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Nuclear Physics B36 (1972) 436-444. North-Holland Publishing Company

DETERMINATION OF πN SIGMA TERM FROM PION NUCLEUS SCATTERING*

Shafik J. HAKIM **
Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan 48104

Received 27 July 1971

Abstract: We make use of the current algebra sum rules developed by Fubini and Furlan for pion-nucleon scattering to evaluate π^--nucleus scattering lengths. Assuming that the pion-nucleus sigma term Σ can be written as a coherent sum of pion-nucleon sigma terms S and using experimental data we determine S for each reaction tested averaging 22 ± 1 MeV. With S = 22 ± 1 we calculate π^--nucleus scattering lengths and our results indicate a good agreement with experimental data.

1. INTRODUCTION

Different estimates of the pion nucleon sigma commutator

$$S = -i \int \mathrm{d}^4x \exp[iq\cdot x] \{\langle \mathrm{N}|[\dot{A}_0^{\alpha}(-\tfrac{1}{2}x), A_0^{\beta}(\tfrac{1}{2}x)]|\mathrm{N}\rangle$$

$$- \langle \mathrm{N}|\mathrm{N}\rangle\langle 0|[\dot{A}_0^{\alpha}(-\tfrac{1}{2}x), A_0^{\beta}(\tfrac{1}{2}x)]|0\rangle\}\, \delta(x_0)\,,$$

where q_μ is the pion four momentum, A^α is the axial vector current (with an SU(3) label α and $|\mathrm{N}\rangle$ is the nucleon state vector) have been recently given [1-4] using meson baryon scattering length data and pion nucleon phase shift data. In this work we report a new method to determine this parameter using π^--nucleus scattering data.

The π^--nucleus scattering length can be obtained using the extrapolation program developed by Fubini and Furlan [5]. Recently the current algebra technique developed by Fubini and Furlan was used by Ericson, Figureau and Molinari [6] to evaluate the π-nucleus antisymmetric scattering lengths, the results of which show an acceptable agreement with experimental data. In this paper we will use the extrapolation program to obtain π^--nucleus scattering length for light nuclei ($A \lesssim 24$). Fubini and Furlan extrapolated

* Supported in part by the U.S. Atomic Energy Commission.
** Fellow, Council for Scientific Research, Lebanon.
Present address: International Atomic Energy Agency Centre for Theoretical Physics, Miramare, P.O.B. 586, 34100 Trieste, Italy.

the soft meson scattering amplitude [7,8] in the collinear representation to a physical pion amplitude at threshold along the extrapolation curve

$$u = \nu^2/M_c^2\,, \tag{1}$$

where $u = q^2$ and $\nu = p\cdot q$. In the collinear representation we have

$$q_0 = M_c x\,, \qquad p_0 = M_c\,, \tag{2}$$

$$\boldsymbol{p} = \boldsymbol{q} = 0,$$

where p_μ is the nucleus four momentum, M_c is the mass of the nucleus and x is a parameter called the mass variable. For $I = 0$ and $I = \frac{1}{2}$, $I_z = -\frac{1}{2}$ nuclei the π^--nucleus scattering length in the canonical field algebra * is given by

$$4\pi f_\pi^2(1+m_\pi/M_c)\,\mathrm{Re}\,a(I=0) = -m_\pi\Sigma + \frac{2}{\pi}\frac{m_\pi^3}{M_c^2}\int_{x_0}^{\infty}\mathrm{d}x\,\frac{\mathrm{Im}\,F(I=0)}{x(x^2-m_\pi^2/M_c^2)}\,, \tag{3}$$

$$4\pi f_\pi^2(1+m_\pi/M_c)\,\mathrm{Re}\,a(I=\tfrac{1}{2},I_z=-\tfrac{1}{2})$$

$$= -\tfrac{1}{2}m_\pi^2 - m_\pi\Sigma + \frac{2}{\pi}\frac{m_\pi^3}{M_c^2}\int_{x_0}^{\infty}\mathrm{d}x\,\frac{\mathrm{Im}\,F(\frac{1}{2},-\frac{1}{2})}{x(x^2-m_\pi^2/M_c^2)}\,, \tag{4}$$

respectively.

Here m_π is the pion mass, f_π is the decay constant ** for the process $\pi^\pm \to \mu^\pm\nu$ and a is the scattering length in units of m_π. The imaginary part of the amplitude for scattering an off mass-shell pion on a nucleus, Im F, is defined by

$$\mathrm{Im}\,F^{\alpha\beta} = \frac{(2\pi)^4}{2m_\pi^4 f_\pi^2}\sum_n \langle \mathrm{N_c}|(\Box+m_\pi^2)\,\partial_\mu A_\mu^\alpha|n\rangle\langle n|(\Box+m_\pi^2)\,\partial_\nu A_\nu^\beta|\mathrm{N}\rangle$$

$$\times\,\delta^{(4)}(p+q-p_n) - \text{cross term}, \tag{5}$$

* Assuming PCAC, the canonical field algebra, reduces to

$$[\partial_\mu A_\mu^\alpha(x), \partial_\nu A_\nu^\beta(y)]\,\delta(x_0-y_0) = 0,$$

$$(f_\pi m_\pi^2)^{-2}[\partial_\mu \dot{A}_{\mu'}^\alpha(x)\partial_\nu A_\nu^\beta(y)]\,\delta(x_0-y_0) = \delta_{\alpha\beta}\delta^{(4)}(x-y)\,.$$

** For the pion decay constant f_π we will use the value $f_\pi = 0.69\ m_\pi$.

with the property

$$\lim_{x \to m_\pi/M_C} F_{\alpha\beta}(x) = 4\pi f_\pi^2 T_0^{\alpha\beta}(\text{threshold}) , \qquad (6)$$

where α, β are the pion SU(2) labels and T_0 is the physical s-wave pion nucleus scattering amplitude. The pion-nucleus sigma term Σ in eqs. (3) and (4) is given by

$$\Sigma = -i \int d^4x \exp[iq \cdot x]\, \delta(x_0) \{\langle N_C | [\dot{A}_0^\alpha(-\tfrac{1}{2}x), A_0^\beta(\tfrac{1}{2}x)] | N_C\rangle$$

$$- \langle N_C | N_C\rangle \langle 0 | [\dot{A}_0^\alpha(-\tfrac{1}{2}x), A_0^\beta(\tfrac{1}{2}x)] | 0\rangle\} , \qquad (7)$$

where $|N_C\rangle$ represents the nucleus state vector.

The results we obtain in this work are based on the following three assumptions: 1. The pion-nucleus sigma term can be written as a coherent sum of pion nucleon sigma terms. 2. The imaginary part of the amplitude for scattering of an off-mass-shell pion on a nucleus can be approximated by the imaginary part of the physical amplitude near the elastic threshold. 3. The lowest inelastic threshold is a process in which a neutron is released with a residual nucleus after scattering. In sect. 2 we elaborate on these assumptions and use them in eq. (3) and (4) to obtain numerical estimates for S. For the average of the individual determination of S from each reaction tested we obtain $\bar{S}$ = 22 ± 1 MeV (see table 1 and fig. 1), and our results for Re a with S = 22 ± 1 are in good accord with experimental data (see table 1 and fig. 2). In sect. 3 we discuss our results.

Table 1

For each nucleus tested we exhibit the experimental values of the scattering length and the lowest inelastic threshold ($E+B$). The values of Σ/A which best fits the experimental data are shown. Theoretical determinations of Re a using Σ/A = 22 ± 1 MeV are also listed.

Nucleus	Experimental results Re a (m_π^{-1})	Im a (m_π^{-1})	$E+B$	Dispersion integral	Σ/A	Theoretical results Re a with S = 22 ± 1
^{9}Be	-0.293 ± 0.011	0.059 ± 0.005	7.36	-0.007 ± 0.001	0.022 ± 0.002	-0.325 ± 0.010
^{10}B	-0.321 ± 0.01	0.081 ± 0.003	8.41	-0.007 ± 0.001	0.031 ± 0.002	-0.267 ± 0.011
^{11}B	-0.380 ± 0.009	0.090 ± 0.009	12.01	-0.001 ± 0.001	0.027 ± 0.001	-0.371 ± 0.012
^{12}C	-0.341 ± 0.006	0.097 ± 0.004	16.74	-0.009 ± 0.002	0.026 ± 0.001	-0.321 ± 0.014
^{14}N	-0.400 ± 0.011	0.079 ± 0.005	10.80	-0.001 ± 0.000	0.028 ± 0.001	-0.364 ± 0.015
^{16}O	-0.440 ± 0.011	0.114 ± 0.008	12.89	0.001 ± 0.000	0.027 ± 0.001	-0.416 ± 0.017
^{19}F	-0.538 ± 0.013	0.086 ± 0.014	7.82	-0.007 ± 0.001	0.023 ± 0.001	-0.578 ± 0.02
^{23}Na	-0.663 ± 0.013	0.050 ± 0.009	9.59	-0.002 ± 0.001	0.024 ± 0.002	-0.683 ± 0.021
^{24}Mg	-0.599 ± 0.026	0.045 ± 0.029	12.48	-0.001 ± 0.001	0.024 ± 0.002	-0.624 ± 0.024

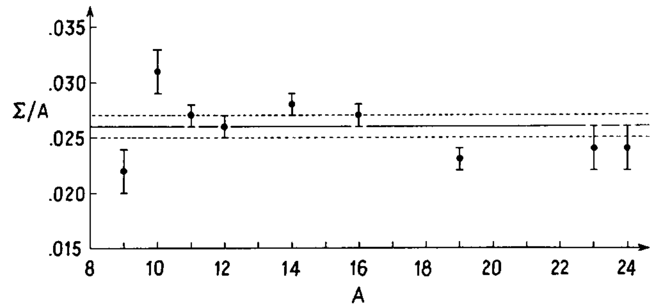

Fig. 1. Values of Σ/A obtained for $9 \lesssim A \lesssim 24$ plotted versus A. Their statistical average $\bar{\Sigma}/A$ is shown.

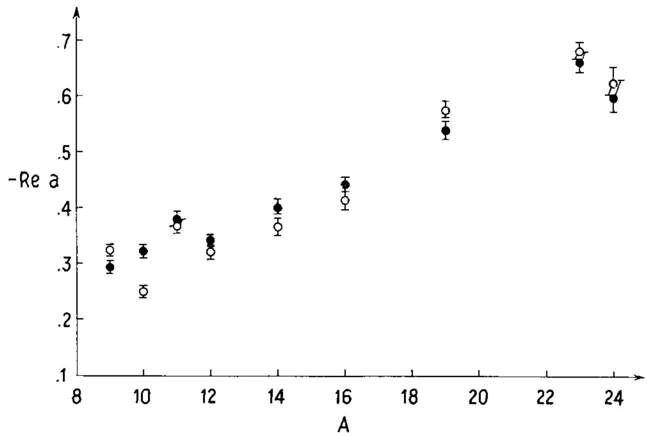

Fig. 2. The experimental (⧳) and the theoretical values (⧲) of $(-\mathrm{Re}\,a)$ obtained using $\Sigma/A = 22 \pm 1$ MeV.

2. DETERMINATION OF THE SIGMA TERM

In this section we will use pion nucleus experimental data for Re a and Im a to obtain numerical estimates for the pion-nucleon sigma term. Averaging these determinations for all reactions tested we use the result to calculate Re a.

The pion-nucleus sigma term Σ eq. (7) can be written as a coherent sum of pion-nucleon sigma terms. This assumption can be easily justified on the basis that since the matrix element of the sigma commutator $[\dot{A}_0^{\alpha}, A_0^{\beta}]$ between nucleon states is proportional to mass terms * it differs from being a coherent sum of nucleon sigma terms by only the binding energy of the particular nucleus under consideration. For nuclei $9 \lesssim A \lesssim 24$ the average binding energy per nucleon is approximately a constant being of the order of

* See ref. [4].

8 MeV. Thus a plot of Σ/A versus A is expected to be a constant independent of A. Thus, in eqs. (3) and (4) we make the substitution

$$\Sigma = AS . \tag{8}$$

The expressions (3) and (4) have been derived in the basis that the lowest possible state contributing to Im F is the pion-nucleus system itself. However, this is not true, for the scattering of a pion on a nucleus has the complex feature of having a large number of inelastic channels open below threshold, for example

$$\pi^- + \mathrm{N_c}(Z, A) \to \mathrm{n} + \mathrm{N_c'^*}(Z-1, A-1) , \tag{9a}$$

$$\to \mathrm{d} + \mathrm{N_c''}(Z-2, A-2) , \tag{9b}$$

$$\to \mathrm{n} + \mathrm{p} + \mathrm{N_c'''}(Z-2, A-2) , \tag{9c}$$

and so on, where Z and A are the atomic and mass number of the parent nucleus respectively. The asterisk in eq. (9a) represents an excited nucleus. Thus we have to deform the extrapolation curve eq. (1) from a point on the ν axis which corresponds to the threshold of the lowest possible open inelastic channel on the x scale *. The deformation we perform amounts to approximating the imaginary part of the amplitude for the unphysical processes (9a-c) (since the pion is off its mass shell) by the imaginary part of the amplitude for the corresponding physical process

$$\mathrm{Im}\, F \approx 4\pi f_\pi^2 \,\mathrm{Im}\, T_0 , \tag{10}$$

where T_0 is the physical s-wave scattering amplitude. The approximation (10) is exact at threshold and is expected to be good near the elastic threshold and has already been employed in our study of $\bar{\mathrm{K}}\mathrm{N}$, $\pi\Sigma$ and $\pi\Lambda$ scattering amplitudes [4]. We found that such a deformation of the extrapolation path and the approximation (10) do not ruin a possible agreement with experiment we might otherwise obtain. The extrapolation technique restricts the intermediate states contributing to Im F to have the same baryon number and total angular momentum as the target, but of opposite parity. The lowest possible intermediate state on the x scale satisfying these restrictions comes from the process in which one neutron is knocked out of the nucleus. Some authors [9] make the statement that this process is improbable even though there is no concrete and conclusive experimental evidence to support this statement.

Koltun [10] pointed out that the experimental results on branching ratio of the processes (9a-c) are inconclusive. For example, consider the π^--absorption by ^{4}He. Here we have the following possibilities

* See refs. [1,4].

$$\pi^- + {}^4\mathrm{He} \to \mathrm{p} + 3\mathrm{n}\,, \tag{11a}$$

$$\to \mathrm{d} + 2\mathrm{n}\,, \tag{11b}$$

$$\to \mathrm{t} + \mathrm{n}\,. \tag{11c}$$

Ammiraju and Lederman [11] found that the process (11a) is the dominant one while Schiff [12] found that the process (11c) amounts to 30%. Bizzari [13] and Block [14] found the triton ratio to be 18.4% and 19.4%, respectively.

The threshold for processes (11a) corresponds to

$$x_B = \frac{M(\mathrm{N_c'}) + M - M(\mathrm{N_c}) + E}{M(\mathrm{N_c})}\,,$$

$$= \frac{B+E}{M(\mathrm{N_c})}\,, \tag{12}$$

where B is the binding energy of the least bound neutron, E is the least possible excitation energy needed to make the neutron and the residual nucleus-system has the same spin and of opposite parity to the target nucleus $\mathrm{N_c}$. Here $M(\mathrm{N_c})$, $M(\mathrm{N_c'})$ and M are the masses of the target, residual nucleus and the nucleon, respectively. The values of $(B+E)$ for the nuclei tested are listed in table 1. For example, consider the process

$$\pi^- + {}^{19}\mathrm{F} \to \mathrm{n} + {}^{18}\mathrm{O}^*\,. \tag{13}$$

Since we are in a system in which both the pion and ^{19}F nucleus are at rest, then the spin of the target nucleus is its intrinsic spin $\frac{1}{2}$, its intrinsic parity is +. But ^{18}O has a spin-0 and positive parity. Thus, if we examine the excitation levels of ^{18}O [15] we find that the lowest possible state satisfying our spin and parity selection rules is the ground state if the neutron and ^{18}O system have a relative orbital angular momentum $l = 1$.

To evaluate the dispersion integrals we have to approximate $\mathrm{Im}\,T_0$. We make two observations, the first is due to Ericson and Locher [16] in which they pointed out that since $\mathrm{Im}\,a \ll \mathrm{Re}\,a$ one can approximate $\mathrm{Im}\,T_0$ by its threshold value and extrapolate that value smoothly below threshold to the lowest inelastic threshold. Secondly, consider the processes eqs. (11a-c) Koltun [10] pointed out that in a two-body breakup as in eq. (11c) triton has a unique energy while the energies of P and d in eqs. (11a-d) are distributed smoothly. Thus the contribution of process (11c) would appear as a sharp peak on a background of protons and deuterons. Since the threshold of the process (11c) is too far from the elastic threshold we expect it not to influence $\mathrm{Re}\,a$. Thus we make the approximation

$$\mathrm{Im}\,T_0 \approx \mathrm{Im}\,a\,. \tag{14}$$

However if we examine our dispersion integrals we see that the integrand for positive values of x has two poles at $x = x_0$ and $x = 0$. The first pole does not give any trouble since its contribution below and above x_0 cancels.

The second pole at $x = 0$ is due to our dispersion integrals being subtracted once at zero. But since the lower integration limit x_B (typically of the order of $0.025/A$, where A is the mass number, as compared with the elastic threshold $x_0 \sim 0.15/A$), is too close to the pole, then this would tend to overestimate the contribution of the dispersion integral to the real part of the scattering length. Since the process is the lowest energetically possible process we should have

$$\operatorname{Im} T_0(x_B) = 0. \tag{15}$$

Thus we make the ansatz

$$\operatorname{Im} T_0(x) \approx \left(\frac{x - x_0}{x_B - x_0}\right)^{\frac{1}{2}} \operatorname{Im} a. \tag{16}$$

Eq. (16) has the advantage over eq. (14) in that it ensures the property (15) and it compensates for the effect of the pole $x = 0$. Thus, we are in a position to evaluate the dispersion integrals. Using eqs. (3), (4), (8) and (16) we obtain

$$4\pi f_\pi^2 \left(1 + \frac{m_\pi}{M_c}\right) \operatorname{Re} a(I=0) = -ASm_\pi + \left\{\frac{1}{\pi} \frac{1}{(x_0 - x_B)^{\frac{1}{2}}} \left[-2(x_0 + x_B)^{\frac{1}{2}} \times \operatorname{arc\,tg}\left(\frac{x_0 - x_B}{x_0 + x_B}\right)^{\frac{1}{2}} + 4x_B^{\frac{1}{2}} \operatorname{arc\,tg}\left(\frac{x_0 - x_B}{x_0}\right)^{\frac{1}{2}}\right] + 4x_B^{\frac{1}{2}} - (x_B + x_0)^{\frac{1}{2}}\right\} \operatorname{Im} a, \tag{17}$$

$$4\pi f_\pi^2 \left(1 + \frac{m_\pi}{M_c}\right) \operatorname{Re} a(\tfrac{1}{2}, -\tfrac{1}{2}) = -\tfrac{1}{2} m_\pi^2 - ASm_\pi + \left\{\frac{1}{\pi} \frac{1}{(x_0 - x_B)^{\frac{1}{2}}} \left[-2(x_0 + x_B)^{\frac{1}{2}} \times \operatorname{arc\,tg}\left(\frac{x_0 - x_B}{x_0 + x_B}\right)^{\frac{1}{2}} + 4x_B^{\frac{1}{2}} \operatorname{arc\,tg}\left(\frac{x_0 - x_B}{x_0}\right)^{\frac{1}{2}}\right] + 4x_B^{\frac{1}{2}} - (x_B + x_0)^{\frac{1}{2}}\right\} \operatorname{Im} a. \tag{18}$$

We are however interested in evaluating the pion-nucleon sigma term, and for that purpose we will use experimental data for Re a and Im a in eq. (17) and (18). Seki [17] evaluated s-wave π^--nucleus scattering lengths for light nuclei ($A \lesssim 24$) using 2p-1s transition energy data. He found different values for the scattering lengths depending on the particular transition data used *. We made a statistical average of the different values of the scattering lengths reported. In table 1 we show the values of $M_c x_B$, the contribution of the dispersion integral and the value of the pion-nucleon sigma term obtained which best fits each reaction we tried (fig. 1). We found that the average $\bar{S}$ of all the pion-nucleon sigma terms obtained is

$$\bar{S} = 22 \pm 1 \text{ MeV}, \tag{19}$$

* See ref. [17] and references cited therein.

and its contribution to the pion-nucleon scattering length in units of m_π is

$$\frac{1}{4}\,\frac{\bar{S}m_\pi^2}{\pi f_\pi^2\left(1+\frac{m_\pi}{M}\right)} = 0.023 \pm 0.001. \tag{20}$$

Using $\bar{S}$ = 22 ± 1 MeV and the experimental values for Im a we calculated Re a. Our results given in table 1 and fig. 2 are in good accord with experimental data.

3. CONCLUSION

We feel at this stage ready to discuss briefly the different determinations of S presented by other authors. VonHippel and Kim and this author obtained S = 26 MeV and 30 MeV respectively in the canonical model. In the SU(3) ⊗ SU(3) chiral symmetry breaking model of Gell-Mann, Oakes and Renner [18] this author obtained S = 26 MeV.

Recently Cheng and Dashen [2] obtained a value of S much larger than reported by the first two papers (S = 110 MeV) while Höhler, Jacob and Straub [3] reported S = 39 ± 8 MeV.

There appear to be certain ambiguities in using meson baryon data and pion nucleon phase-shift data to determine S because of the smallness of the scattering length and the controversy between different experimental determinations. For the meson baryon case the pion-nucleon scattering length reported by Höhler is $a_{\frac{1}{2}}$ = 0.175, $a_{\frac{3}{2}}$ = -0.103 while Lovelace reported [20] $a_{\frac{1}{2}}$ = 0.196, $a_{\frac{3}{2}}$ = -0.069 in units of m_π. This situation deteriorates when considering KN and $\pi\Sigma$ data. In the case of the pion-nucleon phase-shift data, Höhler pointed out that any results obtained using the phase-shift data depends on the particular data and energy range considered.

The nuclear data offers a better, more concrete ground since the scattering lengths are larger in value and the experimental determinations of Re a and Im a are much less controversial than the meson baryon scattering lengths or the phase-shift determinations. The approximations we made in evaluating the dispersion integrals might be questioned. However our experience with meson baryon scattering [4] and the qualitatively good agreement with experiment obtained indicate that such approximations do not influence our results, especially considering the smallness of Im a and the contribution of the dispersion integral.

It is interesting at this stage to study the significance of $S \approx 22$ to the breaking of the chiral symmetry and scale invariance, which is at the present a subject of study we will report on soon.

It is a pleasure to thank Prof. Yukio Tomozawa for suggesting the problem and useful discussions and R. Carroll for reading the manuscript. I benefitted also from discussions with Prof. L. Wolfenstein.

REFERENCES

[1] F.VonHippel and J.K.Kim, Phys. Rev. D1 (1970) 151.
[2] T.P.Cheng and R.Dashen, Phys. Rev. Letters 26 (1971) 594.
[3] G.Höhler, H.P.Jacob and R.Straub, Universities of Bielefeld and Karlsruhe, preprint (1971).
[4] Shafik J.Hakim, University of Michigan preprint 71-21 (1971).
[5] F.Fubini and G.Furlan, Ann. of Phys. 48 (1968) 322;
V.DeAlfaro and C.Rossetti, Nuovo Cimento Suppl. 6 (1968) 515.
[6] M.Ericson, A.Figureau and A.Molinari, Nucl. Phys. B10 (1969) 501.
[7] Y.Tomozawa, Nuovo Cimento 46 (1966) 707.
[8] S.Weinberg, Phys. Rev. Letters 17 (1966) 616.
[9] E.H.S.Burhop, High energy physics (Academic press, New York, 1969) V. 3, p. 109.
[10] D.S.Koltun, Advances in nuclear physics, ed. M.Baranger and E.Vogt (Plenum press, New York, 1969), V. 3, p. 71.
[11] P.Ammiraju and L.M.Lederman, Nuovo Cimento 17 (1960) 726.
[12] M.Schiff, R.H.Hildebrand and C.Giese, Phys. Rev. 122 (1961) 255.
[13] R.Bizzari et al., Nuovo Cimento 33 (1964) 1497.
[14] M.M.Block et al., Phys. Rev. Letters 11 (1963) 301.
[15] National Research Council data sheets (USA) 1957.
[16] T.E.O.Ericson and M.Locher, CERN preprint 69-30 (1969).
[17] R.Seki, Phys. Rev. Letters 23 (1969) 1000.
[18] M.Gell-Mann, R.J.Oakes and B.Renner, Phys. Rev. 175 (1968) 2195.
[19] G.Höhler et al., Z.Phys. 229 (1969) 217.
[20] C.Lovelace, Pion nucleon scattering, ed. G.L.Shaw and D.Y.Young (Wiley, New York, 1969), p. 27.