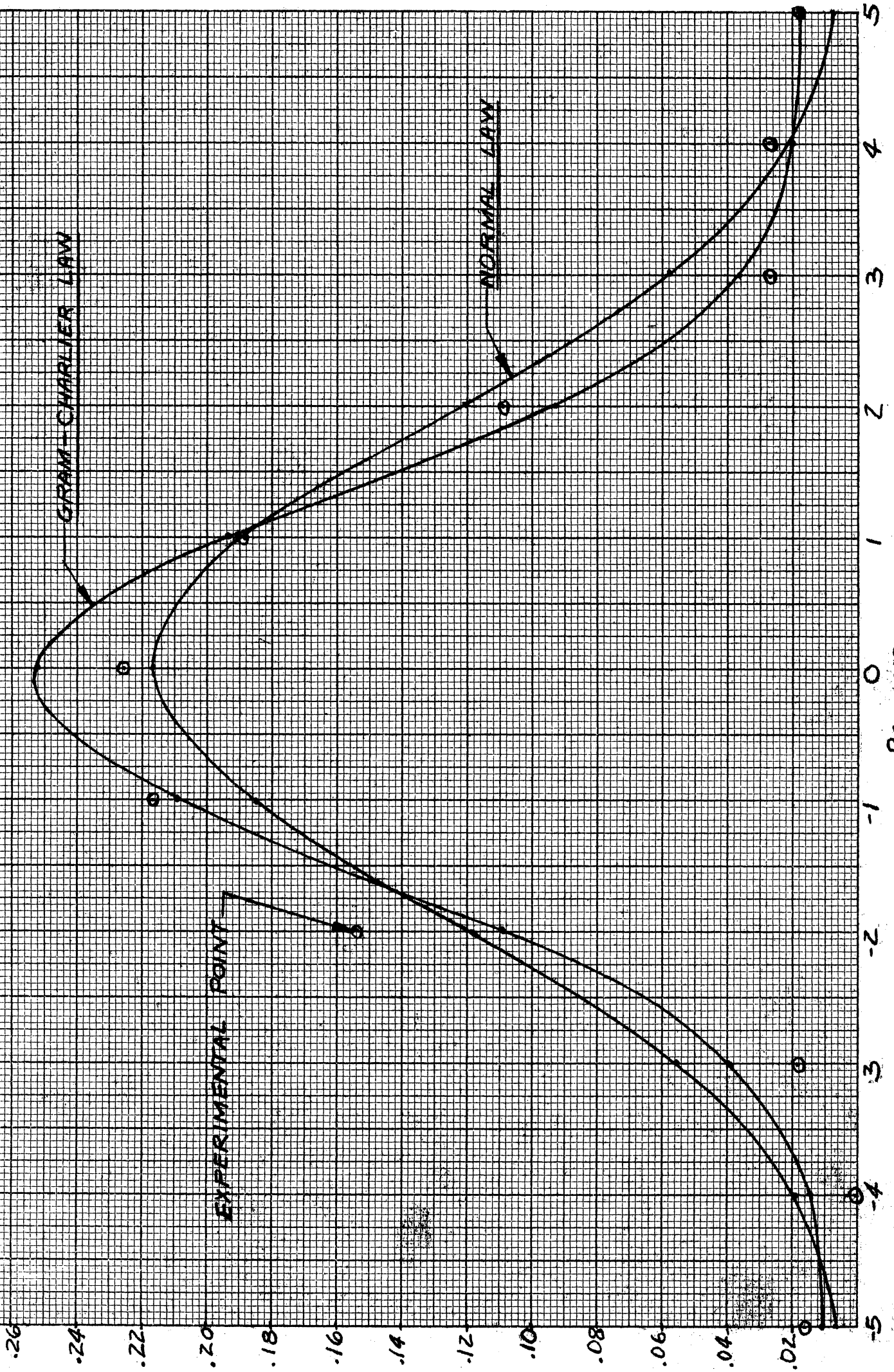


Boundary Layer Total Head Rake  
5 x Actual Size

Fig. 1



# DISTRIBUTION FUNCTIONS



$P/2 \times 0.010$

FIG. 2

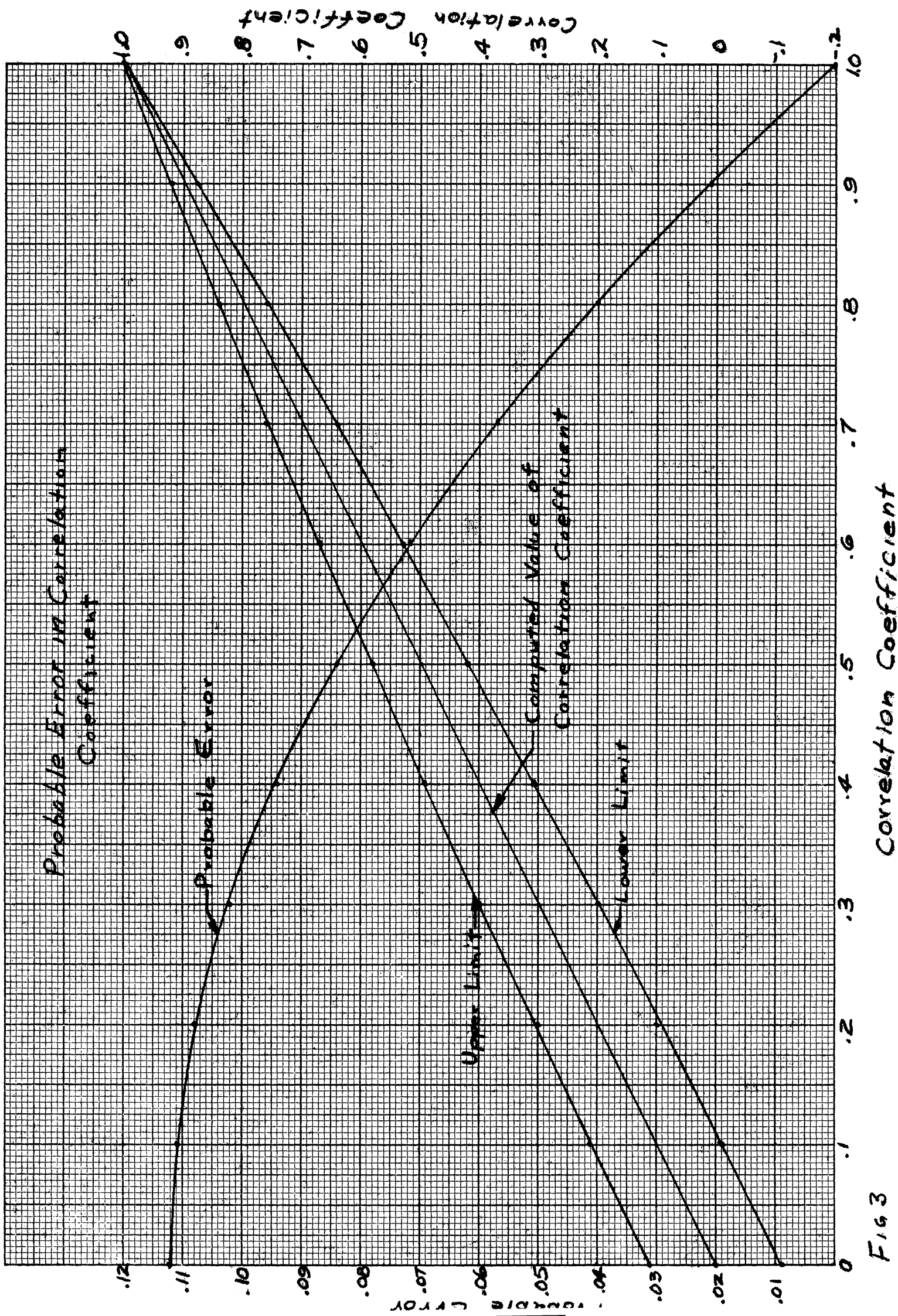


FIG 3

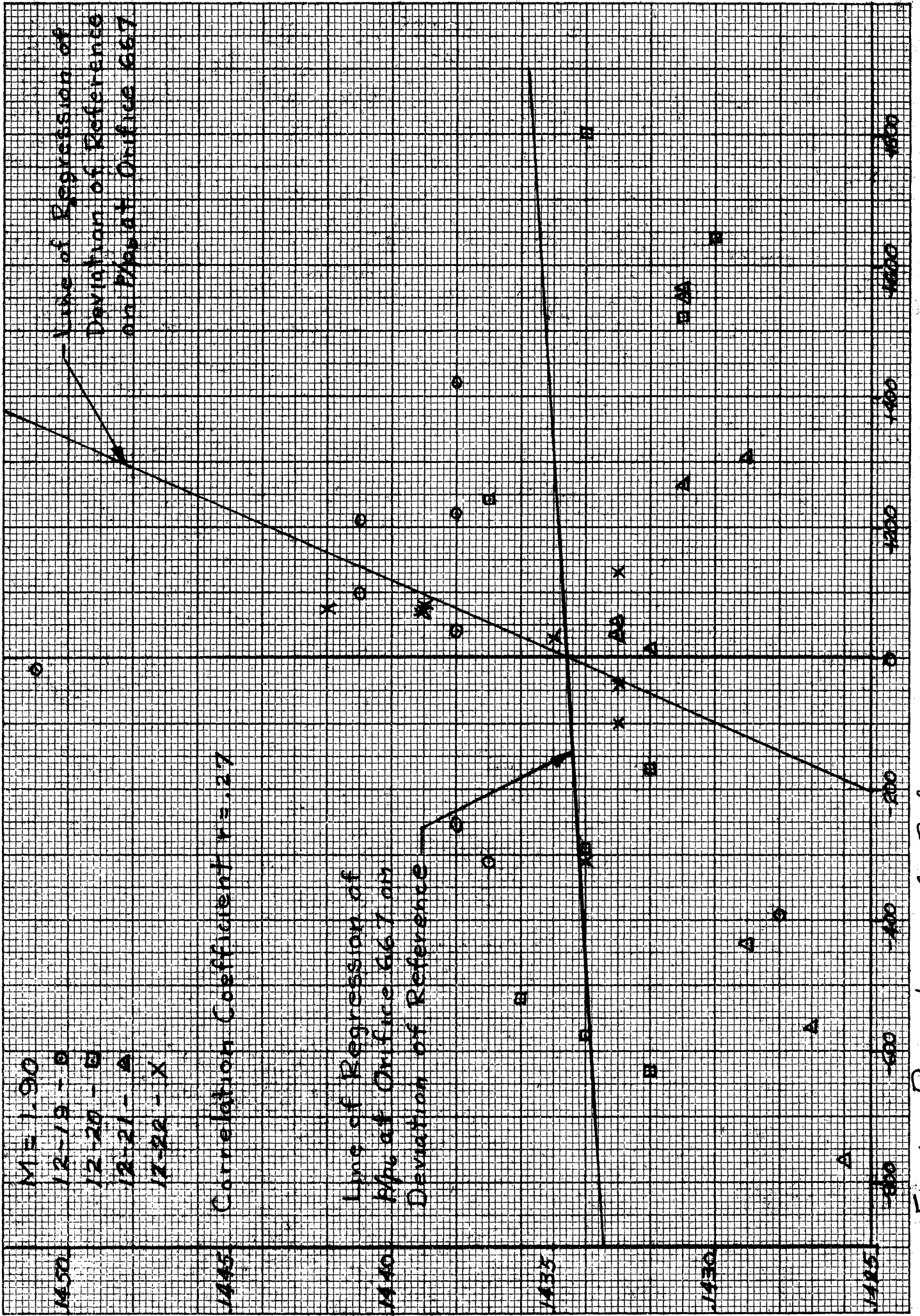


Fig. 4 Deviation of Reference Tube from Mean Value

# Pressure Orifice Numbering System - U.M.E.R.I. Tunnel

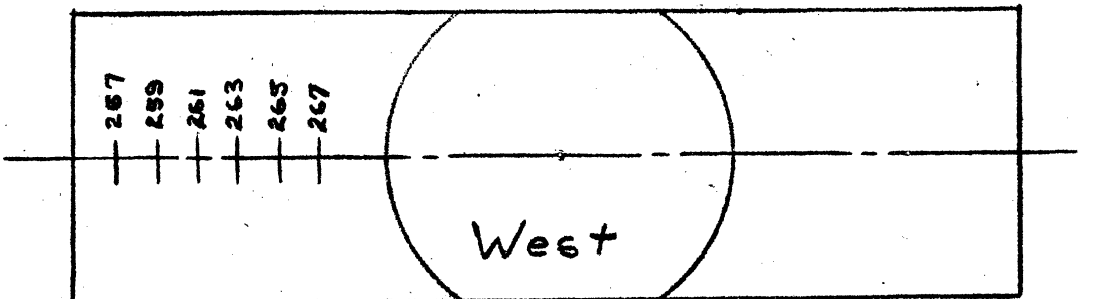
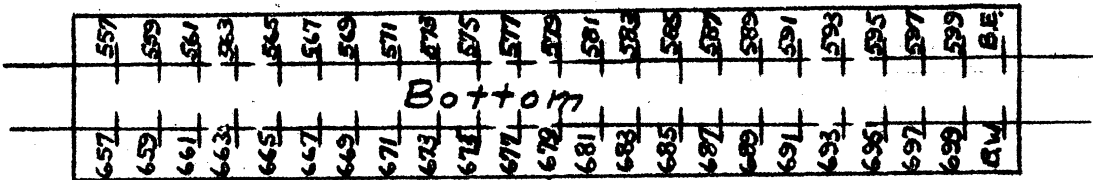
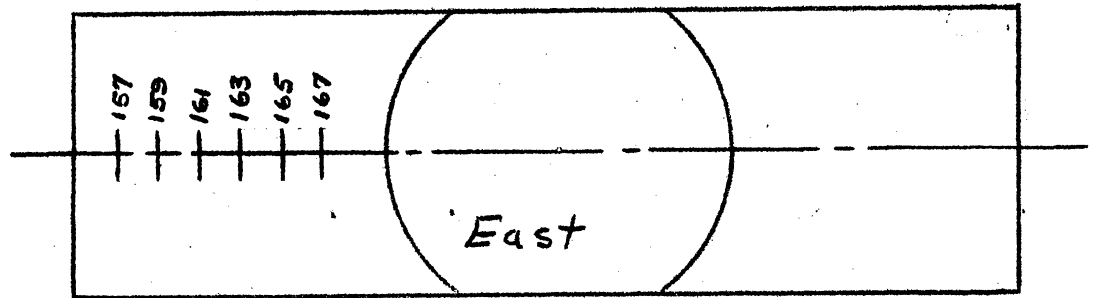
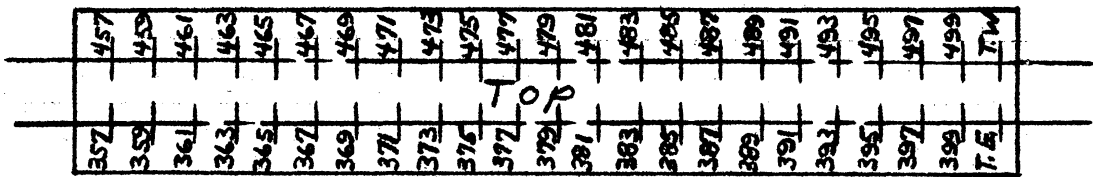
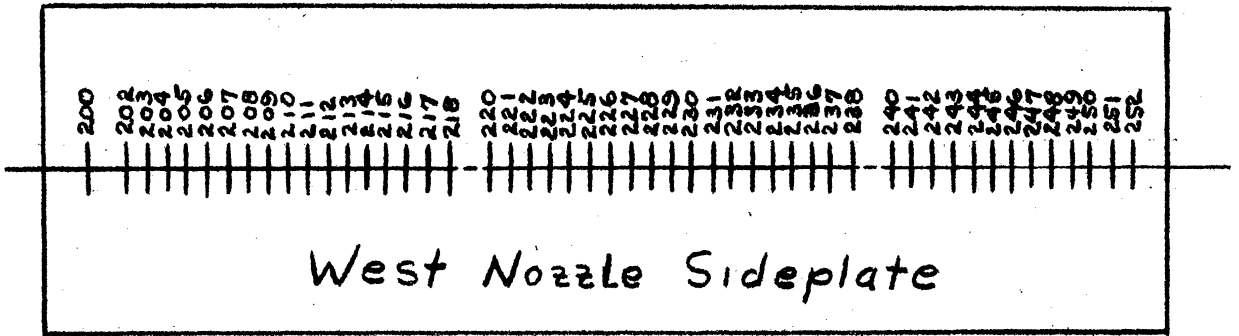
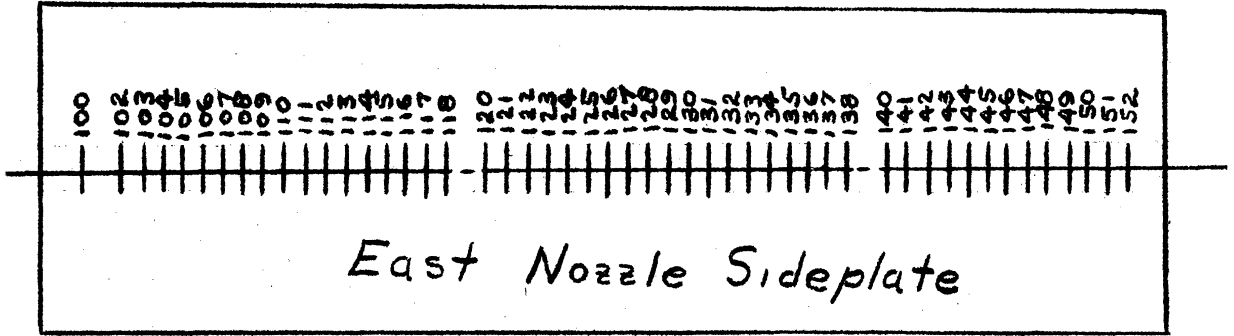


Fig. 5

Office 667 M=1.9

12-19 - O  
12-20 - B  
12-21 - A  
12-22 - X

1450

1445

1440

67

1435

1430

1425

29.5

29.0

P

29.5

30.0

1.5 CM

1.5 CM

650 Mph

650 Mph

Ave. 667

$\mu = 6.48$

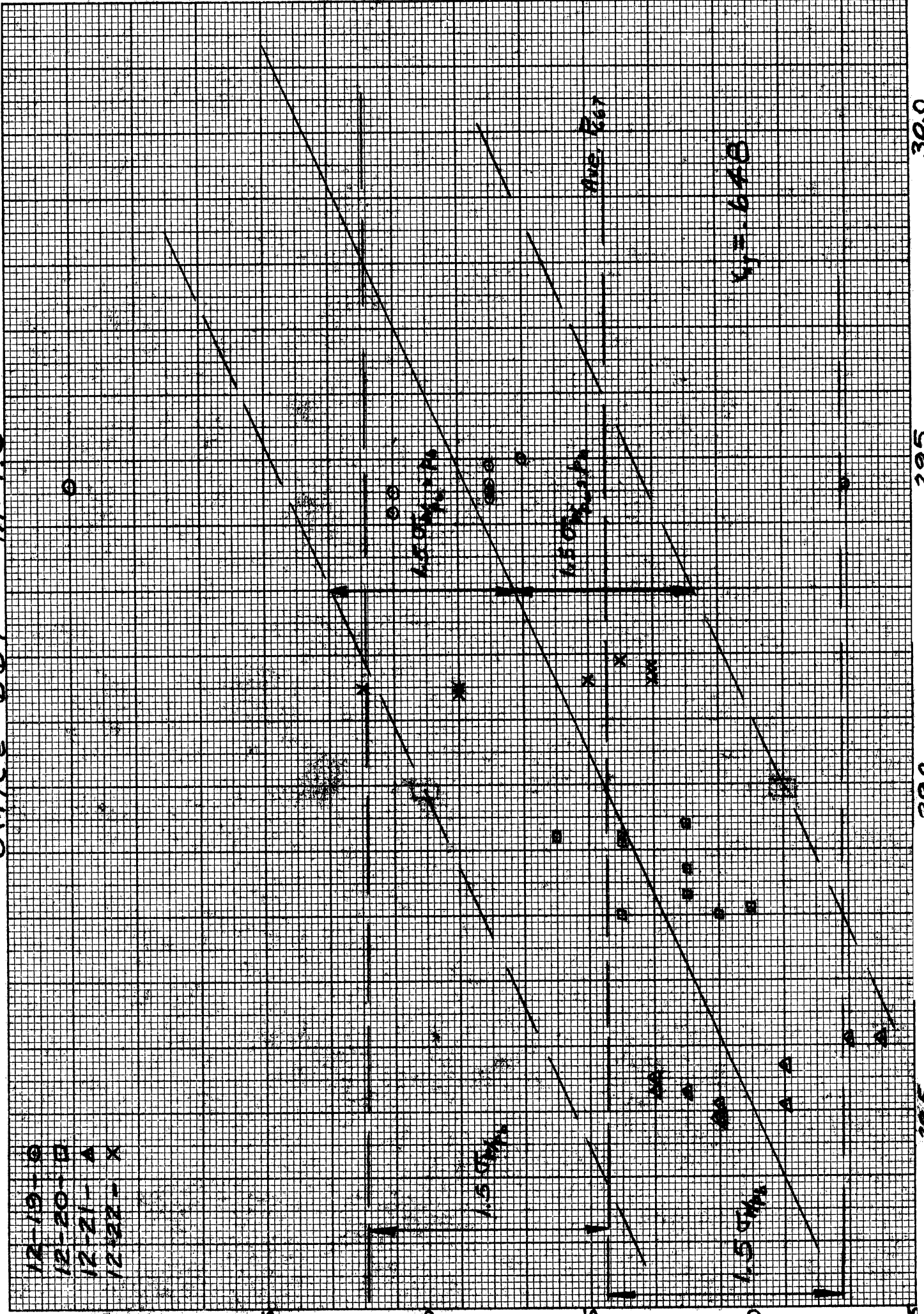


FIG 6

# Correlation of Total Head in Boundary Layer

52-12-19 - 0  
 -20 - 0  
 -21 - 0  
 -22 - X  
 $r_{xy} = .94$

Line of Regression  
 of Tube #5 on Tube #2

Line of Regression  
 of Tube #2 on Tube #5

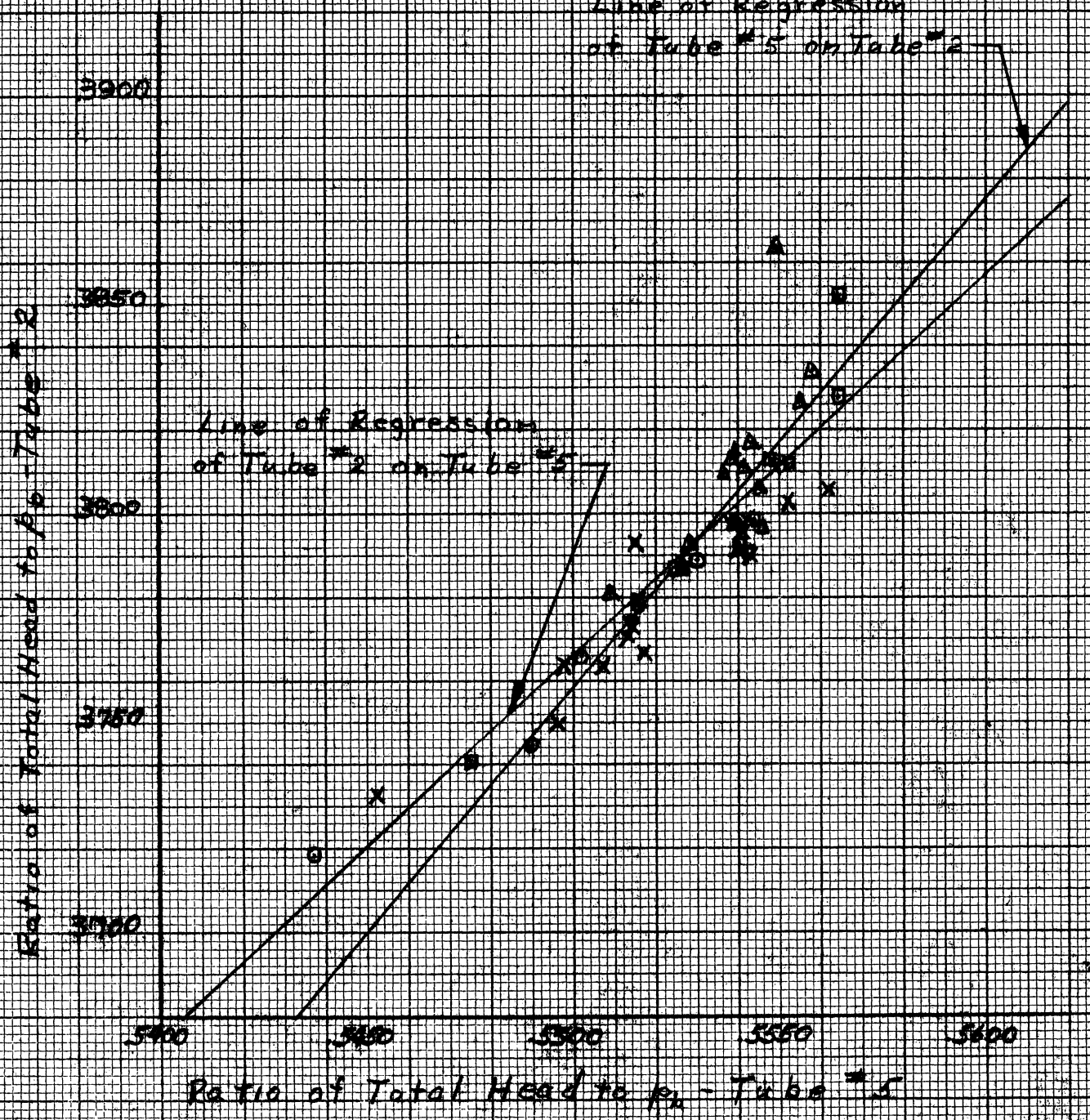


Fig 7