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DETECTION OF ONE-, TWO-, AND THREE-DIMENSIONAL MARKOV CONSTRAINTS IN VISUAL DISPLAYS *

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ABSTRACT

One-, two- and three-dimensional Markov constraints were introduced into visual displays by a common method. To obtain three-dimensional displays, two-dimensional spatially-encoded displays were presented successively in time. Following random initialization, all additional display elements were generated by rule. With non-probabilistic rules, sequences with horizontal (*X*) constraint, or with vertical (*Y*) constraint, alone were easily detected. Sequences with temporal (*T*) constraint alone, and with two-dimensional constraints in *XY*, *XT*, and *YT*, also were detected. Three-dimensional *XYT* constraints, however, could only be detected at chance level. Discrimination thresholds with probabilistic rules also show the relative superiority of one- over two-dimensional constraints. One- and two-dimensional constraints in *T* are sensitive to the rate of presentation of successive displays, whether memory is carried by the display or by the eye.

1. INTRODUCTION

People often are able to appreciate extremely complicated masses of information after encoding in the form of visual displays. Whether a military map, the structure of the DNA molecule, the interaction of demographic, geographic and economic indexes, etc., 'understanding' is often aided by – indeed, understanding often requires – a concrete visual representation.

The present report considers some of the psychophysical limits associated with visual displays of relatively large quantities of binary-coded information. Specifically this paper extends the study of discrimination of one- and two-dimensional Markov constraints, encoded in terms of the spatial coordinates *X* and *Y* (POLLACK, 1971), to three-dimensional Markov constraints. The temporal coordinate,

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time (T), is added to the spatial coordinates. Although the present attempt to develop a display for the detection of three-dimensional Markov constraints was unsuccessful, the approach is here described in order to encourage investigators with other methods of encoding visual information within three-dimensional displays, (size, color, stereoscopic presentation, etc.) to seek a solution for the display of three-dimensional Markov constraints. In addition, the interaction of temporal and individual spatial dimensions was selected for special study.

2. Method

2.1. *General approach*

One-, two- and three-dimensional Markov constraints were obtained by a common method. In each case, the aim was to produce displays with constraints of a given dimensionality in the absence of constraints of lower dimensionality. A display with three-dimensional constraints, for example, showed no constraints within any one dimension alone or for any pair-wise combination of two dimensions. A display with two-dimensional constraints showed no constraints within any one dimension alone.

The method is illustrated in table 1. For each constraint, represented

TABLE 1

Method of generation of displays with one-, two- and three-dimensional constraints.

Constraint	Random generation 1st display	Random generation 2nd+ display	Rule generation 1st display	Rule generation 2nd+ display	Rule calculation
X	column_1	column_1	rows^-	rows^-	1
Y	row_1	row_1	columns^-	columns^-	1
T	entire	none	none	entire	1
XY	c_1 r_1	c_1 r_1	entire^-	entire^-	3
XT	entire	c_1	none	rows^-	3
YT	entire	r_1	none	columns^-	3
XYT	entire	c_1, r_1	none	entire^-	7

within a single row, the following is listed: the randomly-generated elements within the first display, and within the second, and subsequent, displays; the elements generated by rule within the first display, and in successively presented displays; and the number of previously-selected display elements entering into the determination of any given display element.

For example, the top line represents a 1-dimensional constraint in X (horizontal). The first column of the initial display is randomly generated with each display element either a dot or a no-dot. The two states are written here as 0 or 1. One random selection might be: 1 0 0 1 for a 4×4 display. The remainder of the elements within each row are determined entirely by the first element of the row for non-probabilistic rules. If the probability of an even aimed-for sum, $p(\Sigma)_e$, is unity over successive positions, the first row would be *1* 1 1 1, where the italicized element represents the initial random selection. If the probability of an even aimed-for sum is zero over successive positions, the first row would be *1* 0 1 0. Similarly, with $p(\Sigma)_e = 0$, the second row would be *0* 1 0 1. The same procedure would hold for generation of the second and of subsequent displays, since there is no correlation in T. The X-constraint is seen as a set of solid horizontal lines $[p(\Sigma)_e = 1]$ or as a set of dashed horizontal lines $[p(\Sigma)_e = 0]$, which change randomly in position upon successive displays.

The second line of table 1 represents constraints in Y (vertical). All features are identical with constraints in X, except for a reversal of the role of columns and rows.

The third line represents constraints in T (time). The entire initial display is randomly generated. Successive displays are identical with the initially-generated display $[p(\Sigma) = 1]$; or are spatially complementary, point for point, with the initial display $[p(\Sigma)_e = 0]$. At high display rates, the T-constraint is seen as stationary random display $[p(\Sigma)_e = 1]$; or as a solidly-filled display $[p(\Sigma)_e = 0]$.

The fourth line represents constraints in XY. In the initial display, the first column and the first row are randomly generated. The rest of the display is determined by rule. For example, if the three elements in the corner of the display formed by the first row and column were 0, 0, 0, the fourth element would be 0 $[p(\Sigma) = 1]$ or would be 1 $[p(\Sigma)_e = 0]$. For each possible combination of 3 elements, the fourth element can be selected by rule. The element obtained upon the first selection by rule, and the second and third elements of the first row, determine the second selection by rule. The procedure continues until all elements are determined. Successive displays in time are independently generated. The XY-constraint is seen as a pattern of squares and holes $[p(\Sigma)_e = 1]$ or as a lacey pattern $[p(\Sigma)_e = 0]$, which changes on successive displays.

The fifth line represents constraints in XT. The initial display is entirely randomly determined, as are the first columns of successive displays. The second element in the first row of the second display is determined by the first two elements in the first row of the first display, plus the first element of the first row of the second display. With $p(\Sigma)_e = 1$, each row of the second display is identical with its corresponding row of the first display, if the row element of the two displays is of the same state; each row of the second display is complementary to its corresponding row of the first display, if the initial row element is of the other state. With $p(\Sigma)_e = 0$, the complementary relationships are reversed. The XT-constraint is seen at intermediate rates as a series of irregularly-dashed rows.

The sixth line represents constraints in YT and correspond to those of XT, except for interchange of columns and rows.

The seventh line of table 1 represents the three-way XYT constraint. The initial display is entirely randomly generated. On each succeeding display, the first column and the first row are randomly generated. Imagine a cube in which the four corners of one face represent four elements of the first display. On the opposite face, the corners along the top row and left column represent three more elements. Only the bottom corner of the opposite face remains to be determined. The aimed-for sum is calculated for the eight element of a cube, given seven other elements, distributed in X, in Y, and in T.

In summary, successive displays are independently generated in time, for constraints not including time (T). For one-dimensional constraints, a single plane of elements must first be randomly generated in the 'other' two dimensions: a YT-plane for constraints in X; an XT-plane for constraints in Y; and an XY-plane for constraints in T. For two-dimensional constraints, two planes of elements with a common intersection must first be randomly generated, in which each dimension combines with the 'other' dimension. For example, an XT-plane and a YT-plane are required to generate a sequence of displays with XY-constraints; an XY- and a YT-plane are required to generate a sequence of displays with XT-constraints. Finally, for three-dimensional constraints, three planes of elements with two common intersections must first be randomly generated to generate a sequence of displays with XYT-constraints.

2.2. *Procedure*

A PDP-9 computer generated 4 square matrices in a 2×2 format on a display oscilloscope with a low persistence tube (Tektronics 602 with P15 phosphor). For three of the matrices, $p(\Sigma)_e = 0.50$; for one of the matrices, $p(\Sigma)_e = 0$ or 1.0 with non-probabilistic rules and at intermediate values with probabilistic rules. The observer was equipped with a switch box with 4 buttons in the same 2×2 format as the display. His task was to identify which one of the four matrices employed a different rule from the other three. The randomly-determined initial planes were independently selected for each of the four matrices. Twenty-five consecutive trials were run under a fixed condition.

The subjects had previous intensive experience in detecting differences in $p(\Sigma)_e$ for one- and two-dimensional spatial constraints in a 2-dimensional spatial, non-temporal format. Each point represents the proportion of correct responses based upon 400 observations, contributed by 8 observers.

The visual environment was relatively dark with a light level of 0.1 mlux at the display oscilloscope in the absence of a display.

2.3. *Experimental conditions*

There were 849 experimental conditions, grouped within four experimental series. In one series, successive displays were replenished or refreshed at the rate of 30 displays per second. In effect, the burden of memory of the previous display change was carried by the display, rather than the eye. In another series, displays were not refreshed between display changes, so that the burden of memory of the previous display was carried by the eye. Consider, for example, a temporal sequence of 8 display changes separated by 1 sec each. With non-replenished displays, each of the 8 display changes would be separated by 1 sec. With replenished displays, each of the 8 display changes would be repainted 29 consecutive times to fill the interval between successive display changes. The four matrices were painted with a fixed position for the upper-right corner of each matrix. Corresponding corners of adjacent matrices were separated by 3.5 cm vertically and horizontally.

In another series, the four matrices were painted with a constant distance between inside corners (0.6 cm horizontal and vertical separation), irrespective of matrix size. This centering operation served to bring smaller matrices closer together than the 3.5 cm distance used in the non-centered series. In the non-probabilistic series, the probability of an even aimed-for sum was not necessarily zero or one. In all tests, the distance between successive rows and columns within the same matrix was 0.15 cm. The subject was permitted to adjust his viewing distance to the distance to the display, although most subjects employed a viewing distance of about 75 cm. He was also permitted to vary the intensity and focus of the display oscilloscope for comfortable viewing and for resolution.

3. Results

3.1. *Overall evaluation of constraints*

Results for selected display conditions for the several display constraints are presented in table 2. Under almost all conditions of

TABLE 2

Percentage correct responses for selected displays (chance performance = 25 %).

Display						
Size	6 × 6 (centered)		10 × 10		20 × 20	
Flashes	64		32		8	
IDI, msec	16.7		33.3		133	
$p(\Sigma_e) =$	0	1.0	0	1.0	0	1.0
Constraints						
X	94	100	98	98	99	98
Y	98	99	99	99	98	99
T	99	100	98	97	51	83
XY	28	36	36	72	94	99
XT	35	75	36	71	24	38
XT	36	75	35	72	26	37
XYT	28	26	24	29	28	26

testing, near-perfect discrimination is achieved with *X*- or *Y*-constraints. Under almost all conditions, performance with the three-dimensional constraints was near-chance. Constraints in *T* are sensitive to the interval between successive display changes, as are constraints in *XY* (without constraints in *T*), and in *XT* or *YT*.

The subsequent analysis of the results will be confined to three classes of displays which yielded intermediate performance. The classes are represented by XY, T, and XT. Specifically, we shall be concerned with the effect of the interval between successive display changes, the inter-display interval or IDI, and of the number of successive display changes, n.

3.2. *Short-term memory for selected constraints*

Memory for selected constraints was studied in the tests of fig. 1 by varying the blank interval between two successive displays. The three

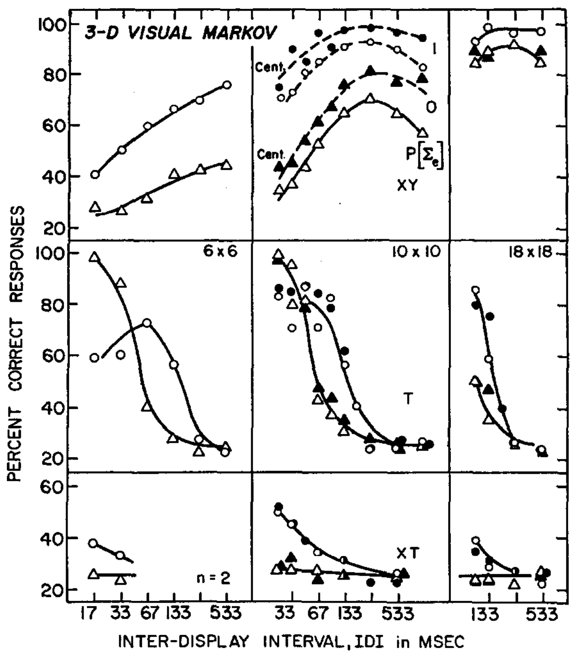

Fig. 1. Discrimination of one- and two-dimensional constraints for two successive displays separated in time by the indicated inter-display interval. The three columns of figs. 1-5 represent 6 × 6, 10 × 10, and 18 × 18 matrices; the three rows represent constraints in XY, in T, and in XT. The circles represent a probability of an aimed-for even sum of unity; the triangles represent a probability of an aimed-for even sum of zero. The filled points of fig. 1 represent centered displays.

columns of figs. 1–5 represent, from left to right: 6 × 6, 10 × 10, and 18 × 18 matrices. The three rows represent, from top to bottom, constraints in XY, T and XT. The triangles represent tests in which $p(\Sigma)_e = 0$; the circles represent tests in which $p(\Sigma)_e = 1$. The filled points represent centered matrices. The shortened range of inter-display times investigated with larger matrices resulted from the longer times required by the computer to generate displays with larger matrices.

There is the anticipated improvement in performance with larger matrices over corresponding IDIs. The improvement in performance with larger matrices results from two features: there is a greater number of display elements available for sampling by the subject, and, for smaller matrices, there is a shorter distance between matrices which facilitates inter-comparison among the matrices.

With one interesting exception, discrimination performance is consistently better with $p(\Sigma)_e = 1$ than $p(\Sigma)_e = 0$. The exception concerns two displays constrained in T separated by the shortest IDIs examined. With $p(\Sigma)_e = 0$ with T-constraint, the critical matrix is seen as solid, since the second display is the point-for-point complement of the first. With $p(\Sigma)_e = 1$ with T-constraint, the critical matrix is seen as a random dot pattern with an average density of superimposed dots of 0.50, while the average density of superimposed dots is 0.75 for the other matrices.

The main point of fig. 1 is that the role of IDI, differs with the form of constraint. Performance improves with IDI for XY-constraint because the point-by-point correlation in time between successive displays is random. The turndown at the longest IDI intervals with the 10×10 matrix is not strongly confirmed with other matrices. (The subjects were reimbursed on a schedule related to the number of completed trials and may have grown restive with longer IDIs.)

By comparison, performance falls rapidly with IDI for one- and two-dimensional constraints employing T. Performance is nearly chance at 250 msec – a figure which is within the range of estimates of the duration of short-term visual memory.

3.3. *Effect of unburdening visual memory*

Transferring the memory requirements from the eye (for the non-replenished displays of fig. 1) to the display (for the replenished displays of fig. 2) resulted in an overall improvement in performance at least for XY- and T-constraints. The improvement is due, in part, to the brighter image of the replenished display; to the lesser interference of subsequent displays upon earlier ones; and to the unburdening of the requirement for visual memory. Presumably the second

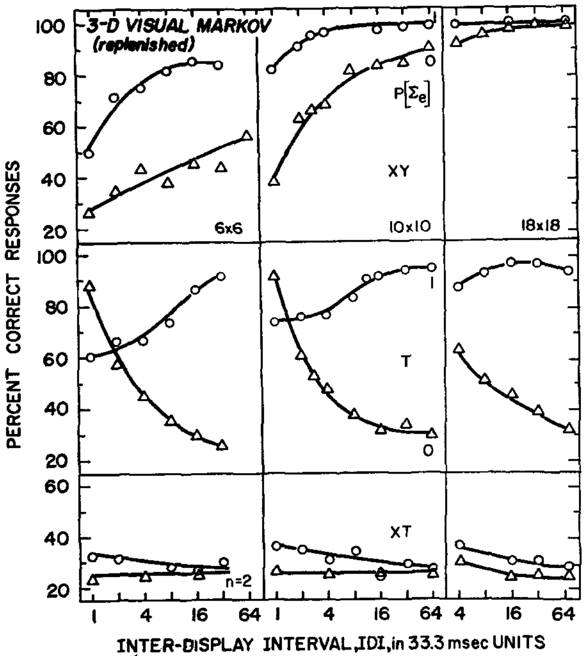

Fig. 2. As fig. 1 for displays which are replenished 30 times per second.

feature was effective for XY-constraints, and the last feature for T-constraints. Specifically, there is a sharp *gain* in performance with IDI for replenished displays with T-constraint, $p(\Sigma)_e = 1$. In that condition, successive display changes are identical. The longer the IDI, the larger is the number of identically painted displays.

3.4. *Integration of information over successive display changes*

Figs. 3 and 4 present the effect of the number of successive display changes for non-replenished displays (open points) and for replenished displays (filled points). The three columns represent the near-minimum IDI obtainable with the three matrices. (IDI has no meaning with an $n = 1$ used for XY-displays.)

The outstanding result of figs. 3 and 4 is that moderate performance levels (over 80 % correct) are attainable with XT-constraints, if a sufficient number of successive rapid display changes are made available, at least with $p(\Sigma)_e = 1$.

The loss in performance for XY-constraints with a small number of

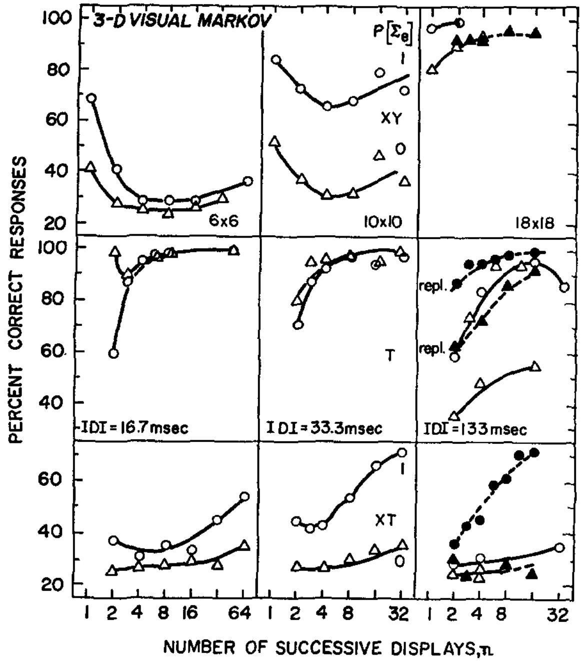

Fig. 3. Integration of one- and two-dimensional constraints over successive display changes. The IDI associated with each matrix is given in the middle row. The filled points represent replenished displays.

successive displays can be reversed with a sufficiently large number of displays. This improvement may result from a longer available inspection time for scanning the four matrices, or from the wider sampling of XY-patterns yielded by independent random selection. Constraints in T benefit sharply from successive displays in time, presumably because the displays are either duplicated or complemented.

3.5. *Tradeoff between number of display changes and the rate of display change*

With replenished displays, we can paint a constant number of displays, vary the number of display changes, and still keep the total duration of presentation constant. For example, 64 displays can be painted with 64 successive display changes, each separated by a single clock-period of 33.3 msec, or 64 displays can be painted with 2 display changes separated by 32 clock-periods, etc.

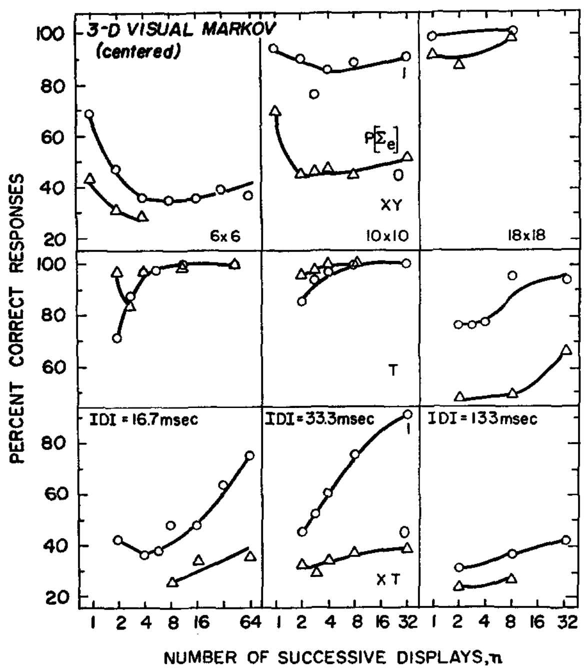

Fig. 4. As fig. 3 for centered displays.

The nature of the tradeoff between the interval between, and the number of, display changes is shown in fig. 5. Highest performance is achieved with T- and XT-constraints with a large number of rapid display changes. With XY-constraints with small matrices, an intermediate range of display changes and speed of change yields highest performance.

3.6. *Probabilistic rules*

The tests of figs. 1–5 employed non-probabilistic generating rules subsequent to initializing random sequences. Are the previous results maintained with probabilistic generating rules? Apparently, yes.

Fig. 6 presents the results of tests with 10×10, 14×14 and 18×18 matrices. The results with extreme probabilities were taken from the earlier tests. It was assumed that chance performance (horizontal dashed line) would have been obtained with all four matrices at $p(\Sigma)_e = 0.50$ (vertical dashed line).

The successive rows of fig. 6 represent constraints in X, XY, T and

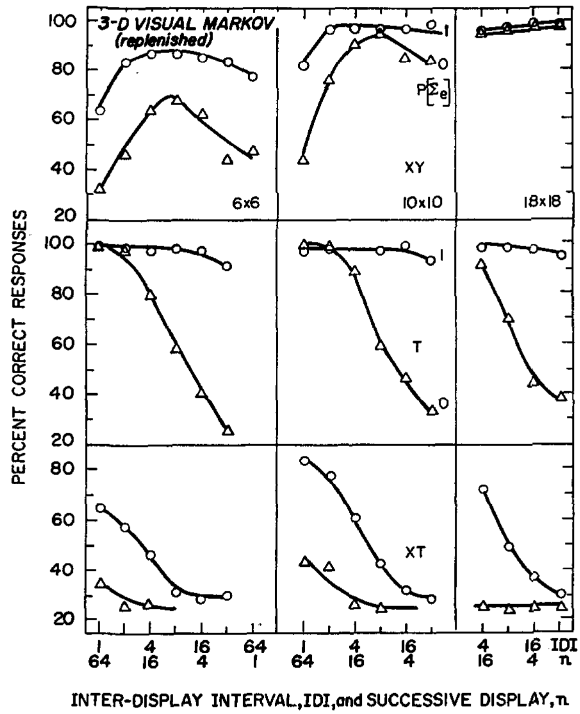

Fig. 5. Tradeoff between number of display changes and the inter-display interval for displays of a total constant duration.

XT. Under the conditions of testing given in the lower right corners of each section, performance is nearly symmetrical about $p(\Sigma)_e = 0.5$ for *X*- and *XT*-constraints. Performance is poorer for *XY* and is nearly chance for *XT* at low $p(\Sigma)_e$ levels, relative to corresponding conditions at high $p(\Sigma)_e$ levels. Intersection of the curves of fig. 6 at arbitrary percent correct response levels will yield sets of detection thresholds from randomly-generated displays. Such thresholds would be lowest for *X*; highest for *XT*; and intermediate for *XY* and *T*, at least for the particular display conditions of fig. 6.

4. Epilogue

Few generalizations have emerged in the present study, despite the very large number of observations (849 conditions $\times$ 400 observations per condition or 3.4×10^5 observations). At best, I have attempted

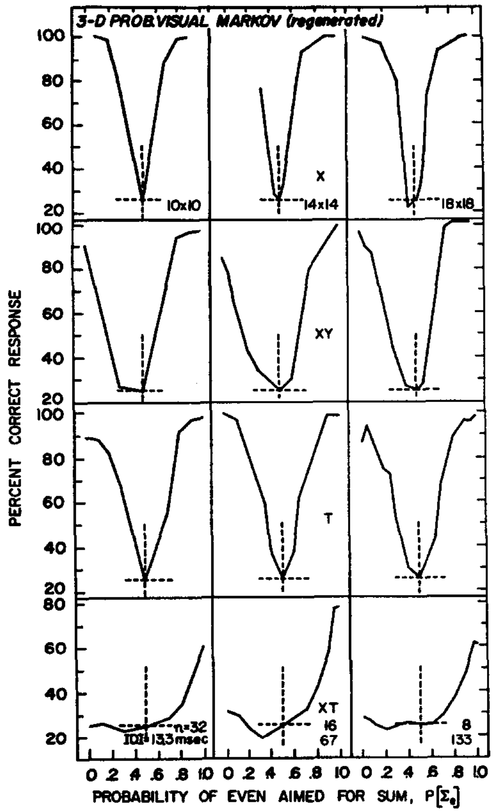

Fig. 6. Discrimination of one- and two-dimensional probabilistic constraints. The display conditions are listed in the lower right corner of each row.

to explore the landscape of conditions over which one-, two-, and, hopefully, three-dimensional constraints can be appreciated within visual displays. The failure to achieve consistent discrimination in the case of three-dimensional constraints may be related to a fundamental limitation of perception, or more likely, may be simply related to the arbitrarily chosen methods of encoding employed. For example, the demonstrations of JULESZ (1960) and others on the rapid perception of depth in the absence of monocular cues suggest that other display features may have succeeded where I have failed. How would you better encode three-dimensional constraints for visual displays?

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