A USER'S GUIDE TO FORTRAN PROGRAMS FOR WIGNER AND RACAH COEFFICIENTS OF SU3

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PROGRAM SUMMARY

Title of program: SU3 WIGNER & RACAH COEFFICIENTS
Catalogue number: ACRM
Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue).
Computer: IBM 360/67; Installation: The University of Michigan, Ann Arbor, Michigan, USA
Operating system: MTS/360
Programming language used: FORTRAN IV
High speed storage required: SU3 ⊃ SU2 × U1 Wigner coefficients, 13008 words
SU3 Racah coefficients, 14654 words;
SU3 ⊃ R3 Wigner coefficients, 14202 words
SU3 Racah coefficients, 14654 words. SU3 ⊃ R3 Wigner coefficients, 14202 words
No. of bits in a word: 32
Is the program overlaid? No.
No. of magnetic tapes required: None.
Other peripherals used: Card reader, line printer
No. of cards in combined program and test deck: 2046
Card punching code: EBCDIC.
Keywords: SU3, Wigner coefficient, Racah coefficient, Clebsch—Gordan coefficient, Recoupling coefficient, Isoscalar factor, U-function, Unitary coupling, Unitary recoupling, K-band projection, Hypercharge.

Nature of physical problem
SU3 ⊃ SU2 × U1 and SU3 ⊃ R3 Wigner coefficients as well as SU3 Racah coefficients are calculated for arbitrary couplings and multiplicity.

Method of solution
A build-up process based on the Biedenharn—Louck prescription for specifying the outer multiplicity is employed to generate SU3 ⊃ SU2 × U1 Wigner coefficients [1]. SU3 Racah coefficients follow through standard recoupling formulae [2]. SU3 ⊃ R3 Wigner coefficients are obtained from the corresponding SU3 ⊃ SU2 × U1 Wigner coefficients via unitary transformation coefficients relating SU3 ⊃ SU2 × U1 and SU3 ⊃ R3 basis states [3].

Restrictions on the complexity of the problem
Factorials M!, M < Mmax = 32, and binomial coefficients \binom{N}{M}, M < N < Nmax = 32, are stored in common. Typically for SU3 ⊃ SU2 × U1 Wigner coefficients \Lambda_1 + \Lambda_2 + \Lambda_3 < Mmax whereas for SU3 ⊃ R3 Wigner coefficients \lambda + \mu + L < Nmax. The limits Mmax and Nmax may be altered by modifying one and only one subprogram.

Typical running time
Running time is a critical function of the complexity of the couplings involved. SU3 ⊃ R3 Wigner coefficients are evaluated as a weighted sum over SU3 ⊃ SU2 × U1 Wigner coefficients and hence are the most costly to calculate.

Unusual features of the program
Four of the fifteen subprograms contain internally dimensioned arrays. To conserve high speed storage the sizes of these

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arrays should be set at a large enough value to accommodate but not over-accommodate the needs of a particular user. A prescription for fixing the array size parameters in terms of representation labels is spelled out within each subprogram with specific examples given as a part of the Long Write-Up.

References

1. Introduction

The FORTRAN IV deck with which this write-up is concerned consists of two main parts. The first part (labelled PART 1: SU3 PACKAGE) consists of codes which can be used to generate SU3 ⊃ SU2 × U1 and SU3 ⊃ R3 Wigner coefficients as well as SU3 Racah coefficients for arbitrary couplings and multiplicities. The second part (labelled PART 2: SAMPLE CONTROL ROUTINES) is comprised of three control routines (one for each type of coefficient) which illustrate usage of the SU3 package subprograms and provide a standard input—output format for the casual user. The purpose of this guide is to (1) define the function served by each of the fifteen subprograms in the SU3 package, (2) illustrate simple alterations that can be made in adapting the programs to a specific need, (3) organize the subprograms into sets for efficient calculation of either SU3 ⊃ SU2 × U1 Wigner coefficients, SU3 Racah coefficients or SU3 ⊃ R3 Wigner coefficients, and (4) tabulate for all three coefficient types output from the sample control routines which illustrates both special and general symmetry properties as well as program checks versatility. A detailed description of the algebraic foundations upon which the codes are based is available in ref. [1].

2. Subprogram description

The fifteen subprograms which comprise Part 1 of the FORTRAN IV deck (carrying identification labels A, B, ..., O, respectively) have been written in a notation which, insofar as practical, parallels that used in ref. [1]. Common equivalences (less subscripts) are as follows:

- \( \lambda \sim \text{LAM, LM, L} \)
- \( \mu \sim \text{MU, M} \)
- \( \rho \sim \text{KRO, KO, K} \)
- \( \kappa \sim \text{KAP, KA, K} \)
- \( \varepsilon \sim \text{IE} \)
- \( 2\Lambda \sim \text{JT} \)
- \( 2M_\Lambda \sim \text{MT} \)
- \( K \sim K \)
- \( L \sim L \)
- \( M \sim M \)

The abbreviated forms (\( \sim \), \( \sim \), etc.) have only been used when a proper interpretation can be readily inferred from context. Caution: \( IP(IQ) \) has been used interchangeably for \( p(q) \) and \( p - q \) (\( \sim = \lambda - p \)). A running index \( \kappa \) has been introduced to distinguish \( \chi \)-bands.

\[ \kappa = (\chi - \chi_{\text{min}})/2 + 1. \]

The statement function \( \text{KSTART}(LAM, MU, L) \) \( \{ \text{KSTART}(MU, LAM, L) \} \) defines \( \chi_{\text{min}} \), the minimum \( \chi \)-value for each \( L \) as given by eq. (4a) [eq. (4b)] of ref. [1]. Other statement functions include MULT (LAM, MU, L) for determining the number of occurrences of a given \( L \) in the \( (\lambda \mu) \) irreducible representation of SU3 (zero if \( L \) does not occur), IDM (LAM, MU) which gives the dimension of the \( (\lambda \mu) \) irreducible representation of SU3, ND(M, N) for a linear reference to the binomial coefficients \( \binom{M}{N} \), \( N \leq M \leq M_{\text{max}} \), and finally INDEX (J1TD, LAM1, J1T, J2TD, LAM2, J2T) for a linear reference to the coefficients

\[ \langle (\lambda_1 \mu_1) e_1^1 \Lambda_1, (\lambda_2 \mu_2) e_2^2 \Lambda_2 || (\lambda_3 \mu_3)^E \rangle_\rho \]

with

\[ JTD = |e_1 - e_2|/3. \]

The name of each subprogram, the function it performs, and a reference to where the equivalent algebraic result can be found is given in table 1. The definition of special parameters used in the calling sequence is given by means of comment statements within each subprogram. In particular for

- CEWU3 (fixed \( \lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 \)):
  \[ \langle (\lambda_1 \mu_1) e_1^1 \Lambda_1, (\lambda_2 \mu_2) e_2^2 \Lambda_2 || (\lambda_3 \mu_3)^E \rangle_\rho = \text{DEWU3}(KRO, \text{IND}); \]

  \[ \rho = \text{KRO}, \quad 1 \leq \text{KRO} \leq \text{KROMAX}, \]

  \[ e_1 = e_3 \quad e_2 = 1^E \quad 1 \leq \text{IND} \leq \text{INDMAX}; \]

  \[ 2\Lambda_1 = \text{JITA} \quad 2\Lambda_2 = \text{J2TA} \quad \text{IND} \]

- CWU3 (fixed \( \lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 e_3 \Lambda_3 \)):
  \[ \langle (\lambda_1 \mu_1) e_1^1 \Lambda_1, (\lambda_2 \mu_2) e_2^2 \Lambda_2 || (\lambda_3 \mu_3 e_3 \Lambda_3) \rangle_\rho = \text{DWU3}(KRO, \text{IND}); \]

  \[ \rho = \text{KRO}, \quad 1 \leq \text{KRO} \leq \text{KROMAX}, \]

  \[ e_1 = e_3, \quad e_2 = e_2. \]
Y. Akiyama, J.P. Draayer, Wigner and Racah coefficients of SU₃

Table 1
Subprograms in the SU₃ package.

<table>
<thead>
<tr>
<th>Name</th>
<th>Function</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>BLOCKS Generate common blocks (binomial coefficients and factorials)</td>
<td>[2]</td>
</tr>
<tr>
<td>B.</td>
<td>CEWU₃ Calculate extremal Wigner coefficients for SU₃ ⊗ SU₂ × U₁ coupling</td>
<td>[1], eq(17), eq. (20)</td>
</tr>
<tr>
<td>C.</td>
<td>CEWU₃S CEWU₃ Support routine</td>
<td>[1], eq. (18), eq. (21)</td>
</tr>
<tr>
<td>D.</td>
<td>MULTU₃ Calculate multiplicity for the SU₃ Coupling (λ₁μ₁) × (λ₂μ₂) → (λ₃μ₃)</td>
<td>[3]</td>
</tr>
<tr>
<td>E.</td>
<td>MULTHY Calculate multiplicity (theory) for the SU₃ coupling (λ₁μ₁) × (λ₂μ₂) → (λ₃μ₃)</td>
<td>[3]</td>
</tr>
<tr>
<td>F.</td>
<td>CWU₃ Calculate Wigner coefficients for SU₃ ⊗ SU₂ × U₁ coupling</td>
<td>[1] eq. (19)</td>
</tr>
<tr>
<td>G.</td>
<td>CRU₃ Calculate Racah coefficients for SU₃</td>
<td>[1] eq. (22)</td>
</tr>
<tr>
<td>H.</td>
<td>DBSR Double back substitution routine for solving simultaneous equations</td>
<td>[4]</td>
</tr>
<tr>
<td>I.</td>
<td>DLUT Decompose a real matrix into the product of a lower and an upper triangular matrix</td>
<td>[4]</td>
</tr>
<tr>
<td>J.</td>
<td>DELTA Calculate Delta for R₃ routines [Δ(j/2j3)]</td>
<td>[5]</td>
</tr>
<tr>
<td>K.</td>
<td>DRR₃ Calculate Racah coefficients for R₃ [W(j1/2j2l1;j3j3)]</td>
<td>[5]</td>
</tr>
<tr>
<td>L.</td>
<td>CWU₃R₃ Calculate Wigner coefficients for SU₃ ⊗ R₃ Coupling</td>
<td>[1] eq. (31)</td>
</tr>
<tr>
<td>M.</td>
<td>DTU₃R₃ Calculate transformation coefficients for SU₃ ⊗ R₃ Reduction (G₃G₃KLM)</td>
<td>[1], eq. (26)</td>
</tr>
<tr>
<td>N.</td>
<td>CONMAT Calculate orthogonalization matrix for SU₃ ⊗ R₃ reduction</td>
<td>[1], eq. (6)</td>
</tr>
<tr>
<td>O.</td>
<td>DWR₃ Calculate Wigner coefficients for R₃ ((j1m1;j2m2;j3m3))</td>
<td>[6]</td>
</tr>
</tbody>
</table>

\[ e₂ = IE2MAX-3(IESMAX-IES) \]

\[ 2\Lambda₁ = J1TMAX(IES, J2S)-2(J1S-1) \]

\[ 2\Lambda₂ = J2TMAX(IES)-2(J2S-1) \]

\[ \text{IND} = [\text{INDMAT}(IES, J2S)-J1T]/2, \quad 1 \leq \text{IND} \leq \text{INDMAX}; \]

\[ \text{CRU}₃ = \text{DRU}₃(KA, KB, KC, KD); \]

\[ \rho\_{A} = KA, \quad 1 \leq KA \leq KRO\_A, \]

\[ \rho\_{B} = KB, \quad 1 \leq KB \leq KRO\_B, \]

\[ \rho\_{C} = KC, \quad 1 \leq KC \leq KRO\_C, \]

\[ \rho\_{D} = KD, \quad 1 \leq KD \leq KRO\_D; \]

\[ \text{CWU}₃R₃ = \text{DWU}₃R₃(KRO, KA₁, KA₂, KA₃); \]

\[ \rho = KRO, \quad 1 \leq KRO \leq KRO\_MAX, \]

\[ \chi₁ = \chi\_1\text{min} + 2(KA₁-1), \quad 1 \leq KA₁ \leq KA\_1\text{MAX}, \]

\[ \chi₂ = \chi\_2\text{min} + 2(KA₂-1), \quad 1 \leq KA₂ \leq KA\_2\text{MAX}, \]

\[ \chi₃ = \chi\_3\text{min} + 2(KA₃-1), \quad 1 \leq KA₃ \leq KA\_3\text{MAX}. \]

Parameters not explicitly defined (for example, NEC = μ₃ = (μ₁ + μ₂ - μ₃)/3 in CEWU₃, CEWU₃S, CWU₃) are arguments transferred between subprograms so as to reduce redundant numerical calculation.

3. Subprogram modification

Of the fifteen subprograms in the SU₃ package, routines (C–E, H–K, M–O) (see table 1) are completely general in the sense that they perform operations on arrays whose sizes are fixed externally to the routines themselves (accomplished by reducing all multiply subscripted variables to linear form, the proper indexing being done within each subprogram). Routines (B, F–G, L), on the other hand, contain internally dimensioned arrays the sizes of which must be set at a large enough value to accommodate the needs of a particular user. A prescription for fixing the sizes of these arrays is given by suitable comment statements within each subprogram. Table 2 gives the values of the relevant parameters (external and internal) for four cases of special interest in nuclear structure studies; namely, standard shell model calculations assuming full usage of SU₃ technology (e.g., see ref. [7]) and general two-body effective interactions (e.g., see ref. [8]) in each of the N = 1, 2, 3, 4 shells of the harmonic oscillator.
<table>
<thead>
<tr>
<th>Subprogram</th>
<th>Parameter</th>
<th>Oscillator Shell Number</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$N=1$</td>
</tr>
<tr>
<td>CEWU3</td>
<td>External – N1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>N2</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Internal – X1</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>5</td>
</tr>
<tr>
<td>CWU3*</td>
<td>External – N1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>N2</td>
<td>5(3)</td>
</tr>
<tr>
<td></td>
<td>N3</td>
<td>27(6)</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>35(10)</td>
</tr>
<tr>
<td></td>
<td>NB</td>
<td>5(3)</td>
</tr>
<tr>
<td></td>
<td>Internal – X1</td>
<td>81(18)</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>5(3)</td>
</tr>
<tr>
<td></td>
<td>X3</td>
<td>25(9)</td>
</tr>
<tr>
<td>CRU3</td>
<td>External – NA</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>NB</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>NC</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>ND</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Internal – X1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>X3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>X4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>X5</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X6</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X7</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X8</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>X9</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X11</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>X12</td>
<td>105</td>
</tr>
<tr>
<td></td>
<td>X13</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>X14</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>X15</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>X16</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>X17</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>X18</td>
<td>9</td>
</tr>
<tr>
<td>CWU3R3**</td>
<td>External – N0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>N1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>N2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>N3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Internal – X1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>X2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>X3</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>X4</td>
<td>4</td>
</tr>
</tbody>
</table>

* The values in () apply if CWU3 is used in conjunction with CRU3 only.
** Appropriate for representations of SU3 with $\lambda + 2\mu < N(N + 1)(N + 2)$. 
that within each shell smaller limits are possible if one is dealing with less than mid-shell nuclei.) The routines in the test deck correspond to the case $N = 2$. Adaptations to other usages can be made in an equally straightforward fashion.

Routine A, BLOCKS, together with the block data statements, generates the common blocks required by the other subprograms. These include the binomial coefficients $(\frac{M}{N}), N \leq M \leq M_{\text{max}} (= 32)$ and factorials $N!, N \leq N_{\text{max}} (= 32)$. The upper limit on $M_{\text{max}}$ and $N_{\text{max}}$ are machine dependent (e.g. $\left(\frac{32}{16}\right) = 601,080,390$ as opposed to $\left(\frac{32}{16}\right) = 1,166,803,110$ does not yield a fixed point (integer) overflow condition). They imply restrictions on the “arbitrariness” of the coupling to which the programs apply (e.g. $\lambda + \mu + L \leq M_{\text{max}}$ as can be seen from eq. (26) of ref. [1]). For all practical purposes, however, the limits $M_{\text{max}} = N_{\text{max}} = 32$ are sufficiently high so as to impose no serious limitations. In any case, should a need arise there exist machines (e.g., the IBM 360 series for which $N_{\text{max}} = 50$ is possible) and/or techniques (e.g., the use of logarithms for large number operations) which allow these limits to be extended. Note, however, that the limits are fixed in BLOCKS and extensions can be made by modifying this and only this subprogram.

A conversion to double precision arithmetic may be required for complex couplings involving large multiplicity $(\rho_{\text{max}} \geq 5)$ and/or large $\lambda + \mu$ values ($\lambda + \mu \geq 16$). This is particularly true for the SU3 ⊃ R3 Wigner coefficients because of the alternating nature of the sums required for an evaluation of the transformation coefficients from the SU3 ⊃ SU2 × U1 to the SU3 ⊃ R3 scheme. The conversion can be made by simply removing the “C” from column 1 of the statement “IMPLICIT REAL *8(D)" in each subprogram and replacing

\[
\text{SORT} \rightarrow \text{DSQRT}, \\
\text{FLOAT} \rightarrow \text{DFLOAT}, \\
\text{ABS} \rightarrow \text{DABS}, \\
\text{E-06} \rightarrow \text{.D-12}, \\
\text{EO} \rightarrow \text{.DO},
\]

throughout, It may also be necessary (depending upon the machine and/or compiler) to change the type declarations “IMPLICIT REAL *8(D)” to “DOUBLE PRECISION(D)” and “IMPLICIT INTEGER(X)” to “INTEGER(X)” throughout.

4. Subprogram usage

The three sample control routines which comprise Part 2 of the FORTRAN IV deck (carrying identification lables, X, Y, Z, respectively) can be used in conjunction with the fifteen subprograms of the SU3 package to generate output in the format as given by the illustrative tables X, Y, Z of Test Run Output. For this purpose it is convenient to group the subprograms of the SU3 package into the four sets given in table 3.

<table>
<thead>
<tr>
<th>Set 1</th>
<th>Set 2</th>
<th>Set 3</th>
<th>Set 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. BLOCKS</td>
<td>F. CWU3</td>
<td>G. CRU3</td>
<td>L. CWU3R3</td>
</tr>
<tr>
<td>B. CEWU3</td>
<td>H. DDBS</td>
<td>M. DTU3R3</td>
<td></td>
</tr>
<tr>
<td>C. CEWU3S</td>
<td>I. DLUT</td>
<td>N. CONMAT</td>
<td></td>
</tr>
<tr>
<td>D. MULTU3</td>
<td>J. DELTA</td>
<td>O. DWR3</td>
<td>J. (DELTA)</td>
</tr>
<tr>
<td>E. MULTHY</td>
<td>K. DRR3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that sets 3 and 4 both contain DELTA. A simple combination of 1, 2, 3, 4 together with X, Y, Z can then be used to calculate the desired coefficients:

\[
\text{SU3} \supset \text{SU2} \times \text{U}1 \text{ Wigner coefficients} \quad (X+1+2), \\
\text{SU3} \text{ Racah coefficients} \quad (Y+1+2+3), \\
\text{SU3} \supset \text{R}3 \text{ Wigner coefficients} \quad (Z+1+4).
\]

Since the size of a particular array is normally set at the limit first encountered in the compilation process, it is important that the main program (X, Y, Z) be compiled first and BLOCKS second with the remaining subprograms (1 + 2, 1 + 2 + 3, 1 + 4) in any convenient order. Once properly compiles, the following data cards can be used to obtain required Wigner and Racah coefficients ($N =$ number of cards):

\[
\text{SU3} \supset \text{SU2} \times \text{U}1 \text{ Wigner coefficients} \\
\text{Card(s) 1} - N \\
\lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 \epsilon_3 2\Lambda_3 \\
\text{...} \\
\text{ENDFILE (= negative integer)}
\]

\[
\text{SU3} \text{ Racah coefficients} \\
\text{Card(s) 1} - N \\
\lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 \lambda_1 2 \mu_1 2 \lambda_2 3 \mu_2 3 \\
\text{...} \\
\text{ENDFILE (= negative integer)}
\]
SU_3 \supset R_3 Wigner coefficients

Card 1
Format (1615)
L_1 \text{min} \quad L_1 \text{max} \quad L_3 \text{min} \quad L_3 \text{max}
Card(s) 2 - N

\lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 \lambda_1 \lambda_2 \lambda_3

\ldots

ENDFILE (= negative integer).

Note that for Elliott-like SU_3 \supset R_3 projection (IJ) = (01) if \mu \leq \lambda and (IJ) = (01) if \mu > \lambda.

The examples shown as part of the Test Run Output include numerical checks on all the symmetry properties of the Wigner coefficients. Also illustrated are a number of coefficients which vanish due to the Biedenharn–Louck prescription for specifying the \rho-multiplicity. The statement NON-EXIST denotes an SU_3 coupling violation while PAR-CHECK indicates that the coupling, although allowed, requires higher limits on the size parameters for internally dimensioned arrays. The statement SUB-LABEL indicates an incorrectly specified subgroup label (\epsilon_2, \Lambda_2, L_2, etc.).

Routines X, Y, Z by no means represent the most efficient use of the subprogram; they simply serve to illustrate the required call sequences and provide a standard input-output format for the casual user. More creative users should incorporate the routines into their own programs for more efficient evaluation of the required coefficients.

5. Conclusion

The programs have been written in a form which the authors found convenient for performing nuclear shell model calculations. It is our hope that other users may find the routines equally helpful in their studies of the usefulness of SU_3 in describing physical phenomena.

References

Y. Akiyama, J.P. Draayer, Wigner and Racah coefficients of SU₃

**TEST RUN OUTPUT**

### TABLE X: SU₃ ⊃ SU₂ × U₁ Wigner coefficients

<table>
<thead>
<tr>
<th>L</th>
<th>M₁</th>
<th>L</th>
<th>M₂</th>
<th>J₁</th>
<th>J₂</th>
<th>J₃</th>
<th>Wigner coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 3 11 3 2 0 -2 2 5 2 9 1</td>
<td>-0.4472130</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 2 1 1 9 1</td>
<td>0.6701892</td>
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**TABLE X:** SU₃ ⊃ SU₂ × U₁ Wigner coefficients
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<th>Y. Akiyama, J.P. Draayer, Wigner and Racah coefficients of SU_3</th>
<th>413</th>
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-3 3 6 4 3 1 -0.0919667 0.0984762 -0.1452513 0.0353649 0.0132181 |
-3 3 6 2 3 1 -0.3251516 0.0090789 0.1224855 -0.0406722 -0.0062435 |
-6 6 9 5 3 1 0.0400000 0.0445478 -0.0733276 -0.0568083 0.0244341 |
-6 4 9 5 3 1 0.1507020 -0.2264612 0.0242500 -0.0406722 0.0062435 |
-6 4 9 3 3 1 0.1911489 0.1552420 -0.0416569 -0.0167740 0.0091319 |
-9 5 12 4 3 1 0.4725329 -0.0439957 -0.0324188 0.0124972 -0.0009368 |

\( \langle LM_1|J_1;LM_2|J_2;LM_3|J_3 \rangle \text{ PCs} \quad R = 1, 2 \)

6 1 -5 7 2 1 -4 2 5 2 -9 5 0.2904958 0.8366600 |
-5 5 -4 2 -9 5 -0.3779646 0.0000000 |
-8 6 -1 3 -9 5 0.8169959 -0.4082484 |
-8 6 -1 1 -9 5 0.3651483 0.3651485 |

\( \langle LM_1|J_1;LM_2|J_2;LM_3|J_3 \rangle \text{ PCs} \quad R = 1, 2 \)

2 5 9 5 2 1 -1 3 1 6 8 6 0.0666661 0.3333331 |
9 5 -1 1 8 6 -0.2981412 0.2981421 |
6 6 2 2 8 6 -0.2981417 -0.4472139 |
6 6 2 2 8 6 0.3333327 0.0000001 |
6 6 2 0 8 6 0.3651480 -0.3651479 |
3 7 5 1 8 6 -0.1951799 0.6812984 |
3 5 5 1 8 6 -0.3086062 0.0000012 |

\( \langle LM_1|J_1;LM_2|J_2;LM_3|J_3 \rangle \text{ PCs} \quad R = 1, 2 \)

9 3 5 5 6 6 PAR-CHECK |

\( \langle LM_1|J_1;LM_2|J_2;LM_3|J_3 \rangle \text{ PCs} \quad R = 1, 2 \)

9 3 3 3 6 6 SUB-LABEL |

\( \langle LM_1|J_1;LM_2|J_2;LM_3|J_3 \rangle \text{ PCS} \quad R = 1, 2 \)

9 3 1 1 6 6 NON-EXIST |

| TABLE Y: SU_3 Racah coefficients |
|---|---|
| U (|L1,|M1);L2,P2;|LM,MU.|L3,M3) ; (|LA,|RA,|RB,|LC,|MC,|RC,|RP) |
| 6 4 2 0 5 5 2 0 6 5 1 1 4 0 1 1 0.4513354 |
| 6 4 2 0 5 5 2 0 6 5 1 1 0 2 1 1 0.530269 |
| 6 4 2 0 5 5 2 0 6 5 1 1 2 1 1 1 -0.0610819 |
| 6 4 2 0 5 5 2 0 5 4 1 1 4 0 1 1 0.7123057 |
| 6 4 2 0 5 5 2 0 5 4 1 1 2 1 1 1 0.6598879 |
| 6 4 2 0 5 5 2 0 5 4 1 1 0 2 1 1 0.2932209 |
| 6 4 2 0 5 5 2 0 5 4 1 1 2 1 1 1 0.0283791 |
| 6 4 2 0 5 5 2 0 7 3 1 1 4 0 1 1 0.4082477 |
| 6 4 2 0 5 5 2 0 7 3 1 1 0 2 1 1 -0.5520518 |
TABLE Z: SU3 ⊗ R3 Wigner coefficients

<LM1MU1> ξ, ξ1 | LM2MU2> ξ2, ξ21 | LM3MU3> ξ3, ξ31

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<th>K</th>
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Y. Akizama, J.P. Draayer, Wigner and Racah coefficients of SU3
\begin{tabular}{cccccccccccccccc}
2 & 5 & 10 & 1 & 2 & 2 & 0 & 01 & 1 & 2 & 3 & 4 & 10 & 1 & 2 & 0.3220300 \\
1 & 3 & 1 & 2 & 1 & 2 & -0.7629643 & 2 & 3 & 1 & 2 & -0.1594033 & 1 & 2 & 0.2849273 \\
1 & 2 & 1 & 2 & 3 & 2 & 0.6106462 & 1 & 3 & 1 & 2 & -0.3843608 & 2 & 3 & 1 & 2 & -0.2193345 \\
1 & 3 & 1 & 2 & 2 & 3 & -0.0000009 & 2 & 3 & 1 & 2 & -0.6541080 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
2 & 0 & 01 & 1 & 2 & 2 & 5 & 10 & 1 & 3 & 3 & 4 & 10 & 1 & 2 & -0.7629647 \\
1 & 2 & 2 & 3 & 1 & 2 & 0.1594033 & 1 & 2 & 3 & 1 & 3 & -0.3843611 \\
1 & 2 & 2 & 3 & 1 & 3 & 0.2193345 & 1 & 2 & 2 & 3 & 2 & 3 & -0.6541080 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RO = 1, \ldots, 5 \\
8 & 5 & 01 & 1 & 2 & 4 & 4 & 01 & 1 & 2 & 8 & 5 & 01 & 1 & 2 & 0.0373651 \\
1 & 2 & 2 & 2 & 1 & 2 & -0.0715400 & 1 & 2 & 1 & 2 & -0.0655601 \\
1 & 3 & 1 & 2 & -0.0878139 & 1 & 2 & 1 & 2 & -0.0458331 \\
2 & 3 & 1 & 2 & -0.0280274 & 1 & 2 & 1 & 2 & -0.0865656 \\
1 & 2 & 2 & 1 & 3 & 0.0665656 & 1 & 2 & 2 & 1 & 3 & 0.0665656 \\
1 & 2 & 1 & 2 & 2 & 3 & -0.0191812 & 1 & 2 & 2 & 1 & 3 & -0.0191812 \\
2 & 3 & 2 & 1 & 2 & -0.0780274 & 1 & 2 & 2 & 1 & 3 & -0.0780274 \\
2 & 3 & 2 & 1 & 3 & -0.0191812 & 1 & 2 & 2 & 1 & 3 & -0.0191812 \\
1 & 2 & 1 & 2 & 2 & 3 & -0.0191812 & 1 & 2 & 2 & 1 & 3 & -0.0191812 \\
1 & 3 & 1 & 2 & 2 & 3 & 0.0000009 & 1 & 2 & 2 & 1 & 3 & -0.0000009 \\
2 & 3 & 1 & 2 & 2 & 3 & -0.0000009 & 1 & 2 & 2 & 1 & 3 & -0.0000009 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RC = 1, \ldots, 2 \\
6 & 1 & 01 & 1 & 2 & 2 & 1 & 01 & 1 & 3 & 5 & 2 & 01 & 1 & 2 & -0.2285706 \\
1 & 2 & 1 & 3 & -0.1975533 & 1 & 2 & 2 & 3 & -0.4602582 & 1 & 2 & 3 & 2 & -0.0659261 \\
1 & 2 & 1 & 3 & -0.1975533 & 1 & 2 & 3 & 2 & -0.0659261 & 1 & 2 & 3 & 2 & -0.0659261 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RC = 1, \ldots, 2 \\
1 & 6 & 10 & 1 & 2 & 1 & 2 & 10 & 1 & 3 & 2 & 5 & 10 & 1 & 2 & -0.2285706 \\
1 & 3 & 1 & 2 & -0.0808313 & 1 & 3 & 1 & 2 & -0.0808313 \\
1 & 3 & 1 & 2 & -0.0808313 & 1 & 3 & 1 & 2 & -0.0808313 \\
1 & 3 & 1 & 2 & -0.0808313 & 1 & 3 & 1 & 2 & -0.0808313 \\
1 & 3 & 1 & 2 & -0.0808313 & 1 & 3 & 1 & 2 & -0.0808313 \\
1 & 3 & 1 & 2 & -0.0808313 & 1 & 3 & 1 & 2 & -0.0808313 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RO = 1, \ldots, 2 \\
2 & 5 & 10 & 1 & 2 & 2 & 1 & 01 & 1 & 3 & 1 & 6 & 10 & 1 & 2 & 0.2015804 \\
1 & 3 & 1 & 3 & -0.2015804 & 2 & 3 & 1 & 3 & -0.2015804 \\
2 & 3 & 1 & 3 & -0.2015804 & 2 & 3 & 1 & 3 & -0.2015804 \\
2 & 3 & 1 & 3 & -0.2015804 & 2 & 3 & 1 & 3 & -0.2015804 \\
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\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RO = 1, \ldots, 2 \\
2 & 1 & 01 & 1 & 2 & 2 & 5 & 10 & 1 & 3 & 1 & 6 & 10 & 1 & 2 & -0.2780242 \\
1 & 3 & 1 & 2 & -0.0715400 & 1 & 3 & 1 & 2 & -0.0715400 \\
1 & 3 & 1 & 2 & -0.0715400 & 1 & 3 & 1 & 2 & -0.0715400 \\
1 & 3 & 1 & 2 & -0.0715400 & 1 & 3 & 1 & 2 & -0.0715400 \\
1 & 3 & 1 & 2 & -0.0715400 & 1 & 3 & 1 & 2 & -0.0715400 \\
\end{tabular}

\begin{tabular}{cccccccccccccccc}
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RO = 1, \ldots, 4 \\
9 & 3 & 5 & 5 & 1 & 1 & 6 & 6 & PAR-CHECK \\
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RO = 1, \ldots, 2 \\
9 & 3 & 3 & 6 & 6 & SUB-LABEL \\
\langle LM1MU11J,K1,l11LM2MU21J,K2,l21LM3MU31J,K3,l3 \rangle & RC = 1, \ldots, 0 \\
9 & 3 & 1 & 1 & 6 & 6 & NCH-EXIST \\
\end{tabular}