STUDIES IN RADAR CROSS SECTIONS XLII - ON MICROWAVE BREMSSTRAHLUNG FROM A COOL PLASMA

by

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PREFACE

This is the forty-second in a series of reports growing out of the study of radar cross sections at The Radiation Laboratory of The University of Michigan. Titles of the reports already published or presently in process of publication are listed on the preceding pages.

When the study was first begun, the primary aim was to show that radar cross sections can be determined theoretically, the results being in good agreement with experiment. It is believed that by and large this aim has been achieved.

In continuing this study, the objective is to determine means for computing the radar cross section of objects in a variety of different environments. This has led to an extension of the investigation to include not only the standard boundary-value problems, but also such topics as the emission and propagation of electromagnetic and acoustic waves, and phenomena connected with ionized media.

Associated with the theoretical work is an experimental program which embraces (a) measurement of antennas and radar scatterers in order to verify data determined theoretically; (b) investigation of antenna behavior and cross section problems not amenable to theoretical solution; (c) problems associated with the design and development of microwave absorbers; and (d) low and high density ionization phenomena.

K. M. Siegel

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SUMMARY

Microwave Bremsstrahlung from, and free-free absorption in, a cool, partially-ionized plasma are treated. Electron-ion encounters are treated by the Born approximation and the classical impulse approximation, a Debye-shielded potential being used. Bremsstrahlung from electron-neutral collisions is treated by the Born approximation. The potential here is obtained by fitting a shielded Coulomb form to the Thomas-Fermi potential for distances less than about 7 atomic radii.

For the plasma parameters chosen ($T = 5000^{o}K$, $n_e = 10^{13}/cm^3$, $Z = 8$) and microwave frequencies of the order of 50 KMc, it would appear that at the correspondingly low degree of ionization, the neutrals are most significant. An effective Z for the oxygen atoms is determined by matching the free-free absorption to Kramers' law. Its value, Z = .17, compares reasonably with the results of previous investigators.

I. Introduction

Among the classes of problems studied in the Radiation Laboratory have been those dealing with plasmas as either a source or absorber of radiation. One of the specific radiation mechanisms considered has been Bremsstrahlung, while its inverse, free-free absorption, contributes to propagation losses in the plasma.

Bremsstrahlung is a process in which a free electron is scattered by a potential into a free state of lower energy, the energy difference appearing as a radiated photon. In the inverse process, free-free absorption, a free electron in a potential absorbs a photon and makes a transition to a free state of higher energy.

Most treatments of these effects have been for a fully ionized gas, in which the Coulomb potential of the positive ions was the potential of interest. Since we at the Radiation Laboratory are more frequently interested in plasmas of aerodynamic than of thermonuclear origin, the assumption of complete ionization is not useful for us, and we must consider the effect of electron encounters with neutral atoms too. (Since the orbital electron distribution of the atom is extended in space, a free plasma electron may penetrate it and thus "see" an incompletely shielded nuclear charge.) For comparison with other work, we choose the neutrals to be oxygen atoms.

Since the work reported here is partially, but certainly not entirely, original in approach and technique, a brief review of the literature and its relation to the present work seems in order.

An exact non-relativistic Bremsstrahlung cross section for pure Coulomb fields was given by Sommerfeld (Ref. 1). The result is in terms of hypergeometric functions, the numerical approximation of which in various cases has been discussed by many authors. Since we will consider the effect of Debye shielding on the Coulomb fields of the ions, and the atomic fields are also shielded, we cannot use the Sommerfeld result. The Born approximation has been applied to the shielded Coulomb potential by Dewitt (Refs. 2, 3), who discusses its limits of validity quite carefully. We follow him in its use for the faster part of the electron velocity distribution in electron-ion scattering. For the slower electrons we use a classical impulse approximation with the shielded potential; this has been used for the pure Coulomb potential by, for example, Roberts (Ref. 4). Bremsstrahlung from neutrals is here treated by matching a Thomas-Fermi potential roughly to a screened Coulomb form, and using the Born approximation for the faster electrons, neglecting the slower ones. This result compares reasonably with the "effective Z" obtained by Breen and Nardone (Ref. 5) for free-free absorption by oxygen atoms, using machine wave functions. Finally, the idea of using detailed balance to obtain the free-free absorption coefficient from the Bremsstrahlung cross section is hardly original (see, e.g., Reference 3); only the use of our cross sections and the detailed evaluation is new.

II. Electron-Ion Bremsstrahlung from the Faster Electrons

Since the potential here, because of Debye screening, will depend on electron density n_e and temperature T, we must pick specific values of these parameters. Such values, fairly realistic for an aerodynamic plasma of possible practical interest, are

$$n_e = 10^{13}/cm^3$$
$$T = 5 \times 10^3 \, {}^oK \quad .$$

We take, as an example, the plasma to be composed of oxygen atoms, O^+ ions, and electrons. The validity of many of the approximations will depend on the choice of radiation frequency investigated. We are most interested in microwave frequencies near the plasma frequency, which is roughly 28 KMc. The computations will therefore be performed at

$$\nu = 50 \text{ KMc} \quad .$$

They will subsequently be extended to 15, 35, and 125 KMc, to obtain some idea of frequency variation in this region.

In this section we refer extensively to the work of Dewitt (Refs. 2, 3), who has applied the Born approximation to the Debye-screened Coulomb potential to treat Bremsstrahlung in a fully-ionized gas. He has investigated the validity of this approximation in detail. Let us refer to incident and scattered quantities by the subscripts 1 and 2. It is useful to talk in terms of $n_{1,2} = \frac{Ze^2}{\hbar v_{1,2}}$. For the pure Coulomb field the Sommerfeld exact Bremsstrahlung cross section may be expanded in powers of $n_2 - n_1$; the first term of this expansion is the Born approximation result. Thus for the pure Coulomb field, the Born approximation describes well the situation of low-frequency radiation ($v_1 \sim v_2$) from not-too-slow electrons. That is, $n_1 >> 1$ is permissable as long as $n_2 - n_1 << 1$. For a screened potential, although there is no exact solution to compare it with, the Born approximation is shown by Dewitt to be even better for the low-frequency part of the spectrum than for the unscreened one, i. e. it is valid for smaller v_1. Since we note that for $v_1 = \sqrt{\frac{kT}{m}}$ and $\nu = 50$ KMc, $n_1 \sim 8$, and $n_2 - n_1 \sim 10^{-4}$, we shall use it down to this value of v_1. For faster electrons, it is of course even better, but we restrict ourselves to non-relativistic electrons, naturally. These limits of validity are quite crude, but since the Born approximation usually works better than it should, we shall use them.

In what follows we use the following notation:

P = momentum in energy units = mvc

K = photon energy = $h\nu$

$\mu = mc^2$

$r_o = e^2/mc^2$

$\alpha = e^2/\hbar c$

$\sigma_o = \frac{16}{3} \alpha Z^2 r_o^2 \mu^2 / P_1^2$

$\gamma = \hbar c/\lambda$

λ, the Debye length = $\sqrt{\frac{kT}{4\pi n_e e^2}}$.

In terms of these, the Born differential Bremsstrahlung cross section may be written

$$d\sigma(K, P_1) = \sigma_0 \frac{dK}{K} \left\{ \frac{1}{2} \ln \frac{(P_1+P_2)^2+\gamma^2}{(P_1-P_2)^2+\gamma^2} - \frac{2\gamma^2 P_1 P_2}{\left[(P_1+P_2)^2+\gamma^2\right]\left[(P_1-P_2)^2+\gamma^2\right]} \right\} \tag{2.1}$$

in which, of course, P_2 is to be eliminated by conservation of energy,

$$P_1^2 - P_2^2 = 2K\mu \ . \tag{2.2}$$

Since the power is obtained by multiplying the cross section by the incident flux and photon energy and ion density n_i, and integrating over the electron velocity distribution, the contribution from this velocity range to the power/unit volume/ circular frequency interval may be written, where $Z = 1$ for the ions always, so $n_e = n_i$,

$$\underline{P}\, d\omega = n_e^2 \frac{16}{3} Z^2 e^6 \frac{4\pi}{m^4 c^5} \left(\frac{m}{2\pi kT}\right)^{3/2} d\omega \int_{c\sqrt{kTm}}^{\infty} dP_1\, P_1 e^{-P_1^2/2\mu kT}$$

$$\cdot \left\{ \frac{1}{2} \ln \frac{(P_1+P_2)^2+\gamma^2}{(P_1-P_2)^2+\gamma^2} - \frac{2\gamma^2 P_1 P_2}{\left[(P_1+P_2)^2+\gamma^2\right]\left[(P_1-P_2)^2+\gamma^2\right]} \right\} . \tag{2.3}$$

It is not possible to introduce approximations to the bracketed term valid over the whole range of integration. $P_1 >> \gamma$ is always valid, and thus $(P_1+P_2) >> \gamma$. However, $P_1 - P_2 = \gamma$ at about $P_{1C} = 15c\sqrt{kTm}$, and since $P_1 - P_2 \sim K\mu/P_1$, $P_1 - P_2 < \gamma$ for greater P_1. Since most of the contribution to this integral comes from $P_1 < P_{1C}$, a fair approximation to the bracket is, if one is required, $\ln\left(\frac{2P_1^2}{K\mu}\right) - \frac{\gamma^2 P_1^2}{2(K\mu)^2}$.

Now for the very slowest electrons with $P_1^2 > 2K\mu$, the microwave radiation is the high-frequency limit of their spectrum. The Born approximation fails here for the pure Coulomb field, but the Sommerfeld solution is reproduced excellently when the Born approximation is modified by the Elwert factor. Although its use cannot clearly be justified for the screened potential, we might hope the screening is weak enough to approximate the Coulomb case, and that it would be correct order-of-magnitude.

The expression is

$$d\sigma_{B-E}(K, P_1) = \sigma_o \frac{dK}{K} \frac{P_1}{P_2} \frac{1-e^{-2\pi n_1}}{1-e^{-2\pi n_2}}$$

$$\left\{ \frac{1}{2} \ln \frac{(P_1+P_2)^2+\gamma^2}{(P_1-P_2)^2+\gamma^2} - \frac{2\gamma^2 P_1 P_2}{\left[(P_1+P_2)^2+\gamma^2\right]\left[(P_1-P_2)^2+\gamma^2\right]} \right\} . \qquad (2.4)$$

The contribution from the very lowest velocities is described by the limit here $P_2/P_1 \longrightarrow 0$, $P_1 >> \gamma$, or

$$d\sigma_{B-E}(K, P_1) \longrightarrow 2\sigma_o \frac{dK}{K} , \qquad (2.5)$$

which differs from the Kramers' result in having a factor of 2 rather than $\frac{\pi}{\sqrt{3}}$. These slowest electrons then contribute (again $Z = 1$ for O^+).

$$\underline{P}d\omega = n_e^2 \frac{32}{3} Z^2 e^6 \frac{4\pi}{m^4c^5} \left(\frac{m}{2\pi kT}\right)^{3/2} d\omega \int P\,dP\,e^{-P^2/2\mu kT} . \qquad (2.6)$$

This is valid for only a very small range, since for P_1 = even $2c\sqrt{2mK}$, $P_2/P_1 = \sqrt{3/2}$, which is certainly not the high-frequency limit.

We would like now a cross section valid for electron velocities between the very lowest and the thermal range. The classical impulse approximation will be used to furnish such a result. (In general, we expect that when the incident particle becomes too slow for a quantum-mechanical description of the scattering by the Born approximation, a classical description, in which the particle is regarded as having a definite orbit and is continuously subject to scattering forces, becomes increasingly valid.)

III. The Impulse Approximation

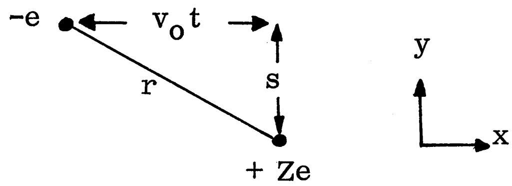
Ideally, an exact classical trajectory treatment should be used to fill in the gap here. However, this cannot be carried out, and we are forced to resort to the impulse approximation, which has been used elsewhere (Ref. 4) for Bremsstrahlung in a pure Coulomb field. Since a screened field causes less acceleration at large distances, the impulse approximation should be better for a given electron velocity here than for the Coulomb case.

Now in a classical treatment we obtain a particle trajectory which is a function of the impact parameter s and the initial velocity v_o. For fixed values of these, the radiated power from one electron in $d\omega$ is (Ref. 6)

$$P_{s,v_o} d\omega = \frac{8\pi}{3} \frac{e^2}{c^3} \left|\vec{a}(\omega)\right|^2 d\omega \quad , \tag{3.1}$$

where $\vec{a}(\omega)$ is the Fourier transform of the vector acceleration, and $\left|\vec{a}(\omega)\right|^2 = \left[a_x(\omega)\right]^2 + \left[a_y(\omega)\right]^2$. This is then multiplied by $n_i dn(v_o) v_o$ and averaged over annuli of radius s. Finally, of course, we average over that part of the electron velocity spectrum for which the expression is valid.

The impulse approximation consists in taking the acceleration which would be associated with an undeviated straight-line trajectory, i. e. acceleration but not displacement results from the presence of the scattering center, which is like the effect of an impulse. The geometry for the calculation is given by the following sketch:



Then

$$\left.\begin{aligned} r^2 &= s^2 + v_o^2 t^2 & \qquad x &= v_o t \quad , \quad y = s \\ \vec{a} &= -\frac{1}{m} \nabla V(r) & \qquad V(r) &= -Ze^2 \frac{e^{-r/\lambda}}{r} = -Ze^2 W \\ a_x &= -\frac{Zq^2}{m} \frac{x}{r} \frac{\partial W}{\partial r} & \qquad a_y &= -\frac{Zq^2}{m} \frac{y}{r} \frac{\partial W}{\partial r} \end{aligned}\right\} \tag{3.2}$$

and, where the bar here indicates a Fourier transform on a, or $a(\omega)$,

$$\bar{a}_x = -\frac{Ze^2 v_o}{2\pi m} \int_{-\infty}^{\infty} \frac{te^{i\omega t}}{r} \frac{\partial W}{\partial r} \, dt \, , \qquad \bar{a}_y = -\frac{Ze^2 s}{2\pi m} \int_{-\infty}^{\infty} \frac{e^{i\omega t}}{r} \frac{\partial W}{\partial r} \, dt \, . \tag{3.3}$$

Since $g(r) = -\frac{\partial W}{\partial r}$ is even in t, we have

$$\begin{aligned} \bar{a}_x &= \frac{Ze^2 v_o i}{\pi m} \int_0^{\infty} t \sin \omega t \; g(r) \frac{dt}{r} \quad , \\ \bar{a}_y &= \frac{Ze^2 s}{\pi m} \int_0^{\infty} \cos \omega t \; g(r) \frac{dt}{r} \quad . \end{aligned} \tag{3.4}$$

Consider the integral in $\bar{a}_x$, which we will call $I(\omega)$. We change the integration variable to r from (3.2), and integrate once by parts, obtaining

$$I(\omega) = \frac{\omega}{v_o^3} \int_0^{\infty} \frac{\cos \frac{\omega}{v_o} \sqrt{r^2 - s^2} \; e^{-r/\lambda}}{\sqrt{r^2 - s^2}} \, dr \tag{3.5}$$

and with $r = s \cosh t$, we have finally

$$I(\omega) = \frac{\omega}{v_o^3} \int_0^\infty \cos\left[\frac{\omega s}{v_o} \sinh t\right] e^{-s/\lambda \cosh t} \, dt$$

$$= \frac{\omega}{v_o^3} K_o \left[\sqrt{\left(\frac{s}{\lambda}\right)^2 + \left(\frac{s\omega}{v_o}\right)^2} \right] , \qquad (3.6)$$

in which K is a modified Hankel function. Then

$$\bar{a}_x = \frac{i\, Ze^2 \omega}{\pi m v_o^2} K_o \left[\sqrt{\left(\frac{s}{\lambda}\right)^2 + \left(\frac{s\omega}{v_o}\right)^2} \right] . \qquad (3.7)$$

But

$$\frac{\partial \bar{a}_y}{\partial \omega} = - \frac{Ze^2 s}{\pi m} I(\omega) , \qquad (3.8)$$

so

$$\bar{a}_y = \frac{Ze^2 s}{\pi m} \left[- \int_0^\omega I(x)\, dx + \int_0^\infty g(r) \frac{dt}{r} \right] . \qquad (3.9)$$

Now consider

$$J(\omega) = \int_0^\omega I(x)\, dx = \frac{1}{v_o^3} \int_0^\omega x K_o \left[\sqrt{a^2 + b^2 x^2} \right] dx , \qquad (3.10)$$

where

$$a = \frac{s}{\lambda} , \quad b = \frac{s}{v_o}$$

$$= \frac{1}{b^2 v_o^3} \int_a^{\sqrt{a^2+b^2\omega^2}} u K_o(u)\, du$$

$$= \frac{1}{b^2 v_o^3} \left[a K_1(a) - \sqrt{a^2+b^2\omega^2}\; K_1 \left(\sqrt{a^2+b^2\omega^2} \right) \right] . \qquad (3.11)$$

We now need only

$$F = \int_0^{\infty} g(r) \frac{dt}{r} = \frac{1}{v_o} \int_s^{\infty} \frac{g(r)dr}{\sqrt{r^2-s^2}} ,$$

which with the substitution r = s cosh t becomes

$$F = \frac{1}{v_o s^2} \int_0^{\infty} e^{-a \cosh t} \left[\frac{1}{\cosh^2 t} + \frac{a}{\cosh t} \right] dt . \qquad (3.12)$$

Consider the integral here, which is a function of a only; call it f(a). Then

$$f'(a) = -a \int_0^{\infty} e^{-a \cosh t} dt = -a K_o(a) \qquad (3.13)$$

so that

$$f(a) = -\int_{\infty}^{a} x K_o(x)\, dx - \lim_{x \to \infty} x K_1(x) = a K_1(a) \qquad (3.14)$$

and

$$F = \frac{aK_1(a)}{v_o s^2} . \qquad (3.15)$$

Then

$$\bar{a}_y = \frac{Ze^2}{\pi m v_o} \sqrt{(\frac{1}{\lambda})^2 + (\frac{\omega}{v_o})^2} \; K_1 \left[s \sqrt{(\frac{1}{\lambda})^2 + (\frac{\omega}{v_o})^2} \right] \qquad (3.16)$$

$$\bar{a}_x = \frac{i Ze^2 \omega}{\pi m v_o^2} K_o \left[s \sqrt{(\frac{1}{\lambda})^2 + (\frac{\omega}{v_o})^2} \right] . \qquad (3.17)$$

It should be noted that for $\lambda \to \infty$ (no shielding), these agree with the corresponding quantities deduced by Roberts (Ref. 4). Then we have for the quantity $|\vec{a}(\omega)|^2$, which we may call A^2,

$$A^2 = C^2 \left[\delta^2 K_1^2(\delta s) - \frac{\omega^2}{v_o^2} K_o^2(\delta s) \right] \qquad (3.18)$$

with

$$C = \frac{Ze^2}{\pi m v_o} , \qquad \delta^2 = (\frac{1}{\lambda})^2 + (\frac{\omega}{v_o})^2 .$$

Next we want to average over impact parameter s. We have no improvement to suggest over the usual procedure of taking the lower limit at $s_{min} = \lambda_{DeBroglie}$, the distance within which the electron cannot be localized, so that it makes no sense to talk about closer approaches to the nucleus. Then we must evaluate the integral

$$B = 2\pi \int_{\epsilon}^{\infty} s\, ds\; C^2 \left[\delta^2 K_1^2(\delta s) - \frac{\omega^2}{v_o^2} K_o^2(\delta s) \right]$$

$$= 2\pi C^2 \left[\int_x^{\infty} + K_1^2(t)\, dt - \frac{\omega^2}{\delta^2 v_o^2} \int_x^{\infty} t\, K_o^2(t)\, dt \right] \qquad (3.19)$$

in which we have used the symbol ϵ for $\lambda_{DeBroglie}$ to eliminate confusion with the screening radius, and $x = \delta \epsilon$. Let the first of these integrals be designated F(x), the second G(x). Now an integration by parts shows that

$$F(x) = x K_o(x) K_1(x) - G(x) , \qquad (3.20)$$

so we can concentrate on the second integral. As may be verified by differentiation, this is simply

$$G(x) = \frac{x^2}{2}\left[K_1^2(x) - K_o^2(x)\right] + \lim_{x\to\infty}\frac{x^2}{2}\left[K_o^2(x) - K_1^2(x)\right] = \frac{x^2}{2}\left[K_1^2(x) - K_o^2(x)\right]. \tag{3.21}$$

Then

$$B = 2\pi C^2\left[\delta\epsilon K_o(\delta\epsilon)K_1(\delta\epsilon) - \frac{(\delta\epsilon)^2}{2}\left\{K_1^2(\delta\epsilon) - K_o^2(\delta\epsilon)\right\}\right.$$
$$\left. - \frac{\omega^2}{2v_o^2}\epsilon^2\left\{K_1^2(\delta\epsilon) - K_o^2(\delta\epsilon)\right\}\right] \tag{3.22}$$

and

$$\frac{dP_{v_o}(\omega)}{d\omega} = \frac{8\pi}{3}\frac{e^2}{c^3} n_i v_o dn_e(v_o) B = \frac{16}{3}\frac{Z^2 e^6}{m^2 v_o c^3} n_i dn_e(v_o) \tag{3.23}$$

$$\left[\eta K\ \eta K_o(\eta)K_1(\eta) - \frac{\eta^2}{2}\left\{K_1^2(\eta) - K_o^2(\eta)\right\} - \frac{1}{2}\left(\frac{\omega\epsilon}{v_o}\right)^2\left\{K_1^2(\eta) - K_o^2(\eta)\right\}\right],$$

in which

$$\eta = \sqrt{\left(\frac{\epsilon}{\lambda}\right)^2 + \left(\frac{\omega}{v_o}\right)^2} \quad \text{and} \quad \epsilon = \lambda_{DeBroglie} = \frac{\hbar}{m v_o}.$$

In order to apply this result, we need a criterion for the validity of the classical description. This can be obtained in rough form by following Bohm (Ref. 7). We require that the size of a wave packet representing the electron be $\lesssim$ the impact parameter, and that the momentum uncertainty involved in forming this packet be much smaller than that transferred during the collision. The impulse approximation should be better for the shielded than the pure Coulomb potential, so we will combine it with the criterion of Bohm, which is

$$\frac{2s^2}{\hbar v} \int_{-\infty}^{\infty} F(r) \frac{dx}{r} >> 1 \quad . \tag{3.24}$$

Inserting the impulse approximation for the trajectory into (3.24) leads to an integral previously evaluated, and yields ($Z = 1$ for us)

$$\frac{4 Ze^2 s}{\hbar v \lambda} K_1 \left(\frac{s}{\lambda}\right) >> 1 \quad . \tag{3.25}$$

Thus, a limiting impact parameter is determined as a function of n_1 or v_1. For $v_{threshold} = \sqrt{\frac{2K}{m}}$, (3.25) is satisfied out to $s/\lambda = 5$ or 6, which should include most of the effect of the potential, while for $v = \sqrt{\frac{kT}{m}}$, $s/\lambda \sim 2.5$ is the limit. However, computation in both cases shows that integrating out to $s = \infty$ is justified because of the rapid decrease of the K functions. Incidentally, another requirement that the classical description be valid for this potential is that the relative variation of the potential over the size of the equivalent wave packet be small. That is,

$$\lambda_{DeBroglie} \frac{\frac{\partial V}{\partial r}}{V} < 1 \quad , \tag{3.26}$$

or

$$\frac{\hbar}{m v_o} \left(\frac{1}{r} + \frac{1}{\lambda}\right) < 1 \quad . \tag{3.27}$$

Since $r \gg s \gg \lambda_{DeBroglie}$ always in this description, and $\frac{\lambda_{DeBroglie}}{\lambda} < .01$, this will be satisfactory in general.

The contribution from the range $\sqrt{\frac{2K}{m}} < v \leq \sqrt{\frac{kT}{m}}$ should then be given by

$$\underline{P}\, d\omega = \frac{16}{3}\, n_e^2\, \frac{Z^2 e^6 d\omega}{m^4\, c^5}\, 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_{\sqrt{2K\mu}}^{\sqrt{kT\mu}} PdPe^{-P^2/2\mu kT}$$

$$\left\{ \sqrt{\left(\frac{\epsilon}{\lambda}\right)^2 + \left(\frac{\mu\omega\epsilon}{Pc}\right)^2}\, K_o(\sqrt{})\, K_1(\sqrt{}) - \frac{1}{2} \left[\left(\frac{\epsilon}{\lambda}\right)^2 + 2\left(\frac{\mu\omega\epsilon}{Pc}\right)^2\right]\left[K_1^2(\sqrt{}) - K_o^2(\sqrt{})\right]\right\}. \tag{3.28}$$

It may be noted that the ratio of the impulse approximation to the Kramers' result at the lower limit is approximately 1/7. At the upper limit, the agreement with the Born approximation result is much better, the ratio being ~ 0.83. Since the faster electron is deviated less from its trajectory, the superior agreement at the upper end may be interpreted as resulting from better validity of the impulse approximation, and is thus in agreement with expectation. It may also be noted that Dewitt gives a "classical low-frequency" expression (eq. 18 of Ref. 3) valid for weak shielding. This is applicable to only a very narrow velocity range, just about $\sqrt{\frac{kT}{m}}$, for our ω. At that limit, the ratio of the impulse cross section to his result is 1.31, which is quite reasonable agreement.

IV. Electron-Neutral Bremsstrahlung

As stated in the introduction, we base our calculations here on a screening radius derived from the Thomas-Fermi atom. While this is admittedly not very good for Z as small as 8, it is hoped that the model is still more physical than that of Nedelsky's (Ref. 8) frequently-quoted paper, which uses the potential

$$\left.\begin{aligned} V(r) &= \frac{Ze^2}{a} - \frac{Ze^2}{r} , \qquad r < a \\ &= 0 , \qquad r > a \end{aligned}\right\} \tag{4.1}$$

and must determine Z and a by recourse to experiment.

We therefore take for the potential seen by an incident electron

$$V(r) = -\frac{Ze^2}{r}\,\phi(r) , \tag{4.2}$$

with ϕ a solution of

$$x^{1/2}\frac{d^2\phi}{dx^2} = \phi^{3/2} , \tag{4.3}$$

in which $x = r/b$, $b = \frac{0.885\, a_o}{Z^{1/3}}$, and a_o is the Bohr radius, $\hbar^2/me^2$. $\phi(x)$ has been tabulated by Bush and Caldwell (Ref. 9). It may be fitted quite well by $e^{-r/\lambda}$, with $\lambda = 1.33\, b = 3.13 \times 10^{-9}$ cm for O. We give here a plot of $V(x) = -\frac{e^{-bx/\lambda}}{x}$ and $\frac{1}{x}\phi(x)$.

Again, the Born approximation cross section may be used for $P > c\sqrt{mkT}$, but different approximations are permitted with the stronger screening here. We write, then, for this part of the electron velocity distribution,

$$P_1 + P_2 \sim 2P_1 \qquad \text{and} \qquad P_1 - P_2 = \frac{P_1^2 - P_2^2}{P_1 + P_2} \sim \frac{\mu K}{P_1} \tag{4.4}$$

so that

$$\underline{P}\, d\omega = n_n\, n_e\, \frac{16}{3}\, e^6\, Z^2\, \frac{4\pi}{m^4 c^5} \left(\frac{m}{2\pi kT} \right)^{3/2} d\omega \int_{c\sqrt{mkT}}^{\infty} d\,P\,P\,e^{-P^2/2\mu kT}$$

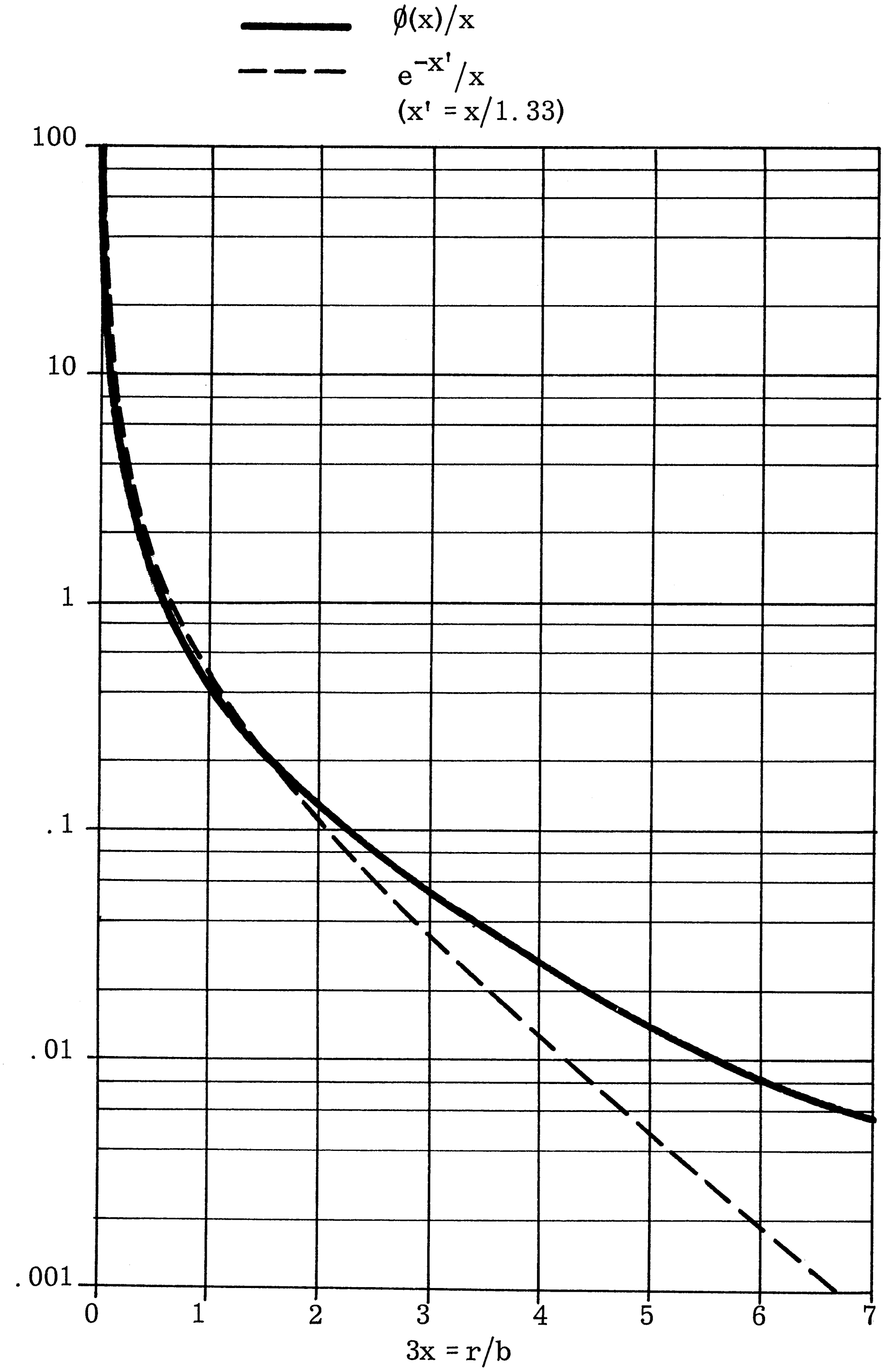
$$\left\{ \frac{1}{2}\, \ln \left(1 + \frac{4P^2}{\gamma^2} \right) - \frac{2P^2}{\gamma^2 (1 + 4P^2/\gamma^2)} \right\} . \tag{4.5}$$

Although, because the strong screening makes this case much different from the pure Coulomb field, we cannot justify the Born-Elwert result, we may use it to get some indication of the contribution of the slowest electrons. We need now the limit $P_2/P_1 \longrightarrow 0$, $P_1 << \gamma$, and find

$$d\sigma(K) = 2\sigma_o \, \frac{dK}{K} \left(\frac{P_1}{\gamma} \right)^4 , \tag{4.6}$$

which because of the shielding is quite a small result compared to the contribution from the thermal range.

The classical impulse approximation is found not to be valid for neutrals, and no method has been found for treating the contribution of the intermediate part of the electron velocity distribution. However, there is no physical reason to expect any special phenomena to characterize this range, so that we still expect the contribution to the total power to be given essentially by (4. 5); the power rising with velocity from that given by (4. 6) to this value.



COMPARISON OF THOMAS-FERMI AND SCREENED COULOMB FORMS OUT TO 7 "ATOMIC RADII"

In fact, the extreme screening limit $P_1 - P_2 << \gamma$ of the Born approximation holds up to velocities of the order of $\sqrt{\frac{kT}{m}}$. Since in this limit the Born approximation has the same form, $\sigma_o \frac{dK}{K} \, 4(P/\gamma)^4$, as the Born-Elwert limit, differing only by the factor of 2, it seems plausible that a correct cross section in this velocity region $\sqrt{\frac{2K}{m}} < v < \sqrt{\frac{kT}{m}}$ might have this form. In this case the ratio of power/cm^3 in equal frequency ranges from thermal electrons to that from slow ones is essentially

$$\frac{P_{therm}}{P_{slow}} \sim \frac{v_{th}}{v_{sl}} \, e^{-\frac{mv_{th}^2}{2kT}} \left(\frac{v_{th}}{v_{sl}}\right)^4 , >> 1 \qquad (4.7)$$

so the slowest electrons can be neglected with respect to those at $v = \sqrt{\frac{kT}{m}}$. Likewise, the high-energy tail $P_1 \gtrsim \gamma$ contributes little, and in fact, the integrand of (4.5) peaks near $v_M = \sqrt{\frac{kT}{m}}$, dropping off faster for $v > v_M$ than for $v < v_M$, but such that (4.5) does give the essential contribution.

V. Evaluation of Results, Extension to Other Frequencies

The integration indicated in (4.5) for the Bremsstrahlung power from neutrals may be carried out, leading to a closed form. Consider

$$I_1 = \int_{\sqrt{\mu kT}}^{\infty} P dP e^{-P^2/2\mu kT} \left\{ \frac{1}{2} \ln\left(1 + 4\frac{P^2}{\gamma^2}\right) - \frac{2P^2}{\gamma^2(1+4P^2/\gamma^2)} \right\} . \tag{5.1}$$

With $y = P^2/2\mu kT$, $\beta = 8\mu kT/\gamma^2$, we find

$$I_1 = \frac{1}{2} \mu kT \int_{Y_2}^{\infty} e^{-y} \left\{ \ln(1+\beta y) - \frac{\beta y}{1+\beta y} \right\} dy , \tag{5.2}$$

and two integrations by parts yield finally

$$I_1 = \frac{1}{2} \mu kT \left\{ e^{-1/2} \left[\ln(1+\beta/2) - 1 \right] - e^{1/\beta} (1+\beta)\, Ei\left(-\frac{1}{2} - \frac{1}{\beta}\right) \right\} . \tag{5.3}$$

Here Ei is the exponential integral, defined by

$$- Ei(-x) = \int_x^{\infty} \frac{e^{-t}}{t} dt . \tag{5.4}$$

Numerically,

$$I_1 = 4.65 \times 10^{-22} \text{ erg}^2 \tag{5.5}$$

and (4.5) becomes, for the power/cm^3 in $d\omega$ from neutrals at 50 KMc,

$$\underline{P}\, d\omega = n_n n_e \frac{16}{3} e^6 Z^2 \frac{4\pi}{m^4 c^5} \left(\frac{m}{2\pi kT}\right)^{3/2} \times 4.65 \times 10^{-22} d\omega \text{ ergs/cm}^3 \text{ sec}$$

$$= n_n\, d\omega\, 4.45 \times 10^{-29} \text{ ergs/sec cm}^3 , \quad n_n \text{ being given in cm}^{-3}, \tag{5.6}$$

and $d\omega$ in sec^{-1}. $Z = 8$ has been taken.

For the ion Bremsstrahlung, the higher velocity electrons yielded the power expression (with $n_i = n_e$)

$$\underline{P}d\omega = n_e^2 \frac{16}{3} Z^2 e^6 \frac{4\pi}{m^4c^5} \left(\frac{m}{2\pi kT}\right)^{3/2} d\omega$$
$$\int_{\sqrt{\mu kT}}^{\infty} \mathbf{P} d\mathbf{P} e^{-\mathbf{P}^2/2\mu kT} \left\{ \ln\left(\frac{2\mathbf{P}^2}{k\mu}\right) - \frac{\gamma^2 \mathbf{P}^2}{2(K\mu)^2} \right\}. \qquad (5.7)$$

The integral I_2 appearing here may also be evaluated in closed form, and yields, with $\delta = (4kT/K)$ and $\epsilon = \frac{\gamma^2 kT}{K^2 \mu}$,

$$I_2 = \mu kT \left[e^{-1/2} \left\{ \ln \delta/2 - 3\frac{\epsilon}{2} \right\} - Ei\,(-\tfrac{1}{2}) \right] = 3.01 \times 10^{-18} \text{erg}^2, \qquad (5.8)$$

so that, with Z = 1 of course, and $n_e = 10^{13}/cm^3$,

$$\underline{P}\, d\omega = 4.5 \times 10^{-14}\, d\omega \text{ erg/cm}^3 \text{ sec} . \qquad (5.9)$$

The integral arising from the impulse approximation must be evaluated numerically. We find for it

$$I_3 = 1.24 \times 10^{-18} \text{ erg}^2 , \qquad (5.10)$$

so that the slower electrons contribute to the ion Bremsstrahlung

$$\underline{P}\, d\omega = 1.83 \times 10^{-14}\, d\omega \text{ erg/cm}^3 \text{ sec} . \qquad (5.11)$$

Let us now try to extend these results to a few other frequencies. This will require investigation of the approximations upon which the validity of integrals, I_1, I_2 and I_3 is based, as well as those entering directly into the forms of their integrands.

First of all, it seems reasonable to take the Born approximation as valid for $n_2 - n_1 \sim 10^{-2}$. Then, if we use the same lower limit on **P** in I_1, and I_2 as before, we can use the Born approximation for ions and neutrals at frequencies such as 15 KMc and 35 KMc. Of course it is fine for higher frequencies, such as 125 KMc.

The criterion 3.25 for validity of a classical description (in the impulse approximation), is better satisfied at the low velocity end for lower frequencies, since the lowest $v \sim \sqrt{2h\nu/m}$, and we have the $1/v$ variation in 3.25. For 125 KMc, it is still satisfied well enough. The variation of potential argument 3.27 is likewise still reasonably good. Then the Born and impulse approximations will be used in the same velocity ranges as previously, for convenience in computation. (It is clear that new ranges of validity could be determined if desired, corresponding to the choices of ν .)

Now the changes in form of the integrals must be investigated. The impulse approximation integral I_3 is explicitly frequency-dependent and need simply be recomputed for the new values of ω. The manner in which the Born approximation has been approximated must, however, be examined.

We have the form

$$\sigma \propto \frac{1}{2} \ln \frac{(P_1+P_2)^2+\gamma^2}{(P_1-P_2)^2+\gamma^2} - \frac{2\gamma^2 P_1 P_2}{\left[(P_1+P_2)^2+\gamma^2\right]\left[(P_1-P_2)^2+\gamma^2\right]} \tag{5.12}$$

to approximate. Here

$$P_1^2 - P_2^2 = 2K\mu \;,\quad P_2 = \sqrt{P_1^2 - 2K\mu} \;,\quad (P_1+P_2)^2 = 2P_1^2 - 2K\mu$$

$$+ 2P_1^2\left(1 - \frac{K\mu}{P_1^2} - \frac{1}{2}\,\frac{K^2\mu^2}{P^4}\right) \sim 4P_1^2 - 4K\mu - \frac{K^2\mu^2}{P_1^2}$$

and

$$(P_1 - P_2)^2 \sim \frac{K^2\mu^2}{P_1^2} \quad .$$

For ions, $\frac{\gamma}{P_1} < \frac{\hbar c}{\lambda c \sqrt{mkT}} \sim 4 \times 10^{-4}$, $\frac{\sqrt{K\mu}}{P_1} \sim 4.67 \times 10^{-2}$

at 125 KMc and the same $P = P_{min}$, varying as $\nu^{1/2}$. Inserting the expansions into (5.11), we have

$$\sigma \propto \frac{1}{2}\ln\frac{4P_1^2 - 4K\mu - \frac{K^2\mu^2}{P_1^2} + \gamma^2}{\frac{K^2\mu^2}{P_1^2} + \gamma^2} - \frac{2\gamma^2 P_1^2\left(1 - \frac{K\mu}{P_1^2} - \frac{1}{2}\frac{K^2\mu^2}{P_1^4}\right)}{\left[4P_1^2 - 4K\mu - \frac{K^2\mu^2}{P_1^2} + \gamma^2\right]\left[\frac{K^2\mu^2}{P_1^2} + \gamma^2\right]} \qquad (5.13)$$

$$\longrightarrow \frac{1}{2}\ln\frac{1 - \frac{K\mu}{P_1^2} - \frac{1}{4}\left(\frac{K\mu}{P_1^2}\right)^2 + \frac{1}{4}\left(\frac{\gamma}{P_1^2}\right)^2}{\frac{1}{4}\left(\frac{K\mu}{P_1^2}\right)^2 + \frac{1}{4}\left(\frac{\gamma}{P_1}\right)^2} - \frac{\frac{1}{2}\gamma^2\left(1 - \frac{K\mu}{P_1^2} - \frac{1}{2}\frac{K^2\mu^2}{P_1^4}\right)}{\left[1 - \frac{K\mu}{P_1^2} - \frac{1}{4}\left(\frac{K\mu}{P_1^2}\right)^2 + \gamma^2\right]\left[\gamma^2 + \left(\gamma^2 + \left(\frac{K\mu}{P_1^2}\right)\right)\right]}$$

(5.14)

In view of the magnitudes of the parameters, as given, it is required to use a more detailed expansion than before, namely

$$\sigma \propto \frac{1}{2} \ln \frac{4P_1^4}{(K\mu)^2 + \gamma^2 P_1^2} - \frac{1}{2} \frac{1}{1 + \left(\frac{K\mu}{\gamma P_1}\right)^2} , \qquad (5.15)$$

which reduces only at higher frequencies and lower P_1 values to the old form, which was

$$\sigma \propto \ln \left(\frac{2P_1^2}{K\mu} \right) - \frac{1}{2} \left(\frac{\gamma P_1}{K\mu} \right)^2 . \qquad (5.16)$$

The integral I_2 must be recomputed with this form for the new frequencies.

Now let us look at I_1, the Born approximation integral for the neutrals. Now γ is very large, because of the severe screening. We have

$\left(\frac{P_1}{\gamma}\right)^2 \sim 5.5 \times 10^{-3}$ at the lowest P_1

$\frac{K\mu}{\gamma^2} < 6.5 \times 10^{-6}$, the value at $\nu = 125$ KMc.

This leads to

$$\sigma \propto \frac{1}{2} \left[\ln \left(1 + \frac{4P_1^2}{\gamma^2} \right) - \frac{4P_1^2/\gamma^2}{1 + 4P_1^2/\gamma^2} \right] . \qquad (5.17)$$

However, more care must be exercised before accepting this. Since $\left(\frac{P_1}{\gamma}\right)^2$ is small, the bracketed quantity may be expanded as

$$\frac{4P_1^2}{\gamma^2} - 8(\frac{P_1}{\gamma})^4 - \frac{4P_1^2}{\gamma^2}\,(1 - \frac{4P_1^2}{\gamma^2}) \longrightarrow 8(\frac{P_1}{\gamma})^4 \tag{5.18}$$

so that lowest-order terms vanish, leaving those of order $(P_1/\gamma)^4$. Then we must be sure that terms in $K\mu/\gamma^2$ have not been neglected; they are of the same order. We then write

$$\sigma \propto \frac{1}{2}\ \ln\left(\frac{4P_1^2 - 4K\mu + \gamma^2}{\gamma^2}\right) - \frac{2\gamma^2\, P_1^2(1 - \frac{K\mu}{P_1^2})}{\gamma^2\left[1 + 4(\frac{P_1}{\gamma})^2 - 4\frac{K\mu}{\gamma^2}\right]} \tag{5.19}$$

$$\longrightarrow \frac{1}{2}\left[\ln\left(1 + 4(\frac{P_1}{\gamma})^2 - 4\frac{K\mu}{\gamma^2}\right) - 4(\frac{P_1}{\gamma})^2\left(1 - \frac{K\mu}{P_1^2} - 4(\frac{P_1}{\gamma})^4 + 4\frac{K\mu}{\gamma^2}\right)\right] \tag{5.20}$$

$$\longrightarrow \frac{1}{2}\left[4(\frac{P_1}{\gamma})^2 - 4\frac{K\mu}{\gamma^2} - 8(\frac{P_1}{\gamma})^4 - 4(\frac{P_1}{\gamma})^2 + 4\frac{K\mu}{\gamma^2} + 16(\frac{P_1}{\gamma})^6 - 16\,P_1^2\,\frac{K\mu}{\gamma^4}\right] \tag{5.21}$$

$$\longrightarrow 4(\frac{P_1}{\gamma})^4\ ,\ \text{as in (5.18).}$$

Thus, the terms in $K\mu/\gamma^2$ cancel when those leading to them are kept, and by luck we may use the expansion (5.17), as previously. Therefore, the power for neutrals is frequency-independent in this range of ν .

The results of the re-computations are:

ν	=	15 KMc	35 KMc	50 KMc	125 KMc
I_2	=	3.04×10^{-18}	3.02×10^{-18}	3.00×10^{-18}	2.81×10^{-18}
I_3	=	1.48×10^{-18}	1.33×10^{-18}	1.25×10^{-18}	1.05×10^{-18}

Incidentally, the forms previously given for the contribution of the slowest electrons (the Born-Elwert limit) remain unaltered for both ion and electron Bremsstrahlung. Thus, finally, we conclude that with our approximations the ion Bremsstrahlung power varies only very slowly with frequency, and that from neutrals is frequency-independent, in the microwave range covered here.

VI. Use of Detailed Balance to Obtain Free-Free Absorption Coefficients from Bremsstrahlung Cross Sections

In order to assess the importance of Bremsstrahlung at frequencies above the plasma frequencies, a knowledge of the free-free absorption coefficient is required, since radiation is absorbed in the plasma by this process.

Now in Bremsstrahlung an electron of momentum $\mathbf{P}_1$ (in Heitler's units) releases a photon of energy K (in range dK), and itself ends up with momentum $\mathbf{P}_2$ (range $\mu/P_2\,dK$) such that $\mathbf{P}_1^2 - \mathbf{P}_2^2 = 2K\mu$, where $\mu = mc^2$. In free-free absorption, we can consider an electron of momentum $\mathbf{P}_2$ to absorb a photon of energy K to K + dK, so the electron ends up with $\mathbf{P}_1$ to $\mathbf{P}_1 + \mu/\mathbf{P}_1\,dK$. Thus these two processes are inverse to each other. We have obtained Bremsstrahlung cross sections previously and want the free-free absorption coefficients. The idea immediately presents itself to use the detailed balance theorem of statistical mechanics to relate these two quantities.

The detailed balance theorem requires that in equilibrium the probability of a transition from state 1 ⟶ state 2 of a closed system is equal to that from state 2 ⟶ state 1. (Note that this is detailed balance, a stronger statement than constancy of population of a state; the accounts balance between all pairs of states, not only in total income and outgo of population of each state.) Then, basically, the detailed balance principle requires that

$$\frac{\text{probability of Bremsstrahlung with } P_1 \rightarrow (P_2,\ P_2 + \frac{\mu}{P_2}\, dK),\ \text{photon with } (K,\ K+dK) \text{ emitted}}{\text{number of electron states in } \frac{\mu}{P_2}\, dK \text{ at } P_2 \times \text{number of photon states in } dK \text{ at } K}$$

$$= \frac{\text{probability of free-free absorption of photon } (K,\ K+dK),\ \text{electron has } P_2 \rightarrow (P_1, P_1 + \frac{\mu}{P_1}\, dK)}{\text{number of electron states in } \frac{\mu}{P_1}\, dK \text{ at } P_1} \qquad (6.1)$$

The denominators arise from the assumption that all degenerate states are equally probable within an energy level. The degeneracy exists because electron energy is a function of $|P|$, not $\vec{P}$, and is taken independent of spin, while photon direction of propagation and polarization do not affect the photon energy. Then, since the probability of entering any state varies inversely with the number of co-degenerate ones available to be entered, these numbers appear in the denominator. The quantities $\mu/P\, dK$ are the ranges dP corresponding to dK, obtained from conservation of energy, $P_1^2 - P_2^2 = 2\mu K$.

Let us consider all process to take place in a volume V, which will cancel as it should in the final result. Then

Number of electron states at P_1 in $dP_1 = \frac{\mu}{P_1}\, dK$ is $\quad \frac{8\pi P_1^2}{(hc)^3} \frac{\mu}{P_1}\, dK\, V \qquad (6.2)$

Number of electron states at P_2 in $dP_2 = \frac{\mu}{P_2}\, dK$ is $\quad \frac{8\pi P_2^2}{(hc)^3} \frac{\mu}{P_2}\, dK\, V \qquad (6.3)$

Number of photon states at K in dK is $\quad \frac{8\pi K^2 dK}{(hc)^3}\, V\, . \qquad (6.4)$

Next we want to relate the probabilities involved to the cross section of interest. There is a general relation between these two descriptions of a process. For consider a volume V in which there is a volume density n_t of target systems, capable of making the transition $1 \longrightarrow 2$ when struck by another type of particle, the latter being present in density n_R. Let $\sigma(1,2)$ be the cross section for the transition, and $\mathbf{P}(1,2)$ be the probability/unit time of the process.

By the definition of cross section, we say

$$\sigma(1,2) = \frac{\text{events/unit time/target system}}{\text{incident particles/unit area/unit time}} \tag{6.5}$$

$$= \frac{\mathbf{P}(1,2) \times \dfrac{\text{Number of target systems in V}}{\text{Number of target systems in V}}}{n_R \; v_R} \tag{6.6}$$

$$= \frac{\mathbf{P}(1,2)}{N_R/V \, v_R} \tag{6.7}$$

where there are N_R incident particles in V . So

$$\mathbf{P}(1,2) = \frac{N_R v_R \, \sigma(1,2)}{V} \quad . \tag{6.8}$$

Finally, choose V such that $N_R = 1$ or $V = (n_R)^{-1}$ (all results will be independent of V in the end). Then $\mathbf{P}(1,2) = v_R \, \sigma(1,2) / V$, which is the usual relation.

Then, for the Bremsstrahlung process $P_1 \longrightarrow P_2$,

$$P(P_1 \longrightarrow P_2) = \frac{v_1 \, \sigma_{Br} (P_1 , K) \, dK}{V} \tag{6.9}$$

(the dK occurs because $\sigma(K, P_1)$ obtained previously is a "cross section density" such that $\sigma(K, P_1)\, dK$ is really the cross section for emission by an electron with P_1 of a Bremsstrahlung photon in K to K + dK). But in the case of Bremsstrahlung it is the electrons which are the target systems, v_1 still being the relative velocity, while the ions are the bombarding particles. So we must choose $V = 1/n_i$ as the volume within which our conceptual processes occur, and

$$P(P_1 \longrightarrow P_2) = v_1 \, n_i \, \sigma_B \, (K, P_1) \, dK \quad . \tag{6.10}$$

For the free-free transitions, correspondingly,

$$P(P_2 \longrightarrow P_1) = \frac{c \, \sigma_{ff} \, (K, P_2) \, dK}{V} \quad . \tag{6.11}$$

Inserting all of these relations into the statement of detailed balance, we obtain

$$\frac{\sigma_B(K_1P_1) \, dK \, n_i v_1}{V \frac{8\pi P_2^2}{(hc)^3} \, \frac{\mu}{P_2} \, dK \; V \frac{8\pi \, K^2 dK}{(hc)^3}} = \frac{c \, \sigma_{ff} (P_2 , K) \, dK}{V \times V \frac{8\pi P_1^2}{(hc)^3} \, \frac{\mu}{P_1} \, dK} , \tag{6.12}$$

whence

$$\sigma_{ff} (P_2 , K) \, dK = \frac{\sigma_B (K, P_1) \, n_i \, v_1 (hc)^3}{8\pi \, K^2 c} \left(\frac{P_1}{P_2} \right) , \tag{6.13}$$

in which $\sigma_{ff}(P_2, K)\,dK$ is, as with Bremsstrahlung, to be interpreted as the absorption cross section. (We will use the symbol σ for true absorption cross section from now on, dropping the dK).

We may write $\sigma_B(K, P_1) = \sigma_o \frac{\mu^2}{P_1^2} K^{-1} F(K, P_1, \lambda)$ for those cases in which the Born approximation is valid, in which F is the quantity which appeared in brackets previously and was approximated in various ways. Then, with $\sigma_o = 16/3\, Z^2 \alpha r_o^2$ as before,

$$\sigma_{ff}(P_2, \omega) = \frac{\sigma_o \mu n_i}{P_2} \left(\frac{c}{\omega}\right)^3 (2\pi)^3 F(K, P_1, \lambda) . \qquad (6.14)$$

We may multiply by the electron velocity distribution $dn_e(P_2)$ and integrate over P_2 to obtain the absorption coefficient

$$\alpha(\omega) = n_e n_i \left(\frac{16}{3} Z^2 \alpha r_o^2 \pi^2\right) \left(\frac{c}{\omega}\right)^3 \int_0^\infty \frac{4\mu}{\sqrt{\pi}} - \frac{P_2 dP_2 e^{-P_2^2/s^2}}{s^3} F(\omega, P_1, P_2) \qquad (6.15)$$

where $s^2 = 2\mu kT$.

When the Born approximation is valid,

$$F = \left[\frac{1}{2} \ln \frac{(P_1+P_2)^2+\gamma^2}{(P_1-P_2)^2+\gamma^2} - \frac{2\gamma^2 P_1 P_2}{\left[(P_1+P_2)^2+\gamma^2\right]\left[(P_1-P_2)^2+\gamma^2\right]}\right] , \qquad (6.16)$$

and in the integration, P_1 is to be expressed as a function of P_2 by conservation of energy, $P_1^2 = P_2^2 + 2\mu\hbar\omega$.

This is precisely the result obtained by Dewitt in Reference 3; thus this conclusion is not original. However, it was felt desirable for the sake of clarity to include a detailed derivation of (6.15) in this report.

In the case of electron-neutral Bremsstrahlung and absorption, it is clear that the only change in the formulation is to replace n_i by n_n, the density of neutrals. When the impulse approximation is used for electron-ion Bremsstrahlung, the quantity F is replaced by

$$F = \left\{ \sqrt{(\frac{\epsilon}{\lambda})^2 + (\frac{\mu\omega\epsilon}{Pc})^2}\; K_o(\sqrt{})\, K_1(\sqrt{}) - \frac{1}{2}\left[(\frac{\epsilon}{\lambda})^2 + 2(\frac{\mu\omega\epsilon}{Pc})^2\right]\left[K_1^2(\sqrt{}) - K_o^2(\sqrt{})\right]\right\} \tag{6.17}$$

which occurred in (3.28).

For practical purposes, we must again determine the range in P_2 of validity of various approximations, for a given choice of K or ω. For the neutrals, we find that the Born approximation which was used before for $P_1^2 > \mu kT$, correspondingly is used for $P_2^2 > \mu kT - 2K\mu$. It may be written

$$F = \frac{1}{2} \ln \frac{\left(P_2 + \sqrt{P_2^2 + 2K\mu}\right)^2 + \gamma^2}{\left(\sqrt{P_2^2 + 2K\mu} - P_2\right)^2 + \gamma^2} - \frac{1}{2} \frac{2P_2\sqrt{P_2^2 + 2K\mu}}{\left[(P_2 + \sqrt{})^2 + \gamma^2\right]\left[(\sqrt{} - P_2)^2 + \gamma^2\right]} \tag{6.18}$$

and again, in this range, approximated as

$$F \sim \frac{1}{2}\left[\ln(1 + \frac{4P_2^2}{\gamma^2}) - \frac{4P_2^2}{\gamma^2(1 + 4P_2^2/\gamma^2)}\right] \tag{6.19}$$

so that for the contribution of this part of the electron velocity spectrum to the absorption coefficient from neutrals we have

$$\alpha(\omega) = n_n n_e \left(\frac{16}{3} Z^2 \alpha r_o^2 \pi^2\right) \left(\frac{c}{\omega}\right)^3 \frac{4\mu}{\sqrt{\pi}} \left(\frac{1}{2\mu kT}\right)^{3/2} \times \frac{1}{2} \mu kT$$

$$\left\{ e^{\left(-\frac{1}{2} - \frac{\hbar\omega}{kT}\right)} \left[\ln\left(1 + \frac{4\mu kT}{\gamma^2} - \frac{8\mu\hbar\omega}{\gamma^2}\right) - 1\right] - e^{\frac{\gamma^2}{8\mu kT}} \left(1 + \frac{\gamma^2}{8\mu kT}\right) \mathrm{Ei}\left(-\frac{1}{2} + \frac{\hbar\omega}{kT} - \frac{\gamma^2}{8\mu kT}\right) \right\} \tag{6.20}$$

We note that, since $\hbar\omega/kT \ll 1$ and $\hbar\omega \ll \gamma^2/8\mu$ for the microwave frequencies we consider, the quantity in the large bracket is essentially frequency-independent and simplifies to the form used previously, and we do obtain in this range a Kramers-type absorption coefficient, $\sim 1/\omega^3$. The contribution of the lower part of the velocity spectrum would appear to be much smaller; in any case neglecting it yields a lower bound on $\alpha(\omega)$, which is of practical importance.

Now if Kramers' law really held for all P_2, we would have $F = \pi/\sqrt{3}$. We may define an effective Z by equating our $\alpha(\omega)$ with $Z = 8$ to that obtained from Kramers' law, in which the "effective Z" is assumed to appear. We had, evaluating (6.20)

$$\alpha(\omega) = n_n n_e Z^2 \frac{16}{3} \alpha r_o^2 \pi^2 \left(\frac{c}{\omega}\right)^3 \frac{4\mu}{\sqrt{\pi}} \frac{4.65 \times 10^{-22}}{(2\mu kT)^{3/2}} \qquad (6.21)$$

in which the numerical factor arising from the integration, containing μkT, has been explicitly evaluated rather than left as in (6.20). Now Kramers' law yields

$$\alpha(\omega) = n_n n_e Z^2 \frac{16}{3} \alpha r_o^2 \pi^2 \left(\frac{c}{\omega}\right)^3 \frac{4\mu}{\sqrt{\pi}} \frac{1}{(2\mu kT)^{3/2}} \int_0^\infty \frac{\pi}{\sqrt{3}} \mathbf{P} d\mathbf{P} e^{-\mathbf{P}^2/2\mu kT} \qquad (6.22)$$

Equating these two, regarding the Z in Kramers' law as Z effective, we obtain

$$Z \text{ effective} = 0.17 \ . \qquad (6.23)$$

This compares reasonably with the result of Breen and Nardone, Z $\sim$ .31 at 10,000 Å and 15,000 Å, for T = 8000°K. The higher frequency radiation they consider should result from electrons which see a greater Z.

Incidentally, the reasonable agreement of the effective Z with that of previous investigators may be regarded as a practical justification of the fitting of the potential for small r only. Although a formal analysis of the effect of underestimating the potential for r $>$7b , roughly, is difficult, it would appear that the potential has dropped off sufficiently by this radius so that, with the given velocity distribution, there are simply not enough scatterings giving rise to microwave Bremsstrahlung from large r to affect the power significantly. That is, since our effective Z differs from theirs by a factor of roughly 2, but should differ by some factor presumedly between 1 and 2, it would seem that a large error has not been made.

Now, for the ion contribution to free-free absorption, let us first look at the Born-approximation part. Here, where again P_2 runs from $\sqrt{\mu kT - 2K\mu}$ to ∞, we find that $F = \frac{1}{2} \ln \frac{4P_2^4}{(K\mu)^2 + \gamma^2 P_2^2} - \frac{1}{2} \frac{1}{1 + K^2\mu^2/P_2^2\gamma^2}$ is the correct approximation, and a new integration is required, the integral being a function of frequency.

For the classical part, since the function corresponding to F contains only a single momentum (classical scattering being a continuous process rather than a transition between states), we simply label this momentum $\mathbf{P}_2$. Rather than trying to derive a lower lower limit of integration, we note that the integrand drops off rapidly enough so that we ignore any contribution for smaller **P**. (That is, the integrand has dropped off so much by $\sqrt{2K\mu}$ that any extension to $\mathbf{P} = 0$ makes negligible difference.) Then this result stands unchanged, and must be integrated numerically as before.

In view of the fairly smooth match when the Born and impulse approximations were joined at $\mathbf{P} = \sqrt{\mu kT}$, little error is incurred by continuing to join them there, rather than at $\sqrt{\mu kT - 2K\mu}$, which procedure saves a new computation.

We then obtain, combining the previous numerical results, for the free-free absorption coefficient of electrons in the field of ions,

$$\alpha(\omega) = n_e n_i \frac{16}{3} \alpha r_o^2 \pi^2 \left(\frac{c}{\omega}\right)^3 \frac{4\mu \times 10^{-18}}{\sqrt{\pi}(2\mu kT)^{3/2}} \begin{cases} 4.52 & \text{at } 15 \text{ kmc} \\ 4.35 & \text{at } 35 \text{ kmc} \\ 4.26 & \text{at } 50 \text{ kmc} \\ 3.86 & \text{at } 125 \text{ kmc} \end{cases} \tag{6.21}$$

For that due to electrons in the field of neutrals, we had

$$\alpha(\omega) = n_n n_e Z^2 \frac{16}{3} \alpha r_o^2 \pi^2 \left(\frac{c}{\omega}\right)^3 \frac{4\mu}{\sqrt{\pi}(2\mu kT)^{3/2}} \times 4.65 \times 10^{-22} \qquad (6.22)$$

for all these frequencies. In both these results, all quantities are in cgs units, α in cm^{-1}.

For the low degree of ionization corresponding to T = 5000^{o}K in air, the absorption due to neutrals should dominate strongly that due to ions. While we need not concern ourselves with a specific value of n_n and n_i, which would point too directly to a specific aerodynamic situation, we may comment that this free-free absorption may be extremely severe at the frequencies discussed, for reasonable practical values of n_n.

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