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DETECTION OF INFORMATIONAL CONSTRAINTS RELATED TO MULTI-VARIATE VISUAL DISPLAYS*

Irwin POLLACK **

Mental Health Research Institute, University of Michigan, Ann Arbor, Michigan 48104 U.S.A.

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A contingent uncertainty of one bit is perceptible when imposed upon a combination of two binary-coded visual display variables, but not when imposed upon a combination of three variables. Why? The limitation may be sought in the average amount of the constraint, in the form of the constraint, or in the particular selection of display variables. Tests were carried out in which apparently equivalent informational constraints were imposed upon a single display variable. Such constraints were highly discriminable. Further tests reveal that the limiting feature for the detection of multi-variate constraints is probably the mean constraint level, averaged over all display elements, rather than the constraint level imposed upon the constrained display elements.

1. Introduction

The present report considers the discriminable limits associated with visual displays of relatively large quantities of binary-coded information. In a previous report to this journal (Pollack, 1971a), I explored the discrimination of one-, two- and three-dimensional Markov constraints which were encoded within one, two, and three independently-constrained display variables. The major result of that study was that one-dimensional constraints in X (horizontal), Y (vertical), and T (time) were clearly discerned; as were two-dimensional combinations of XY,

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XT and YT; but, discrimination of the three-dimensional combinations of XYT hardly exceeded chance level.

Why did the discrimination of three-dimensional constraints fail? Three alternative hypotheses appear reasonable. First, a poor set of three display variables was selected. This is not likely because selection of the same variables within pairs of two resulted in excellent discrimination. Second, the form of the imposed constraints – a one-bit constraint imposed upon the combinations of one, two and three binary-encoded variables – was not optimal. Might we not discriminate informationally-equivalent constraints imposed on one of the variables alone? And, third, the strength of the imposed constraint was too weak to be detected. Specifically, a one-bit parity sum (Julesz, 1962) or a contingent uncertainty of 1 bit (Garner, 1962) imposed on the combination of one, two, and three display variables imposes an average constraint of 1, 0.5, and 0.33 bits upon each display variable. On this basis, the level of discrimination is to be sought in the average constraint level.

The present study attempts to test the second of the alternatives directly, and thereby, tests the third of the alternatives.

2. Method

2.1. General approach

Two requirements were placed upon the tests: (1) the method of display should employ dimensions which proved most effective in the earlier tests in order that the results should not be display-limited; and (2) equivalent informational constraints should be introduced. The previous tests showed that performance with the individual spatial dimensions, X and Y, yielded better performance than the temporal dimension, T. Since X and Y were essentially equivalent, only one was chosen in the present tests, Y. In order to achieve equivalent informational constraints with a single display variable, another procedure was adapted from independent tests (Pollack, 1971b). Essentially, constraints appropriate to the simultaneous presentation of several display variables were translated to sequential constraints with respect to a single display variable.

2.2. Shifting block procedure

The shifting block procedure is illustrated in fig. 1. The top line considers a block B of 2 binary-coded elements with a shift of 1 element between successive blocks. The sequence is first initialized with $(B-1)$ unconstrained random selections, as indicated by the underlined elements. A sequence of parity sums is also developed by controlling the probability of an even parity sum, $P(\Sigma_e)$. Given an initial item of 1 and an even parity sum, the next item must be a 1

B	s	Initialization	Sequences
2	1	1	1 0 1 1 . . .
3	1	1 1	0 0 1 1 . . .
2	2	1	1 0 1 1 0 1 1 . . . .
Parity Sum			e o o e

Fig. 1. Schematic illustration of the shifting block parity procedure.

(1 + 1 = even); we now shift 1 element so that the initial element is a 1 (as determined by the previous parity calculation) and since an odd parity is indicated, the next item of the sequence must be a 0 (1 + 0 = odd), ... etc. The process continues without interruption.

The second line of fig. 1 illustrates a block size, $B = 3$, again with a shift, $s = 1$. The sequence is again initialized by $(B-1)$ unconstrained random selections and a sequence of parity sums is developed. Given two initial items of 11 and an even parity sum, the next selection must be 0 (1 + 1 + 0 = even); with a shift of 1 and an odd parity sum, the next selection must be 0 (1 + 0 + 0 = odd) ... etc. In general, in this method, the parity restriction over B elements, with $s = 1$, involves $(B-1)$ previously determined elements and the indicated parity sum. In the previously-cited tests with one-, two-, and three-dimensional binary-encoded displays, a parity sum was imposed over the combination 2, 4, and 8 display elements, respectively. The equivalent informational constraint upon a single variable sliding block procedure employs $B = 2$, 4, and 8 with $s = 1$, respectively.

The third line of fig. 1 illustrates the shift parameter with $s = 2$. A random selection of $(B-1)$ elements initializes the sequence. The first calculated item (1 + 1 = even) is obtained as before. With $s = 2$, however, we shift beyond the just-determined element and must re-initialize the sequence with another unconstrained random selection, shown with underlining. In general, the proportion of unconstrained random selections is $(s-1)/s$.

2.3. *Apparatus and procedure*

Binary-coded sequences were generated by a PDP–9 computer (Digital Equipment Corp.). The binary sequences were translated into sequences of dots and of no-dots and displayed upon a fast display surface (Tektronix 602 equipped with a P15 phosphor).

A given observation consisted of the simultaneous presentation of 4 dot matrices: three of the matrices with $P(\Sigma_e)_R = 0.5$ or at a chance level, and one matrix with a variable, $P(\Sigma_e)_v$. The observer's task was to identify which one of the four matrices was the odd one. An adaptive stimulus programming procedure (Taylor and Creelman, 1967) varied $P(\Sigma_e)_v$ to converge upon 50% correct responses in the four-alternative forced-choice test. About one-half of the tests explored increment thresholds with $P(\Sigma_e)_v > 0.5$; and half explored decrement thresholds with $P(\Sigma_e)_v < 0.5$.

Display duration was varied by repeatedly refreshing the display. A single painting of the display took 18 msec. With 128, 32 and 8 paintings, the approximate durations were 2.4, 0.6, and 0.15 sec. respectively. Typically, for a given combination of B, s, and direction of threshold, successive tests were run at decreasing durations.

Natural binocular viewing was used. The observer varied his distance from the 8 × 8 cm display for maximum comfort. His median distance to the display was about 75 cm. The inter-column and inter-row spacing was 1.17 mm.

2.4. Display formats

Three display formats were employed. Independent linear sequences, illustrated in fig. 1, were plotted as sequences of dots and no-dots in successive columns of a 16 (column) × 64 (row) matrix, in successive columns of a 32 × 32 matrix; and in alternate columns of a 32 × 32 matrix. The last format was a modification of the first format. The odd-numbered positions within a single column of 64 elements were laid down in an odd-numbered column of 32 elements, and the even-numbered positions within a single column of 64 elements were laid down in an adjacent even-numbered column of 32 elements. The offset procedure was employed in order to reduce the linear separation among elements entering a large block parity calculation.

Each column of the non-offset displays, and each pair of columns of the offset display, was initialized by $(B-1)$ unconstrained random elements. Therefore the proportion of unconstrained elements uninfluenced by the parity constraint is $(B-1)/L$, where L is the length of the matrix measured by the number of rows. A longer format, e.g. 16 × 64, has the advantage of a smaller proportion of unconstrained elements. A wider format, e.g. 32 × 32, has the advantage of a larger diversity of initial random sequences, in which easily recognized linear patterns, e.g. all dots, can be propagated by the parity restriction.

2.5. Experimental conditions

Each of 16–18 observers contributed two thresholds under 99 experimental conditions for each of the three display formats, representing a total of about 1×10^5 individually-determined thresholds. Under conditions of difficult discrimination, the adaptive procedure sought $P(\Sigma_e)_v$ levels below 0 and above 1.0. When this occurred, a single observation was provided at the extreme conditions. An incorrect response terminated the trial. When the proportion of terminated trials, $P(T) \leqslant 0.10$, the reported thresholds are arithmetic means, excluding the terminated trials; with $P(T) > 0.10$, thresholds are medians including the terminated trials. The former are plotted as open points; the latter either as half-filled points, when $0.10 < P(T) \leqslant 0.20$; or as filled points, when $P(T) > 0.20$. The degree of shading of the points thus represents a crude display of discriminability.

3. Results

3.1. Comparison among display formats

The offset display yielded more sensitive thresholds, or lower absolute differences between the threshold and reference constraints, than the other formats. Of 41–42 experimental conditions, common between the offset display and the other displays, with $s = 1$, the offset display yielded lower threshold differences for 83% of the experimental conditions. Of 60 experimental conditions, with $s = 1$ between the 16 × 64 and the 32 × 32 formats, the 32 × 32 format yielded lower threshold differences at shorter block lengths; the 16 × 64 format yielded lower threshold differences at longer block lengths. Since the test conditions were heavily weighted toward shorter block lengths, the 32 × 32 format yielded lower threshold differences for 73% of the experimental conditions.

3.2. Minimal shift, $s = 1$

The three panels of fig. 2 represent the three displays: linear constraints on the 16 × 64

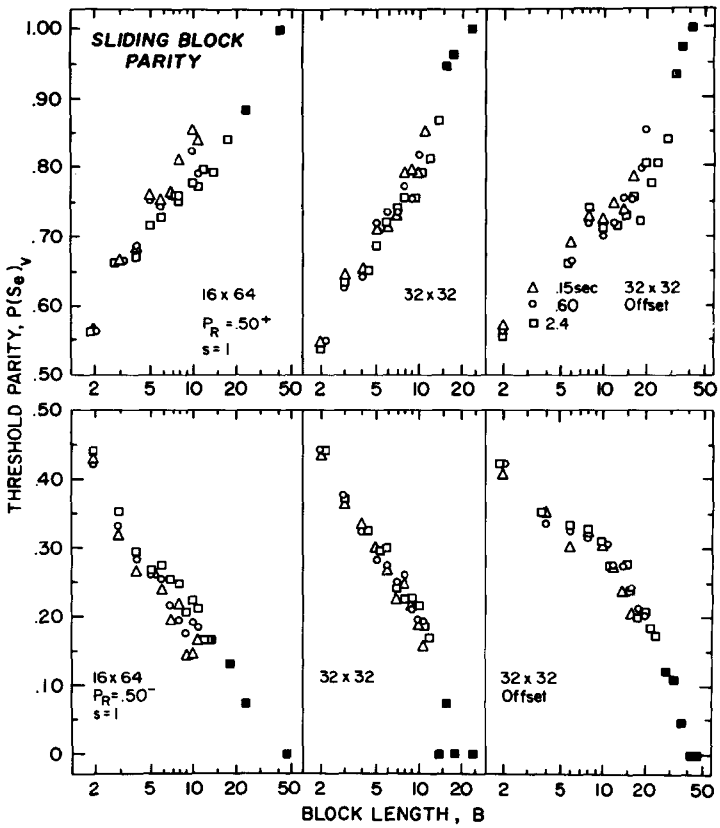

Fig. 2. Increment (top section) and decrement (bottom section) parity constraint thresholds relative to a chance constraint for three display formats: 16 × 64 (left panel), 32 × 32 (middle panel), and 32 × 32 offset (right panel). The abscissa is the block length; all tests employed $s = 1$. The shape of the points reflects the display duration. The shading of the points provides a crude measure of discriminability.

display (left panel), linear constraints on the 32 × 32 display (middle panel), and offset constraints (right panel) on the 32 × 32 display. The upper portion of each panel represents increment constraint thresholds above a chance reference constraint; the lower portion of each panel represents decrement constraint thresholds below a chance reference constraint. The shape of the points represents display duration. The degree of shading represents a crude measure of discriminability. Only data at $s = 1$ are represented in fig. 2.

Except near the upper limit of block length, there were only small differences in favor of the longest display duration. The upper discriminable limits for the three displays for decrement thresholds, and for increment thresholds, are obtained at block-lengths of about 13–21, 13–15, and 22–30, respectively. For all displays, clearly defined thresholds are obtained for a block size of eight, which is informationally equivalent to a constraint imposed upon the combination of three binary-coded variables.

Table 1
Increment and decrement constraint thresholds, relative to a chance reference constraint level, under the sliding block procedure.

n	B	Thresholds	
2	4	0.55	0.30
3	9	0.63	0.29
4	16	0.61	0.29
5	25	0.65	0.28
6	36	0.65	0.27
7	49	0.71	0.24
8	64	0.71	0.24
9	81	0.73	0.20
10	100	0.72	0.19
11	121	0.76	0.19

The superiority of the offset display suggests that a critical display feature with the sliding block procedure is the linear distance among constrained elements. In order to evaluate the offset feature more fully, additional tests were carried out with $n \times n$ offsets at $B = n^2$, $s = 1$ for n between 2 and 11 at a display duration of 2.4 sec. A new group of observers contributed 38 thresholds under each of 20 experimental conditions. The mean thresholds are presented in table 1 for increments above a chance reference level, and for decrements below a chance reference level. The maximum proportion of terminated trials was 7.5% at $B = 121$. Clearly, discrimination is possible under the sliding block procedure with extremely large block sizes with offset displays.

3.3. Tentative conclusion

The success at the longer block-lengths with a single variable appeared to provide a strong test of the initial goals of the study. The apparently same informational constraint, previously non-discriminable when imposed over the combination of three display variables, was clearly discriminable when imposed upon a single display variable. This result encouraged me to write a detailed scenario about the difficulties of processing multi-variate contingencies. However, the multi-variate procedure and the sliding block procedure yielded such different constraint thresholds that a detailed reexamination of the procedures appeared to be needed. Before reconsidering the analysis of single vs. multi-variate displays, let us examine the effect of longer shifts.

3.4. Higher shifts, $s > 1$

As noted in the lowest line of fig. 1, with shifts greater than unity, additional unconstrained random units are introduced outside of the constraints of the parity sum. With the exception of the initial starting sequence, the proportion of additional unconstrained random units introduced at different shift levels is $(s-1)/s$. Table 2 presents constraint thresholds under the three procedures. The left columns describe the displays in terms of block length, shift, and format ($64 \equiv 16 \times 64$; $32 \equiv 32 \times 32$; $32_0 \equiv 32 \times 32$ offset). The middle entries represent increment thresholds (T) and the proportion of terminated trials (%) for three display durations; the right side of table 2 presents corresponding decrement thresholds. Not shown is the consistently poor showing under $B = 5, s = 2$ for the two non-offset displays.

Table 2
Increment and decrement constraint thresholds (T) and the percentage of terminated trials (%) for conditions $s > 1$.

B	s	Display	Increment thresholds						Decrement thresholds					
			0.15 sec		0.6 sec		2.4 sec		0.15 sec		0.6 sec		2.4 sec	
			T	%	T	%	T	%	T	%	T	%	T	%
2	2	64	0.65	0	0.60	0	0.61	0	0.34	0	0.36	0	0.35	0
		32	0.62	0	0.59	0	0.57	0	0.37	0	0.40	0	0.42	0
		32_0	0.62	0	0.61	0	0.59	0	0.37	6	0.37	0	0.37	0
3	2	64	0.95	29	1.0	63	0.85	26	0.06	36	0.01	50	0.14	15
		32	0.90	17	0.89	11	0.81	21	0.02	47	0.08	22	0.16	11
4	2	64	0.89	13	0.84	10	0.85	4	0	77	0	68	0	71
		32	0.94	33	0.86	14	0.87	14	0	71	0	69	0	71
		32_0	0.73	6	0.71	0	0.67	0	0	62	0.02	45	0.22	22
6	2	32					0.93	23					0.04	39
3	3	64	1.0	67	1.0	54	1.0	50	0	68	0	58	0	66
		32	1.0	77	1.0	69	1.0	57	0	57	0.01	46	0.02	39
4	3	32_0	1.0	70	0.97	44	0.94	28	0	84	0	65	0	65

Discrimination is achieved for $s = 2$ for block lengths of only about 3 for decrement constraint thresholds. Discrimination is consistently poor for $s = 3$. There is a hint in table 2, as well as in table 1, that even block lengths may yield lower thresholds than the just-shorter odd block lengths. This apparent anomaly is probably related to the fact that, with even block lengths, long strings of either dots or of no-dots could be obtained at high constraint levels; with odd block lengths, long strings of only no-dots could be obtained at high constraint levels. Nevertheless, the striking result is the sharp drop in the upper discriminable limit on block length with $s > 1$.

4. Discussion

We are now in a position to reevaluate the tentative conclusions of section 3.3. Fig. 2 and table 1 clearly showed that an informational restriction of less that 1 bit can be imposed upon the combination of eight successive binary elements, representing a single display variable, with excellent discrimination. Previously, it was shown that the apparently equivalent informational restriction imposed upon the combination of three binary-coded variables was non-discriminable. This result encouraged the conclusion of section 3.3 that the crucial difference was to be sought in the multi-variate combination.

The extremely poor discrimination associated with higher shift levels with a single display variable provides a clue that the sought-for features might be found in the shift variable. To this end, let us reexamine the operation of informational constraints within the multi-variate display procedure and in the sliding block procedure. In the multi-variate procedure with n display variables, the binary states of $(n-1)$ display variables are randomly chosen upon each selection, *without* reference to earlier selections. The binary state of the nth display variable is determined by a binary restriction upon the combination across the three variables. Thus, the proportion of unconstrained variables is $(n-1)/n$ with multi-variable displays. In the sliding block procedure, the proportion of unconstrained elements with different shifts with the single variable display is $(s-1)/s$. The sharp failure at $n = 3$ in the multi-variate tests and at $s = 3$ in the sliding block procedure suggests that performance is limited in both tests by the proportion of unconstrained elements. Equating this factor, as well as the apparent block size, says that appropriate comparison for $n = 2$ in the multi-variate tests is $B = 4$, $s = 2$ in the sliding block procedure; and that the appropriate comparison for $n = 3$ in the multi-variate tests is $B = 8$, $s = 3$ in the sliding block procedure. On the basis of these comparisons, the apparent discrepancy between the two procedures disappears.

An alternative view of the results is to consider the mean constraint level, averaged over all display elements. The mean constraint level, given in eq. (1), is the proportion of unconstrained elements of $(n-1)/n$ multiplied by their constraint level of $P(\Sigma_e) = 0.5$; plus the proportion of unconstrained elements of $1/n$ multiplied by their constraint level of $P(\Sigma_e)_\nu$.

$$C = [(n-1)/n]\,0.5 + [1/n]\,P(\Sigma_e)_\nu \tag{1}$$

For the extreme case, where $P(\Sigma_e)_\nu = 1.0$; $\bar{C}_{max}$ $(n = 2) = 0.75$; and, $\bar{C}_{max}$ $(n = 3) = 0.67$. Stated differently, the maximum-attainable constraint level, averaged over all display elements, is only 0.67 for a multi-variate display of three variables. In essence, the important variable is not the constraint level imposed upon constrained elements, but, rather, the mean constraint level imposed upon all elements. Since constrained and unconstrained elements are not marked for the observer, the mean constraint level is the appropriate measure.

5. Conclusion

Constraints imposed upon combinations of variables within multivariate visual displays are difficult to detect probably because the average constraint level, imposed upon all display elements, is insufficient; and, not because of special difficulties in processing multi-variate information.

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