Missile Plume Radiation Characteristics

First Quarterly Report
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ABSTRACT

This is the first quarterly report on DA 28-043 AMC-01378(E) the objective of which is the theoretical prediction of intensity of radiation from the exhaust of rocket engines in the frequency range 30–300 Gcs. Mechanisms producing both line and continuum radiation are discussed and some problems brought forth which are proving difficult to date. Experimental results on radiation intensity are not quoted in detail since, at this stage, they serve only as a guide to the choice of theoretical approaches. It would be premature to test research at this time, by comparison with experiment. The goal of this contract is not merely a survey and evaluation of existing research on the subject, but also an investigation aimed at deriving satisfactory theoretical predictions of measurements of the radiation of interest.
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DEFINITION OF PROBLEM AND STATUS OF PROBLEM AREAS AFTER
FIRST QUARTER

1.1 Statement of Work as Specified by the Contract.

(1) This scope of work covers the study of specific missile fuel configurations to determine their theoretical electromagnetic characteristics in the 30–300 Gcs frequency range. The radiation characteristics of interest occur during the boost phase for each configuration.

(2) The missile fuel configurations to be studied are:
   a. Pershing booster
   b. Nike Hercules booster
   c. Nike Zeus booster
   d. Nike X booster
   e. Honest John
   f. Sergeant.

The fuels for each missile may change during the period of the contract but only changes during the first six months will affect this scope of work. The fuels in use at the end of the six month period will apply to this study. The responsibility for obtaining the necessary information and data for this study will be the Contractor's. The Government will establish the need-to-know of the Contractor at other installations when requested by the Contractor.

(3) The results required from this study include, but are not limited to:
   a. Spectral distribution of radiation within the 30–300 Gcs region.
   b. Type of spectra involved and the mechanism causing same.
   c. Estimate of power levels involved.
   d. Changes in the characteristics during boost phase due to altitude effects, acceleration effects, velocity effects, etc.
   e. Comparison of theoretical results with known experimental or field measurements available to the Contractor during the contract period.
1.2 Division of Effort into Research Tasks

1.2.1: Task A: Formulation of Exhaust Composition

Investigation of the mechanisms controlling the radiated intensity has demonstrated, as expected, that this quantity depends on the abundance of various molecules, ions and free electrons in the exhaust. It has been discovered that computer programs intended to generate the information exist, as do handbooks giving fuel composition and combustion chamber parameters, both of which are usually required as inputs for these computer programs. Information on the programs, together with the input data is now being obtained. It is anticipated that small amounts of impurities in the fuels will contribute much to the radiation, but it will be essentially impossible to predict this contribution.

1.2.2: Task B: Radiative Transfer Problem

The ultimate goal of this contract is the prediction of the radiation intensity emerging from the exhaust plume at all frequencies in the range of interest. To do this, one must solve the "radiative transfer equation" within the exhaust. This equation includes the effects of generation of radiation, absorption and scattering, and geometrical fall-off (such as the inverse-square decrease in spherically symmetric situations). In order to set up the equation it is necessary, in principle at least, to have some information on the geometry (i.e. shape of surfaces of constant concentration or temperature in the exhaust). Also entering into the formulation of the equation are the volumetric emissive power at a given frequency (power per unit volume and unit frequency range, generated in the exhaust) and the frequency-dependent absorption coefficient.
It is expected that the effects of the variation of these quantities with position inside the exhaust will not be severe, and that information on this variation will not be available with accuracy sufficient to justify the time and expense of incorporating it into a numerical integration of the radiation transfer.

The realistic and sensible procedure is, therefore, to attempt to construct an "engineering approximation" to the actual situation in which average values of the important parameters, rather than their detailed (and unknown) variation within the exhaust will determine a useful solution to the problem of predicting the emergent radiation intensity. This approach is being followed at the present time.

1.2.3: Task C: Line Radiation

It is not known whether experimental work is contemplated with radio-meters capable of detecting relatively narrow lines resulting from molecular transitions. Since predictions of such lines might influence future work and inasmuch as a line might at least be detected as an enhanced power level over a broader band in the frequency region of interest, an effort will be made to investigate power levels associated with lines in the region 30–300 Gcs. Some of the lines selected are discussed below. Comments are also made on other molecules considered less important.

CO$_2$. This molecule, being a linear symmetric triatomic molecule, has no pure rotational spectrum (e.g. page 29, Townes and Schawlow, 1955). Since this type of spectrum is usually the source of any significant microwave line radiation from a gaseous substance and is not present for CO$_2$, one of the most promising mechanisms for radiation from an exhaust is thus eliminated. The fundamental vibration-rotation bands of CO$_2$ are near 667 cm$^{-1}$ and 2349 cm$^{-1}$, or 20,000 and 70,000 Gcs. Most of the vibration-rotation spectrum is restricted to regions near these bands (Ketelaar, et al, 1953).
which are above the frequency range of interest. At quite high pressures, 
CO₂ might conceivably have some weak pure rotational spectrum, perhaps 
because of induced asymmetries, but this is not likely to occur in situations 
of interest under this study.

\( \text{H}_2\text{O} \). This molecule is an asymmetric rotator. As such, it possesses 
a spectrum difficult to analyze theoretically. One line has been observed 
in absorption (King and Gordy, 1954) at 183.3 Gcs., within the frequency range 
upon which we are directed to concentrate. Unfortunately, no intensity is 
reported for this line, which is assigned on the basis of its frequency to a 
transition between the levels 3₁,₃ and 2₂,₀. Even if an intensity in 
absorption were determined, it might not be accurate to use this to compute 
a dipole moment and corresponding spontaneous dipole emission probability. 
This is true since the large change in the pseudoquantum number \( K_{-1} \), 
\( |ΔK_{-1}| = 3 \), relegates it to a weak, almost "forbidden" sub-branch rather 
than an "allowed" transition governed by dipole radiation laws. It is further 
true that, because of this fact, no entry can be found in tabulations of the 
"line strength factor", \( S \), corresponding to the transition. An attempt 
to compute this factor has failed because of complexities and ambiguities 
in the notation for asymmetric top molecules as given in Townes and Schawlow 
(Ch. IV, 1955) or by the original source (Cross et al., 1944).

A further difficulty is that use of the tabulations (valid if the dipole 
assumption on which such procedure is based, is correct) requires 
specification of a molecular parameter such as the "asymmetry parameter 
of Ray", \( K \), and there appears to be some question about this parameter. 
According to the National Bureau of Standard tables (p. 79, Kisliuk and 
Townes, 1952), the value for \( \text{H}_2\text{O} \) is \( K = -0.696 \). This value is ascribed 
to Strandberg (1949) but the examination of his paper reveals that \( K = -0.696 \)
refers to HDO, deuterated water, rather than H₂O. Another circumstance casting doubt on the value of K is that a computation based on moments of inertia given on p. 634 of Townes and Schawlow (1955) reveals a value for K of roughly -0.5. It seems then, that the basic parameters required for calculation of transition probabilities are not well known, and that the available formulation of the theory is difficult to use.

CO. The lowest rotational transition of this molecule, J = 0 → 1, corresponds to a frequency of 115.3 Gcs. The computation of volumetric emissive power and absorption coefficient will present no difficulties as far as the application of quantum mechanics to molecular spectroscopy goes, and has, accordingly, been deferred until Tasks A and B have been completed, so that numerical results for this line can then be obtained.

O₂. This molecule has bands around 4 - 6 mm (see p. III of Ingram, 1955). The transition, being magnetic dipole rather than electric, may not be very strong, but in the event that a rocket of interest uses LOX as an oxidizer, so that O₂ might be abundant in the exhaust, the bands will be investigated.

OH. This radical is occasionally present in flames of hydrocarbon fuels, and does have a spectrum in the region of interest which has been observed by workers at the Airborne Instruments Laboratory (Peyton, et al, 1965). These lines result from transitions between levels of \( \Lambda \)-doublets; the lowest is near 37 Gcs. They have been studied extensively by Peyton and his coworkers and also by the Radiation Laboratory under a previous contract (Barasch, et al, 1964).

1.2.4: Task D: Continuum Radiation

Bremsstrahlung Radiation: This phenomenon, the emission by electrons scattered by ions or neutrals, generates a continuum which varies only weakly with frequency. The volumetric emissive power for Bremsstrahlung depends on abundance of electrons and scatterers, and on temperature (if this can be defined by assuming equilibrium or at least by a local thermodynamic equili-
brium approximation). The corresponding absorption coefficient is associated with the effect inverse to Bremsstrahlung, which is "free-free absorption". The same comments as for emission apply to the absorption coefficient. Expressions for Bremsstrahlung cross sections and free-free absorption cross sections valid in various situations abound in the literature and need not be reproduced here. The subject has been studied thoroughly because of the importance of Bremsstrahlung as a source of continuum radiation.

Recombination Radiation. Since radiative recombination of ions and electrons often gives rise to a significant continuum, it is necessary to make at least a preliminary investigation of this process.

Now the frequency of the quantum radiated when a free electron of velocity \( v \) recombines with an ion of ionization potential \( V \) volts, is given by

\[
h \nu = eV + \frac{1}{2} mv^2,
\]

or the minimum frequency radiated is

\[
\nu_{\text{min}} = \frac{eV}{h}.
\]

But the requirement that this be lower than the upper limit of the range of interest is \( \nu_{\text{min}} < 300 \text{ Gcs} \), or

\[
\frac{eV}{h} < 300 \text{ Gcs},
\]

whence the requirement is

\[
V < 10^{-3} \text{ volts},
\]

a value far below those found for ionization potentials in practice.

It is therefore clear that radiative recombination to the ground state of the resulting neutral will give rise to a continuum containing frequencies much higher than the range of interest.
An objection might be raised that a process which forms neutrals in a high-lying excited state, rather than the ground state, could yield frequencies below 300 Gcs (since less energy is available for radiation). Such a process is of approximately equal probability as recombination to the ground state in systems sufficiently simple to have allowed calculation of the cross sections for recombination, such as the oxygen atomic ion according to Bates as quoted by Mitra (1952). However, the final state would need to be one of high excitation in order to bring the frequencies radiated down to the range below 300 Gcs. Since for such high-lying states the final-state wave function would approximate that of a free electron, this effect should be included within the Bremsstrahlung calculations. Recombination radiation will therefore not require special attention.

Radiation from Soot. Solid carbon particles (soot), if present in the exhaust and sufficiently hot, are expected to radiate a continuum. The gross intensity of this continuum will depend, of course, on the concentration and size distribution of the soot particles. In particular, the spectral distribution of radiation will be determined by the temperature of the soot and its frequency-dependent emissivity. Many of these effects have been investigated elsewhere (Ludwig and Ferriso, 1965).
REFERENCES


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