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THEORETICAL AND EXPERIMENTAL ANALYSIS OF FLUID FLOW SEPARATION

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PREFACE

The problem of flow separation has been recognized as an important one for many years. In contrast to this importance there has been little work done in this area of fluid mechanics. One of the major reasons is the difficulty of formulating the problem so that it will be amenable to analysis. The flow behavior, in general, is complicated especially in the regions which lie downstream of separation. When the author initially undertook this study, it was believed that little work had been done on this particular class of flow geometries. Further investigation revealed the work of M. J. Lighthill. Still further study convinced the author that an extension and evaluation of Lighthill's work would be beneficial, as many questions remained unanswered. It is believed that the following work will give insight into the problem and the limitations of the analysis. There are many valuable areas of work still remaining untouched in the general problem of corner flows, as the following analysis treats the flow outside the separated region.

The completion of this work marks the end of a formal period of study for the author. There are many people to whom the author owes a great deal. Although space does not permit that everyone can be mentioned, a few deserve special recognition. The author owes much to his parents for guidance and assistance throughout his entire period of formal education. The author would like to thank Professor A. G. Hansen, his advisor, for his encouragement and help in all stages of this work. Special thanks must go to the author's wife and children for their loving devotion, patience, and sacrifice.

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NOMENCLATURE

Symbols used for potential solution

A, B, C, D, E, F	points in the flow field
a	vertical distance from corner to the reattachment point
b	distance along horizontal plate from separation point to corner
c	distance along horizontal plate from leading edge to separation point
h	vertical height of obstacle
i	$\sqrt{-1}$
$\bar{K}$	constant of integration
$\bar{L}$	constant of integration
K	complete elliptic integral $K = \int_0^{\pi/2} \frac{d\theta'}{\sqrt{1-k^2\sin^2\theta'}}$
K'	complete elliptic integral with modulus k'
k	modulus of elliptic integral $k = \sqrt{\frac{N}{M}}$
k'	complimentary modulus $k' = \sqrt{1-k^2}$
l	length of horizontal plate a + b
M	$\alpha - 1$
N	$\beta - 1$
P	constant
q	velocity magnitude
S	distance from B to C along the free streamline B-C
nS	distance from B to D along the line B-C-D
s	distance from point B to any point on the line B-C-D-E
t	transformation variable $w = \frac{UY}{\pi}\ell nt$
U	velocity of flow in upstream direction
u	velocity component in x direction

V	velocity along the free streamline B-C
v	velocity component in y direction
W	velocity along the free streamline D-E
w	complex potential $\phi + i\Psi$
X	$X = (\int_0^{nS} q ds)/nS$
x	coordinate direction along plate
x'	variable $x' = (t-1)^{1/2}$
Y	channel depth
y	coordinate direction normal to plate
y'	variable
Z	distance between points E and F in x-y coordinate plane
z	complex variable $z = x + iy$
z'	variable $\ell n\, q/V$
α, β, γ	points in t plane corresponding to points B, C, D respectively in x-y plane
$\lambda, \mu, \eta, \epsilon$	exterior angles in Ω plans
θ	angle of flow measured with respect to the positive x axis
θ'	variable $\sin^{-1} x'/\sqrt{N}$
Γ	angle of the obstacle measured with respect to the positive x axis
ϕ	potential function
Ψ	stream function
Ω	transformation variable $\ell n\, dw/dz$

Symbols used by Görtler in his series solution

F($\xi\eta$) function $F(\xi, \eta) = \dfrac{\Psi(x,y)}{\{2\nu\int_0^x U(x)dx\}^{1/2}}$

NOMENCLATURE (CONT'D)

$\overline{F}_k$	dimensionless function $F_k Re^k$
$\overline{g}(\overline{x})$	dimensionless function $\overline{U}(\overline{x})/\sqrt{2\overline{\xi}}(\overline{x})$
k	exponents $(k = 0, 1, 2, \ldots \infty)$
Re	Reynold's number $U_o h/\nu$
$U(x)$	velocity of outer flow
U_o	velocity at leading edge
$\overline{U}(\overline{x})$	dimensionless velocity $\overline{U}(\overline{x}) = \frac{u(x)}{U_o}$
u_k	coefficients of the polynomial $\sum_{k=o}^{\infty} u_k x^k$
$\overline{u}_k$	dimensionless coefficient $u_k h^k/U_o$
$\overline{x}$	dimensionless variable x/h
$\overline{y}$	dimensionless variable $yRe^{1/2}/h$
$\beta(\xi)$	function $\beta(\xi) = 2U'(x) \int_o^x U(x)dx/U^2(x)$
$\overline{\beta}_k$	dimensionless coefficient $\beta_k Re^k$
η	variable $\eta = U(x)y/\{2\nu\int_o^x U(x)dx\}^{1/2}$
$\overline{\eta}$	dimensionless variable $\overline{g}(\overline{x})\overline{y}$
ν	kinematic viscosity
ξ	variable $\xi = \frac{1}{\nu} \int_o^x U(x)dx$
$\overline{\xi}$	dimensionless variable ξ/Re

Symbols used by Witting in his finite difference method

A_r	$(r = 0,1 \ldots 5)$ constant coefficients
B_r	$(r = 0,1 \ldots 5)$ constant coefficients
h	dimension of lattice in x direction
i	position from startingpoint in x direction $(i = 0, \pm 1, \pm 2 \ldots h>0)$
k	position from plate surface in y direction $(k = 0, 1, 2 \ldots \ell>0)$

NOMENCLATURE (CONT'D)

ℓ	dimension of lattice in y direction
$L_{i,k}$	$\nabla_{i-1,0} - k/6\ (\nabla^2_{i+1,0} - \nabla^2_{i-1,0})$
$N_{i,k}$	see equation 81
p	pressure
$S_{i,k}$	$\sum_{r=1}^{k-1} \Delta_{i,r}$
$u_{i,k}$	velocity component in x direction located by $u(x_i,\ y_k)$
$\hat{u}_{i(y)}$	polynomial approximation to velocity in vicinity of plate surface
$v_{i,k}$	velocity component in y direction located by $v(x_i,\ y_k)$
x_i	x coordinate point at i th section
y_k	y coordinate point at k th line above plate surface
$Z_{i,k}$	see equation 82
$\Delta_{i,k}$	$u_{i+1,k} - u_{i-1,k}$
$\nabla_{i,k}$	$u_{i,k+1} - u_{i,k-1}$
$\nabla^2_{i,k}$	$\nabla_{i,k+1} - \nabla_{i,k-1}$
ζ	density

CHAPTER I

INTRODUCTION

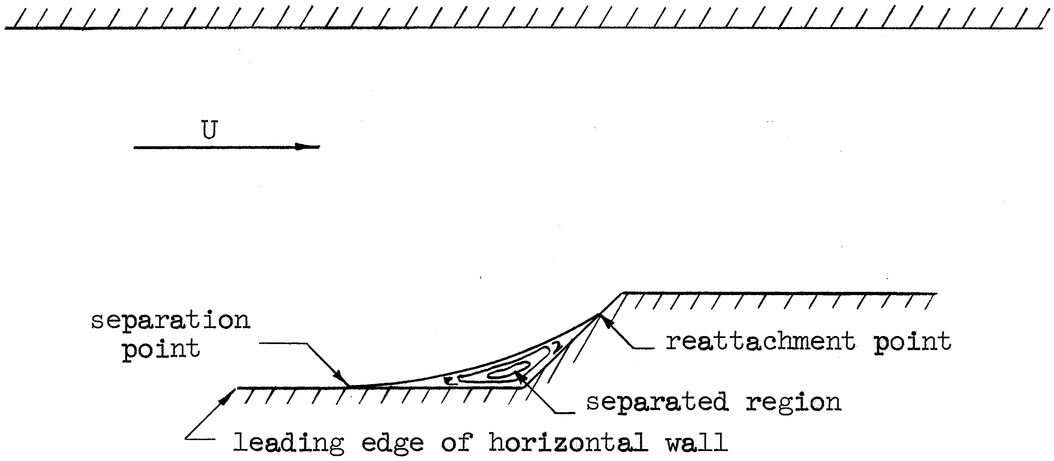
The study of fluid flow separation has been an important area of fluid mechanic research for a number of years. Whenever a moving fluid is contained, there exists the possibility that at some point in the flow separation may occur. When this happens, the normal flow behavior breaks down and the flow can become erratic and unstable. The fluid flow parameter which is generally used to quantitatively describe this behavior is the wall shear stress. Where the wall shear stress equals zero, the flow is said to have separated at that point. This fluid flow behavior is common to a majority of fluid handling devices. Directly or indirectly flow separation leads to loss of efficiency in turbomachines, increased drag on submerged objects, generation of noise in flow processes, and alteration of heat transfer characteristics between fluid and solid boundaries.

Some of the earlier investigations into this general problem dealt with the flow separation from the downstream surface of an infinite cylinder. Schubaurer(2) presented an analytical solution for the separting laminar boundary layer of an infinite cylinder. Von Kármán and Millikan(1) presented a mathematical discussion of the boundary layer equations with the view of facilitating the investigation of separation. Nikuradse(3) has studied flow in variable wall two dimensional converging and diverging channels. Abramowitz(4) extended this configuration by considering the backflow resulting from diverging channels. Because of confusion in the meaning of the term separation, Maskell(5) has defined the phenomenon in terms of a limiting streamline which lies infinitesimally close to the boundary surface before separation and "leaves" the surface at separation. With this definition it is possible to consider three dimensional separation in a broader and more meaningful sense.

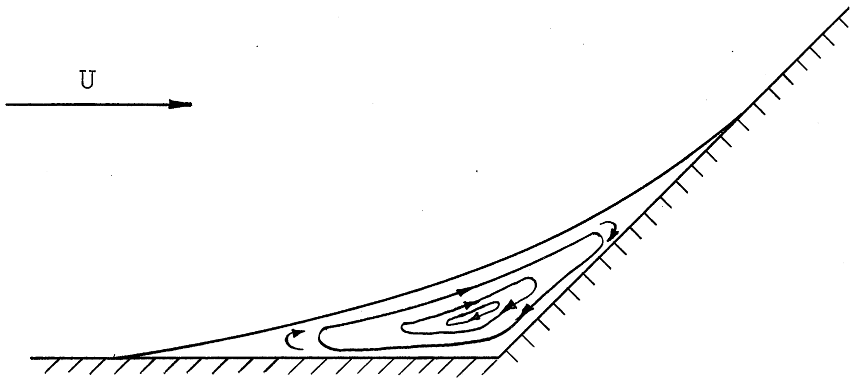
The problem here is to study a two-dimensional steady laminar fluid flow which is directed into a two-dimensional corner. The flow direction is normal to the line of intersection of the two walls making up the corner. In particular, the aim will be to predict the separation point of the fluid flow along the wall section which is parallel to the flow direction. As the flow proceeds into the corner it moves against an increasing pressure. This adverse pressure gradient causes a thickening of the boundary layer and will eventually cause separation. The usual methods of predicting separation require a knowledge of the mainstream pressure gradient in advance. The attempt is made to eliminate this step for simple configurations.

In regard to problems of corner flow with incompressible fluids, work has been done for the case of the mainstream flow being parallel to the wall intersection. References 6, 7, 8, and 9 deal with the flow in straight and curved ducts. References 10, 11, 12, 13, and 14 are concerned with flow parallel to the intersection of two semi-infinite walls. However, with the exception of the case of flow in curved ducts, where secondary flow plays an important part, none of the above cases will result in flow separation.

There are two possible avenues of approach to the problem under study. The flow configuration and behavior is shown below. As the flow proceeds along the duct toward the corner, it engages the leading edge



of the horizontal plate. The momentum change in the mainstream will be felt in the boundary layer which forms along the horizontal wall. The result is that the increasing pressure will cause a thickening of the boundary layer and separation results. The flow can be divided into two regimes - the outer flow where viscous action of the fluid flow is negligible, and the boundary layer and separated region where viscous action is very important. The outer flow moves over the horizontal plate and up and over the back wall. At the same time the fluid inside the boundary layer will decelerate and finally separate from the wall. The fluid inside the separated region will in general move in a vortex motion. This is experimentally verified by Reference 6 and the author's photographs (Figure 13). As mentioned, two approaches may be taken. One approach is to consider the outside flow and boundary layer as controlling the location of the separation point, and at the same time approximate the flow behavior inside the separated region so as to simplify the problem. Another approach is to try to describe the flow in the separated region and use this to predict the upstream separation point and the downstream reattachment point. The latter approach could be attacked by considering that the separated region is filled with one steadily flowing vortex, as shown.



First the stream function for the vortex region must be found. This must then be matched to the outside flow by considering that the velocities along the separation line must be equal. Batchelor(15) has considered this general type of problem, where he shows that the closed streamlines for steady laminar flow at large Reynold's numbers must satisfy Poisson's equation. He points out the difficulty that Poisson's equation must be matched at one boundary to a streamline whose shape and velocity distribution are unknown. Second the rotational speed and size of the vortex region must be found. To do this the viscosity of the fluid must be taken into account. One approach would be to solve the boundary layer equations for the flow adjacent to the walls. This assumes that the boundary layer equations will describe the flow accurately which is very doubtful, especially in the vicinity of the corner. It would probably be more realistic in an analysis of this type to make only minor simplifications in the