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# LETTERS TO THE EDITOR

Comment on: **A theoretical analysis of the sandwich rolling process** by A. A. AFONJA and D. H. SANSOME, *Int. J. mech. Sci.* **15**, 1 (1973)

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I WOULD like to bring to the attention of Drs. Afonja and Sansome the paper entitled "The Deformation of Sandwich Materials"[1] that was published in Vol. 12 of this journal in 1970, in which rolling was discussed along with other forming processes. The mathematical theory for sandwich rolling presented by Afonja and Sansome is *identical* with the analysis in ref. 1, except that they neglect work-hardening in the differentiation of their equation (10d).

The apparently new contribution of Afonja and Sansome is to show that if the differential equation representing the distribution of roll pressure along the arc of contact (their equation (16)) is numerically integrated, the resulting values of rolling load and torque are closer to experimental values than those obtained by using the "equivalent yield stress" in Bland and Ford's theory.[2] It is stated in the Discussion that "... the values obtained by the equivalent mean yield stress method are less accurate in most cases than the values obtained from this (i.e. Afonja and Sansome's) analysis ...", the implication being that there is something erroneous in the use of an equivalent yield stress. However, equation (16) formally contains the equivalent stress, and after transposition is identical with the following equation taken from ref. (1), which in Afonja and Sansome's notation is

$$\mathrm{d}q = \frac{\mathrm{d}h}{h}\left(K_e \pm \frac{\mu q}{\theta_c}\right).$$

If the Bland and Ford approximation concerning the constancy of the product $[K_e . h]$ is expressed in the form $q \approx K_e$, we obtain[1]

$$\frac{\mathrm{d}q}{q} = \frac{\mathrm{d}h}{h} \pm \frac{\mu R' \,\mathrm{d}\theta_c}{H_0 + \frac{1}{2}R'\theta_c^2}.$$

This integrates in closed form to the conventional Bland and Ford expression with $K_e$ representing the overall flow behaviour of the sandwich.

Consequently any discrepancy between the closed form solution and numerical integration arises from the artefacts used to put the differential equation in a form suitable for integration and *not* from any inherent error in using $K_e$. The work of Jortner *et al.*[3] reported in this journal in 1960 is relevant in this connection. They numerically integrated Coulomb slipping friction rolling problems and compared the results with the data of Hessenberg and Sims. Although they did not quote equivalent results from a Bland and Ford analysis, they imply that the numerical integration results were a little closer to experiment.

Finally, it should be noted that the maximum discrepancy in roll force between Bland and Ford's theory, and Afonja and Sansome's experiments is no more than 14 per cent (Table 1). Consequently the statement in the Discussion about "... the often acceptable 15–20 per cent (errors) derived from other theories employing the equivalent mean yield stress ..." is somewhat gratuitous. There is nothing wrong with the use of an "equivalent flow stress", a concept established in general terms in 1942 by Pomp and Lueg[4] and by others later. Bland and Ford calculations for Afonja and Sansome's sandwich experiments are not that bad. The differences between the two sets of answers are due to the method of integration in any case.

*Department of Mechanical Engineering,*
*University of Michigan,*
2046 *East Engineering Building,*
*Ann Arbor, Michigan* 48104, *U.S.A.*

A. G. ATKINS