

A USER'S GUIDE TO FORTRAN PROGRAMS FOR WIGNER AND RACAH COEFFICIENTS OF SU_3 *

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PROGRAM SUMMARY

Title of program: SU3 WIGNER & RACAH COEFFICIENTS

Catalogue number: ACRM

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue).

Computer: IBM 360/67; *Installation:* The University of Michigan, Ann Arbor, Michigan, USA

Operating system: MTS/360

Programming language used: FORTRAN IV

High speed storage required: $SU_3 \supset SU_2 \times U_1$ Wigner coefficients, 13 008 words

SU_3 Racah coefficients, 14 654 words;

$SU_3 \supset R_3$ Wigner coefficients, 14 202 words

SU_3 Racah coefficients, 14 654 words. $SU_3 \supset R_3$ Wigner coefficients, 14 202 words

No. of bits in a word: 32

Is the program overlaid? No.

No. of magnetic tapes required: None.

Other peripherals used: Card reader, line printer

No. of cards in combined program and test deck: 2046

Card punching code: EBCDIC.

Keywords: SU_3 , Wigner coefficient, Racah coefficient, Clebsch-Gordan coefficient, Recoupling coefficient, Isoscalar factor, U-function, Unitary coupling, Unitary recoupling, K-band projection, Hypercharge.

Nature of physical problem

$SU_3 \supset SU_2 \times U_1$ and $SU_3 \supset R_3$ Wigner coefficients as well as SU_3 Racah coefficients are calculated for arbitrary couplings and multiplicity.

Method of solution

A build-up process based on the Biedenharn-Louck prescription for specifying the outer multiplicity is employed to generate $SU_3 \supset SU_2 \times U_1$ Wigner coefficients [1]. SU_3 Racah coefficients follow through standard recoupling formulae [2]. $SU_3 \supset R_3$ Wigner coefficients are obtained from the corresponding $SU_3 \supset SU_2 \times U_1$ Wigner coefficients via unitary transformation coefficients relating $SU_3 \supset SU_2 \times U_1$ and $SU_3 \supset R_3$ basis states [3].

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Restrictions on the complexity of the problem

Factorials $M!$, $M \leq M_{\max} = 32$, and binomial coefficients $\binom{N}{M}$, $M \leq N \leq N_{\max} = 32$, are stored in common. Typically for $SU_3 \supset SU_2 \times U_1$ Wigner coefficients $\Lambda_1 + \Lambda_2 + \Lambda_3 \leq M_{\max}$ whereas for $SU_3 \supset R_3$ Wigner coefficients $\lambda + \mu + L \leq N_{\max}$. The limits M_{\max} and N_{\max} may be altered by modifying one and only one subprogram.

Typical running time

Running time is a critical function of the complexity of the couplings involved. $SU_3 \supset R_3$ Wigner coefficients are evaluated as a weighted sum over $SU_3 \supset SU_2 \times U_1$ Wigner coefficients and hence are the most costly to calculate.

Unusual features of the program

Four of the fifteen subprograms contain internally dimensioned arrays. To conserve high speed storage the sizes of these

arrays should be set at a large enough value to accommodate but not over-accommodate the needs of a particular user. A prescription for fixing the array size parameters in terms of representation labels is spelled out within each subprogram with specific examples given as a part of the Long Write-Up.

References

- [1] J.P. Draayer and Yoshimi Akiyama *J. Math. Phys.*, in press.
- [2] K.T. Hecht, *Nuclear Physics* 62 (1965) 1.
- [3] J.P. Draayer, *Nuclear Physics A129* (1969) 647.

LONG WRITE-UP

1. Introduction

The FORTRAN IV deck with which this write-up is concerned consists of two main parts. The first part (labelled PART 1: SU₃ PACKAGE) consists of codes which can be used to generate SU₃ ⊃ SU₂ × U₁ and SU₃ ⊃ R₃ Wigner coefficients as well as SU₃ Racah coefficients for *arbitrary* couplings and multiplicities. The second part (labelled PART 2: SAMPLE CONTROL ROUTINES) is comprised of three control routines (one for each type of coefficient) which illustrate usage of the SU₃ package subprograms and provide a standard input-output format for the casual user. The purpose of this guide is to (1) define the function served by each of the fifteen subprograms in the SU₃ package, (2) illustrate simple alterations that can be made in adapting the programs to a specific need, (3) organize the subprograms into sets for efficient calculation of either SU₃ ⊃ SU₂ × U₁ Wigner coefficients, SU₃ Racah coefficients or SU₃ ⊃ R₃ Wigner coefficients, and (4) tabulate for all three coefficient types output from the sample control routines which illustrates both special and general symmetry properties as well as program checks versatility. A detailed description of the algebraic foundations upon which the codes are based is available in ref. [1].

2. Subprogram description

The fifteen subprograms which comprise Part 1 of the FORTRAN IV deck (carrying identification labels A, B, ..., O, respectively) have been written in a notation which, insofar as practical, parallels that used in ref. [1]. Common equivalences (less subscripts) are as follows:

λ ~ LAM, LM, L
 μ ~ MU, M
 ρ ~ KRO, KO, K
 κ ~ KAP, KA, K
 ε ~ IE
 2Λ ~ JT
 2M_Λ ~ MT
 K ~ K
 L ~ L
 M ~ M

The abbreviated forms (λ ~ L, μ ~ M, etc.) have only been used when a proper interpretation can be readily inferred from context. Caution: IP(IQ) has been used interchangeably for p(q) and $\tilde{p} = \mu - q$ ($\tilde{q} = \lambda - p$). A running index κ has been introduced to distinguish \mathcal{X} -bands.

$$\kappa = (\mathcal{X} - \mathcal{X}_{\min})/2 + 1.$$

The statement function KSTART (LAM, MU, L) [KSTART (MU, LAM, L)] defines \mathcal{X}_{\min} , the minimum \mathcal{X} -value for each L as given by eq. (4a) [eq. (4b)] of ref. [1]. Other statement functions include MULT (LAM, MU, L) for determining the number of occurrences of a given L in the (λμ) irreducible representation of SU₃ (zero if L does not occur), IDM (LAM, MU) which gives the dimension of the (λμ) irreducible representation of SU₃, ND(M, N) for a linear reference to the binomial coefficients $\binom{M}{N}$, $N \leq M \leq M_{\max}$, and finally INDEX (J1TD, LAM1, J1T, J2TD, LAM2, J2T) for a linear reference to the coefficients

$$\langle (\lambda_1 \mu_1) \epsilon_1 \Lambda_1, (\lambda_2 \mu_2) \epsilon_2 \Lambda_2 \| (\lambda_3 \mu_3) E \rangle_\rho$$

with

$$JTD = |\epsilon - \epsilon_E|/3.$$

The name of each subprogram, the function it performs, and a reference to where the equivalent algebraic result can be found is given in table 1.

The definition of special parameters used in the calling sequence is given by means of comment statements within each subprogram. In particular for

$$\begin{aligned} & \text{CEWU3 (fixed } \lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3): \\ & \langle (\lambda_1 \mu_1) \epsilon_1 \Lambda_1; (\lambda_2 \mu_2) \epsilon_2 \Lambda_2 \| (\lambda_3 \mu_3) E \rangle_\rho \\ & = \text{DEWU3 (KRO, IND),} \\ & \left. \begin{aligned} \rho & = \text{KRO,} & 1 \leq \text{KRO} \leq \text{KROMAX,} \\ \epsilon_1 & = \epsilon_{3E} - \epsilon_2, \\ \epsilon_2 & = \text{IEA (IND)} \\ 2\Lambda_1 & = \text{J1TA (IND)} \\ 2\Lambda_2 & = \text{J2TA (IND)} \end{aligned} \right\} 1 \leq \text{IND} \leq \text{INDMAX;} \end{aligned}$$

$$\begin{aligned} & \text{CWU3 (fixed } \lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 \epsilon_3 \Lambda_3): \\ & \langle (\lambda_1 \mu_1) \epsilon_1 \Lambda_1; (\lambda_2 \mu_2) \epsilon_2 \Lambda_2 \| (\lambda_3 \mu_3) \epsilon_3 \Lambda_3 \rangle_\rho \\ & = \text{DWU3 (KRO, IND);} \\ & \left. \begin{aligned} \rho & = \text{KRO,} & 1 \leq \text{KRO} \leq \text{KROMAX,} \\ \epsilon_1 & = \epsilon_3 - \epsilon_2, \end{aligned} \right\} \end{aligned}$$

Table 1
Subprograms in the SU_3 package.

Name	Function	Ref.
A. BLOCKS	Generate common <i>blocks</i> (binomial coefficients and factorials)	[2]
B. CEWU ₃	Calculate extremal Wigner coefficients for $SU_3 \supset SU_2 \times U_1$ coupling	[1], eq.(17), eq. (20)
C. CEWU ₃ S	CEWU ₃ Support routine	[1], eq. (18), eq. (21)
D. MULTU ₃	Calculate multiplicity for the SU_3 Coupling $(\lambda_1\mu_1) \times (\lambda_2\mu_2) \rightarrow (\lambda_3\mu_3)$	[3]
E. MULTHY	Calculate multiplicity (theory) for the SU_3 coupling $(\lambda_1\mu_1) \times (\lambda_2\mu_2) \rightarrow (\lambda_3\mu_3)$	[3]
F. CWU ₃	Calculate Wigner coefficients for $SU_3 \supset SU_2 \times U_1$ coupling	[1] eq. (19)
G. CRU ₃	Calculate Racah coefficients for SU_3	[1] eq. (22)
H. DBSR	Double back substitution routine for solving simultaneous equations	[4]
I. DLUT	Decompose a real matrix into the product of a lower and an upper triangular matrix	[4]
J. DELTA	Calculate <i>Delta</i> for R_3 routines $[\Delta(j_1j_2j_3)]$	[5]
K. DRR ₃	Calculate Racah coefficients for R_3 $[W(j_1j_2l_1; j_3l_3)]$	[5]
L. CWU ₃ R ₃	Calculate Wigner coefficients for $SU_3 \supset R_3$ Coupling	[1] eq. (31)
M. DTU ₃ R ₃	Calculate transformation coefficients for $SU_3 \supset R_3$ Reduction $\langle\langle G G_E\rangle\rangle KLM$	[1], eq. (26)
N. CONMAT	Calculate orthonormalization matrix for $SU_3 \supset R_3$ reduction	[1], eq. (6)
O. DWR ₃	Calculate Wigner coefficients for R_3 $\langle\langle j_1m_1; j_2m_2 j_3m_3 \rangle\rangle$	[6]

$$\begin{aligned} \epsilon_2 &= \text{IE2MAX}-3(\text{IESMAX}-\text{IES}) \\ 2\Lambda_1 &= \text{J1TMAX}(\text{IES}, \text{J2S})-2(\text{J1S}-1) \\ 2\Lambda_2 &= \text{J2TMAX}(\text{IES})-2(\text{J2S}-1) \\ \text{IND} &= [\text{INDMAT}(\text{IES}, \text{J2S})-\text{J1T}]/2, \end{aligned} \left. \begin{array}{l} 1 \leq \text{IES} \leq \\ \text{IESMAX}, \\ 1 \leq \text{J2S} \leq \\ \text{J2SMAX}(\text{IES}), \\ 1 \leq \text{J1S} \leq \\ \text{J1SMAX}(\text{IES}, \\ \text{J2S}). \end{array} \right\}$$

Parameters not explicitly defined (for example, $\text{NEC} = \nu_3 = \{(\lambda_1 + \lambda_2 - \lambda_3) + 2(\mu_1 + \mu_2 - \mu_3)\}/3$ in CEWU3 , CEWU3S , CWU3) are arguments transferred between subprograms so as to reduce redundant numerical calculation.

3. Subprogram modification

CRU3 (fixed $\lambda_1\mu_1, \lambda_2\mu_2, \lambda_3\mu_3, \lambda_{12}\mu_{12}, \lambda_{23}\mu_{23}$):
 $\bar{U}((\lambda_1\mu_1), (\lambda_2\mu_2), (\lambda\mu), (\lambda_3\mu_3);$
 $(\lambda_{12}\mu_{12})\rho_A\rho_B(\lambda_{34}\mu_{34})\rho_C\rho_D)$
 $= \text{DRU3}(\text{KA}, \text{KB}, \text{KC}, \text{KD})$
 $\rho_A = \text{KA}, \quad 1 \leq \text{KA} \leq \text{KROA},$
 $\rho_B = \text{KB}, \quad 1 \leq \text{KB} \leq \text{KROB},$
 $\rho_C = \text{KC}, \quad 1 \leq \text{KC} \leq \text{KROC},$
 $\rho_D = \text{KD}, \quad 1 \leq \text{KD} \leq \text{KROD};$

CWU3R3 (fixed $\lambda_1\mu_1, \lambda_2\mu_2, \lambda_3\mu_3, L_1, L_2, L_3$):
 $\langle\langle (\lambda_1\mu_1)\chi_1 L_1; (\lambda_2\mu_2)\chi_2 L_2 \parallel (\lambda_3\mu_3)\chi_3 L_3 \rangle\rangle_\rho$
 $= \text{DWU3R3}(\text{KRO}, \text{KA1}, \text{KA2}, \text{KA3}),$
 $\rho = \text{KRO}, \quad 1 \leq \text{KRO} \leq \text{KROMAX},$
 $\chi_1 = \chi_{1\text{min}} + 2(\text{KA1}-1), \quad 1 \leq \text{KA1} \leq \text{KA1MAX},$
 $\chi_2 = \chi_{2\text{min}} + 2(\text{KA2}-1), \quad 1 \leq \text{KA2} \leq \text{KA2MAX},$
 $\chi_3 = \chi_{3\text{min}} + 2(\text{KA3}-1), \quad 1 \leq \text{KA3} \leq \text{KA3MAX}.$

Of the fifteen subprograms in the SU_3 package, routines (C–E, H–K, M–O) (see table 1) are completely general in the sense that they perform operations on arrays whose sizes are fixed externally to the routines themselves (accomplished by reducing all multiply subscripted variables to linear form, the proper indexing being done within each subprogram). Routines (B, F–G, L), on the other hand, contain internally dimensioned arrays the sizes of which must be set at a large enough value to accommodate the needs of a particular user. A prescription for fixing the sizes of these arrays is given by suitable comment statements within each subprogram. Table 2 gives the values of the relevant parameters (external and internal) for four cases of special interest in nuclear structure studies; namely, standard shell model calculations assuming full usage of SU_3 technology (e.g., see ref. [7]) and general two-body effective interactions (e.g., see ref. [8]) in each of the $N = 1, 2, 3, 4$ shells of the harmonic oscillator. (Note

Table 2
Array size parameters.

Subprogram	Parameter	Oscillator Shell Number			
		N=1	N=2	N=3	N=4
CEWU3	External-N1	3	5	7	9
	N2	35	165	455	969
	Internal- X1	15	45	91	153
	X2	5	9	13	17
CWU3*	External-N1	3	5	7	9
	N2	5(3)	9(5)	13(7)	17(9)
	N3	27(6)	125(15)	343(35)	729(63)
	NA	35(10)	165(35)	455(84)	969(165)
	NB	5(3)	9(5)	13(7)	17(9)
	Internal- X1	81(18)	625(75)	2401(245)	6561(540)
	X2	5(3)	9(5)	13(7)	17(9)
	X3	25(9)	81(25)	169(49)	289(81)
	CRU3	External-NA	1	2	3
NB		1	2	3	3
NC		1	2	3	3
ND		3	5	7	9
Internal- X1		1	2	3	3
X2		1	2	3	3
X3		1	2	3	3
X4		3	5	7	9
X5		10	35	84	165
X6		10	35	84	165
X7		10	35	84	165
X8		35	165	455	969
X9		10	70	252	495
X10		10	70	252	495
X11		10	70	252	495
X12		105	825	3185	8721
X13		5	9	13	17
X14		15	45	91	153
X15	6	15	35	63	
X16	6	30	105	189	
X17	3	5	7	9	
X18	9	25	49	81	
CWU3R3**	External-N0	3	5	7	9
	N1	2	5	11	21
	N2	2	2	2	2
	N3	2	5	11	21
	NA	35	165	455	969
	Internal- X1	2	5	11	21
	X2	2	2	2	2
	X3	8	50	242	882
	X4	4	10	22	42

* The values in () apply if CWU3 is used in conjunction with CRU3 only.

** Appropriate for representations of SU_3 with $\lambda + 2\mu < N(N+1)(N+2)$.

that within each shell smaller limits are possible if one is dealing with less than mid-shell nuclei.) The routines in the test deck correspond to the case $N = 2$. Adaptations to other usages can be made in an equally straightforward fashion.

Routine A, BLOCKS, together with the block data statements, generates the common blocks required by the other subprograms. These include the binomial coefficients $\binom{M}{N}$, $N \leq M \leq M_{\max}$ ($= 32$) and factorials $N!$, $N \leq N_{\max}$ ($= 32$). The upper limit on M_{\max} and N_{\max} are machine dependent (e.g. $\binom{32}{16} = 601\,080\,390$ as opposed to $\binom{33}{16} = 1\,166\,803\,110$ does not yield a fixed point (integer) overflow condition). They imply restrictions on the "arbitrariness" of the coupling to which the programs apply (e.g. $\lambda + \mu + L \leq M_{\max}$ as can be seen from eq. (26) of ref. [1]). For all practical purposes, however, the limits $M_{\max} = N_{\max} = 32$ are sufficiently high so as to impose no serious limitations. In any case, should a need arise there exist machines (e.g., the IBM 360 series for which $N_{\max} = 50$ is possible) and/or techniques (e.g., the use of logarithms for large number operations) which allow these limits to be extended. Note, however, that the limits are fixed in BLOCKS and extensions can be made by modifying this and only this subprogram.

A conversion to double precision arithmetic may be required for complex couplings involving large ρ -multiplicities ($\rho_{\max} \gtrsim 5$) and/or large $\lambda + \mu$ values ($\lambda + \mu \gtrsim 16$). This is particularly true for the $SU_3 \supset R_3$ Wigner coefficients because of the alternating nature of the sums required for an evaluation of the transformation coefficients from the $SU_3 \supset SU_2 \times U_1$ to the $SU_3 \supset R_3$ scheme. The conversion can be made by simply removing the "C" from column 1 of the statement "IMPLICIT REAL *8(D)" in each subprogram and replacing

```
SQRT  → DSQRT,
FLOAT → DFLOAT,
ABS   → DABS,
.E-06 → .D-12,
.E0   → .D0,
```

throughout. It may also be necessary (depending upon the machine and/or compiler) to change the type declarations "IMPLICIT REAL *8(D)" to "DOUBLE PRECISION(D)" and "IMPLICIT INTEGER(X)" to "INTEGER(X)" throughout.

4. Subprogram usage

The three sample control routines which comprise Part 2 of the FORTRAN IV deck (carrying identification labels, X, Y, Z, respectively) can be used in conjunction with the fifteen subprograms of the SU_3 package to generate output in the format as given by the illustrative tables X, Y, Z of Test Run Output. For this purpose it is convenient to group the subprograms of the SU_3 package into the four sets given in table 3.

Table 3
Subprogram for the SU_3 package.

SET 1	SET 2	SET 3	SET 4
A. BLOCKS	F. CWU3	G. CRU3	L. CWU3R3
B. CEWU3		H. DDBS	M. DTU3R3
C. CEWU3S		I. DLUT	N. CONMAT
D. MULTU3		J. DELTA	O. DWR3
E. MULTHY		K. DRR3	J. (DELTA)

Note that sets 3 and 4 both contain DELTA. A simple combination of 1, 2, 3, 4 together with X, Y, Z can then be used to calculate the desired coefficients:

$SU_3 \supset SU_2 \times U_1$ Wigner coefficients (X+1+2),
 SU_3 Racah coefficients (Y+1+2+3),
 $SU_3 \supset R_3$ Wigner coefficients (Z+1+4).

Since the size of a particular array is normally set at the limit first encountered in the compilation process, it is important that the main program (X, Y, Z) be compiled first and BLOCKS second with the remaining subprograms (1 + 2, 1 + 2 + 3, 1 + 4) in any convenient order. Once properly compiled, the following data cards can be used to obtain required Wigner and Racah coefficients ($N =$ number of cards):

```
SU3 ⊃ SU2 × U1 Wigner coefficients
Card(s) 1 – N                               Format (1615)
λ1 μ1 λ2 μ2 λ3 μ3 ε3 2Λ3
...
ENDFILE (= negative integer)

SU3 Racah coefficients
Card(s) 1 – N                               Format (1615)
λ1 μ1 λ2 μ2 λ μ λ3 μ3 λ12 μ12 λ23 μ23
...
ENDFILE (= negative integer)
```

$SU_3 \supset R_3$ Wigner coefficients

Card 1 Format (1615)

$L_1 \min L_1 \max L_3 \min L_3 \max$
Card(s) 2 - N Format (1615)

$\lambda_1 \mu_1 \lambda_2 \mu_2 \lambda_3 \mu_3 I_1 J_1 I_2 J_2 I_3 J_3 L_2$
...

ENDFILE (= negative integer).

Note that for Elliott-like $SU_3 \supset R_3$ projection (IJ) = (01) if $\mu \leq \lambda$ and (IJ) = (01) if $\mu > \lambda$.

The examples shown as part of the Test Run Output include numerical checks on all the symmetry properties of the Wigner coefficients. Also illustrated are a number of coefficients which vanish due to the Biedenharn-Louck prescription for specifying the ρ -multiplicity. The statement NON-EXIST denotes an SU_3 coupling violation while PAR-CHECK indicates that the coupling, although allowed, requires higher limits on the size parameters for internally dimensioned arrays. The statement SUB-LABEL indicates an incorrectly specified subgroup label ($\epsilon_3, \Lambda_3, L_2$, etc.).

Routines X, Y, Z by no means represent the most efficient use of the subprogram; they simply serve to illustrate the required call sequences and provide a standard input-output format for the casual user. More creative users should incorporate the routines

into their own programs for more efficient evaluation of the required coefficients.

5. Conclusion

The programs have been written in a form which the authors found convenient for performing nuclear shell model calculations. It is our hope that other users may find the routines equally helpful in their studies of the usefulness of SU_3 in describing physical phenomena.

References

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- [8] T.T. Huo and G.E. Brown, *Nucl. Phys.* 85 (1966) 40.

TEST RUN OUTPUT

TABLE X: $SU_3 \supset SU_2 \times U_1$ Wigner coefficients

<(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3>

4	3	11	3	2	0	-2	2	5	2	9	1	-0.4472130
		8	2			1	1			9	1	0.6708192
		5	1			4	0			9	1	0.5916078

<(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3>

3	4	-5	1	0	2	-4	0	2	5	-9	1	0.5916081
		-8	2			-1	1			-9	1	-0.6708204
		-11	3			2	2			-9	1	-0.4472136

<(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3>

2	5	-9	3	2	0	-2	2	3	4	-11	3	0.4714043
		-9	1			-2	2			-11	3	-0.3333331
		-12	2			1	1			-11	3	0.8164963

<(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3>

2	0	1	1	2	5	-12	2	3	4	-11	3	0.8164966
		-2	2			-9	3			-11	3	-0.4714044
		-2	2			-9	1			-11	3	-0.3333333

<(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3> RD = 1, ..., 5

8	5	15	5	4	4	-12	4	8	5	3	1	0.1175695	0.1423146	0.1096674	0.0443789	0.0018869
		15	3			-12	4			3	1	-0.3450901	-0.1365644	-0.0184610	0.0167523	0.0074424
		12	6			-9	5			3	1	0.0000000	0.1655406	0.2816388	0.2310127	0.0701511
		12	4			-9	5			3	1	0.0564783	-0.2752138	-0.2112512	-0.0084270	0.0499113
		12	4			-9	3			3	1	-0.1844581	-0.1531486	-0.0091124	0.0894425	0.0532419
		12	2			-9	3			3	1	0.3320628	-0.0200952	-0.0829114	-0.0072025	0.0239624
		9	7			-6	6			3	1	0.0000000	0.0000002	0.2022517	0.4024984	0.2813261
		9	5			-6	6			3	1	0.0000000	0.1051989	-0.1794031	-0.2076343	0.0249458
		9	5			-6	4			3	1	0.0996186	-0.0904384	-0.0876961	0.1535112	0.2342139
		9	3			-6	4			3	1	-0.2065165	0.1963400	-0.0400202	-0.1430856	0.0352273
		9	3			-6	2			3	1	-0.0640483	-0.1125946	-0.1315216	0.0252074	0.1306762
		9	1			-6	2			3	1	0.1634971	0.1579462	-0.0337038	-0.0678923	0.0367159
		6	8			-3	7			3	1	0.0000001	0.0000001	0.0000002	0.2424927	0.4396580
		6	6			-3	7			3	1	-0.0000001	-0.0000002	0.1374027	-0.1435394	-0.2045853
		6	6			-3	5			3	1	-0.0000003	0.2231607	0.1218487	0.0779112	0.2948456
		6	4			-3	5			3	1	0.0621235	-0.2892222	0.0844727	-0.0347555	-0.1428876
		6	4			-3	3			3	1	-0.1679206	-0.1062852	-0.1136591	-0.0208115	0.2714724
		6	2			-3	3			3	1	0.0668970	-0.0197446	0.1448354	-0.0704561	-0.0628080
		6	2			-3	1			3	1	0.1466495	0.0044746	-0.1198090	-0.0951678	0.1840702
		6	0			-3	1			3	1	-0.1337924	0.0996605	0.0942513	-0.1028597	0.0295924
		3	9			0	8			3	1	0.0000001	0.0000000	0.0000000	0.0000001	0.2867444
		3	7			0	8			3	1	-0.0000001	-0.0000000	0.0000000	0.1426699	-0.2797034
		3	7			0	6			3	1	-0.0000000	-0.0000000	0.3238611	0.2165026	-0.0146409
		3	5			0	6			3	1	0.0000005	0.1487740	-0.2625284	0.0667632	-0.1226110
		3	5			0	4			3	1	0.1141281	-0.0476189	0.0754315	0.1197792	0.0571560
		3	3			0	4			3	1	-0.1414666	0.1091161	-0.1482300	0.1253772	-0.1449734
		3	3			0	2			3	1	-0.1884385	-0.0769747	-0.0762355	-0.0198023	0.1454155
		3	1			0	2			3	1	0.1751731	-0.0377186	-0.0605741	0.1637287	-0.1494466
		3	1			0	0			3	1	0.1824744	0.1209748	-0.0895329	-0.1307549	0.1614395
		0	8			3	7			3	1	-0.0000002	0.0000001	0.0	0.3526897	-0.2040735
		0	6			3	7			3	1	0.0000002	0.0000002	0.1869814	-0.3143120	0.1319153
		0	6			3	5			3	1	0.0000007	0.3036838	0.1118390	0.1888314	-0.1579979
		0	4			3	5			3	1	0.0842883	-0.2367578	0.1572902	-0.2032034	0.0893505
		0	4			3	3			3	1	-0.0225273	-0.1080241	-0.0137818	0.2023816	-0.1064696
		0	2			3	3			3	1	-0.0625755	0.0099035	0.1338661	-0.1473395	0.0495516
		0	2			3	1			3	1	0.0236511	-0.1794822	-0.0638043	0.1302010	0.0474012
		-3	7			6	6			3	1	0.0000001	-0.0000004	0.4003033	-0.1438671	-0.0236966
		-3	5			6	6			3	1	0.0000002	0.1942371	-0.2852132	0.0664855	0.0256631
		-3	5			6	4			3	1	0.1839325	0.1027606	0.2410648	-0.1085403	-0.0099629

-3 3	6 4	3 1	-0.0919667	0.0984762	-0.1452513	0.0353649	0.0132181
-3 3	6 2	3 1	-0.3251516	0.0090789	0.1224855	-0.0406722	-0.0062435
-6 6	9 5	3 1	0.0000000	0.4445478	-0.0733276	-0.0560823	0.0244341
-6 4	9 5	3 1	0.1560720	-0.2264612	0.0242500	0.0308638	-0.0119579
-6 4	9 3	3 1	0.1911489	0.1552420	-0.0416569	-0.0167740	0.0091319
-9 5	12 4	3 1	0.4725329	-0.0439957	-0.0324188	0.0124972	-0.0009368
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 2							
6 1	-5 7	2 1	-4 2	5 2	-9 5	0.2390458	0.8366600
	-5 5		-4 2		-9 5	-0.3779646	0.0
	-8 6		-1 3		-9 5	0.8164969	-0.4082484
	-8 6		-1 1		-9 5	0.3651483	0.3651485
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 2							
1 6	8 6	1 2	1 3	2 5	9 5	0.8164959	0.4082484
	8 6		1 1		9 5	-0.3651482	0.3651479
	5 7		4 2		9 5	0.2390453	-0.8366588
	5 5		4 2		9 5	0.3779625	-0.0000018
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 2							
2 5	9 5	2 1	-1 3	1 6	8 6	0.6666661	0.3333331
	9 5		-1 1		8 6	-0.2981412	0.2981421
	6 6		2 2		8 6	-0.2981417	-0.4472139
	6 4		2 2		8 6	0.3333327	0.0000001
	6 6		2 0		8 6	0.3651480	-0.3651479
	3 7		5 1		8 6	-0.1951799	0.6831298
	3 5		5 1		8 6	-0.3086062	-0.0000012
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 2							
2 1	5 1	2 5	3 7	1 6	8 6	0.5413314	0.4601318
	5 1		3 5		8 6	-0.2567768	0.1711834
	2 2		6 6		8 6	-0.0000005	-0.5374836
	2 0		6 6		8 6	0.5063686	0.1012735
	2 2		6 4		8 6	0.2773494	-0.1848992
	-1 3		9 5		8 6	-0.3697958	0.6471494
	-1 1		9 5		8 6	-0.4134487	-0.0826893
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 4							
9 3	5 5	6 6	PAR-CHECK				
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 2							
9 3	3 3	6 6	9 4	SUB-LABEL			
$\langle(LM1MU1)E1,J1;(LM2MU2)E2,J2::(LM3MU3)E3,J3\rangle$ RO = 1, ..., 0							
9 3	1 1	6 6	NON-EXIST				

TABLE Y: SU_3 Racah coefficients

$U((L1,M1)(L2,M2)(LM,MU)(L3,M3);(LA,MA)RA,RE(LC,MC)RC,RD)$	
6 4 2 0 5 5 2 0 6 5 1 1 4 0 1 1	0.4513354
6 4 2 0 5 5 2 0 6 5 1 1 0 2 1 1	0.5340269
6 4 2 0 5 5 2 0 6 5 1 1 2 1 1 1	-0.0610819
	0.7123057
6 4 2 0 5 5 2 0 5 4 1 1 4 0 1 1	0.6938879
6 4 2 0 5 5 2 0 5 4 1 1 0 2 1 1	0.2932209
6 4 2 0 5 5 2 0 5 4 1 1 2 1 1 1	0.0283791
	-0.6570641
6 4 2 0 5 5 2 0 7 3 1 1 4 0 1 1	0.4082477
6 4 2 0 5 5 2 0 7 3 1 1 0 2 1 1	-0.5520518

6	4	2	0	5	5	2	0	7	3	1	1	2	1	1	1	0.6945825
										1	1		1	2		0.2147684
6	4	2	0	5	5	2	0	4	6	1	1	4	0	1	1	0.3849001
6	4	2	0	5	5	2	0	4	6	1	1	0	2	1	1	-0.5692743
6	4	2	0	5	5	2	0	4	6	1	1	2	1	1	1	-0.7162519
										1	1		1	2		0.1214912
6	6	0	4	6	6	4	0	8	3	1	1	4	4	1	1	-0.0231100
										1	1		1	2		-0.1105537
										1	1		1	3		-0.1472464
										1	1		1	4		-0.0984493
										1	1		1	5		0.4153821
9	3	1	2	8	2	2	1	8	3	1	1	2	2	1	1	0.2168387
										1	1		1	2		-0.0399911
										1	1		2	1		-0.2052838
										1	1		2	2		0.1964977
										1	2		1	1		-0.0382751
										1	2		1	2		0.1965769
										1	2		2	1		0.2263088
										1	2		2	2		0.4226531
										2	1		1	1		0.1321138
										2	1		1	2		-0.0243655
										2	1		2	1		0.4405778
										2	1		2	2		-0.0707791
										2	2		1	1		0.0422222
										2	2		1	2		0.4751489
										2	2		2	1		0.0972563
										2	2		2	2		-0.2836112
7	4	1	2	5	5	0	2	4	5	1	1	1	1	1	1	0.4360309
5	5	2	0	7	4	2	1	4	5	1	1	1	1	1	1	0.4360307
9	3	1	1	6	6	2	2	9	3	1	1	3	3	1	1	0.4364350
										1	1		1	2		0.0000003
										2	1		1	1		0.1390705
										2	1		1	2		0.5916793
9	3	1	1	6	6	3	3	9	3			4	4			PAR-CHECK
8	2	0	4	5	3	2	1	7	2	0	0	2	5	0	0	NCN-EXIST

TABLE Z: $SU_3 \supset R_3$ Wigner coefficients

<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2::(LM3MU3)IJ,K3,L3>

4	3	01	1	2	2	0	01	1	2	5	2	01	1	2	0.3055046
			1	3				1	2				1	2	-0.3243206
			2	3				1	2				1	2	-0.6856724
			1	2				1	2				1	3	-0.6117334
			1	2				1	2				2	3	0.1279683
			1	3				1	2				1	3	-0.3646375
			2	3				1	2				1	3	-0.0000002
			1	3				1	2				2	3	0.2080802
			2	3				1	2				2	3	-0.6205428

<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2::(LM3MU3)IJ,K3,L3>

3	4	10	1	2	0	2	10	1	2	2	5	10	1	2	0.3055046
			1	3				1	2				1	2	-0.3243206
			2	3				1	2				1	2	-0.6856724
			1	2				1	2				1	3	-0.6117334
			1	2				1	2				2	3	0.1279683
			1	3				1	2				1	3	-0.3646375
			2	3				1	2				1	3	-0.0000002
			1	3				1	2				2	3	0.2080802
			2	3				1	2				2	3	-0.6205428

<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2::(LM3MU3)IJ,K3,L3>

2 5 10	1 2	2 0 01	1 2	3 4 10	1 2	0.3220300				
	1 3		1 2		1 2	0.7629643				
	2 3		1 2		1 2	-0.1596035				
	1 2		1 2		1 3	0.2889273				
	1 2		1 2		2 3	0.6108462				
	1 3		1 2		1 3	-0.3843608				
	2 3		1 2		1 3	0.2193345				
	1 3		1 2		2 3	-0.0000009				
	2 3		1 2		2 3	-0.6541088				
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3>										
2 0 01	1 2	2 5 10	1 3	3 4 10	1 2	-0.7629647				
	1 2		2 3		1 2	0.1596033				
	1 2		1 3		1 3	-0.3843611				
	1 2		2 3		1 3	0.2193345				
	1 2		1 3		2 3	-0.0000009				
	1 2		2 3		2 3	-0.6541088				
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 5										
8 5 01	1 2	4 4 01	1 2	8 5 01	1 2	0.0373651	0.0439388	0.0066230	0.0500954	-0.0197199
	1 2		2 2		1 2	-0.0715490	-0.0841364	-0.0126820	-0.0959254	0.0377607
	1 3		1 2		1 2	0.0878139	0.0663584	-0.0058050	0.0696993	-0.0043999
	2 3		1 2		1 2	-0.0458331	-0.0572700	-0.0797430	-0.1007875	-0.1361247
	1 3		2 2		1 2	0.0348150	-0.0435452	0.0030094	-0.0660166	-0.0728580
	2 3		2 2		1 2	0.0780247	0.0302061	-0.0994340	-0.0886091	-0.1148131
	1 2		1 2		1 3	0.0665561	0.0018454	-0.0007162	-0.0121268	-0.0526576
	1 2		2 2		1 3	0.0440922	0.0670548	-0.0054795	0.0802245	0.0321348
	1 2		1 2		2 3	0.0319812	-0.0067081	-0.1074766	-0.0546460	-0.1453726
	1 2		2 2		2 3	-0.0694705	-0.0543092	-0.0072868	-0.0657465	-0.0389647
	1 3		1 2		1 3	-0.0301820	-0.0161238	0.0304046	0.0073204	0.0376334
	2 3		1 2		1 3	0.0045619	0.0181886	-0.0662000	-0.0334105	-0.0775405
	1 3		2 2		1 3	0.0577942	0.0308752	-0.0582204	-0.0140175	-0.0720625
	2 3		2 2		1 3	0.0475206	0.0149339	-0.1066674	0.0205866	-0.1196150
	1 3		1 2		2 3	-0.0416047	-0.0226490	0.1253651	-0.0060092	0.1424709
	2 3		1 2		2 3	0.0637214	-0.0163902	0.0600707	0.0712655	0.0523129
	1 3		2 2		2 3	0.0234109	-0.0063930	-0.0066255	0.0448966	-0.0047176
	2 3		2 2		2 3	-0.1220173	0.0313848	-0.1150265	-0.1364630	-0.1001715
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RC = 1, ..., 2										
6 1 01	1 2	2 1 01	1 3	5 2 01	1 2	-0.2285706	0.5428551			
	1 3		1 3		1 2	0.6408313	0.2746418			
	1 2		1 3		1 3	-0.1975533	-0.0238425			
	1 2		1 3		2 3	-0.4630282	-0.3367475			
	1 3		1 3		1 3	0.2612962	0.2228701			
	1 3		1 3		2 3	-0.0652961	-0.5223668			
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RU = 1, ..., 2										
1 6 10	1 2	1 2 10	1 3	2 5 10	1 2	-0.2285706	-0.5428551			
	1 3		1 3		1 2	0.6408313	0.2746418			
	1 2		1 3		1 3	-0.1975533	0.0238425			
	1 2		1 3		2 3	-0.4630282	0.3367475			
	1 3		1 3		1 3	0.2612962	0.2228701			
	1 3		1 3		2 3	-0.0652961	0.5223668			
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 2										
2 5 10	1 2	2 1 01	1 3	1 6 10	1 2	0.2015804	0.4787537			
	1 3		1 3		1 2	-0.2061467	0.0248799			
	2 3		1 3		1 2	-0.4831694	0.3513960			
	1 2		1 3		1 3	0.4776473	-0.2047058			
	1 3		1 3		1 3	-0.2304413	-0.1965532			
	2 3		1 3		1 3	0.0575853	-0.4606845			
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 2										
2 1 01	1 2	2 5 10	1 3	1 6 10	1 2	-0.4276024	0.0038171			
	1 2		2 3		1 2	0.0471822	-0.2359120			
	1 3		1 3		1 2	-0.1853252	0.0936484			
	1 3		2 3		1 2	-0.5969408	-0.0243648			
	1 2		1 3		1 3	-0.2970074	0.0798906			
	1 2		2 3		1 3	-0.1277711	-0.1916558			
	1 3		1 3		1 3	0.0827109	-0.2913679			
	1 3		2 3		1 3	-0.3034552	-0.3513695			
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 4										
9 3		5 5	1 1	6 6		PAR-CHECK				
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 2										
9 3		3 3	0 0	6 6		SUB-LABEL				
<(LM1MU1)IJ,K1,L1:(LM2MU2)IJ,K2,L2:(LM3MU3)IJ,K3,L3> RO = 1, ..., 0										
9 3		1 1		6 6		NCN-EXIST				