PION MULTIPLICITY AND REGGE TRAJECTORIES*

Y. TOMOZAWA
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48104, *USA*

Received 10 April 1971

Assuming that the zero multiplicity limit of the Poisson distribution in multiparticle production is related to the non diffractive part of the elastic scattering cross section, we obtain the pion multiplicity expressed in terms of Regge parameters and other observable quantities. The agreement with the experimental data for the proton proton reaction is reasonable.

A recent cosmic ray experiment [1] indicates that a) the charged particle multiplicity $\bar{n}_c$ in the proton-proton collision is a linear function of $\ln s$, s being the total c.m.s. energy squared, and that b) the cross sections σ_n for multiparticle production exhibit a Poisson or quasi-Poisson distribution [2]. These results were expected in the multiperipheral model [3,4], in the statistical model [5,6] or in the parton model [7,8] and in some cases with the right order of magnitude for the slope in the $\bar{n}_c - \ln s$ relation, although the constant term is an adjustable parameter in these models.

An interesting question concerning the Poisson distribution is multiparticle production processes is that of the limit of zero multiplicity of the cross sections

$$\sigma_n = \sigma_{\text{inel}} \exp(-\bar{n})(\bar{n})^n/n! \,. \tag{1}$$

In eq. (1), σ_{inel} stands for the total inelastic cross section, n for the number of the pions produced and $\bar{n}$ for their average (the pion multiplicity)‡. At 25 GeV lab. energy

$$\sigma_{\text{inel}} = 30 \text{ mb} \,, \quad \bar{n} = 4 \sim 5 \tag{2}$$

and we thus obtain

$$\lim_{n\to 0} \sigma_n = \sigma_0 = 0.55 \sim 0.20 \text{ mb.} \tag{3}$$

This value is much smaller than the observed elastic total cross section [9]

$$\sigma_{\text{el}} = 9 \text{ mb} \,, \quad \text{at 25 GeV.} \tag{4}$$

Therefore, we cannot identify σ_0 with the elastic total cross section. If the former has anything to do with the elastic amplitude, it must be associated with a part of the latter, but not with the full amplitude.

We present a model which resolves the question raised above and leads to the observed relation

$$\bar{n}_c = A \ln s + B \tag{5}$$

with reasonable values for the parameters A and B.

The elastic amplitude may be divided into two parts, the diffractive and the non-diffractive amplitudes:

$$f_{\text{el}} = f_{\text{d}} + f_{\text{nd}} \,. \tag{6}$$

The diffractive amplitude f_{d} is produced as a result of inelastic processes through unitarity and probably has nothing to do with the zero multiplicity limit of the inelastic processes. Therefore, it is natural to relate the limiting cross section σ_0 to the non-diffractive amplitude f_{nd}.

We assume that

$$\text{(I)} \qquad \sigma_0 = \int_{-s/2}^{0} \left(\frac{d\sigma}{dt}\right)_{\text{nd}} dt \,, \tag{7}$$

(II) the diffractive amplitude is represented by the exchange of the Pomeron and the cut associated with it, and

(III) the non-diffractive amplitude is constructed by the exchange of Regge particles and the cuts associated with them. For simplicity we assume that the non-diffractive p-p scattering is dominated by ω-exchange.

The ρ and A_2 exchange are known to give a smaller contribution than that of the ω-exchange

* Work supported in part by the US Atomic Energy Commission.
‡ We neglect the production of particles other than pions.

in nucleon-nucleon scattering [10,12] as is exhibited in an approximate equality of the pp and np total cross sections. The contribution of the f^0-exchange is not well known since its effect is overshadowed by the Pomeron and its cut. Should a more accurate analysis of the scattering data in the future reveal the relative importance of the f^0-exchange, we would have to take its effect into consideration.

Neglecting the cut contribution, we may, then, write down the non-diffractive differential cross section as

$$\left(\frac{d\sigma}{dt}\right)_{nd} = \frac{g_\omega^4}{4\pi} \frac{(F(t))^4}{m_\omega^4} \left(\frac{s}{s_0}\right)^{2\alpha(t)-2}, \tag{8}$$

where g_ω and m_ω are the ω-p-p coupling constant and the mass of the ω particle, respectively, the parameter s_0 is taken to be 1 (GeV)^2, and the Regge trajectory function $\alpha(t)$ of the ω is

$$\alpha(t) = \alpha_0 + \alpha_1 t \tag{9}$$

with

$$\alpha_0 = 0.4 \text{ and, } \alpha_1 = 1 \text{ (GeV)}^{-2}. \tag{10}$$

The form factor $F(t)$ of the ω-p-p vertex is

$$F(t) = \exp(at), \tag{11}$$

where the value of a is fixed as

$$a = 2.8 \tag{12A}$$

if $F(t)$ is identified with the electromagnetic form factor † or

$$a \approx 1 \tag{12B}$$

if the effective form factor pp scattering at 8 GeV is used [11,12] :

$$\exp(2\alpha_1 \ln(s/s_0) + 4a)t = \exp 9t. \tag{13}$$

The difference between (12A) and (12B) for the evaluation of the slope a may be due to the fact that the contribution of the cut is likely to give an underestimate for the slope in eq. (13).

From eqs. (1), (7)-(11), it follows that

$$\sigma_{inel} \exp(-\bar{n}) = \frac{g_\omega^4}{4\pi} \frac{1}{m_\omega^4} \left(\frac{s}{s_0}\right)^{2\alpha_0 - 2} \frac{1}{2\alpha \ln(s/s_0) + 4a} \tag{14}$$

which leads to the expression for the pion multiplicity

$$\bar{n} = (2-2\alpha_0)\ln\frac{s}{s_0} + \ln\left\{(2\alpha_1 \ln\frac{s}{s_0} + 4a)\frac{4\pi\sigma_{inel} m_\omega^4}{g_\omega^4}\right\}. \tag{15}$$

The first term in eq. (15) was suggested by Feynman [7], although he did not specify which trajectory should be used.
If we had assumed that

$$\sigma_0 = \sigma_{el} \tag{16}$$

we would have had to use the Pomeran intercept $\alpha_0 = 1$. Then the first term of eq. (15) disappears, and we would have

$$\bar{n} = \ln\ln s + \text{const.} \tag{17}$$

This result corresponds to the case considered by Kaiser [13] who has concluded that $\bar{n}$ is bounded by 3 $\ln\ln s$. As was pointed out in the preceding section, the assumption (16) is inconsistent with the experimental data.

We now assume that the charged pion multiplicity is $\frac{2}{3}$ of the total pion multiplicity $\bar{n}$ and that average proton number $\bar{n}_p$ is a constant over a wide range of the incident energy. Then the charged multiplicity is given by

$$\bar{n}_c = \frac{2}{3}(2-2\alpha_0)\ln s/s_0 + \frac{2}{3}\ln\left\{2\alpha_1(\ln\frac{s}{s_0} + 4a)\frac{4\pi\sigma_{inel} m_\omega^4}{g_\omega^4}\right\} + \bar{n}_p. \tag{18}$$

The coefficient of $\ln s$ in eq. (18) is estimated by using the value of eq. (10),

$$\frac{2}{3}(2-2\alpha_0) = 0.8, \tag{19}$$

which should be compared with the empirical formula of ref. [1],

$$\bar{n}_c = (0.71 \pm 0.10)\ln s + 2.04 \pm 0.19. \tag{20}$$

Although all parameters in eq. (18) can be determined, in principle, by other experiments, the average number of protons, which is less than 2, is difficult to measure, especially at high energies. In table 1, we list the values for the pair of parameters $\bar{n}_p$ and $g_\omega^2/4\pi$ which give a good fit to the experimental data (shown in fig. 1). We notice that the second term in eq. (18) is almost constant in the range of energies considered. Since the value of the coupling constant $g_\omega^2/4\pi$ obtained in our analysis is close to the estimate given elsewhere [12,14]

$$g_\omega^2/4\pi = 4 \sim 8, \tag{21}$$

we may conclude that our assumptions (I)-(III) are not far from reality.

† It is easily shown that the dipole form factor gives the same result, as far as our problem is concerned.

Table 1
The values of parameters which give the best fit for the charged multiplicity (fig. 1)

$\bar{n}_p$	$g_\omega^2/4\pi$	
	(A) $F(t) = \exp 2.8t$	(B) $F(t) = \exp t$
0.6	4.3	3.5
0.8	5.0	4.0
1.0	5.8	4.7
1.2	6.8	5.4
1.4	7.9	6.3
1.6	9.1	7.4
1.8	10.6	8.5
2	12.3	9.9

As for the value of $\bar{n}_p$, we may quote the estimate of ref. [15] which suggests $\bar{n}_p = 1.4$ at 19 GeV. A similar value is obtained at 30 GeV if one uses the parameterization [8] for the $\pi^\pm$ spectrum in the experiment pp $\rightarrow \pi^\pm$ + anything and the formula

$$\bar{n}_p = (\tfrac{1}{2}\bar{n}_c + 1) - (\tfrac{1}{2}\bar{n}_c - 1)\, \bar{n}_{\pi+}/\bar{n}_{\pi-} \ , \qquad (22)$$

where $\bar{n}_{\pi\pm}$ stands for the average number of charged pions*. At 12 GeV, one gets $\bar{n}_p \approx 1$ by integrating the $\pi^\pm$ spectrum [16] **. Since, however, these parameterizations of the $\pi^\pm$ spectrum are based on insufficient experimental data, the above mentioned estimate of $\bar{n}_p$ is not conclusive. It would be very useful if the $\pi^\pm$ spectra were measured over a wider range of the momentum transfer (especially for small Δp^2, which gives a large contribution to the average number $\bar{n}_{\pi\pm}$). This would afford a test of the assumption that the variation of $\bar{n}_p$ is small.

The inclusion of the contribution of the cut associated with the ω-exchange would reduce the value of the right hand side of eq. (18): It should be multiplied by the factor

$$(1 - 2\lambda/\ln s) \ , \qquad (23)$$

where λ designates the strength of the cut contribution. This enhances the value of $g_\omega^2/4\pi$ in table 1. It is not clear, however, whether the most important cut, which is an interference effect of the ω-exchange and the Pomeron-exchange should be included in our consideration.

We may use the canonical value for the scale parameter

$$s_0 = 2m_N^2 \qquad (24)$$

which follows from the formula

* Eq. (22) follows from charge conservation, $2 = \bar{n}_p + \bar{n}_{\pi+} - \bar{n}_{\pi-}$ and the definition $\bar{n}_c = \bar{n}_p + \bar{n}_{\pi+} + \bar{n}_{\pi-}$.
** We use the parameterization of ref. [8] or that of Yao (private communication).

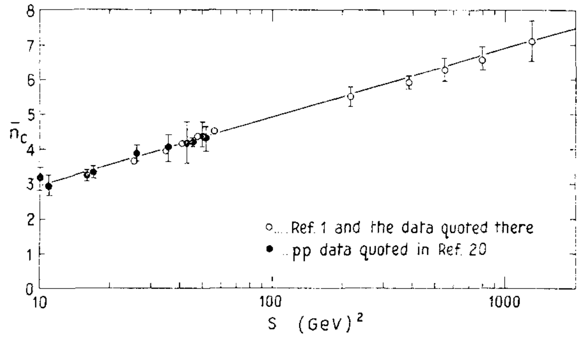

Fig. 1. Charged multiplicity in the proton-proton reaction versus the c.m.s. energy squared in the log scale. The solid line is the theoretical prediction, eq. (18), with the values of parameters given in table 1.

$$-\cos\theta_t = \frac{2s}{4m_N^2 - t} \approx \frac{s}{2m_N^2} \ , \quad \text{for small } t \ . \qquad (25)$$

The value of $\bar{n}_p$ in table 1, then, should be increased by

$$\frac{2}{3}(2-2\alpha_0)\ln s_0 = 0.5 \ . \qquad (26)$$

We present some interpretations of the result of the model which is discussed in the preceeding section.

(a) Multiperipheral production

If the pions are produced by the pair as in fig. (2), the multiplicity in eq. (18) must be the number of pairs. Therefore, the pion multiplicity is

$$\bar{n} = 2(2-2\alpha_0)\ln s + \ldots \qquad (27)$$

This formula can be consistent with experiment only if

$$\alpha_0 \approx 0.7 \ . \qquad (28)$$

This would be the case if the f^0-trajectory has the intercept of eq. (28) and dominates in the p-p scattering. The magnitude of the coupling constant g_f of the f^0-p-p vertex must be also appropriate in order to fit the experimental data. However, this model predicts that the production of an even number of pions dominates over that of an odd number which seems an unlikely case.

On the other hand, the multiperipheral model in which the ω and ρ in the cross channel alternate as is shown in fig. 3, would predict eq. (18), provided the relation between the coupling constants

$$g_\omega = g_\rho$$

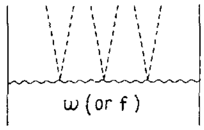

Fig. 2. Multiperipheral production of pions due to ω or f Regge exchange. The dotted lines are pions.

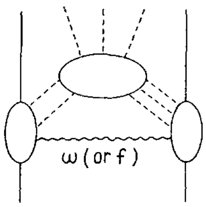

Fig. 4. Nonperipheral multipion production by pp collision.

is satisfied. As pointed out before this seems not to be the case.

Based on the discussion of this subsection we may conclude that the assumptions (I)-(III) may be incompatible with a naive form of multiperipheral models such as exhibited in figs. 2 and 3.

(b) Our assumption (I)-(III) would be compatible with diagrams in figs. 4 and 5, provided the pions are produced in an uncorrelated way so that the Poisson distribution is observed. Of course, the detailed analysis of these mechanisms of multiparticle production is yet to be studied. We note that the diagrams (4) and (5) may correspond to nonmultiperipheral processes or a more complicated form of multiperipheral model in which both the fermion and boson Regge particles are exchanged.

The author is indebted to F. Henyey, L. W. Jones and his group, G. L. Kane, I. Kimel, A. D. Krisch, D. E. Lyon, B. Roe, M. H. Ross, J. Vander Velde, L. Wolfenstein and Y. P. Yao for useful suggestions and discussions. Thanks are also due to D. N. Williams for reading the manuscript.

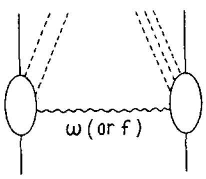

Fig. 5. Multiperipheral production of pions due to the fermion and boson Regge exchange or nonmultiperipheral production.

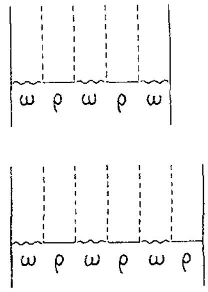

Fig. 3. Multiperipheral production of pions due to alternating ω and ρ Regge exchange.

References

[1] L.W.Jones et al., Phys.Rev.Letters 25 (1970) 1679; E.26 (1971) 213;
D.E.Lyon Jr., Proton-Proton Interactions between 90 and 800 GeV/ , Symposium on High Energy Multiple production at Argonne National Laboratory, Nov.1970.
[2] C.P.Wang, Phys.Rev. 180 (1969) 1463;
L.Caneschi and A.Schwimmer, Charge Distributions and Multiperipheralism:
See, however, A.Wroblewski, Phys.Letters 32B (1970) 145.
[3] D.Amati, A.Stanghellini and S.Fubini, Nuovo Cimento 26 (1962) 896.
[4] G.F.Chew and A.Pignotti, Phys.Rev.176 (1968) 2112.
[5] A.D.Krisch, Phys.Rev. 135 (1964) B1456.
[6] R.Hagadorn and J.Ranft, Nuovo Cimento Suppl. 6 (1968) 169.
[7] R.P.Feynman, Phys.Rev.Letters 23 (1969) 1415; Proc.Stony Brook Conf. on High energy collisions (Gordon and Beach 1969) ed. C.N.Yang et al., p.237.
[8] N.F.Bali, L.S.Brown, R.D.Peccei and A. Pignotti, Phys.Rev.Letters 25 (1970) 557.
[9] O.Benary, L.R.Price and Alexander, UCRL - 20000 NN (1970).
[10] W.Rarita, R.J.Riddell Jr., C.B.Chiu and R.J.N. Phillips, Phys.Rev. 165 (1968) 1615;
D.M.Austin, W.H.Greiman and W.Rarita, Phys. Rev.2D (1970) 2613.
[11] V.D.Barger and D.B.Cline, Phenomenological theories of high energy scattering, (W.A. Benjamin, N.Y., 1969), p.91.
[12] G.L.Kane, F.Henyey, D.R.Richards, M.Ross and G.Williamson, Phys.Rev.Letters 25 (1970) 1519
[13] G.D.Kaiser, Multiplicities in particle production processes, Cambridge HEP 70-8 (1970). Note, however, that this article uses a definition of the multiplicity different from ours.
[14] G.Ebel, H.Pilkuhn and F.Steiner, Nucl.Phys. B17 (1970) 1.
[15] Scandinavian Collaboration, Some Features of Particle Multiplicities and Pion Momentum Spectra in Inelastic p-p Collisions at 19 GeV. Internal report (1969).
[16] C.W.Akerlof et al., Inelastic high energy proton-proton collisions, Phys.Rev., to be published.
[17] V.S.Barashenkov et al., Fortschritte Phys. 14 (1966) 357.