Most predictions of growth are based on some mathematical model of the growth process, two kinds of which can be distinguished: (1) the transformed coordinate method of Dague Thompson, applied to human growth by Medawar and to craniofacial growth by Moorrees and Lebret, and (2) equations producing curves descriptive of processes—some intended to be of general application (for example, Gompertz’ equation, Weiss and Kavanaugh’s model, and the compound Gompertz-linear equation of Laird). Others were intended only for specific kinds of growth, such as Huxley’s allometric equation and Tanner’s equation. Useful though these equations may be for the analysis and description of growth, they have in common one unavoidable shortcoming so far as prediction is concerned: They do not adequately describe any single growth series. The orthodontist wishes to predict the future sizes and relationships of parts of a single person whose growth is usually abnormal and who may not be represented in any populations thus far studied.

This article considers the state of the art in quantitative prediction of craniofacial measures of particular interest to orthodontists.

**Methods of prediction**

Several predictive methods are used in industry and science. We may group these under the following four headings: (1) theoretical, (2) regression, (3) experiential, and (4) time series.

*Theoretical methods of prediction.* Astronomers recently discovered an earth-sized planet several thousand light years away from us by collecting a series of inexplicable, apparently random data on the behavior of celestial bodies until a
A theoretical model could be constructed mathematically which might explain all the unusual activity observed, and a test for a hypothesis was devised. The model assumed the existence of an unknown planet of a certain size in a certain orbit, which was subsequently found precisely in the theoretically predicted location.

No one had suggested the existence of such a planet until the model was formulated; the model began theoretically and was proved practically. Theoretical models of craniofacial growth have not yet been defined mathematically in terms precise enough to permit the application of the method to prediction.

**Regression methods.** These methods serve to calculate a value for one variable, called *dependent*, on the basis of its initial state and the degree of its correlations with one or more *independent* variables.

Johnston has recently evaluated and reviewed the regression methods of approach to craniofacial prediction. Among his conclusions are that (1) the ultimate accuracy of cephalometric prediction may be limited to some extent by intrinsic errors within the cephalometric method itself and (2) contemporary methods seem inadequate to provide an efficient estimate of individual changes attributable only to growth. Burstone has reviewed some of the problems of attack and of selection of independent variables with regard to growth prediction.

For a number of years the regression methods had been used by several workers, in addition to Johnston, in analyzing the serial growth data available at the University of Michigan. Our familiarity with this work leads us to agree with Johnston that, useful as regression methods are in the study of growth, there are both theoretical and practical implications in the use of these methods in predicting the growth of a single person. Chiefly, these are the following:

1. The assumption within the method that the coefficients remain constant over the whole time period. Growth does not proceed in this manner in the real world.

2. An individual whose growth is to be predicted in clinical practice may not even be a member of the population upon which the regression equation was based.

**Experiential method.** Experiential methods are based on the clinical experience of a single investigator who attempts to quantify his observations of practice in such a way that they can be codified for use by others. The best-known example of the experiential method in craniofacial growth prediction is that of Ricketts, whose estimates of growth prediction for the individual utilize means derived from a large sample of treated orthodontic patients. The method is popular and widely used, but its theoretical base is shaky on two counts. First, the assumption must be made that the individual being predicted will behave as the mean of a population of which he is not a member. Second, the morphology of the mandible and other parts is a clue to the future growth of the face, a point disputed by Horowitz and Hixon, Balbach, and Woodside.

**Time-series methods.** Because of the great interest in prediction of craniofacial growth and the limitations of the methods thus far tried, it seems pertinent to ask whether there might be some other method of prediction, as yet untried on growth problems, which would provide the desired accuracy, efficiency, and individuality for clinical application.
Operations research (problem solving through applied mathematics) has been concerned over the past decade or so with the development of methods of prediction which are based on individual, not population, behavior. These procedures deal with a "random walk" type of process, as exemplified by the sales volume of a particular product, the demand for a certain item of inventory, the market price of a specific stock, or the position of a missile to be intercepted.

The methods are of essentially two types: (1) time-series analysis which extracts in a mathematical form the fundamental nature of the process as it relates to time, and (2) smoothing methods, either moving averages or exponential, which operate to give representative or average values to the parameters of a previously derived time-series equation. Although time-series methods have been recognized before by Meredith as an approach to facial growth analysis, they have not been combined, to our knowledge, with the predictive techniques developed by operations research.

For purposes of analysis, a time series is considered to be composed of four parts. These are trend or long-term movement, oscillations about a trend, cyclic or periodic events, and random (unsystematic) components. The analysis consists of the assessment of each of these parts by means of specific statistical tests, the details for which have been reported by Hirschfeld in a separate article. In craniofacial growth prediction, it is necessary as the first part of the analysis to determine which components of the time series are present in the specific measure to be predicted; that is, whether its growth behavior is best described as a trend, a cyclic event, or some combination, etc. Thus, the time-series analysis can reveal the nature of the changes of state of the process with time and describe each change mathematically by means of appropriate models.

Exponential smoothing is a way of estimating the current value of a parameter by means of some sort of average of past values of that parameter. Prediction is then based on coefficients derived from the smoothed parameters. Since the coefficients will change in accord with changes in the parameter, the predicted value will reflect the past behavior of the specific variable for which the prediction is to be made. It is possible with this method to make the initial value of the smoothed statistic equal to the mean value of the variable at that time as derived from some other source. Subsequent statistics are, of course, obtained from the cephalograms in the one subject's series. In craniofacial growth prediction, this means that one could predict from a single cephalogram if mean values for an appropriate population are available as well. However, more precise predictions can be obtained if there are at least two successive cephalograms of the individual to be predicted.

Our work with time-series methods leads us to believe that time-series analysis offers more promise for craniofacial growth prediction than any of the methods thus far used.

What are we interested in predicting in the craniofacial complex?

Future size of a part. The prediction of future size, as Burstone has pointed out, is primarily a problem of predicting future increments which are to be added to a size that is already known. Most of the size dimensions of interest to
the orthodontist display a combination growth curve through time. An example would be prediction of the length of the mandible.

**Relationship of parts.** Perhaps the most important prediction for the clinician is the future relationship of parts, that is, the future facial pattern. Pattern, however represented, is a summation of growth and size in several component regions.

**Timing of growth events.** Because growth does not proceed evenly, certain facial dimensions demonstrate marked changes in their velocity curve. These “spurts” make prediction much more difficult. If one were to predict a “spurt,” he might want to predict the time of its onset, the duration of the increased rate of growth, and the rate of growth during the spurt.

**Vectors of growth.** Most predictive methods thus far presume a continuation of the pattern first seen. Therefore, the presumption is made that the vectors of growth present at the time of prediction will remain. There is much documentation that this presumption is not true. Mandibles which grow vertically for a period of time inexplicably start to grow horizontally. Can such changes in growth direction be predicted?

**Velocity of growth.** It would be of use to know the future expected rate of growth. Prediction of velocity is most important during the pubescent spurt.

**The effects of orthodontic therapy on any of the above predicted parameters.** Although Burstone\(^4\) has pointed out that our knowledge of prediction might best proceed by learning to predict untreated growing faces, the clinician must always wonder what effects his therapy is having on the predicted and actual growth of one specific face.

To summarize, it is not unreasonable for the clinician to be interested in predicting future size, future relationship of parts, timing of growth events, vectors of growth, velocity of future growth, and effect of orthodontic therapy on any of these parameters. When one breaks down the problem of facial growth prediction into these component parts (and there is always the possibility of wanting to predict other items not mentioned), the immensity and the difficulty of craniofacial growth prediction are apparent.

**How well can we predict each of these parameters?**

**Future size.** A number of studies of the size-size, size-gain, and gain-gain type have been reported for a variety of craniofacial dimensions,\(^4, 9, 21\) but the findings are of little clinical use as yet, despite an occasional statistically significant value. (One must differentiate between statistical significance and clinical utility.) Craniofacial growth is so complex that it is unlikely that any simple series of size predictions will prove clinically useful.

**Relationship of parts.** Harvold\(^9\) has attempted to predict maxillomandibular relations at one age on the basis of maxillomandibular relations and mandibular size at an earlier age. Johnston\(^17\) found measures of relationship and proportion to be of greater predictive significance than the linear size of anatomic parts. Balbach\(^1\) attempted to predict the future occlusal position of the mandible on the basis of its morphology. None were able to be much more accurate than one would be by assuming the addition of mean increments to the initial value being predicted.
**Timing of growth events.** Björk and Helm\(^2\) related several maturational events to attempt to predict the maximum pubertal spurt in growth. Hunter and Miller\(^{15}\) reported the shape of the face as roughly related to the timing of the pubescent spurt—a most interesting finding, even if identified only in a general way at the moment. Frisancho, an anthropologist working at the Center for Human Growth and Development on problems of malnutrition and growth, has had some success in predicting the individual spurt in stature from noting the timing of calcification of the sesamoid bone.\(^7\) These general findings point to interest in the problems of prediction and offer encouragement, but nothing yet is precise enough for practical prediction of the individual face.

**Vectors of growth.** Ricketts\(^{23}\) assumes that the morphology of the mandible is a clue to the future vectors of growth of the craniofacial complex. Woodside\(^{27}\) recently reported findings from work done on the Burlington sample which showed only part of Ricketts’ assumed vectors to be true. Balbach\(^1\) found support for the idea of relative stability of mandibular morphology which may explain the modest success some have had in prediction by simply adding mean increments to the existing facial pattern. Yet, as both Balbach\(^1\) and Johnston\(^77\) have pointed out, this is only one of three important components. The presence of change in vectors and possible orthodontic intervention make the assumption of vectors from the present pattern a crude estimate at best. Many researchers have discussed the frustrating implications of changes in the growth vector, but no one has yet suggested a means of anticipating such changes in the direction of growth in the face. Predicting vectors is not the same as predicting changes in vectors.

**Velocity.** If one is truly to predict facial growth, one must account for variations in velocity. Hunter and Miller\(^{15}\) found that changes in rate of growth occurred at different times in horizontal maxillary measures than in vertical ones. Not much attention has been given velocity predictions, and the problem seems more complicated than predictions of simple size or relationship of parts.

**The effects of orthodontic therapy on growth.** As our skill in treatment has increased, so has our ability to alter the growth of the facial structures remarkably. Balbach\(^1\) suggests that the most important single factor in prediction for the orthodontist is to be able to predict what effect his therapy will have on the growing face that he is treating. Jenkins, in a personal communication, has suggested a method of predicting the rate of response of one ingenious measure to orthodontic therapy. Perhaps the success of the Ricketts\(^{23}\) method and others like it is due in large part to the clever clinician’s ability to make his treatment harmonize with his predicted goals. He sets his predictions and then works to make them come true.

**Use of time series in craniofacial prediction.** The time-series method, as briefly explained earlier in this article, was applied to a selected list of six craniofacial measures, chosen as representative of the types of measure which orthodontists have been trying to predict. In terms of time-series components, upper face height (A point–SN), maxillary length (PTM–A point) and anterior cranial base length (S–N) behave primarily as linear trends, while ramus height (Art–Go) and mandibular length (Art–Gn) are compound growth curves, composed of linear trends, and the maxillomandibular relationship (AB–FOP angle) is a
constant. Since these measures grow by different patterns of increment, it is necessary, according to time-series theory, to predict with varying mathematical models or combinations of models.

The findings are expressed as the mean percentage differences between predicted and actual values. Fig. 1 shows the average of all 1-year predictions between 6 and 16 years of age, all 2-year predictions possible, etc.; that is, an average of all the possible 1-year predictions including 6-7, 7-8, 8-9, etc. to 15-16, all the two-year predictions (e.g. 6-8, 7-9, etc. up to 14-16), and so on is shown through the 5-year prediction interval. The sample includes ten children from 6-16 years of age; therefore, there is a total of 420 predictions for each measure. These predictions, of course, include the period of the adolescent spurt. However, since all intervals have been averaged, one cannot conclude from this analysis that prediction during the spurt is as good as at other periods; our method of portrayal may have masked out poorer predictability at that time. On the other hand, we cannot yet conclude that our accuracy of prediction is any worse. We simply have not yet determined the accuracy of the method at different developmental ages.

We are most encouraged that both mandibular length and A/B relationship predictions should be so accurate. We think that an error of 2 to 3 per cent difference between predicted and actual values for a 1-year prediction is quite good since, as Johnston points out, the error of the cephalometric method itself is a significant limiting factor.

A direct comparison of time series with regression predictions of three linear measures and one measure of the A-B relationship is shown in Table I, which is
taken from work to be published in full elsewhere. For statistical reasons, the differences are not reported in terms of per cent of actual values but as the numerical value of the difference itself. Therefore, the table, rather than relating the magnitude of a difference to the magnitude of the measure (Fig. 1), compares the differences between actual values and the predictions obtained by the two methods. The magnitude-of-differences tests (Student’s t test) reveal that the time-series predictions of both the linear and the relationship measures come significantly closer to the actual value than do regression predictions. The frequency-of-greater-differences tests (chi-square) reveal that the time-series predictions of linear measures come closer to the actual values significantly more often than do regression predictions.

In summary, we believe that the method shows promise, especially in view of the fact that in time-series analyses there is the built-in possibility of improvement by altering the model, adding new factors, etc.

**Why do we not do better?**

There are many reasons why we do not do a better job of predicting craniofacial growth. Let us examine a few of the possibilities.

*Adequate and proper mathematical models of facial growth have not yet been presented.* Prediction in any field is based on some model; the better the model, the better the prediction. Our efforts in craniofacial growth thus far have been crude and simplistic when compared to the sophisticated mathematics used in many other fields of science and industry.

*We have not always measured the right thing.* Enlow and associates* pointed out the lack of biologic meaning for many traditional cephalometric measures. Pattern, as we know it clinically, is represented mathematically as a number of constants with minimal random variations through time. Such constants lend themselves well to time-series analyses, as we showed in the discussion of A-B relations.
However, we tried five different ways of relating A and B points mathematically before we found one useful for time-series methods. Only one of the five was useful for predicting, and it is not the commonly used method of relating A and B points clinically. It is not enough to say “A-B relations.” One must ask how A-B was measured.

We have not tried all the many predictive procedures that have been developed in other fields. In recent years, the advent of space technology and operations research in industry has brought about a revolution in predictive techniques in such wide-ranging fields as stock market analysis and rocketry. We have continued to rely on clinical hunches when modern computer science obviously might be more practical.

What can we expect in the future?

A series of theses by graduate students at the University of Michigan on the heritability of craniofacial growth and a recent study by Hunter, Balbach, and Lamphiear on the heritability of attained growth in the face have rekindled interest in quantified craniofacial genetics. As such studies occur in the future, we may expect to be able to “plug” into the predictive formula a genetic factor which may increase the accuracy of prediction.

Secular trends in the onset of pubescence and menarche are well known and the period of childhood seems to be shortening. A recent finding by Hunter and Garn of secular trends in craniofacial size and shape is of interest. It seems logical to search now for secular trends in the timing of growth events and dental development. The more frequent earlier appearance of the adolescent spurt during a major period of dental development will make prediction and treatment a different game entirely.

The interesting articles by Ricketts have brought about much more effort by clinicians to quantify the effects of their treatment on facial growth. Although one may fault Ricketts for the lack of a sound theoretical base for his “predictive” methods, he has dramatized the need for quantifying treatment goals.

Research workers may develop mathematical models, devise predictive procedures, and test them statistically, but the practicing orthodontist treating one child at a time will prove the ultimate worth of any suggested methods. The clinician, of course, is interested in how he can learn to apply the time-series method. Work is under way at the Center for Human Growth and Development to develop routine clinical procedures adapting time-series methods to the needs of orthodontic practice.

Summary and conclusions

We have attempted to review progress in the prediction of craniofacial growth. We conclude that:

1. There is much interest in the subject, as shown by the number of articles in the literature.
2. The methods employed by orthodontists, while improving greatly, are not the most sophisticated available.
3. The time series method holds much promise for the prediction of many
craniofacial parameters. It offers the advantages of (a) being based on measures of one individual, (b) being a large and sophisticated theory developed in several other fields of science and industry, (c) having the versatility of choosing different time-series models according to varying growth behavior of different parameters, and (d) the possibility of modifying the model itself to achieve better predictability.

4. We can expect reasonably useful predictive procedures to evolve for most size measures of biologic meaning. Little is yet known about predicting future velocity of growth and the onset of particular growth events, while nothing is understood about predicting changes in growth vector.

REFERENCES

The muscles which tighten the mouth, thus lessening its length, are in the lips themselves; or rather these lips are the actual muscles which close themselves. It is true that the muscle alters the position of the lip below the other muscles which are joined to it, of which one pair are those that distend it and move it to laughter; and that which contracts it is the same muscle of which the lower lip is formed, which restrains it by drawing in its extremities towards its center; and the same process goes on at the same time with the upper lip; and there are other muscles which bring the lips to a point and others that flatten them, others are those which cause them to curl back, others that straighten them, others which twist them all awry, and others that bring them back to their first position; and so always there are found as many muscles as correspond to the various attitudes of these lips and as many others as serve to reverse these attitudes; and these it is my purpose here to describe and represent in full, proving these movements by means of my mathematical principles. (Leonardo da Vinci, 1452-1519, Notebooks on Anatomy.)