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THE DESIGN OF AN EFFICIENT WIDEBAND ANTENNA SYSTEM

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Electronic Defense Group
Department of Electrical Engineering

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ABSTRACT

The details of an efficient broadband antenna system covering the frequency range 6 to 60 mc with two antennas are presented. The antenna system is intended for use with a distributed amplifier, and utilizes monopoles and an inverted vee antenna. Provisions are made for operation at reduced bandwidth when harmonic rejection is desired. The design of suitable power splitting (on a frequency basis) networks and antenna matching networks are summarized. Experimental evaluations of these networks are included in Appendix A. Experimental curves of monopole impedance are presented in Appendix C.

THE DESIGN OF AN EFFICIENT WIDEBAND ANTENNA SYSTEM

I. INTRODUCTION

A study has been made by personnel in the Electronic Defense Group of the technical feasibility of an efficient tactical antenna system comparable in bandwidth to the 2-60 mc bandwidth of a distributed power amplifier under development for the Signal Corps. An objective of the effort was to realize an antenna system with provisions for transmitting a number of simultaneous signals—each independently variable in bandwidth and center frequency over the operating frequency range. The incorporation of circuitry to allow for operation at reduced bandwidth (where harmonic rejection was necessary) was considered subsequently. The details of a proposed prototype antenna system covering the frequency range 6 to 60 mc are presented herein. The system uses monopoles and an inverted vee antenna.

The body of this report has been divided into several sections for convenience. In Section 2, some general remarks on the design problem are presented. A qualitative discussion of the operation of the system proposed by the Electronic Defense Group is presented in Section 3. The details of the synthesis of specific networks performing the functions described in Section 3 are given in Sections 4 and 5. Specifically, Section 4 is concerned with the design of suitable individual antenna matching networks. Assumptions are made concerning the length-to-radius ratio of the monopoles used, ground character, and the antenna base impedance. These limitations are treated to some extent. In addition, some remarks are made on the structural aspects of the problem, such as the switching problem, the use of a ground plane, and the construction of the monopoles and networks. The antenna system is channelized; i.e., each antenna is capable of

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radiating energy over a portion of the total band. Simultaneous operation of the antennas is practical due to the use of power-splitting (on a frequency basis) filters which are derived in Section 5.

The solution of the problems defined in Sections 4 and 5 requires an extensive knowledge in the field of network synthesis. Sections 4 and 5 were written to serve the reader not having synthesis background as a summary of the principles used in the system design. Enough of the numerical procedure is included, however, so that those with sufficient background and interest may, through the use of the references indicated, substantiate the results.

A set of networks for the system was built and tested by the Electronic Defense Group. Data obtained on these networks are summarized in Appendix A.

II. THE DESIGN PROBLEM

As previously stated, a study was made under EDG Task No. 8 of the technical feasibility of an efficient wideband antenna system. A number of antenna parameters (other than efficiency and bandwidth) such as portability, polarization, and directivity are also important in a radiation problem. Although development of a tactically optimum system has not been attempted, certain characteristics have been considered tactically desirable. Portability is regarded as of paramount importance in a front line equipment. For this reason simple antenna types such as monopoles and inverted vees have been considered. The use of a ground plane in this frequency range is considered tactically infeasible.

Provisions have been made in the prototype structure for the use of either an inverted vee or monopole in the 19-60 mc band. It is hoped that valuable information can thus be obtained on the relative merits of a directive,

as compared with an omnidirectional, antenna pattern.

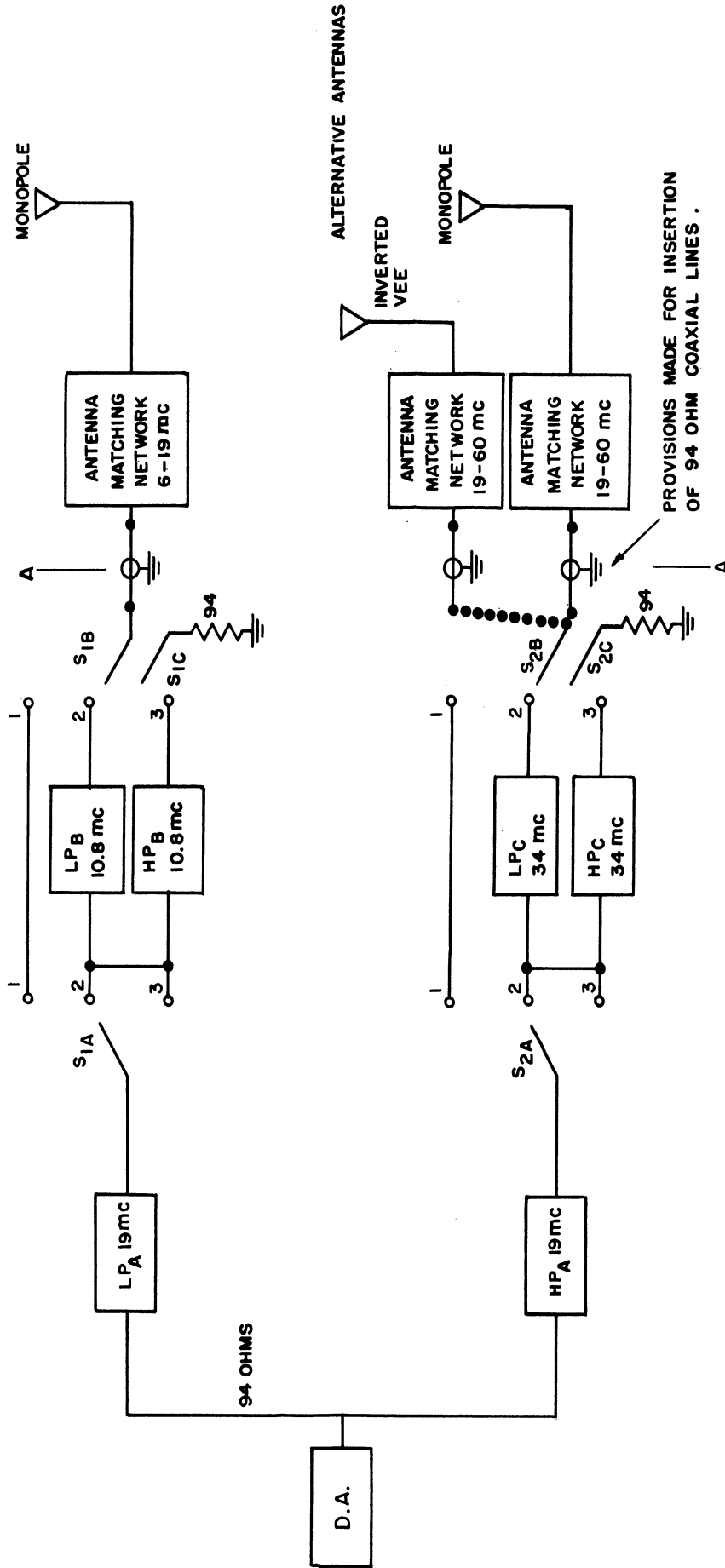
It has been felt that an efficient tactical, single-antenna system with ten-to-one frequency coverage could not be developed in the brief time available. The use of frequency channels and multiple antennas is therefore proposed. It is recognized that in the crossover regions, where two antennas are each radiating significant power, a lobed pattern will be produced. The orientation of the lobes at a given frequency depends on the relative phase and magnitude of the signals applied to the antennas.¹ An attempt has been made to minimize the lobing effect in the prototype by making the

III. DISCUSSION OF PROPOSED SYSTEM OPERATION

The prototype antenna system proposed by Electronic Defense Group is blocked diagrammed in Figure 1. The power delivered by a distributed amplifier over the frequency range 6 to 60 mc is divided by networks LP_A and HP_A on a frequency basis, so that power in the frequency range below 19 mc is channelled through LP_A and power above 19 mc is channelled through HP_A . The design of these networks is considered in Section 5. The networks LP_A and HP_A actually used will perform as desired only if driven in parallel and if each network is terminated in a pure resistance.² To approach the termination requirement, antenna matching networks (AMN) are inserted in the antenna lines. The antenna matching network in, for example, the low frequency band, must be designed to permit efficient delivery of power to its associated antenna over a particular frequency band when driven with a resistive generator; the input impedance of the matching network

¹ Adjustment of the relative phase is feasible.

² Filters of this type are said to be "complementary" and are called "complementary filters".



LPA AND HPA ARE COMPLEMENTARY FILTERS
 LPB AND HPB " " "
 LPC AND HPC " " "

UNUSED NETWORKS TO BE TERMINATED IF
 ITS COMPLEMENTARY NETWORK IS USED.

FIG. 1. PROTOTYPE ANTENNA SYSTEM

approaches a resistance over this band. Actually, as described in Section 4, appreciable mismatch of the complementary filters must exist in the present system. Optimistically, one might hope that approximately the same mismatch would be achieved in the final system.¹ It was found that this result was substantially realized when some relatively simple complementary-type filters are used.² Satisfactory results are also obtained when more complex complementary networks are used.³ Then, power can be delivered efficiently over the entire range 6 to 60 mc with switches S_{1A} , S_{1B} , S_{2A} , and S_{2B} of Fig. 1 in positions 1.

Since each antenna is matched by its antenna matching network over a 3:1 to 1 frequency band, the problem of harmonic radiation exists. The function of network LP_B , HP_B , LP_C , and HP_C is to permit operation over 1.76 to 1 frequency bands when reduction of harmonic radiation is necessary. Radiation of intermodulation power generated in multiple signal operation will also be reduced. Networks LP_B and HP_B are similar to LP_A and HP_A but are designed for the cross-over frequency of 10.8 mc. Proper power transfer in the frequency range 6 to 10.8 mc, for example, is achieved with S_{1A} and S_{1B} in position 2 and S_{1C} in position 3, if network HP_A is terminated. The network HP_A can be terminated in several ways:

- 1) in a resistance if operation over only the 6 to 10.8 mc band is desired;

1

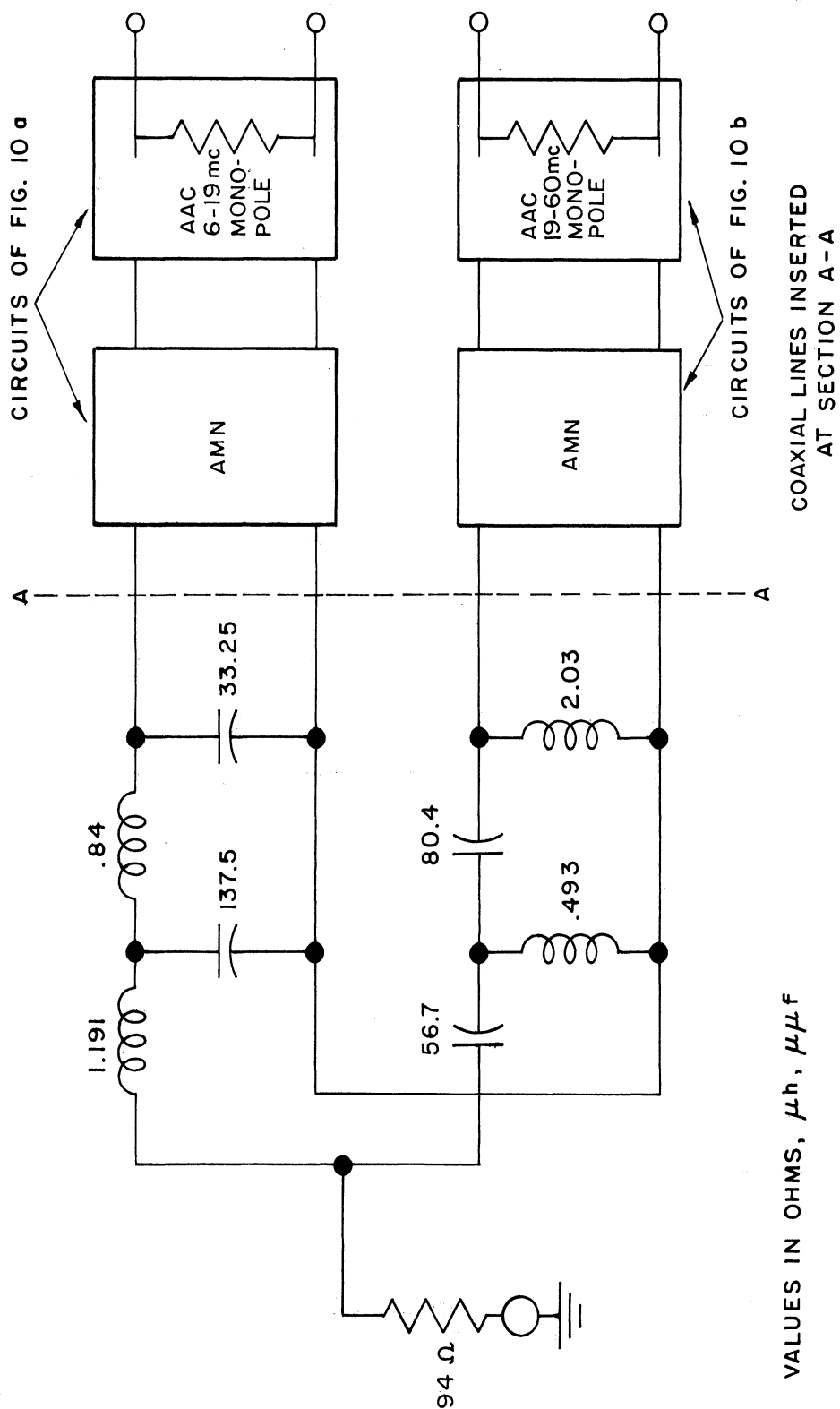
This is on the basis that the networks are like lumped equivalents to transmission lines.

2

The complementary filters referred to are 4-pole Butterworth filters. These networks and the associated results are presented in Fig. 2 and 3.

3

More complex complementary networks such as the 5-pole null frequency Tchebycheff (or equal ripple) networks of Section 5 are needed to achieve harmonic rejection as discussed later.



VALUES IN OHMS, μh , $\mu\mu f$

FIG. 2

EXPERIMENTAL CIRCUIT SIMULATING 6-60 MC ANTENNA SYSTEM AND USING 4-POLE BUTTERWORTH COMPLEMENTARY NETWORKS

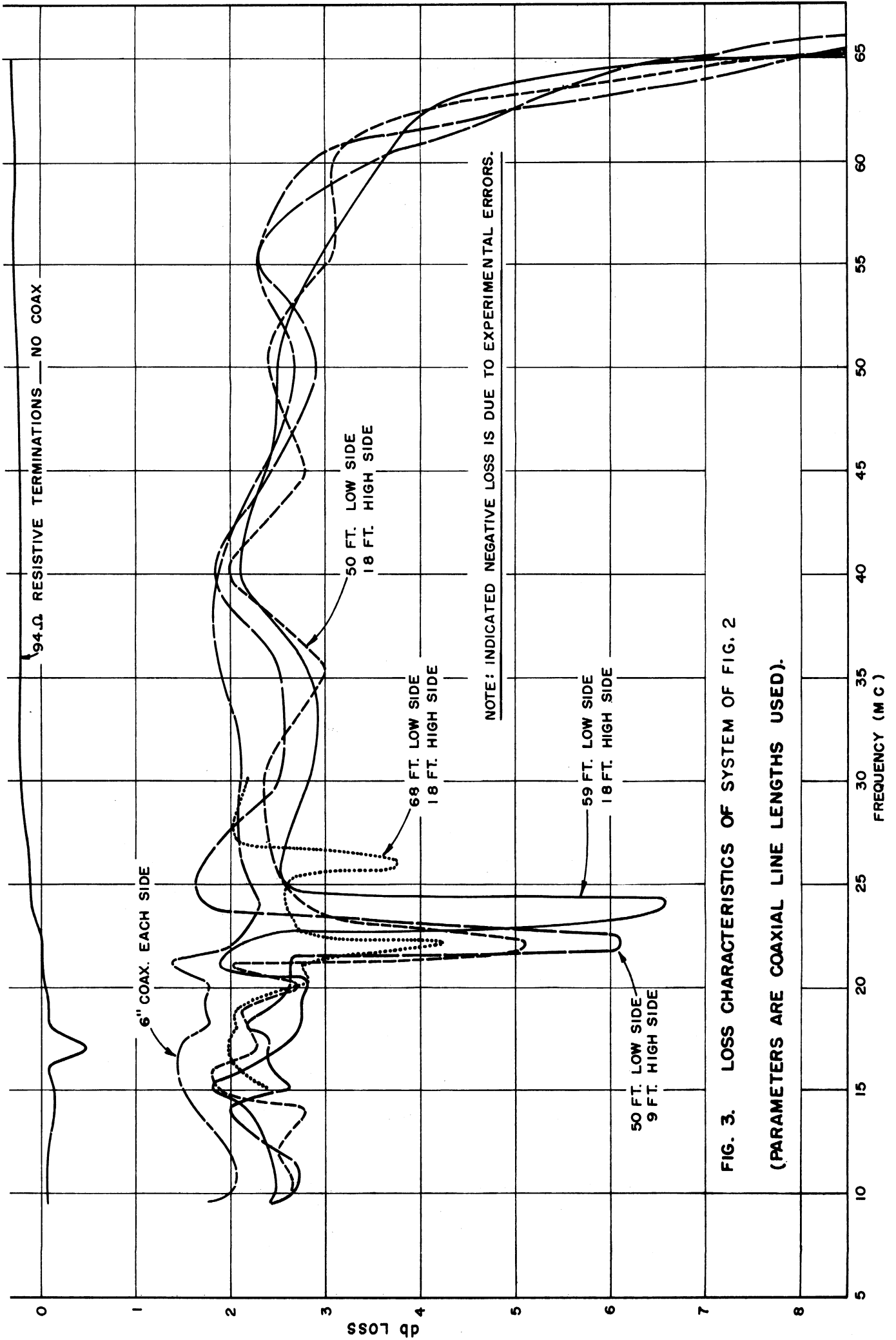


FIG. 3. LOSS CHARACTERISTICS OF SYSTEM OF FIG. 2
(PARAMETERS ARE COAXIAL LINE LENGTHS USED).

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- 2) in either a matched monopole or inverted vee for simultaneous radiation within the 6 to 10.8 mc band and the 19 to 60 mc band;
- 3) through the use of complementary networks LP_C and HP_C for simultaneous operation within the 6 to 10.8 mc band and either 19 to 34 mc band or the 34 to 60 mc band. (Note that termination of the "unused" network LP_C or HP_C is necessary.)

It is clear that through switching at the points indicated, all the desired modes of operation can be achieved. No effort has been made to incorporate switching in the networks built at Electronic Defense Group. The necessary 94 ohm coaxial switches are not readily available. It appears that ganged low capacitance, high current, rotary switches might be preferable.

IV. DESIGN OF ANTENNA MATCHING NETWORKS

Consider the problem of designing a broadband matching network for a monopole antenna. Hallen (Ref. 5) calculated the input impedance of a lossless uniform monopole over a lossless ground (neglecting base impedance). He found that if this impedance is plotted against "the antenna length in radians" ($\beta l = 2\pi \frac{l}{\lambda}$), the impedance has as a parameter only the length-to-radius ratio l/a . Since no general technique is available for designing matching networks for distributed impedances, a practical approach is to approximate the antenna input impedance by a lumped circuit equivalent. It was found that the input impedance of a monopole with $l/a = 60$ is satisfactorily approximated by the input impedance of the circuit of Fig. 4 over a range from $\beta l = 1.52$ radians to $\beta l = 4.71$ radians.¹

¹

The quality of the approximation of Fig. 4 is indicated in Fig. 5. The range $\beta l = 1.52$ to $\beta l = 4.71$ corresponds to 3.1 to 1 frequency coverage by a single antenna. This results in 10 to 1 frequency coverage with two antennas, and also corresponds to the βl range over which the approximation is "good". The meaning of "base capacitance" is discussed in Appendix C.

If the base capacitance (C_b) of the antenna is neglected, the bandpass to lowpass transformation for which bandwidth is conserved yields the antenna lowpass equivalent of Fig. 6a.¹

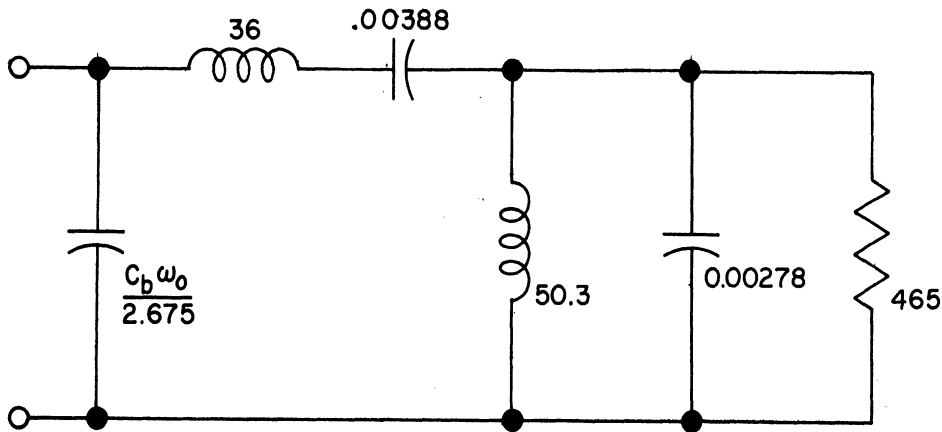
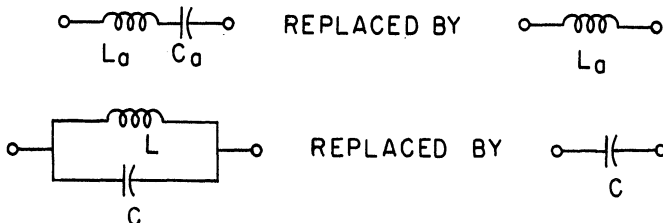


FIG 4
ANTENNA APPROXIMATING CIRCUIT
USE l/λ IN PLACE OF FREQ TO OBTAIN OHMS

¹

The choice of a bandpass approximating circuit is arbitrary. The term "bandpass" implies that all reactive elements occur in either series or parallel LC pairs, and that the associated LC products are all equal. This type of network is advantageous in that an equivalent lowpass network exists. The lowpass equivalent to a bandpass network (with the same bandwidth) is formed by making the following substitutions:



A simplified network results with many properties simply related to the bandpass network properties. The above points are considered in Ref. 4.

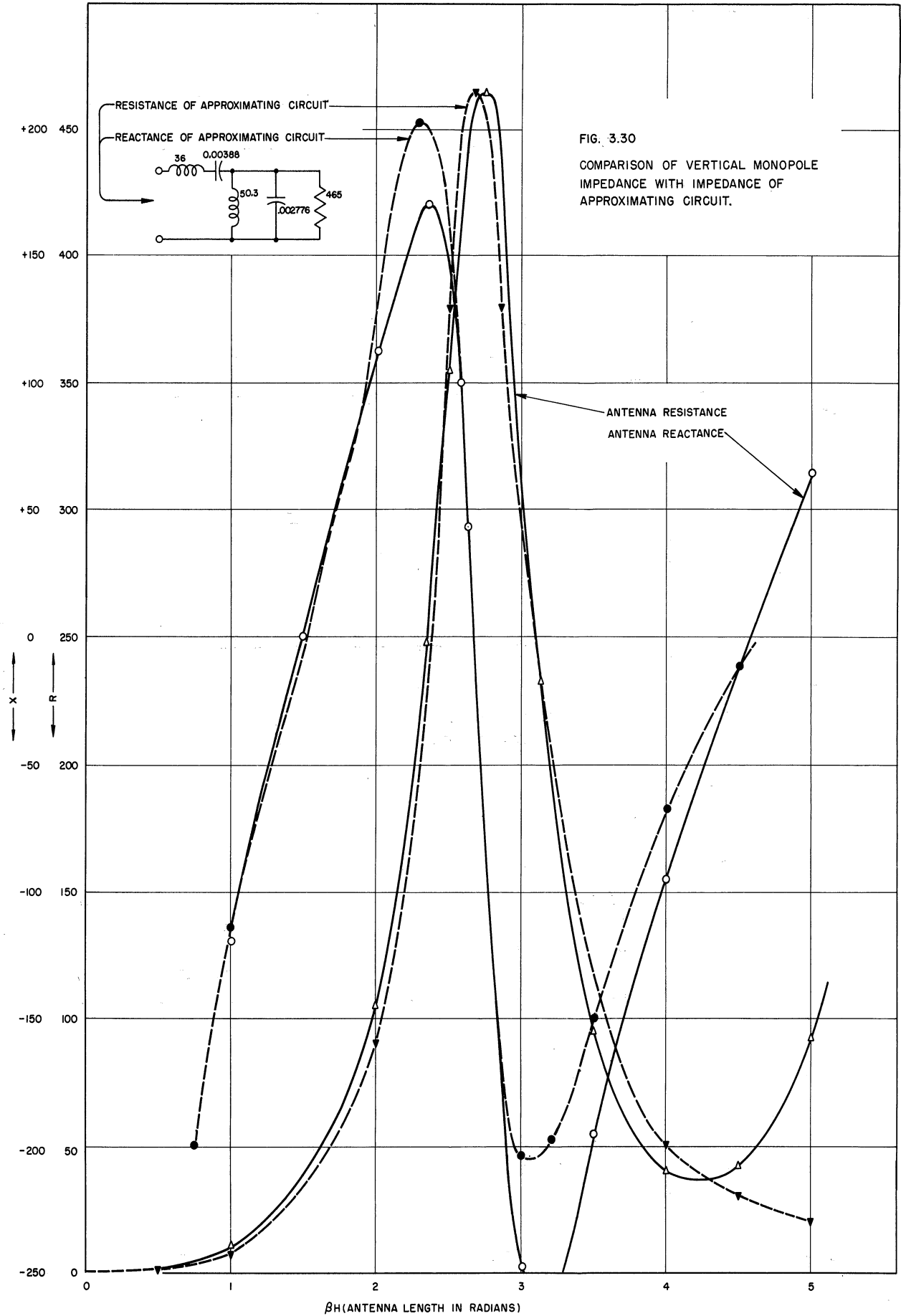


FIG 5

By making an impedance level transformation the network of Fig. 6b with a one ohm termination is obtained.¹ Then the problem of designing a matching network for the antenna is reduced to the problem of specifying a matching network for Fig. 6b over a frequency band from 0 to $\omega_0 = 4.71 - 1.52 = 3.19$ radians per second.

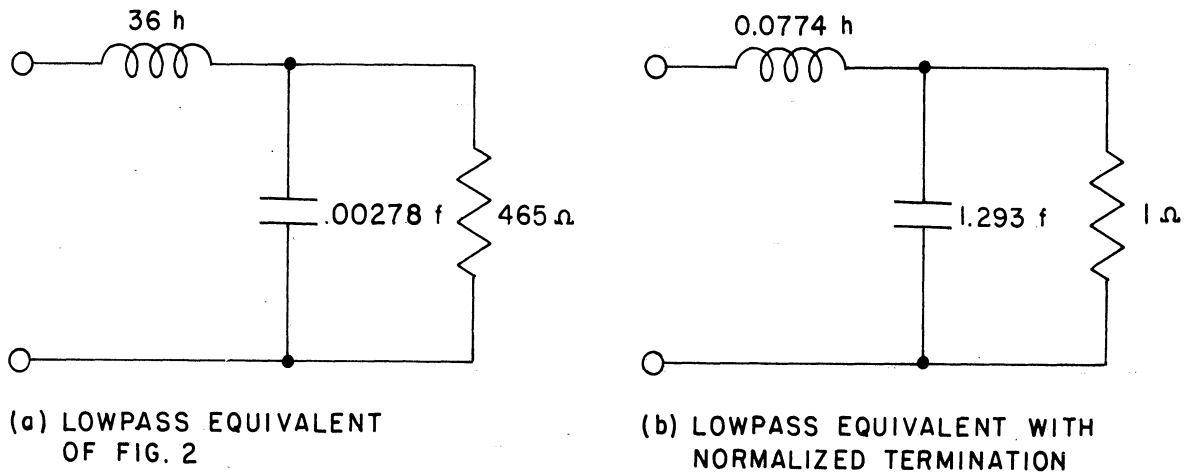
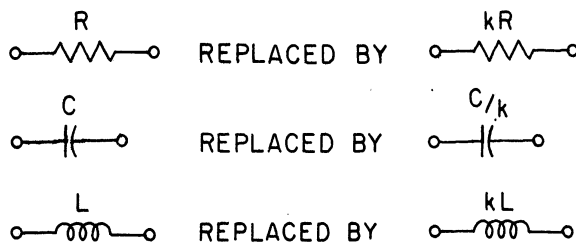


FIG. 6. LOWPASS ANTENNA EQUIVALENT CIRCUITS

The above steps have reduced the problem to one for which a satisfactory solution can be found. The technique will be summarized here. The reader is referred to Electronic Defense Group Technical Report No. 44 (Ref. 6) for a more detailed treatment.²

¹ An impedance level transformation is realized by the following substitutions:



Under this transformation all impedances are changed by the factor k . In EDG Technical Report No. 44 it is shown that efficiency of power transfer is unaffected by this procedure.

²The design of antenna matching networks such as those developed here is discussed in Appendix II of EDG Technical Report No. 44.

Referring to Fig. 6, it is noted that a quantity $\frac{2}{\omega_0 RC}$ can be specified for the RC termination and the desired bandwidth $\omega_0 = 3.19$. It is found to be

$$\frac{2}{\omega_0 RC} = \frac{2}{3.19(1) 1.293} = .485$$

In Ref. 6 optimally efficient networks of a prescribed complexity are defined as a function of the quantity $\frac{2}{\omega_0 RC}$. The efficiency of a network is measured by a function $|t|^2 = \frac{P_{del}}{P_{avail}}$ where P_{del} is the power delivered to the load and P_{avail} is available or matched power from the driving generator.

The 4 pole network selected is Fig. 7. The power transfer characteristic, $|t|^2$, of this network has an equal ripple or Tchebycheff behavior within the band; passband loss oscillates between 1.44 db and 1.22 db. The theoretical minimum loss (the Bode Limit) associated with the above termination is 1.08 db.¹ This loss can be approached with optimum networks of greater complexity.

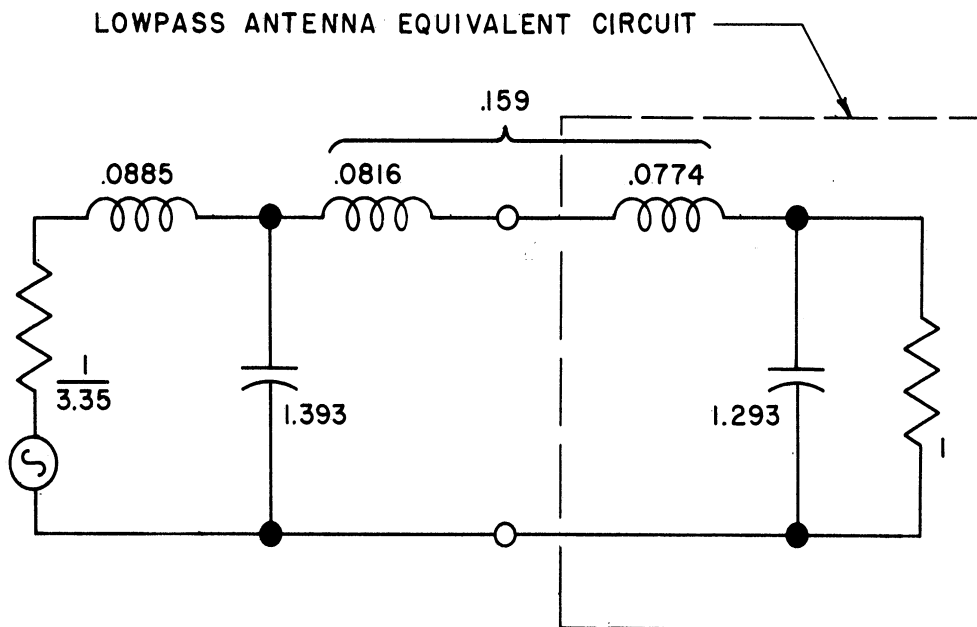


FIG 7
A LOWPASS 4-POLE TCHEBYCHEFF CIRCUIT
(VALUES IN OHMS, HENRYS AND FARADS)

¹ Ref. 3, pg. 362.

It will be noted that the series inductance closest to the one ohm termination in Fig. 7 is .159 h. while the corresponding element of the normalized antenna low pass equivalent circuit is .0774 henry. No simple way is available to adjust this element. This means that the element closest to the antenna, in the antenna matching network, is a series one. It would be desirable to have a shunt capacity at this point to absorb the antenna base capacitance which occurs in any practical installation.

After making impedance level and lowpass to bandpass transformations,¹ the network of Fig. 7 is transformed to that of Fig. 8. Operation with a generator impedance of 94 ohms is desired. This is accomplished through the insertion of an ideal transformer which can be absorbed as indicated in Fig. 9a and 9b.²

The specification of matching networks for monopoles covering the bands 6-19 mc and 19-60 mc is readily made from Fig. 9b. The results are indicated below. (See Fig. 10.) It must be pointed out that certain problems result from the insertion of coaxial lines at A-A in Fig. 1; these cables permit positioning of the antennas at a distance from the distributed amplifier and complementary filter assembly. The complementary filters function as desired only if terminated in 94 ohms. In the antenna system described, the impedance looking into the antenna matching networks is "1.44 db" away from the matched conditions. Since the coaxial line is not perfectly matched by the antenna matching networks and antennas the impedance seen by the complementary filters is a function of line length. Tests conducted at the Electronic Defense Group indicate that the effect of this mismatch on the over all transmission is not pronounced.³ However, some

1

This procedure is simply the reverse of that by which the lowpass equivalent was obtained.

2

This procedure is described in EDG Technical Report No. 44, Appendix II.

3

See Fig. 3.

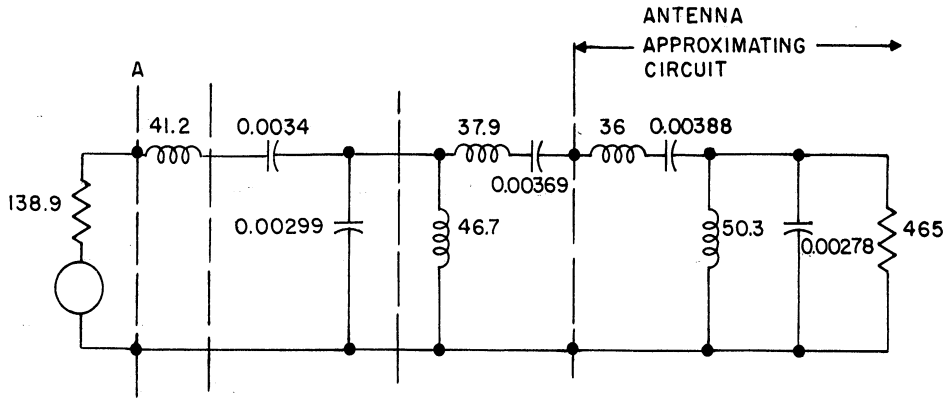


FIG. 8

4 - POLE BANDPASS NETWORK

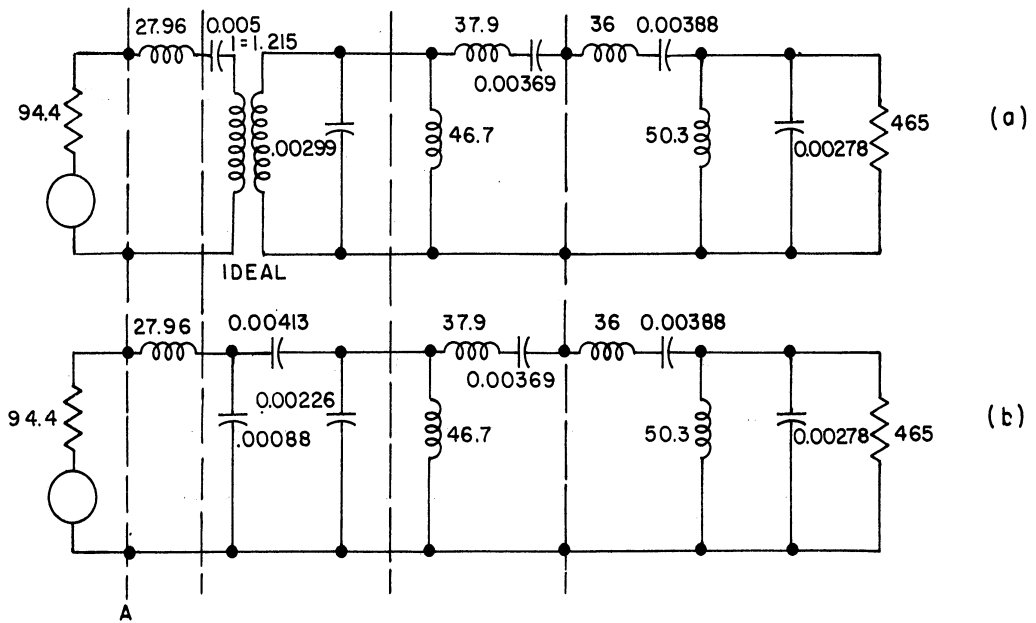
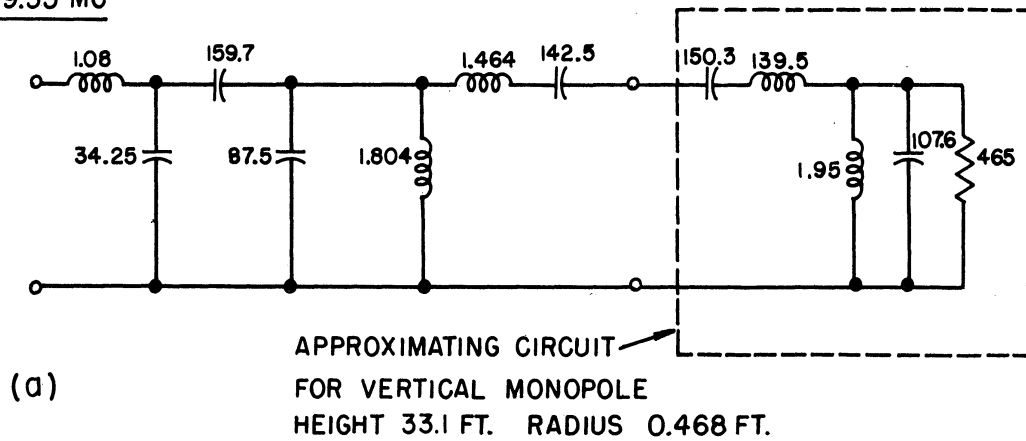


FIG. 9

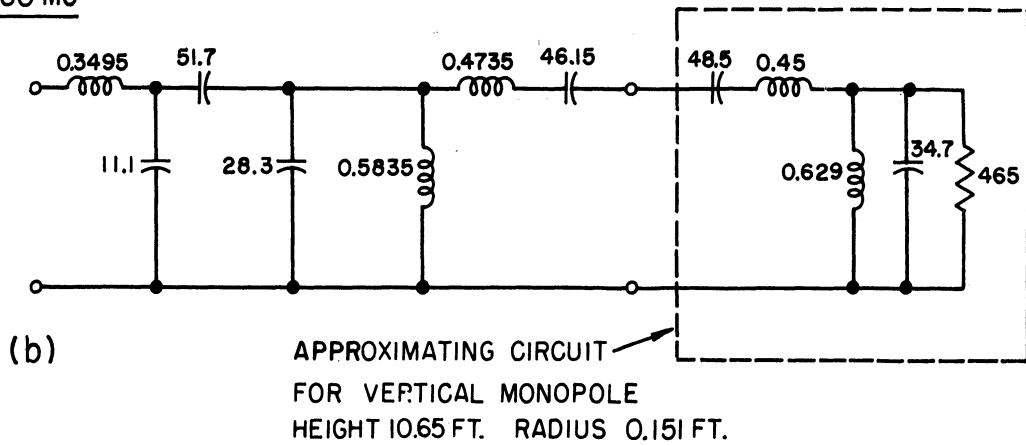
COMPOSITE ANTENNA NETWORK

(NOTE: VALUES IN OHMS, FARADS, AND HENRYS).

6.24 TO 19.35 MC



19.35 TO 60 MC



NOTE :

VALUES IN OHMS, μ h AND $\mu\mu$ f.

FIG. 10. ANTENNA MATCHING NETWORKS AND VERTICAL MONOPOLES RESULTING FROM NORMALIZED NETWORK OF FIG. 7.

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reduction in transmission efficiency over narrow-bands has been observed, and the location of these "suckouts" is a function of the coaxial line lengths used. The choice of optimum line lengths for the prototype can best be made by tests on the actual structure.

The matching networks were defined for a lossless monopole of length to radius ratio $l/a = 60$ over a lossless ground plane. The use of a ground plane of conventional size is considered tactically infeasible. Tests conducted at the Electronic Defense Group indicate that for a wide range of practical ground conditions the above matching networks should be used with antennas of length to radius ratio $l/a = 80$. Further experimental work suggests that practical antennas about 13% shorter than the theoretical lengths should be used.¹ The resulting antenna dimensions are indicated in Fig. 10.

It is felt that some lightweight construction technique is mandatory if the antennas specified in Fig. 10 are to be made at all feasible. No attempt has been made to solve this problem. The use of a mast and spacers to support a wire mesh such as in Fig. 11a may prove satisfactory.

Several other construction details are suggested in Fig. 11. The antenna matching networks must be positioned at the base of the antenna—the total lead inductance to the network should not exceed the series inductances indicated in Fig. 10, and correction of this inductance for whatever lead inductance is used should be made. If a dielectric mast is not used, it is suggested that a dielectric pedestal several inches high be used to reduce base capacitance to a practical figure. Then the matching network may be mounted on a substantial ground post located near the base of the antenna.

¹ The results of these tests are presented in Appendix C.

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A promising alternative construction involves the use of an extendable (telescoping) mast. A crude sketch of the structure is presented below in Fig. 11b. The radiator is made up of a number of metal tapes which are held in tension by the spools located on the bottom section. The tapes are taken up in the spools when the mast is contracted. The metal spacers located near the top of each mast section maintain friction contact with the tapes, reducing vibration and tying the tapes together electrically. When contracted, the structure should be highly portable.

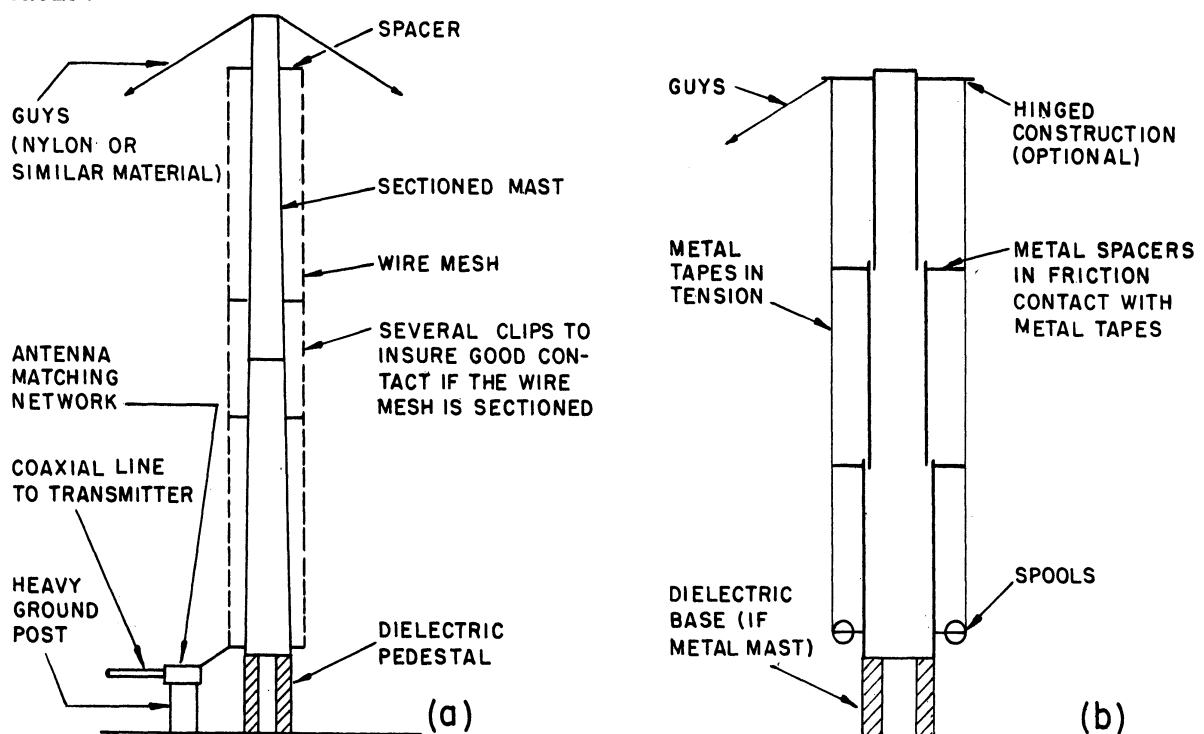


FIG. 11

SKETCHES SHOWING CONSTRUCTION SUGGESTIONS

Both theoretical and experimental data have been used as a guide in obtaining suitable prototype matching networks for the monopole antennas. It must be pointed out, however, that the matching networks used in the prototype are useful for a particular antenna input characteristic. Deviations from the specific antenna dimensions assumed, base impedance, and the presence of parasitic elements in the antenna near-field may materially effect the quality of the antenna match achieved.

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Inverted vee antennas are often terminated to obtain an "end fire" radiation pattern which varies only moderately with frequency. In this case substantially all the incident energy not radiated is absorbed in the termination. For this mode of operation, variation with frequency of the input impedance may be small. The input impedance of typical inverted vees is about 600 ohms. A matching network for the inverted vee antenna was designed to give an efficient match between a 600 ohm load and a 94 ohm generator over the 19-60 mc band. Limitations on time and personnel have prevented a further investigation of the nature of the input impedance of vee antennas.

A matching network for the inverted vee is easily obtained using the techniques developed in Electronic Defense Group Technical Report No. 44. As previously stated, it is assumed that a 19-60 mc bandpass network which couples a 94 ohm generator efficiently to a 600 ohm load is desired. Using the design curves of the above report, the 4 pole low pass network of Fig. 12 may be chosen. This network, with unit load and one radian per second tolerance bandwidth, has an arbitrarily chosen maximum passband loss of .33 db and a ripple of .09 db. After raising the impedance level to give a 600 ohm load and increasing the bandwidth to $2\pi(60-19)10^6 = 258 \times 10^6$ rad/sec, the 19-60 mc bandpass network of Fig. 13 is easily obtained by means of a lowpass to bandpass transformation. To adjust

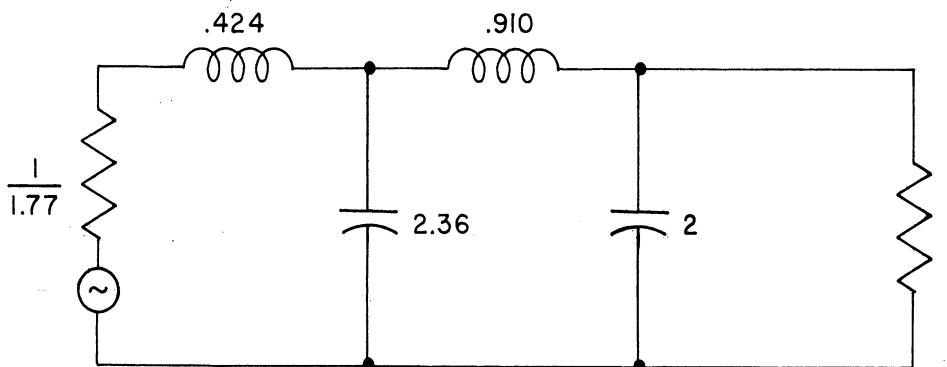


FIG. 12. A 4-POLE TCHEBYCHEFF NETWORK
VALUES IN OHMS, HENRYS & FARADS

the generator impedance, an ideal transformer is inserted and absorbed according

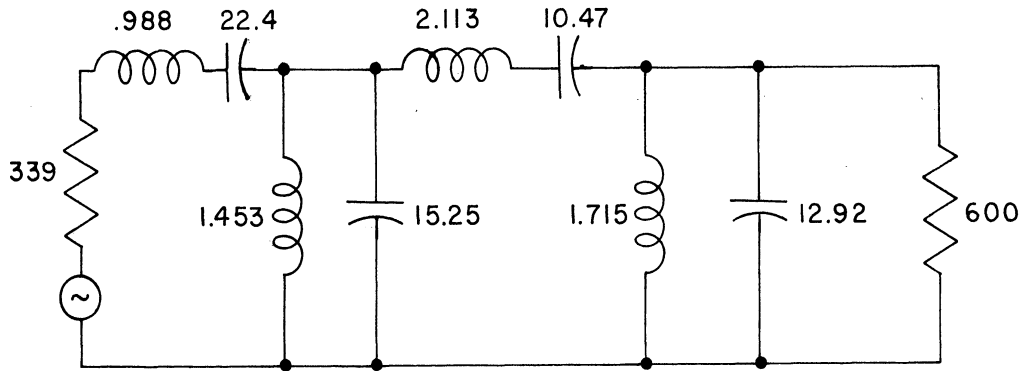


FIG. 13. BANDPASS NETWORK DERIVED FROM FIG. 12

VALUES IN OHMS, μh AND $\mu\mu f$

to principles described in Electronic Defense Group Technical Report No. 44.

The final inverted vee network is indicated in Fig. 14.

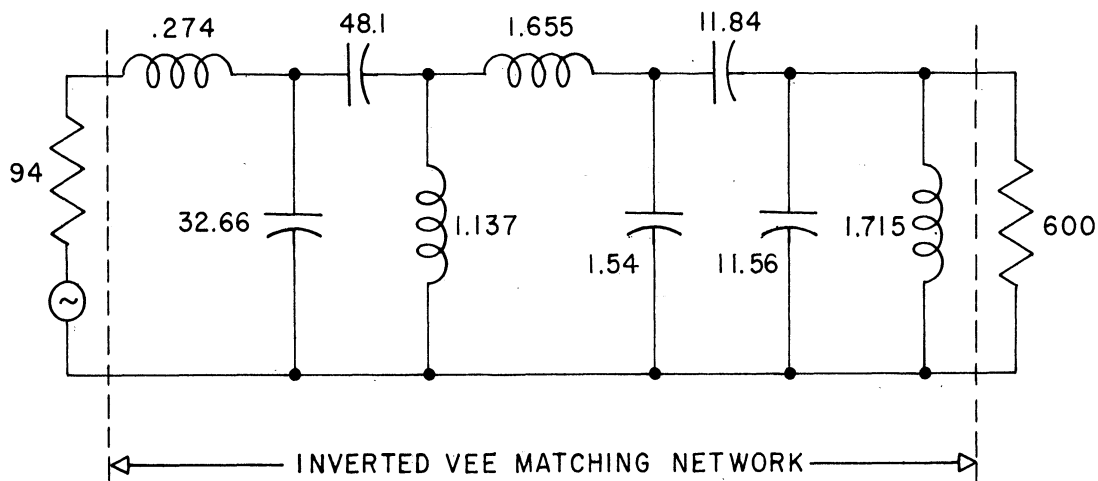


FIG. 14 THE INVERTED VEE ANTENNA MATCHING NETWORK

VALUES IN OHMS, μh AND $\mu\mu f$

The transmission of the network of Fig. 14. drops off very rapidly below 19 mc and above 60 mc. Therefore, this network prevents efficient radiation outside the 19-60 mc band, no matter how broad the frequency coverage of the antenna with which it is used. The use of a network with a sharp cutoff characteristic is desirable on a harmonic rejection basis, since with the lowpass and highpass

complementary networks, the harmonic rejection in the 34-60 mc band must be introduced by the antenna matching networks.

An alternative form of Fig. 14 is defined below. See Fig. 15. This form has several advantages, but seems inferior on the basis that the stray capacitance which can be absorbed in the matching network at the inverted vee input is less than $3.37 \mu\text{mf}$.

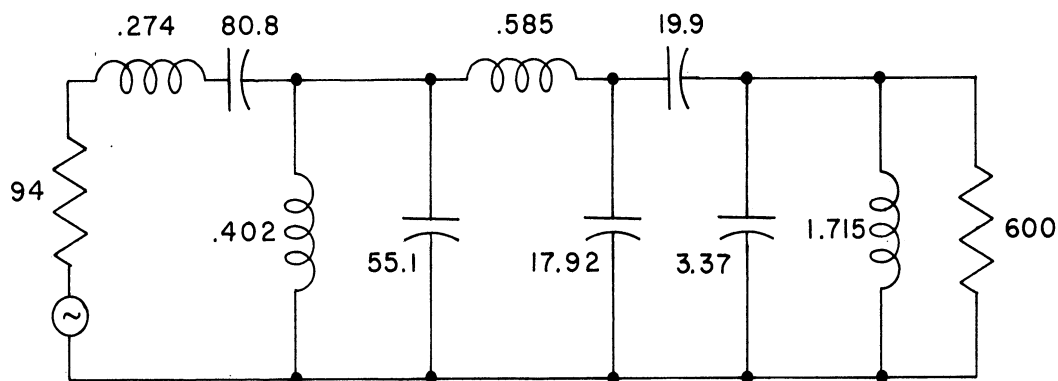


FIG. 15. ALTERNATIVE FORM OF FIG. 14
VALUES IN OHMS, μh AND μmf

V. DESIGN OF COMPLEMENTARY FILTERS

Efficient simultaneous operation of two antennas from a single power source is accomplished through the use of complementary filters. In particular, three complementary pairs are used; each pair consisting of a low pass and a high pass network. When a pair of complimentary networks are driven in parallel by a resistive generator, a division of power between the two network terminations as indicated in Fig. 16 can be achieved. The design procedure used in obtaining the networks is summarized below.

If the networks of a pair are complementary and the generator is matched in the interests of efficiency, two equations can be written which are useful.

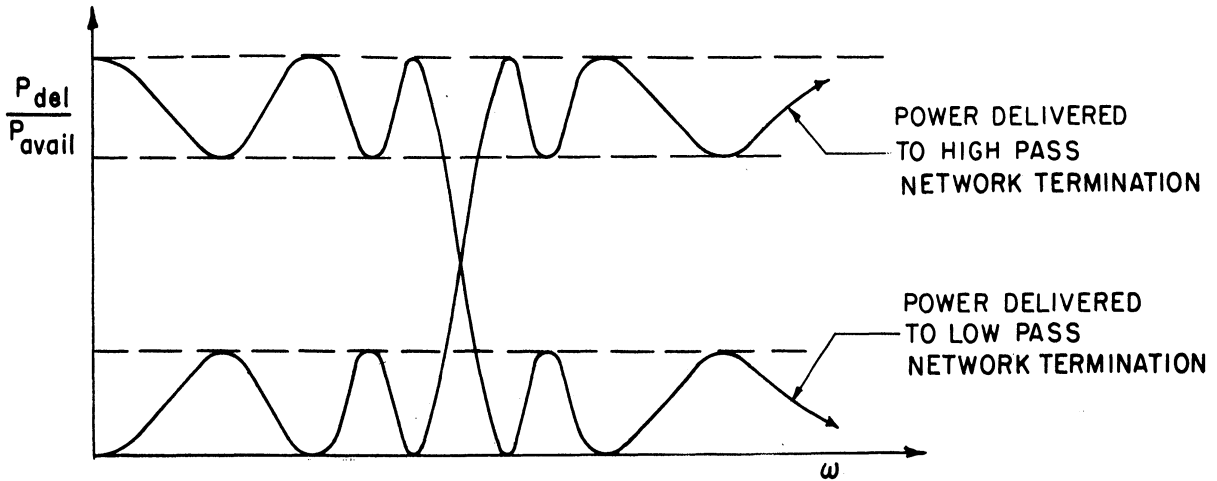


FIG. 16
TCHEBYCHEFF COMPLEMENTARY FILTER CHARACTERISTICS

$$P_{LP}(\omega) + P_{HP}(\omega) = P_{avail} \quad (1)$$

$$Y_{LP}(\omega) + Y_{HP}(\omega) = \frac{1}{R} \quad (2)$$

P_{LP} is the power delivered to the termination of the low pass filter; P_{HP} is similarly defined for the high pass termination. P_{avail} is the power available from the generator; the coupling networks are assumed lossless. Also, referring to Fig. 17, the second equation prescribes that the sum of the input admittances of the two networks is a constant $1/R$ independent of frequency; i.e., the generator is matched. Now, if the generator is matched, a voltage E_1 is applied to both networks at all frequencies.

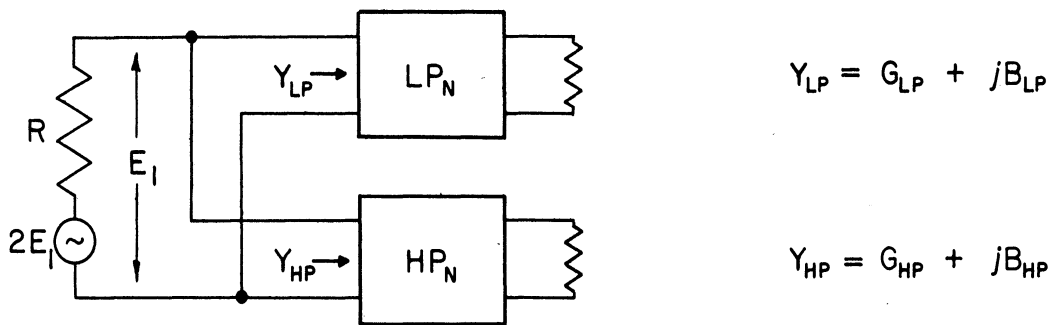


FIG 17
THE COMPLEMENTARY NETWORK PROBLEM

The power delivered to the low pass termination is

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$$P_{LP} = |E_1|^2 G_{LP}(\omega) \quad (3)$$

if LP_N is lossless. Similarly, if HP_N is lossless,

$$P_{HP} = |E_1|^2 G_{HP}(\omega) \quad (4)$$

Thus the power division between the terminations is controlled by choice of $G_{LP}(\omega)$ and $G_{HP}(\omega)$. From Eq 1

$$G_{LP}(\omega) + G_{HP}(\omega) = \frac{1}{R} \quad (5)$$

Equation 5 states that the functions G_{LP} and G_{HP} cannot be chosen independently. To illustrate this, suppose that the power delivered to one termination is 20 db below the available power of the generator. Then 99% of the available power is delivered to the other termination; this is .043 db below the available power. Thus the passband characteristic of one network depends on the rejection characteristic of the other.

The desired properties of the individual conductances $G_{LP}(\omega)$ or $G_{HP}(\omega)$ for use in the antenna system can be summarized in terms of the maximum passband loss, the width of the transition region between the pass and the stop bands, and the minimum attenuation in the stop band. The design of networks meeting requirements of this type has been dealt with in the literature (Refs. 1 and 7). It has been shown that specifications of the above type can be met with a minimum number of network elements through the use of filters which have Tchebycheff or equal ripple character in both the pass and stop bands, as shown in Fig. 16.¹

¹ If a Butterworth complementary pair is used, it is easily demonstrated that, to get 20 db rejection at 12 mc with a loss of 3 db at 10.8 mc, a 22 pole network is required. The use of networks having zeros at finite frequencies is clearly indicated.

The lowest harmonic associated with radiation in the 6 to 10.8 mc band is 12 mc; for a minimum harmonic rejection of 20 db, the requirements for the low pass characteristic are as sketched in Fig. 18.¹ It has been shown that the

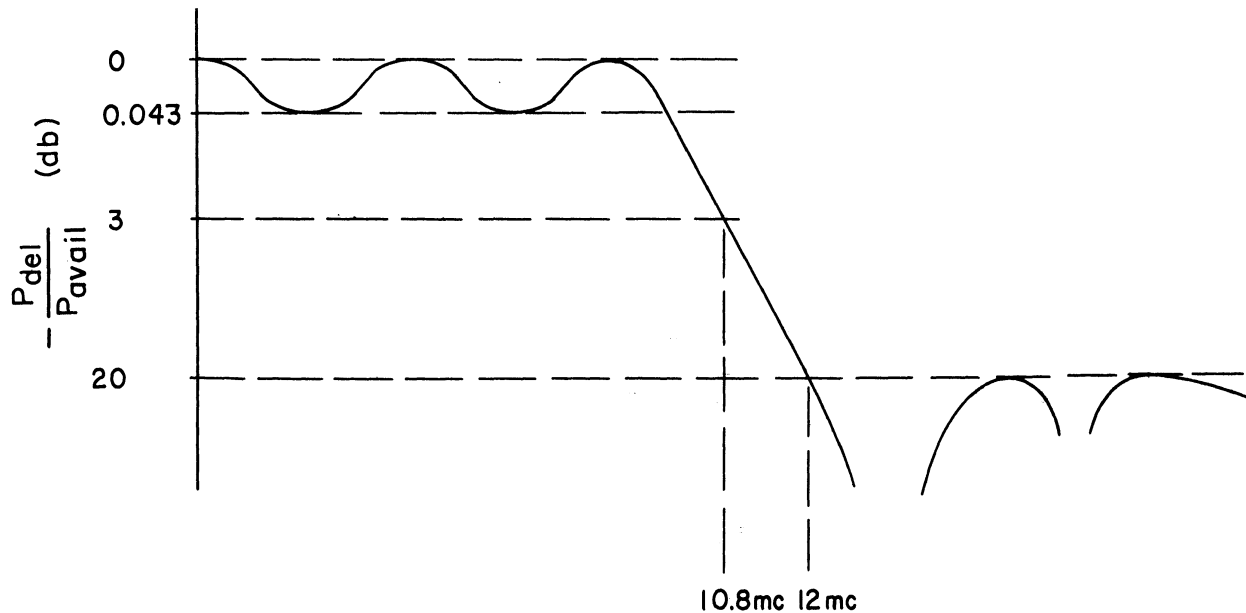


FIG 18
APPROXIMATE DESIRED LOWPASS CHARACTERISTIC

relation between α_p (maximum passband attenuation), α_s (minimum stopband attenuation), the ratio k of the frequencies at the edges of the pass and stop bands (in this case $k = 10.8/12 = .9$), and the complexity of the network n is given approximately by²

$$n = \frac{2K}{nK'} \left[.115\alpha_s + 1.15 \log \left(\frac{4 \times 8.68}{\alpha_p} \right) \right] \begin{matrix} \alpha_s > 15 \text{ db} \\ \alpha_p < 1.5 \text{ db} \end{matrix} \quad (6)$$

where K and K' are complete elliptic integrals and depend only on the parameter k .

From tabulations (Ref. 8)

$$K = 2.2805$$

$$K' = 1.6546$$

¹The use of bandpass networks has been considered. Certain technical problems seem to make the simpler lowpass and highpass networks preferable.

²Based on results in Refs. 1 and 7.

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Therefore,

$$n = .876 \left[.115\alpha_s + 1.15 \log \left(\frac{4 \times 8.68}{\alpha_p} \right) \right]$$

From this equation it can be shown that, assuming the same α_s is required for the low and high pass bands, the following results are obtainable:

$$\begin{array}{lll} n = 5 & \alpha_p = .058 \text{ db} & \alpha_s = 18.8 \text{ db} \\ n = 6 & \alpha_p = .006 \text{ db} & \alpha_s = 27.5 \text{ db} \end{array}$$

In the interests of simplicity, the 5-pole ($n = 5$) values were deemed acceptable. To determine the networks achieving these results, some lengthy computations are required. Following the procedure of Fano (Ref. 7), certain useful parameters are first evaluated.

$$k' = \sqrt{1 - k^2} = .436$$

$$\alpha_p = 10 \log_{10}(1 + \epsilon) \quad \epsilon = 0.135$$

$$q_1 = e^{-\frac{n\pi K_1}{K}} = 11.11 \times 10^{-6}$$

$$k_1 = \frac{4q_1^{1/2}}{(1+2q_1)^2} = .01336$$

$$k'_1 = \sqrt{1 - k_1^2} = .99997$$

Using k_1 and k'_1 , the associated complete elliptic integrals, K_1 and K'_1 , can be evaluated as before.

$$K_1 \cong \frac{\pi}{2}$$

$$K'_1 = 5.71$$

A parameter a is determined from the relation

$$\text{sn} \left(n \frac{K_1}{K} a, k'_1 \right) = \frac{1}{\sqrt{1+\epsilon}} = .9932$$

From tabulations of the elliptic function $\text{sn}(u, k)$ in Ref. 8, one finds

$$a = .825$$

Then the desired poles of the lowpass function with one radian per second bandwidth and $k = .9$ are to be located at¹

$$P_{pl} = j \operatorname{sn} \left\{ \left[2l \frac{K}{n} \pm ja \right], k \right\} \quad \begin{matrix} n & \text{odd} \\ l = 0, 1, 2, \dots, n \end{matrix} \quad (7)$$

Also, the zeroes of transmission are to be located at

$$P_{ol} = \frac{j}{k \operatorname{sn} \left\{ 2l \frac{K}{n}, k \right\}} \quad \begin{matrix} n & \text{odd} \\ l = 0, 1, 2, \dots, n \end{matrix} \quad (8)$$

The elliptic function $\operatorname{sn} \{ u_1 + ju_2, k \}$ is evaluated using

$$\operatorname{sn} \{ u_1 + ju_2, k \} = \frac{\operatorname{sn}(u_1, k) \operatorname{dn}(u_2, k') + j \operatorname{cn}(u_1, k) \operatorname{dn}(u_1, k) \operatorname{sn}(u_2, k') \operatorname{cn}(u_2, k')}{\operatorname{cn}^2(u_2, k) + k^2 \operatorname{sn}^2(u_1, k) \operatorname{sn}^2(u_2, k')} \quad (9)$$

The functions $\operatorname{sn}(u, k)$, and $\operatorname{dn}(u, k)$ are evaluated from tabulations in Refs. 8 or 9². It is found that the poles and zeroes of the lowpass function with unit bandwidth and $k = .9$ are located at

$$P_{o1} = \pm j 1.5105$$

$$P_{o2} = \pm j 1.1350$$

$$P_{p1} = \pm 1.0495$$

$$P_{p2} = \pm .3578 \pm j .9893$$

$$P_{p3} = \pm .05432 \pm j 1.0524$$

The desired lowpass power characteristic with unit tolerance bandwidth is then

$$\frac{P_{LP}}{P_{avail}} = \frac{.19259(p+j1.5105)^2(p-j1.5105)^2(p+j1.135)^2(p-j1.135)^2}{f(p) [-f(-p)]} \quad (10)$$

¹These equations determine the loci of the poles and zeroes of the desired function in the p -plane. Here $p = \sigma + j\omega$ is the complex frequency. See Refs. 2, 6, and 7.

²In Ref. 8, the tabulation is in terms of a parameter m which is simply k^2 in the above notation.

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where

$$f(p) = (p+1.0495)(p+.3578+j.9893)(p+3.578-j.9893)(p+.04532+j1.0524)(p+.05432-j1.0524)$$

and the multiplying constant was chosen for a matched condition at $p = 0$.

Combining,

$$\frac{P_{LP}}{P_{avail}} = \frac{.19259\omega^8 - 1.3750\omega^6 + 3.5836\omega^4 - 4.04139\omega^2 + 1.66372}{\omega^{10} - 2.809\omega^8 + 1.9091\omega^6 + 2.0436\omega^4 - 3.7811\omega^2 + 1.66372} \quad (11)$$

where p^2 is replaced by $-\omega^2$.

Using Eq 1

$$\frac{P_{HP}}{P_{avail}} = \frac{10\omega^{10} - 3.0016\omega^8 + 3.284\omega^6 - 1.5428\omega^4 + .2603\omega^2}{\omega^{10} - 2.809\omega^8 + 1.9091\omega^6 + 2.043\omega^4 - 3.7811\omega^2 + 1.66372} \quad (12)$$

The next problem is to obtain the networks from the desired transfer functions. To do this, use can be made of the equation

$$Y_{12} = \frac{-y_{12}G}{G + y_{22}} \stackrel{d}{=} \frac{I_2}{E_1} \quad (13)$$

where I_2 and E_1 are defined in Fig. 19, and y_{12} and y_{22} are the widely used short circuit parameters of the lossless coupling network N.¹

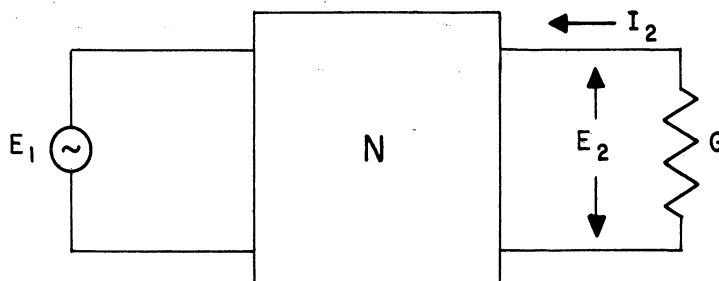


FIG. 19. THE VOLTAGE TRANSFER PROBLEM

¹See Ref. 10 pg. 542.

It is convenient to obtain networks normalized to a one ohm termination ($G = 1$).

Then $I_2 = -E_2$ and Eq 13 becomes

$$\frac{E_2}{E_1} = \frac{+y_{12}}{1 + y_{22}} \quad (14)$$

Now the voltage applied to each of the complementary networks is a constant E_1 as previously defined. The available power from the generator with voltage $2E_1$ (in Fig. 17) and $R = 1$ is

$$P_{\text{avail}} = |E_1|^2 \quad (15)$$

The power delivered to the lowpass termination of one ohm is

$$P_{\text{LP}} = |E_2|^2 \quad (16)$$

where E_2 is the voltage across the lowpass termination. Equation 16 is also applicable to the high pass structure. Then using Eqs 14, 15, and 16

$$\frac{P_{\text{LP}}}{P_{\text{avail}}} = \left| \frac{y_{12_{\text{LP}}}}{1 + y_{22_{\text{LP}}}} \right|^2 \quad (17)$$

$$\frac{P_{\text{HP}}}{P_{\text{avail}}} = \left| \frac{y_{12_{\text{HP}}}}{1 + y_{22_{\text{HP}}}} \right|^2$$

It can be shown that the denominator of $y_{12}/1+y_{22}$ must be made up of the right half plane factors of the denominator in Eq 10. The numerator is simply the square root of the numerator from Eq 10. Thus, for our case,

$$\frac{y_{12_{\text{LP}}}}{1 + y_{22_{\text{LP}}}} = \frac{.4385p^4 + 1.5640p^2 + 1.2898}{p^5 + 1.8737p^4 + 3.1599p^3 + 3.3234p^2 + 2.1892p + 12898} \quad (18)$$

The y_{12} and y_{22} of a lossless lumped parameter network are ratios of odd/even or even/odd polynomials. Further, it can be assumed that y_{12} and y_{22} have identical

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denominators for this discussion. Then, dividing by the odd part of the denominator in Eq 18 for the lowpass case, one can identify

$$y_{12_{LP}} = \frac{.43885p^4 + 1.56404p^2 + 1.2898}{p^5 + 3.1599p^3 + 2.1892p}$$

and

$$y_{22_{LP}} = \frac{1.8737p^4 + 3.3234p^2 + 1.2898}{p^5 + 3.1599p^3 + 2.1892p}$$

To obtain the network it is necessary to synthesize y_{22} in such a way that the desired zeroes of y_{12} are also created. This process is discussed in Appendix B. The network of Fig. 20a is the result of such a synthesis. From Eq 12, and using the procedure followed in the lowpass development above, one finds

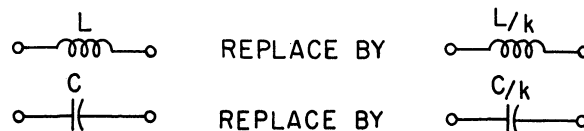
$$y_{12_{HP}} = \frac{p^5 + 1.5008p^3 + .5102p}{1.8737p^4 + 3.3234p^2 + 1.2898}$$

and

$$y_{22_{HP}} = \frac{p^5 + 3.1599p^3 + 2.1892p}{1.8737p^4 + 3.3234p^2 + 1.2898}$$

The resulting network is shown in Fig. 20b. It is now necessary to adjust the networks of Fig. 20 to the desired 94 ohm impedance level and to the desired crossover frequency. At the crossover frequency the power to each network termination is down 3 db; for unit tolerance bandwidth and an 18.8 db loss at $\omega = 1.111(=12/10.8)$ radians per second, the 3 db point is near $\omega = 1.03$ radians per second. It is easily determined¹ that for crossover frequencies of 10.8 mc,

¹The bandwidth of the lowpass characteristic is to be changed from 1.03 radians per second to, for example, 10.8 mc or $2\pi(10.8) \times 10^6$ radians per second. The necessary transformations to modify bandwidth by a factor k are (Ref. 4):



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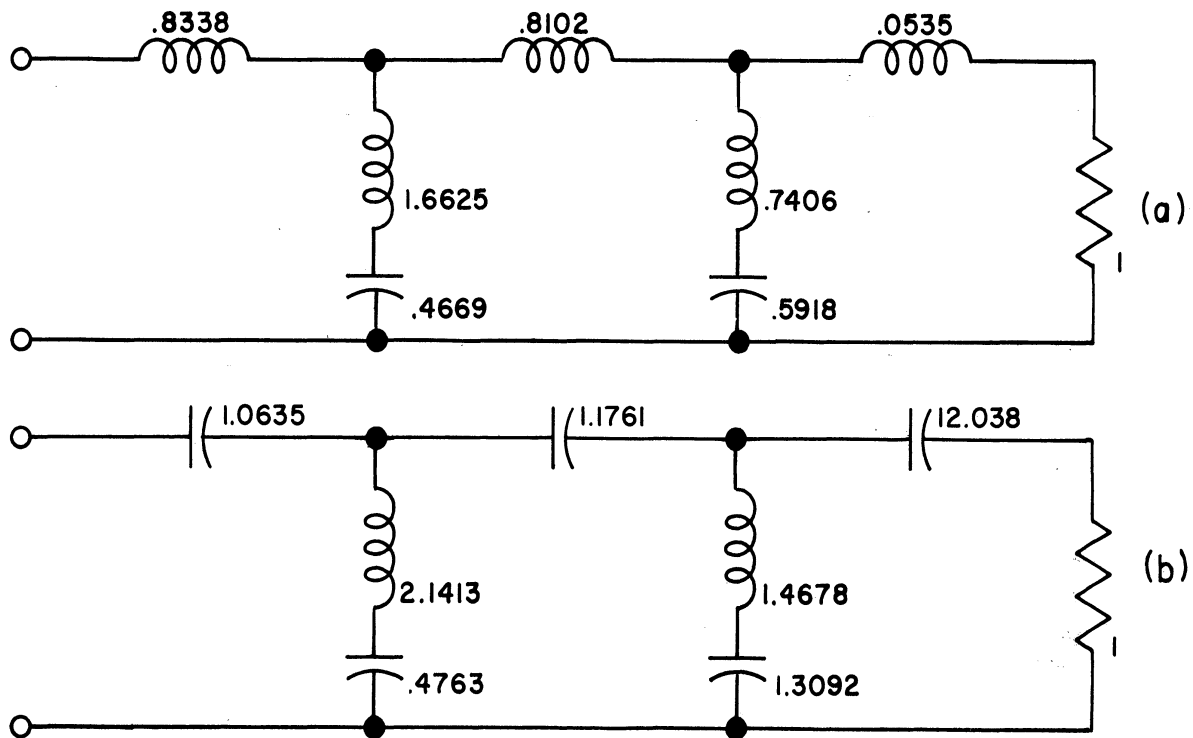
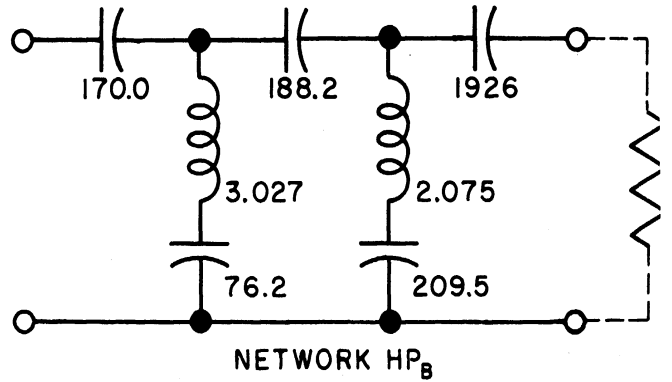
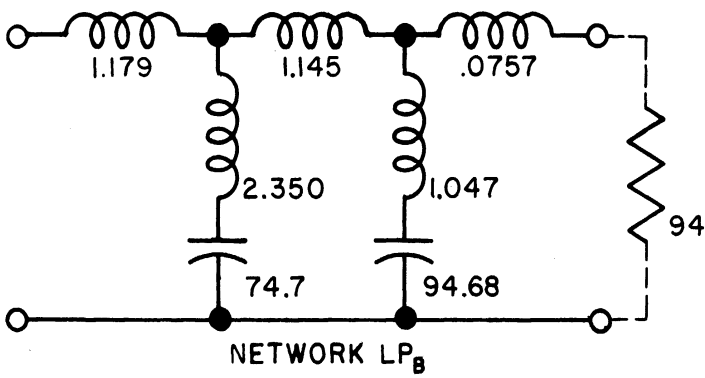
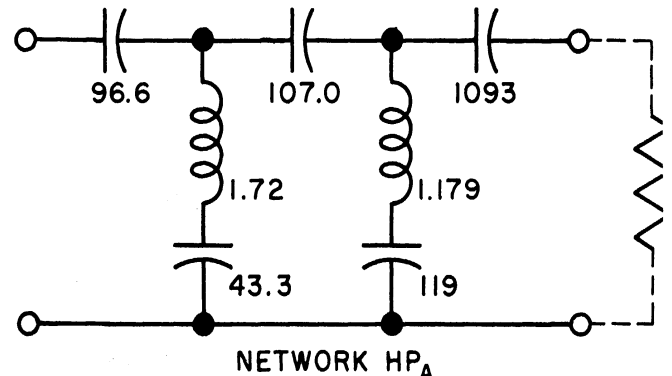
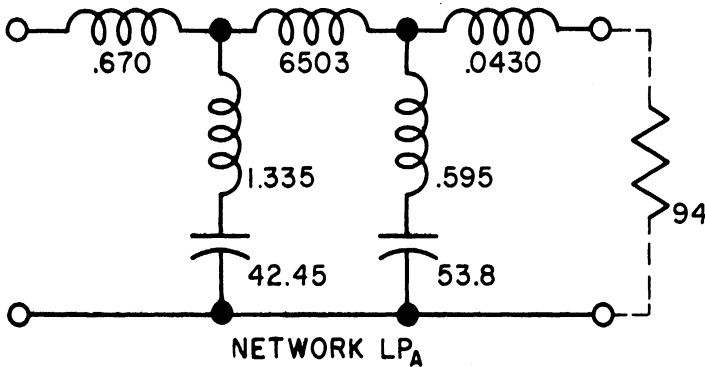


FIG. 20.
 NORMALIZED COMPLEMENTARY TCHEBYCHEFF NETWORKS
 VALUES IN FARADS, HENRYS, AND OHMS

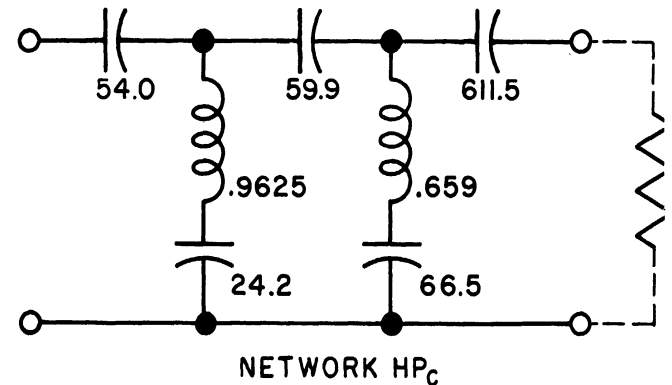
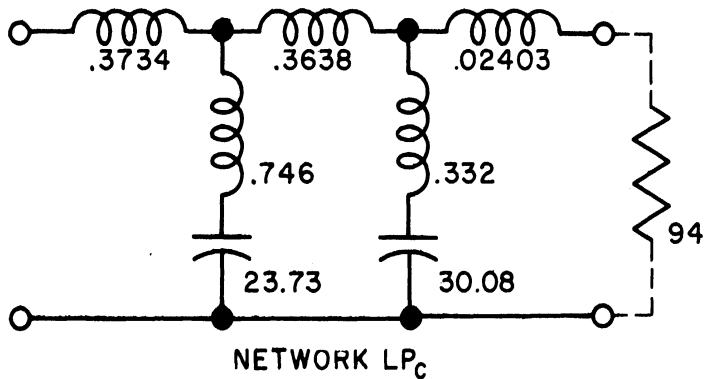
19 mc, and 34 mc, as required in the antenna system, the desired networks are those of Fig. 21.



a) COMPLEMENTARY PAIR: 10.8 mc CROSSOVER



b) COMPLEMENTARY PAIR: 19 mc CROSSOVER



c) COMPLEMENTARY PAIR: 34mc CROSSOVER

FIG 21
COMPLEMENTARY FILTERS OF FIG 1

NOTE: EACH PAIR TO BE DRIVEN IN PARALLEL BY 94 OHM GENERATOR AND EACH NETWORK TO BE TERMINATED IN 94 OHM LOAD AS SHOWN.

APPENDIX A

EXPERIMENTAL DATA ON PROTOTYPE NETWORKS

The antenna system defined in this report is block diagrammed in Fig. 1, and described in general terms in Section 3. The individual networks are defined in Sections 4 and 5, where the design procedures are also summarized. In general, the coupling networks synthesized are lossless; in practice, this can only be approached. Further, it seems reasonable to expect that the advantages of the relatively complex networks defined can be realized only if the element values are accurate. It is therefore expected that the actual system performance will be somewhat poorer than is indicated theoretically. As a guide in estimating this expected deterioration, tests were run on a set of networks built at EDG.

All element values in the networks built by EDG were set on a Q-meter for which 3 percent accuracy is claimed. A minor adjustment of only one element value was made to get the results presented below.¹ The Q's of all capacitances and inductances were about 150 to 200 near the middle of their operating frequency range.

In general, the inductances and capacitances used had self resonant frequencies well above the upper frequency limit of the band. Since in the frequency region substantially below resonance presence of shunt capacity results in an increase in the effective inductance of a coil, and since the presence of capacitor lead length results in an increase in effective capacitance, as indicated

¹

This adjustment was made in the inverted vee matching network and is described later in this appendix.

by the Q-meter, an effort was made to minimize these effects. The results substantiate the opinion that more sophisticated approaches are unnecessary.

Measurements on the overall system achieve only limited insight, because of the system complexity. These data can be made more useful if supplemented with information on the performance of the several subassemblies. These subassemblies are:

- a) the individual matching networks,
- b) the individual complementary pairs.

Erection of antennas has not been feasible. For this reason, experimental data on the performance of the antenna matching networks was obtained with the antennas replaced by the lumped antenna approximating circuits developed in Section

4. The experimental circuit is as indicated in Fig. 22. The output power $\left| \frac{E_2}{R_2} \right|^2$ is measured as a fraction $|t|^2$ of the available power $\frac{|E_1|^2}{4(94)}$ from the

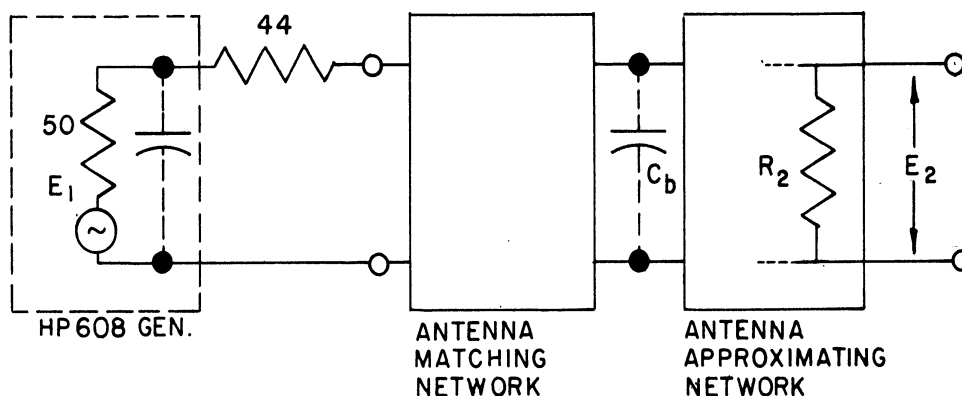


FIG. 22. EXPERIMENTAL CIRCUIT.

resistive generator. In Fig. 23, 24, and 25 the results are plotted for the two monopole and the inverted vee matching networks in the form $-10 \log_{10} |t|^2$ vs. frequency. The theoretical passband losses, and the design frequency ranges are indicated. The results appear reasonable.

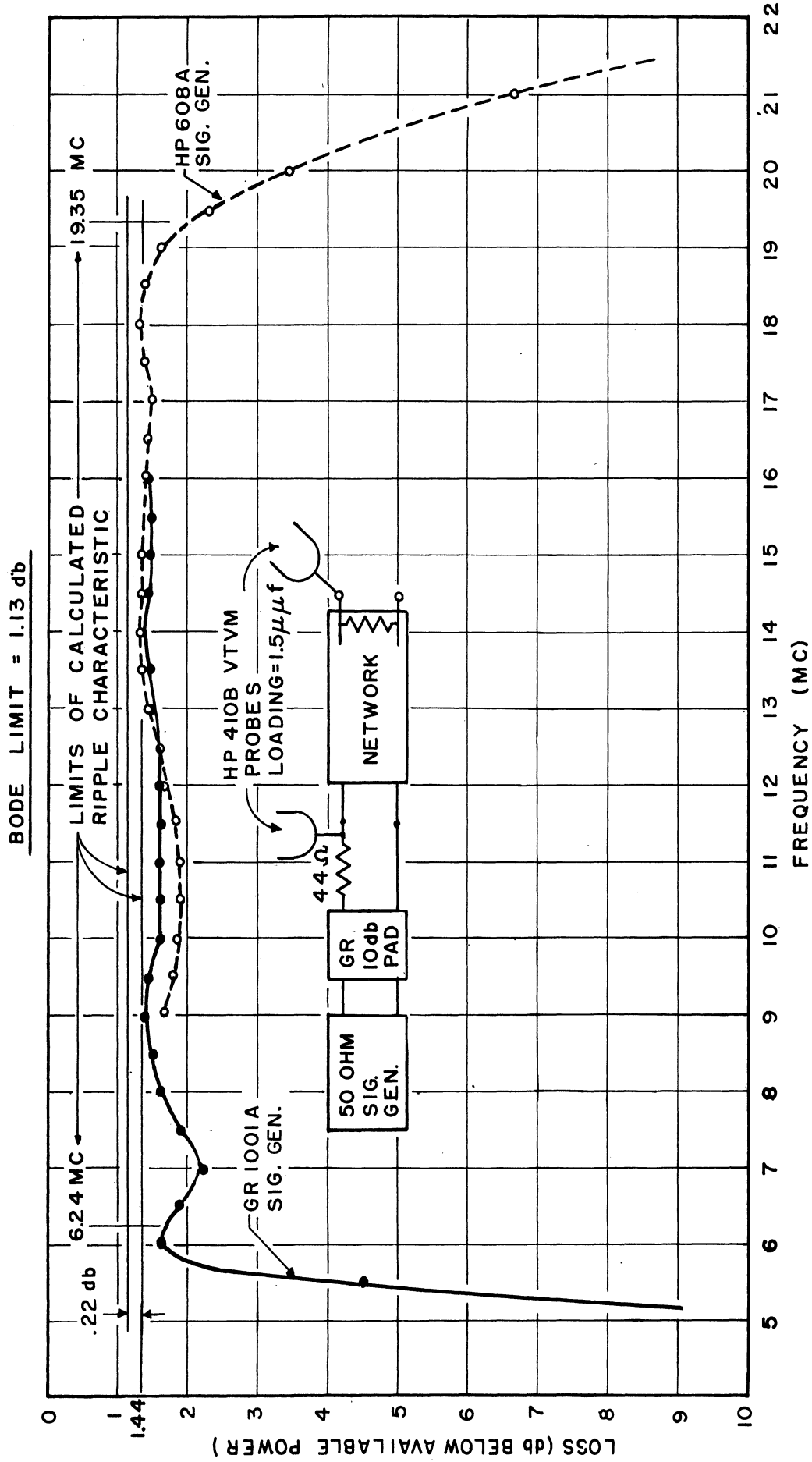


FIG. 23
EXPERIMENTAL BANDPASS CHARACTERISTIC
6-19 mc MONOPOLE AND MATCHING NETWORK

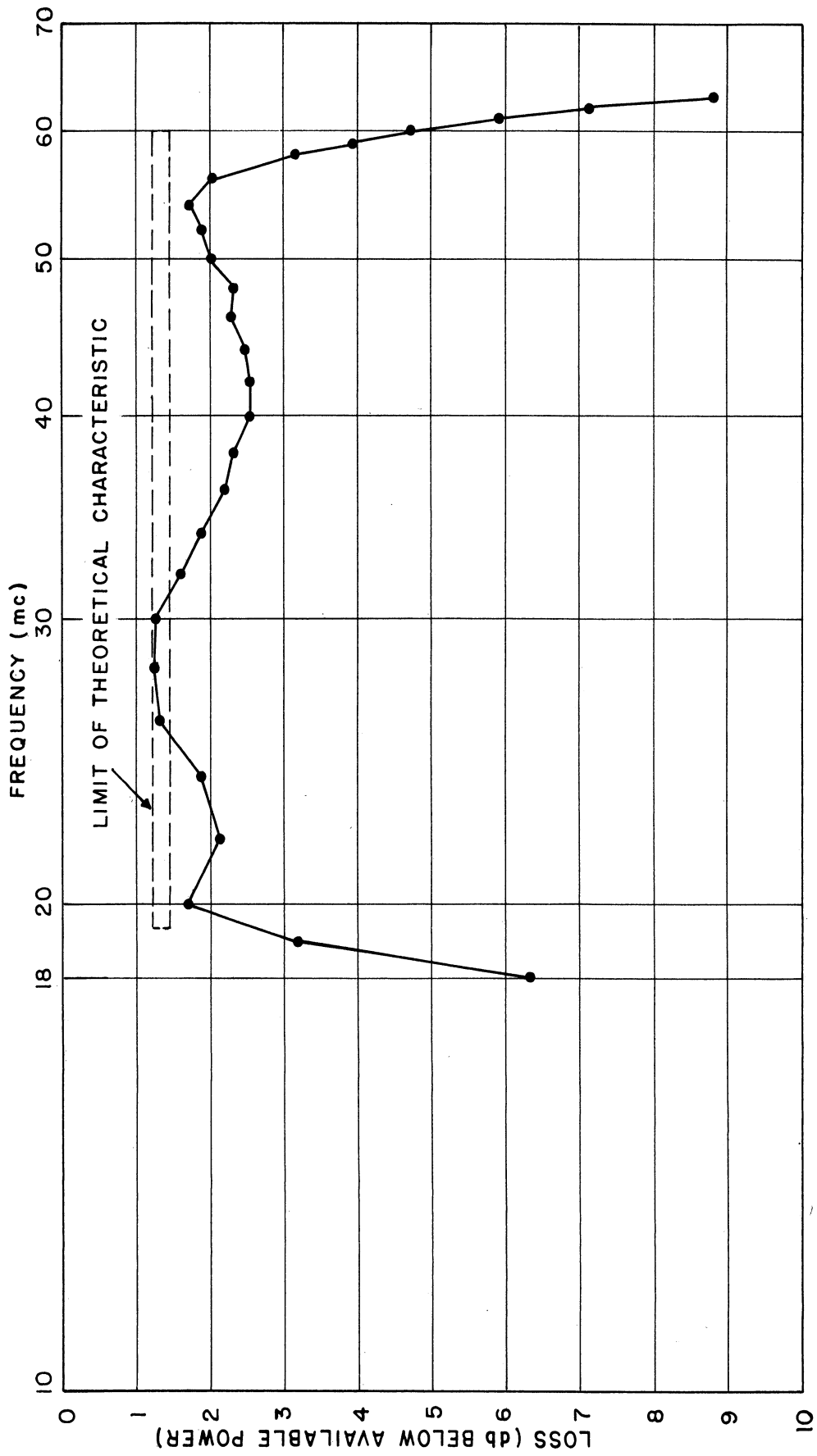


FIG. 24
 EXPERIMENTAL BANDPASS CHARACTERISTIC
 19-60 mc MONOPOLE AND MATCHING NETWORK

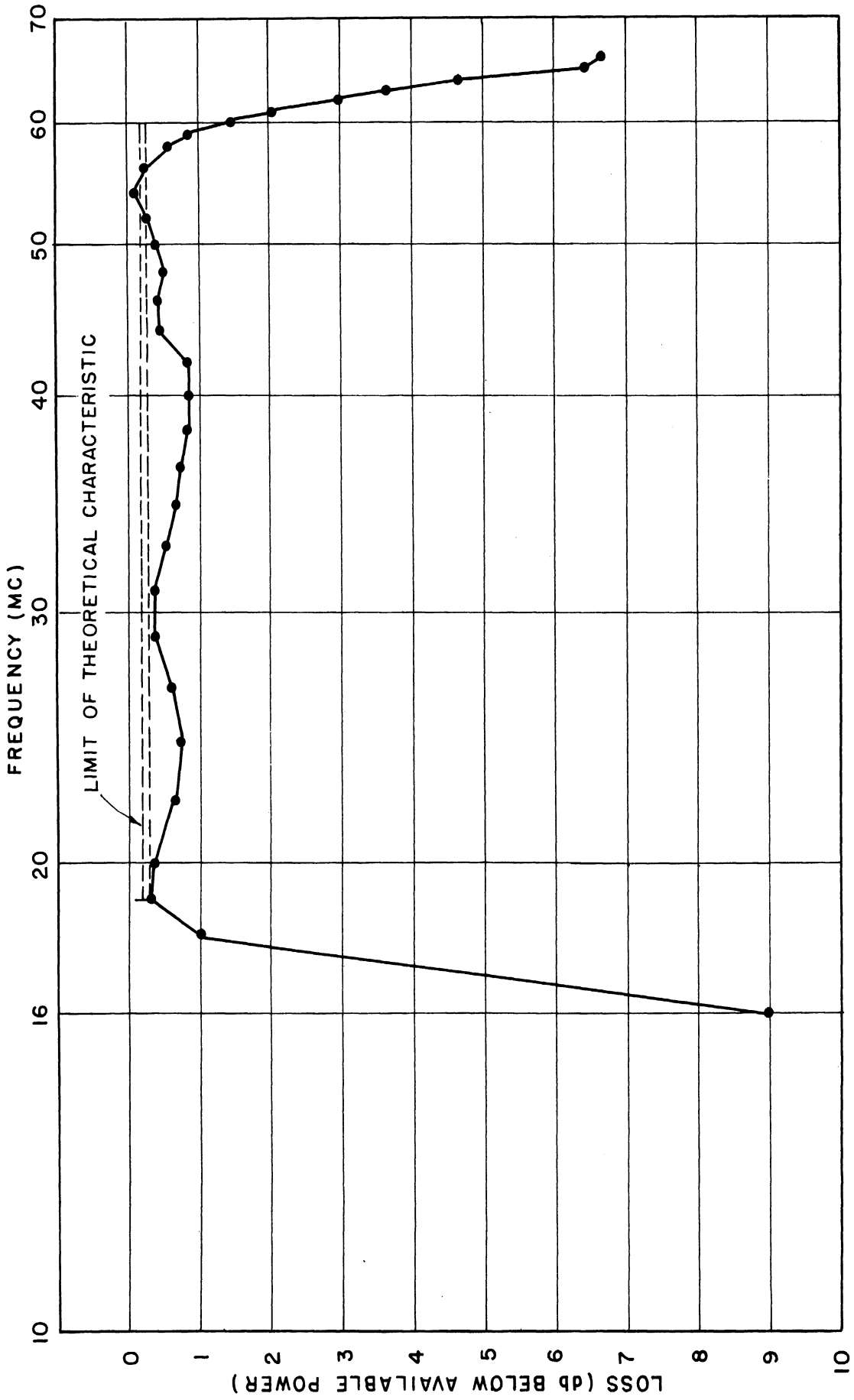


FIG. 25
EXPERIMENTAL BANDPASS CHARACTERISTIC
INVERTED VEE MATCHING NETWORK

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A minor adjustment was made in the inverted vee matching network. It was found that the experimental curve was improved moderately through omission of the 1.5 μf shunt condenser of Fig. 14. Actually, stray capacitance of this magnitude presumably exists at this point in the network. It is noted in passing that the bandwidth of this network can be varied several megacycles/sec through adjustment of the terminal capacitance.

The performance of the complementary pairs was evaluated using the circuit of Fig. 26. The output powers $P_{LP} = |E_{2LP}|^2/94$ and $P_{HP} = |E_{2HP}|^2/94$ are measured as a fraction of the available power $|2E_1|^2/4(94)$ from the resistive generator. The results on the complementary pairs with crossover frequencies of 10.8 mc, 19 mc, and 34 mc are plotted in Figs. 27, 28, and 29. In these figures the data are plotted in the forms $-10 \log_{10} \frac{P_{LP}}{P_{avail}}$, $-10 \log_{10} \frac{P_{HP}}{P_{avail}}$, and $-10 \log_{10} \frac{P_{LP} + P_{HP}}{P_{avail}}$ versus frequency. The theoretical passband losses $-10 \log_{10} \frac{P_{LP}}{P_{avail}}$ and $-10 \log_{10} \frac{P_{HP}}{P_{avail}}$ are $<.058$ db. The plots of $-10 \log_{10} \frac{P_{LP} + P_{HP}}{P_{avail}}$ are a measure of network loss, mismatch due to improper element values, and experimental error. It is especially important to determine whether an appreciable fraction of the total indicated loss is due to dissipation in the network elements. For example, a 0.5 db network loss would lead to a network dissipation

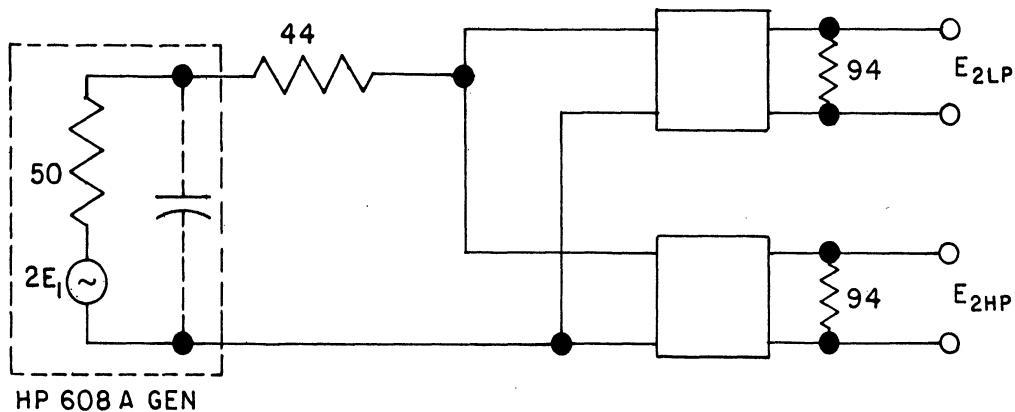
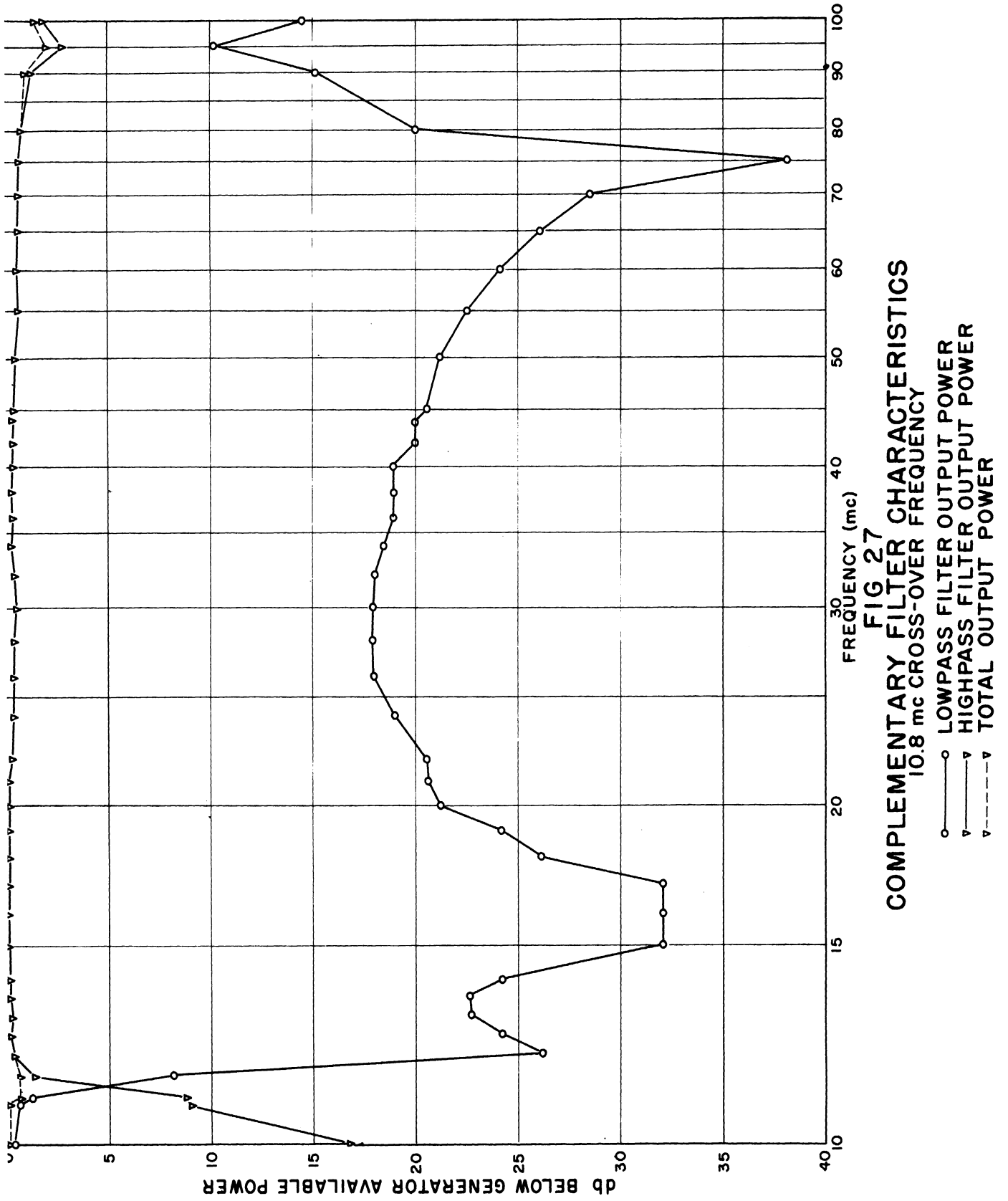
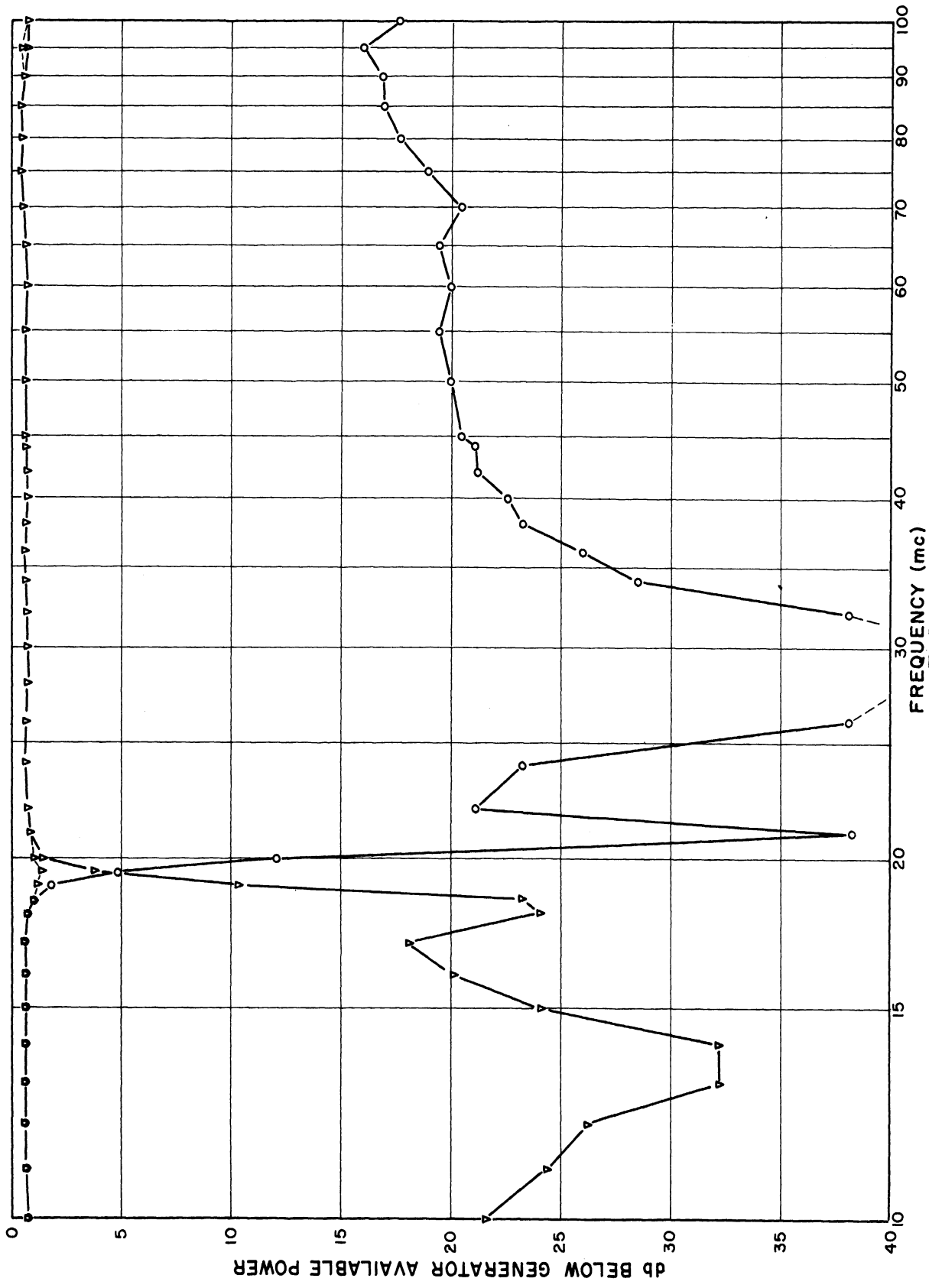
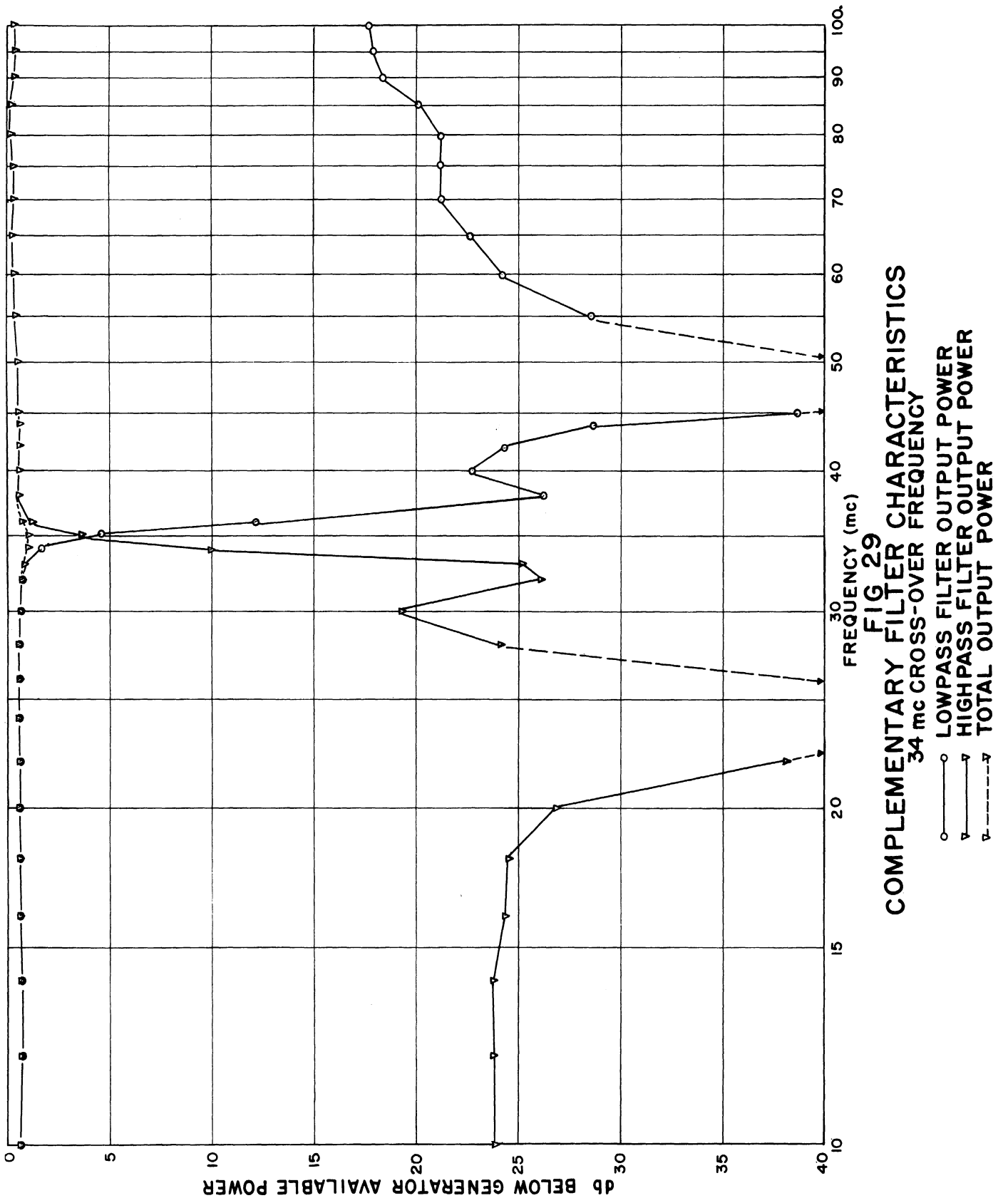


FIG. 26. EXPERIMENTAL CIRCUIT





COMPLEMENTARY FILTER CHARACTERISTICS
FIG 28
 19 mc CROSS-OVER FREQUENCY
 ○ LOWPASS FILTER OUTPUT POWER
 ▼ HIGHPASS FILTER OUTPUT POWER
 - - - - - TOTAL OUTPUT POWER



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requirement of about 11 watts at a 100 watt power level. This dissipation requirement would be an important factor in realizing a workable high power system. The actual loss is seen to be much less than .5 db on the basis of the following experimental evidence.

A complementary pair was driven with a distributed amplifier having a plate line impedance of 90 ohms. The networks were terminated with loads which were nominally 94 ohms. Through the use of an insertion type wattmeter¹ and voltage measurements at the terminations it was ascertained that 50 to 100 watts could be delivered to the terminations at a number of points over the frequency range 6 to 60 mc. Under these conditions, no heating of the network elements of either the lowpass or highpass network of any of the complementary pairs was detected with the hand. It is felt that dissipation of even an appreciable fraction of a watt in an element would cause a detectable (by hand) temperature rise. Due to a lack of quality 94 ohm high power terminations, and a lack of accurate 94 ohm impedance level power measuring equipment, it was not feasible to obtain curves at a high power level. No heating of the elements of the antenna matching networks was detected when subjected to the same test.

It appears that the indicated loss of about .5 db in Figs. 27, 28, and 29 is partially due to the presence of considerable shunt capacitance at the output of the 50 ohm generator used to obtain the data. Referring to Fig. 26 it is noted that when the generator is used in conjunction with a 44 ohm series resistor to obtain a 94 ohm generator, considerable mismatch of the 50 ohm coaxial line at the generator output exists when the composite generator is matched. Therefore, the coaxial line existing at this point has a definite effect on the

¹

The instrument used was a Bird Thru-line Model 43 Wattmeter.

transfer efficiency. In summary, it is felt that the indicated loss is primarily due to imperfections in the experimental circuit. However, the general character of the curve of Figs. 27, 28, and 29 is considered to be reliable. In particular, the 20 db harmonic rejection requirement is substantially met.

The curves of Figs. 30 through 37 show the results obtained on the composite system. It was not considered worthwhile to obtain data for all possible permutations of the structure. The combinations selected are considered representative.

In Fig. 30, the 6-19 mc monopole approximating circuit is fed through its matching network, 25 feet of RG/62U coaxial cable and network LP_A by a 94 ohm generator. The complementary network HP_A is terminated. The power delivered to the approximating circuit is plotted in db below the available power of the 94 ohm source. The power delivered to the termination of network HP_A is also plotted. The indicated negative loss at 10 mc is due to experimental error and does not justify the assumption of a fixed error over the whole curve. The data were obtained by setting the driving generator to a prescribed output meter reading at each frequency, and the several output powers determined. The Thevenin voltage at each frequency was then measured after resetting the generator output meter to the same reading with the generator load removed. Significant sources of error are the inability to reset the output level, meter drift, and variation of the generator internal voltage with load. The overall curves are, however, felt to be of satisfactory accuracy.

It is noted that a number of ripples occur in the power curve over the portion of the 6-19 mc band which is plotted.¹ Referring to Fig. 31, a similar

¹

Data were not obtained below 10 mc due to the lack of a suitable generator.

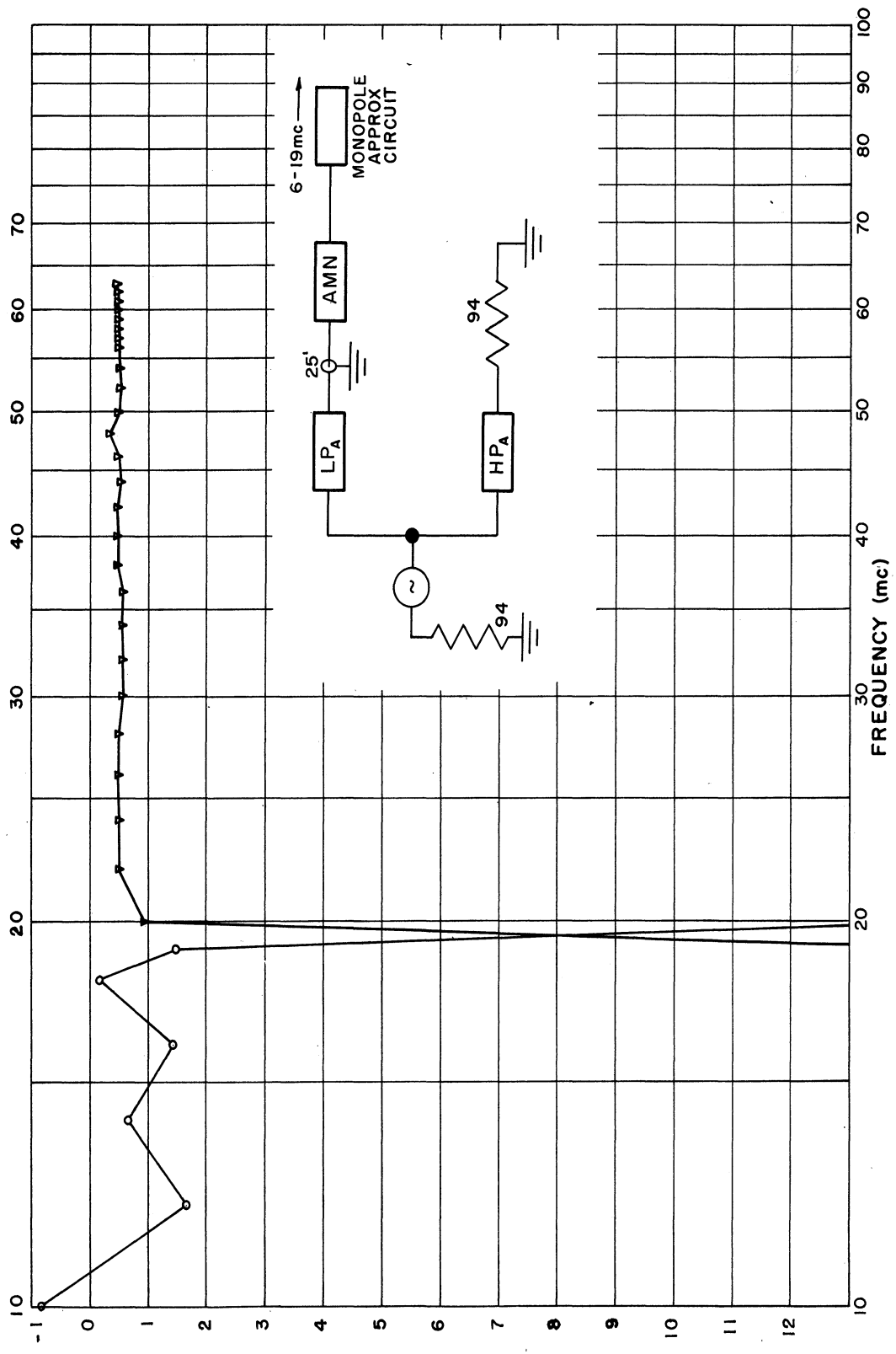


FIG 30
EXPERIMENTAL POWER CHARACTERISTIC

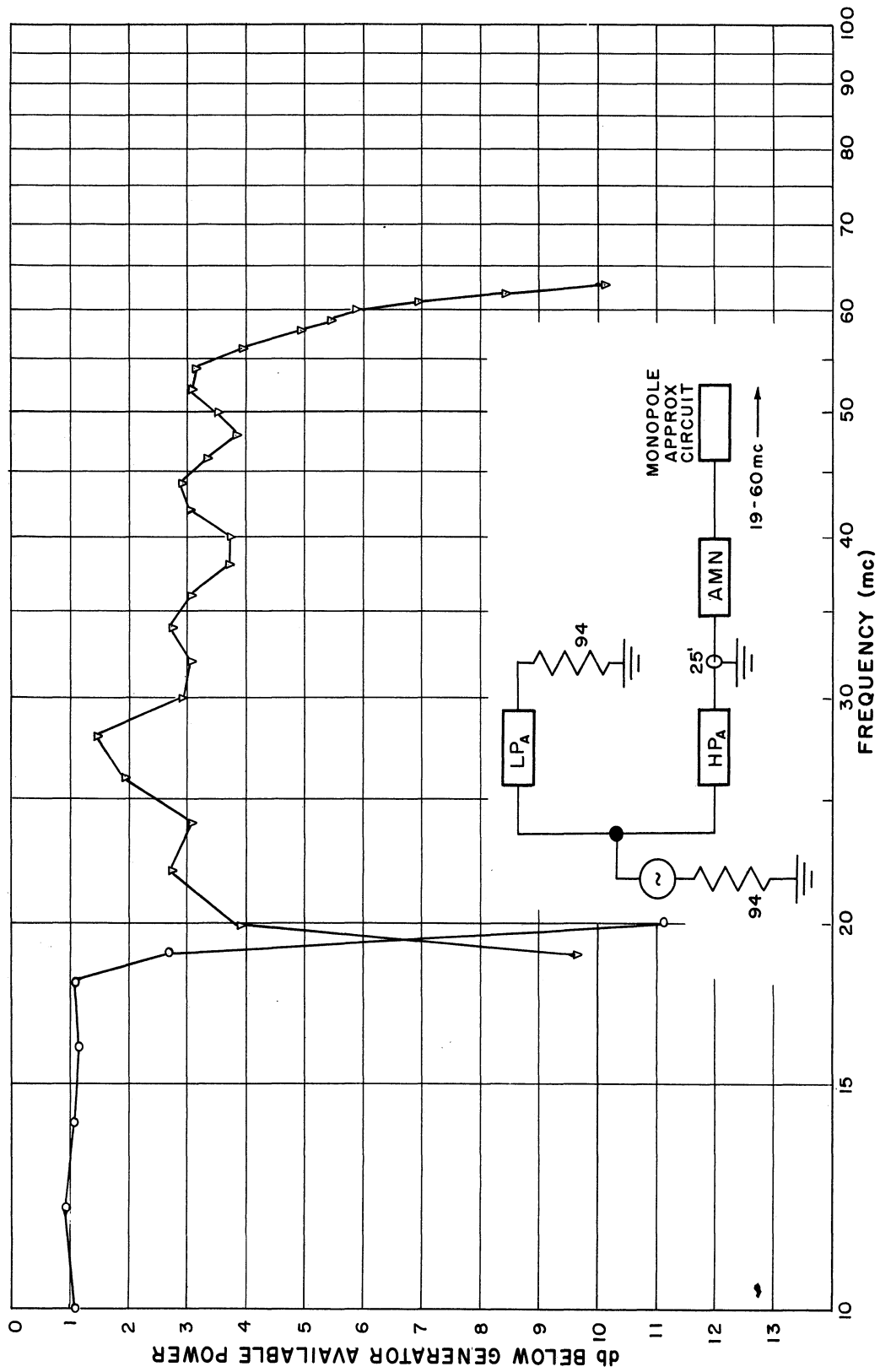


FIG 31
EXPERIMENTAL POWER CHARACTERISTIC

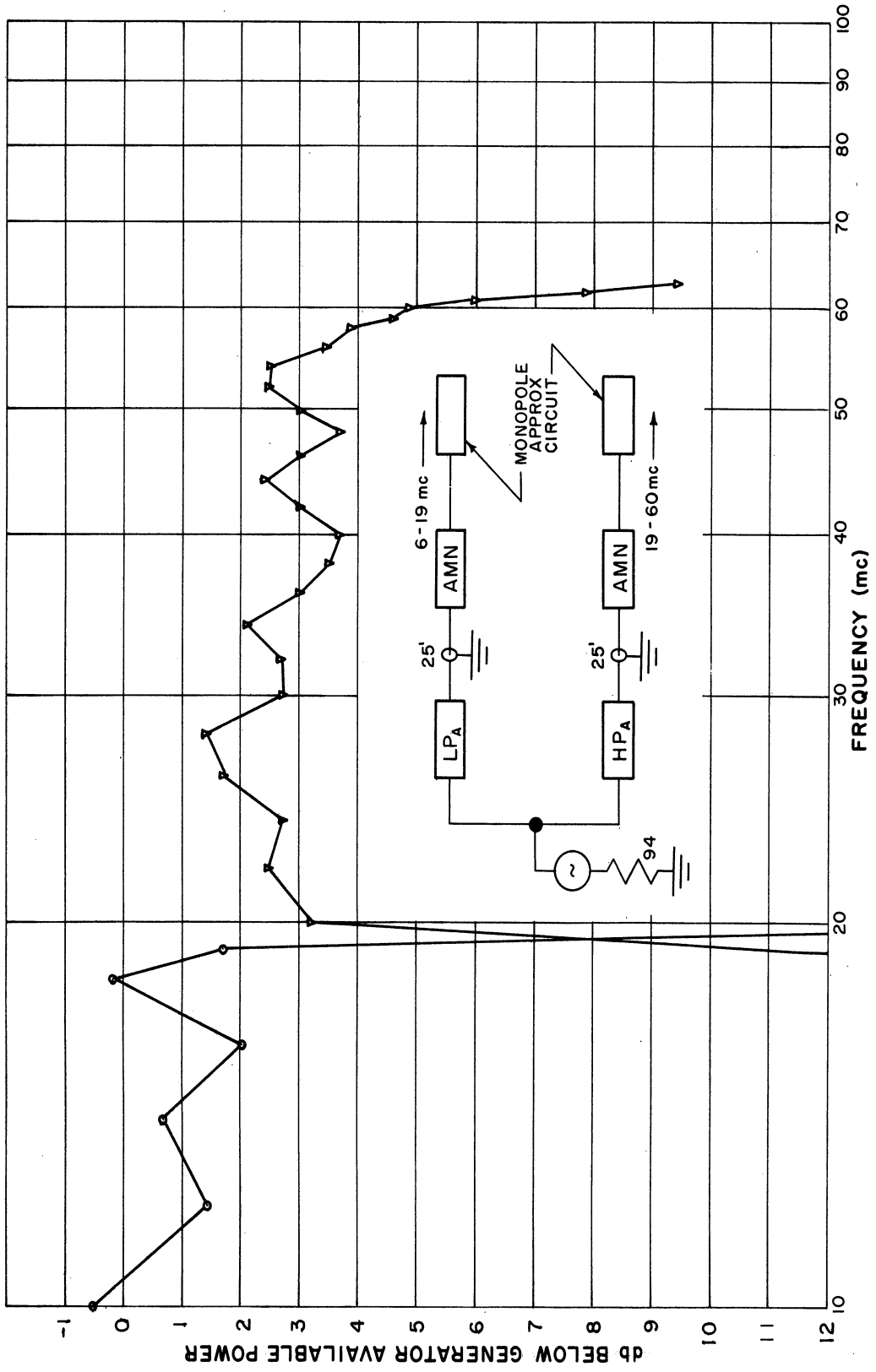


FIG 32
EXPERIMENTAL POWER CHARACTERISTIC

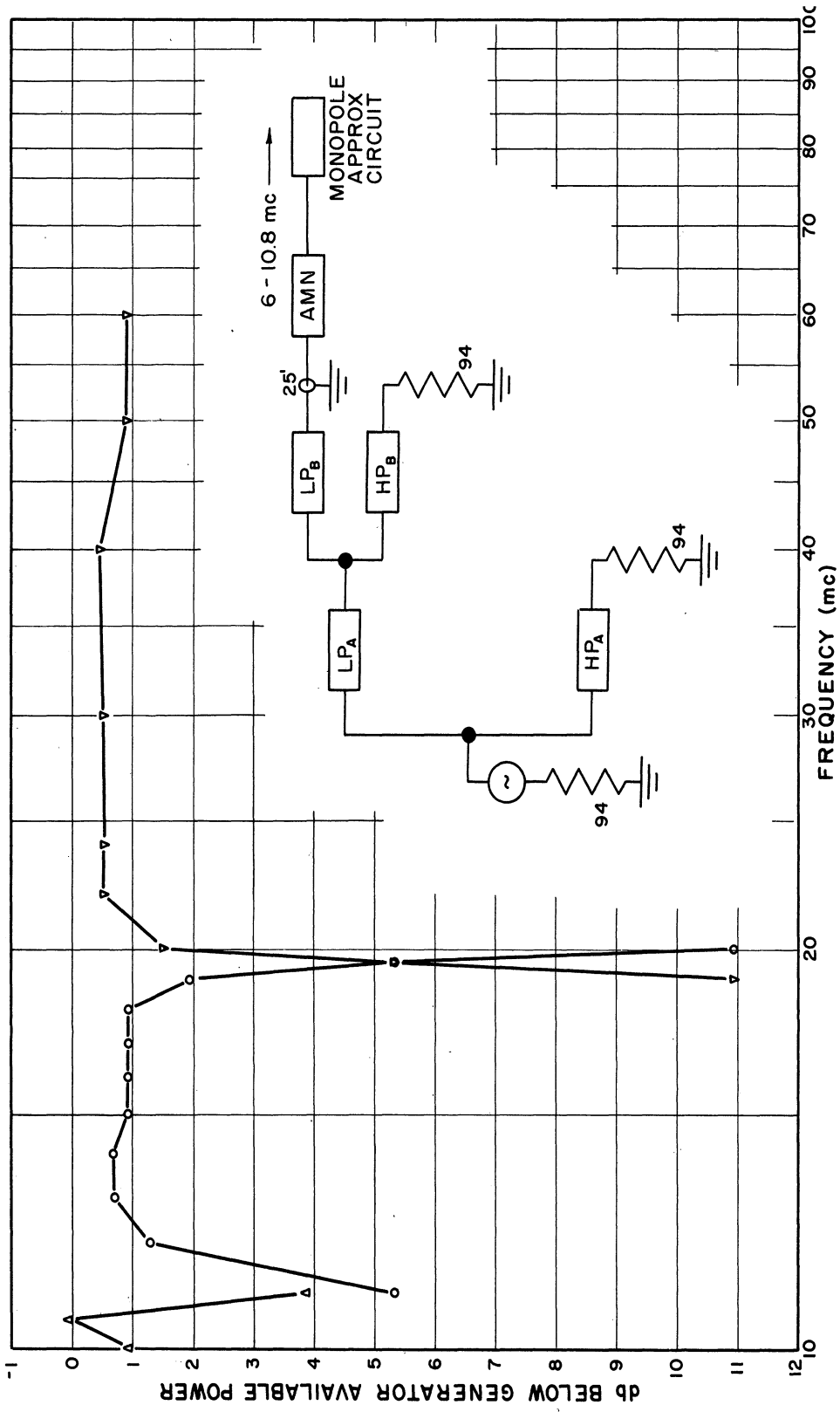


FIG 33
EXPERIMENTAL POWER CHARACTERISTIC

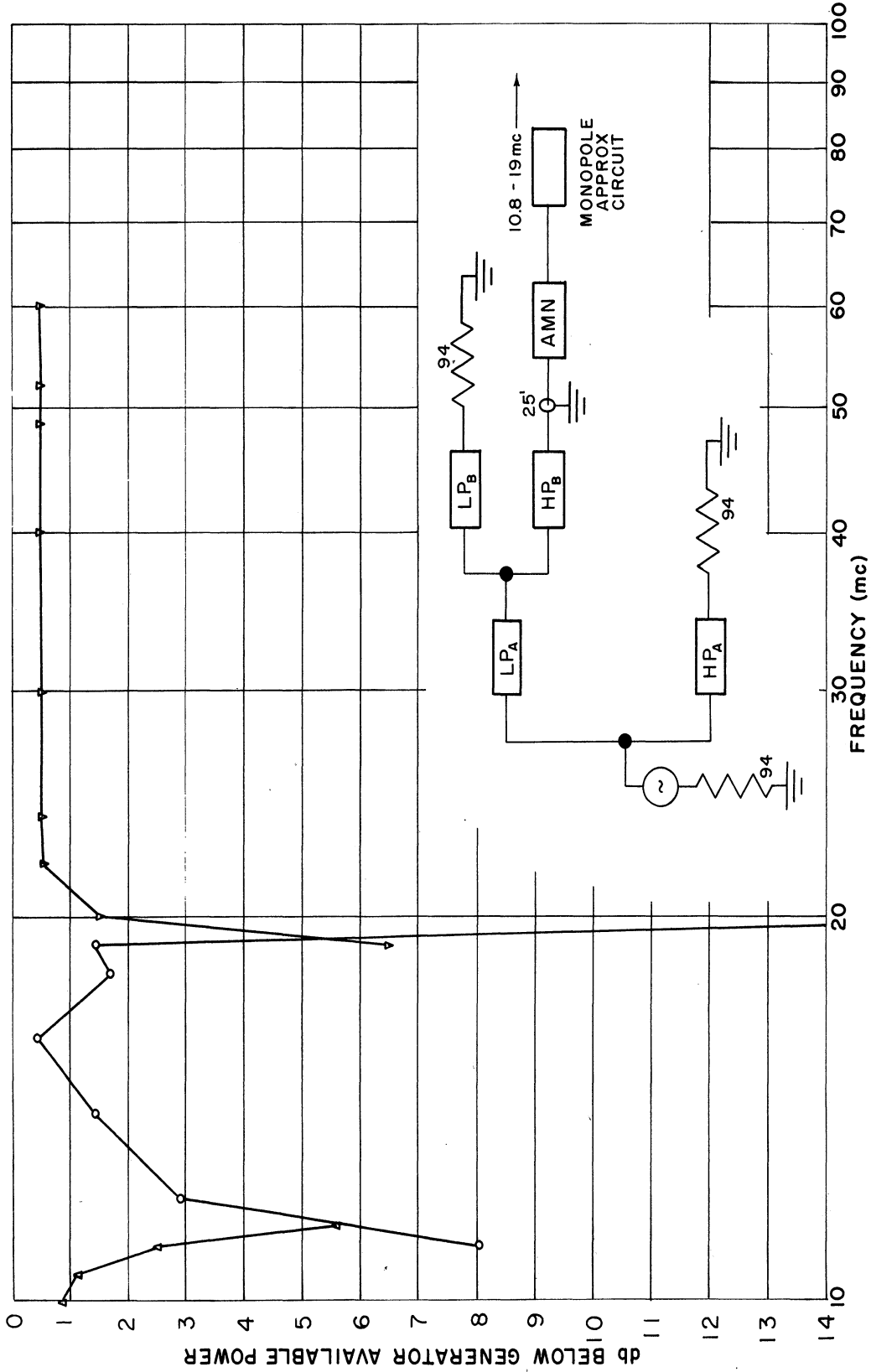


FIG 34
EXPERIMENTAL POWER CHARACTERISTIC

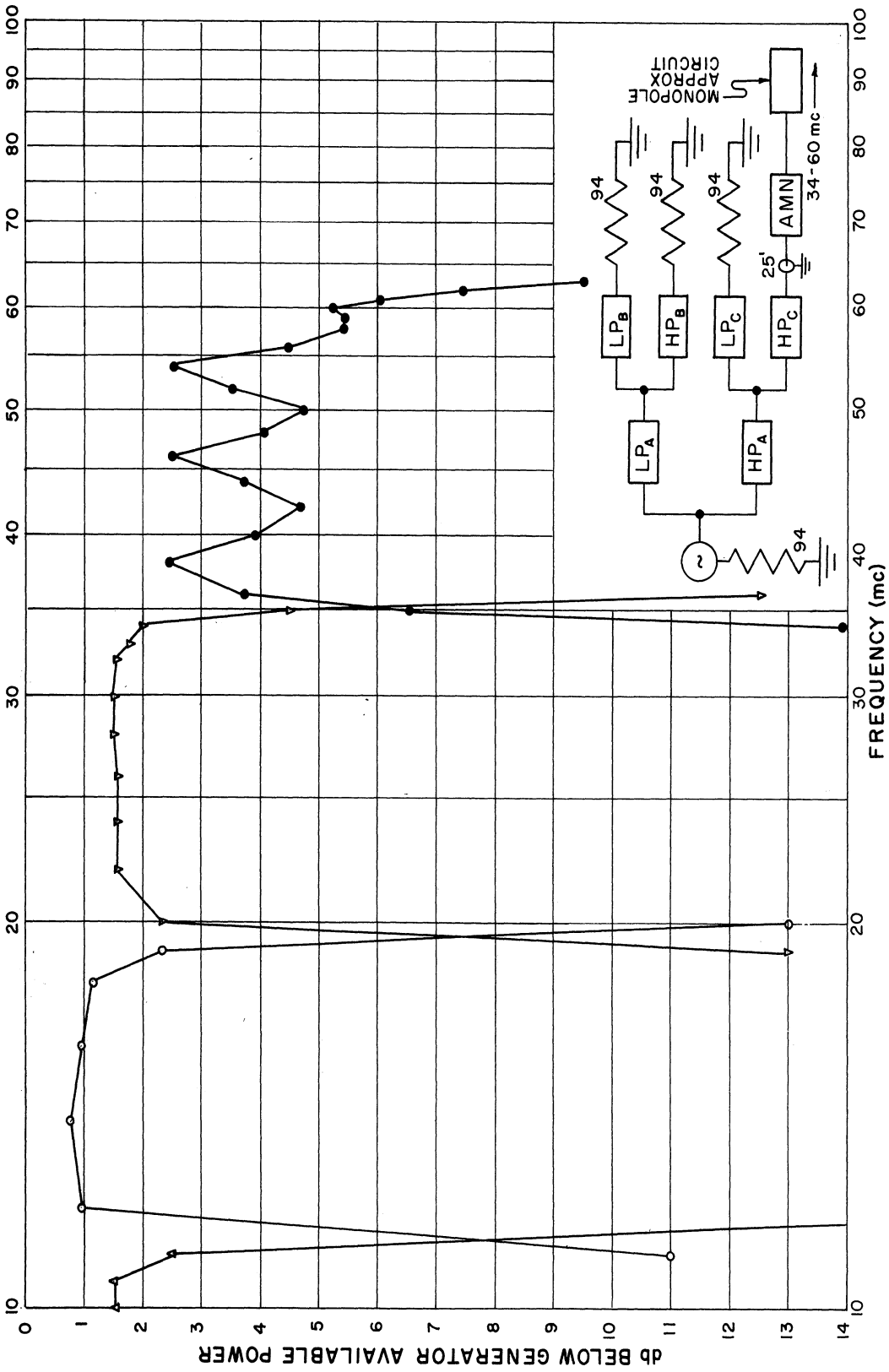


FIG 35
EXPERIMENTAL POWER CHARACTERISTIC

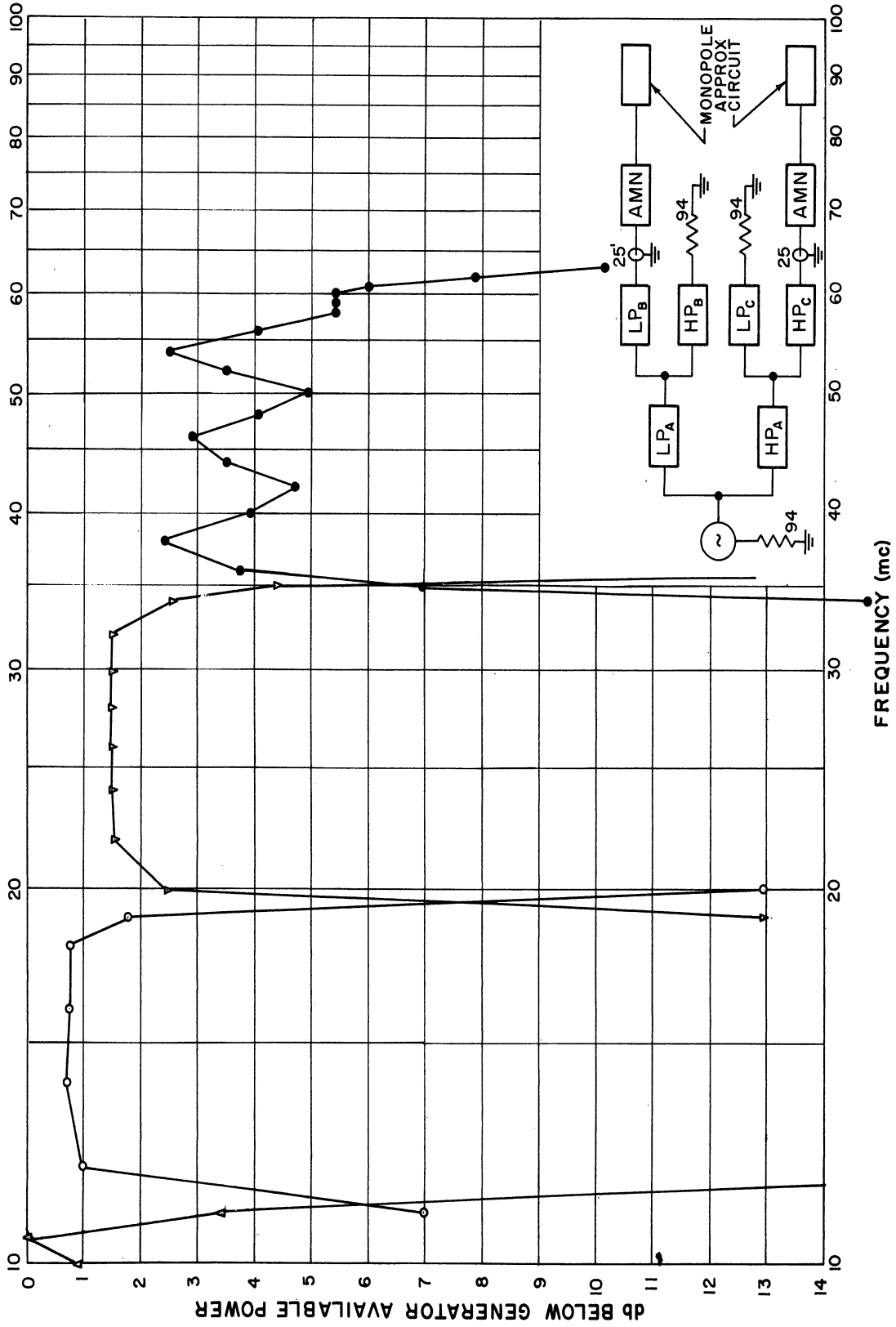


FIG 36
EXPERIMENTAL POWER CHARACTERISTIC

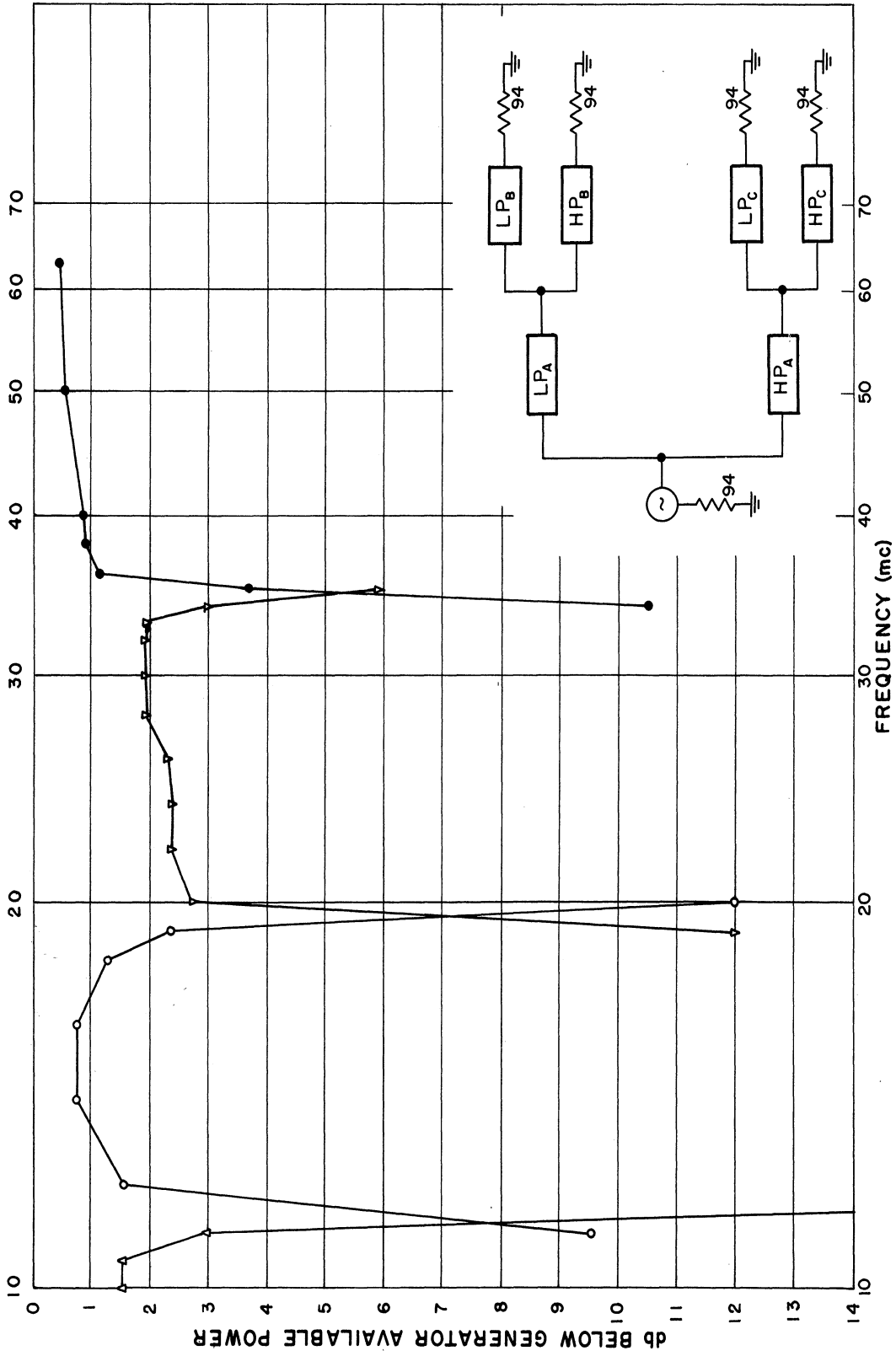


FIG 37
EXPERIMENTAL POWER CHARACTERISTIC

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oscillatory character is observed when the 19-60 mc "matched" monopole circuit is driven through 25 feet of RG/62-U and network HP_A , with LP_A terminated. These ripples occur because of the mismatch which exists at the end of the coaxial line, and the transformation of impedance by the coaxial line.

Somewhat more loss is observed in the 19-60 mc band in Fig. 31 than in the 6-19 mc band of Fig. 30. This is in part due to the increased loss observed for the high band monopole matching network (Fig. 24) compared to its low band counterpart of Fig. 23. Further, the loss of 25 feet of RG/62-U varies from about .2 db at 6 mc to .6 db at 60 mc.

In Fig. 32, networks LP_A and HP_A are each driving a "matched" antenna circuit through 25 feet of cable. The individual curves are very similar to those of Figs. 30 and 31. Since the rhombic antenna matching network presents very little mismatch, the power curves will be similar to that observed with the 94 ohm terminations. For this reason, the inverted vee network was not tested in the composite system.

Additional data are presented in Figs. 32 to 37 where the transmission band is further subdivided through the insertion of additional complementary filters. The results are generally satisfactory. Referring to Fig. 34, the 10.8-19 mc characteristic is narrow on the low side. This is due to the particular line lengths used. Note, for example, that a dip occurs at about the same frequency in the characteristic of Fig. 30. Similarly, a dip near 60 mc (as suggested by Fig. 31) occurs in all curves due to the particular line length chosen.

Antennas exist with improved impedance characteristics compared to the monopoles discussed in this report. With these antennas, a more uniform performance is expected; as a measure, the curves of Fig. 37 were obtained with 94 ohm terminations at all networks.

APPENDIX B

SYNTHESIS OF THE COMPLEMENTARY FILTERS

Consider the problem of obtaining a coupling network for which

$$y_{12_{LP}} = \frac{.43884 p^4 + 1.564 p^2 + 1.2898}{p^5 + 3.1599 p^3 + 2.1892 p}$$

$$y_{22_{LP}} = \frac{1.8737 p^4 + 3.3234 p^2 + 1.2898}{p^5 + 3.1599 p^3 + 2.1892 p}$$

as resulted from the analysis of Section 5. Assume that this can be achieved with a network in the form of Fig. 38. As is indicated by the form of y_{12} and y_{22} ,

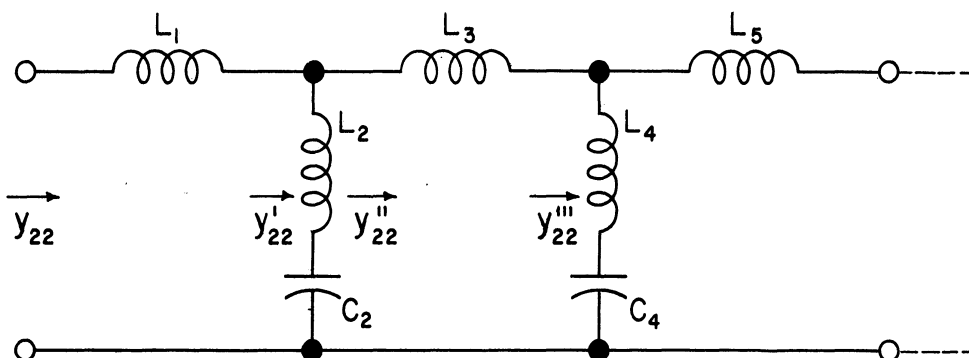


FIG 38
ASSUMED FORM OF LOWPASS COMPLEMENTARY NETWORK

this is a 5-pole structure with a transfer and driving point zero (of admittance) at infinity, and with two transfer zeros at finite frequencies. From the work in Section 5, or the numerator of $y_{12_{LP}}$, the transfer zeroes are to be located at

$$p_{01} = \pm j 1.5105 \tag{B.2}$$

$$p_{02} = \pm j 1.1350 \tag{B.1}$$

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These must be the resonant frequencies of branches 2 and 4 in Fig. 38. Branch 2 is made up of L_2 and C_2 , etc. Then the problem is to pull apart y_{22} in such a way that the resonant frequencies of branches 2 and 4 are consistent with Eqs B.1 and B.2. For example, one wishes to pull out an inductance L_1 , so that y_{22} (See Fig. 38) has poles at p_{02} , i.e., so $1/y_{22}$ has a zero at p_{02} .¹ In this case, we can write

$$\frac{1}{y_{22LP}} = L_1 p + \frac{(p^2 + 2.2816)(ap^3 + bp)}{1.8737 p^4 + 3.3234 p^2 + 1.2898}$$

where

$$p^2 + 2.2816 = (p + j 1.5105)(p - j 1.5105)$$

and L_1 , a , and b are constants which are readily determined by equating coefficients. It is found that

$$\begin{aligned} \frac{1}{y_{22LP}} &= .05353 p + \frac{(p^2 + 2.2816)(.8997 p^3 + .9293p)}{1.8737 p^4 + 3.3234 p^2 + 1.2898} \\ &= .05353 p + \frac{1}{\frac{1.3502 p}{p^2 + 2.2816} + \frac{.6589 p^2 + .5653}{.8997 p^3 + .9293p}} \end{aligned}$$

The procedure is then repeated on

$$\frac{1}{y_{22}} = \frac{.8997 p^3 + .9293p}{.6589p^3 + .5653} = L_{3p} + \frac{B_p(p^2 + 1.2882)}{.6589p^2 + .5653}$$

to create the transfer zero at $p_{01} = j 1.1350$.

¹

The decision to associate branch 2 with p_{02} rather than p_{01} is apparently arbitrary in this discussion. However, if one attempts the alternative procedure it is found that coupled coils are required. Using the chosen procedure, realizations containing coupled coils are readily obtained but are not mandatory to the degree that coupling can be made negligible. Actually, finite coupling will exist in any practical circuit, which will deteriorate the passband characteristic of a circuit such as in Fig. 30, where $k = 0$ between all coils.

From this,

$$\frac{1}{y_{22}''} = .8102p + \frac{.3659p(p^2 + 1.2882)}{.6589p^2 + .5653}$$

Finally,

$$\frac{1}{y_{22}} = .05353p + \frac{1}{\frac{1}{.7406p + \frac{1}{.5918p}} + \frac{1}{.8102p + \frac{1}{\frac{1}{1.6625p + \frac{1}{.4669p}} + \frac{1}{.8338p}}}}$$

and the required circuit is as indicated in Fig. 39.

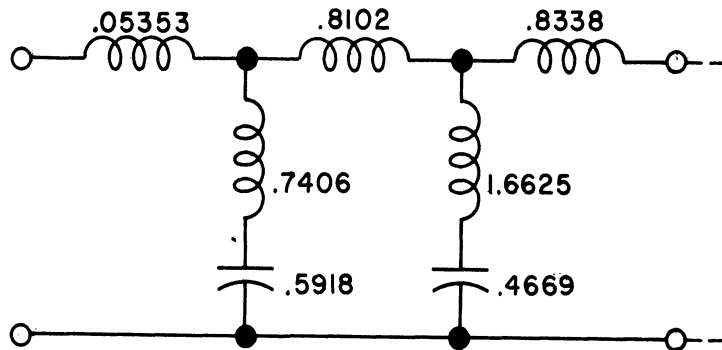


FIG 39
NORMALIZED LOWPASS COMPLEMENTARY NETWORK

The corresponding highpass network is similarly determined. The results are indicated in Section 5.

APPENDIX C

EXPERIMENTAL MONOPOLE IMPEDANCE DATA

The design of antenna matching networks for practical antenna systems is affected by a number of variables not previously considered. The presence of antenna shunt (base) capacitance and ohmic losses, and of a non-ideal ground plane may cause considerable departure of the actual antenna impedance from the idealized antenna input impedance. The conductivity and dielectric constant may vary widely with site-affecting both efficiency and pattern of the antenna.

Data indicating the effect of these parameters were obtained through measurement of the input impedance of an actual monopole installation for a variety of ground conditions. A uniform copper monopole 12.08 ft. long and 3-5/8 inches diameter, corresponding roughly to the 19-60 mc monopole in the system of Fig. 1, was chosen for these tests. A sketch of the antenna mounting arrangement for a set of tests conducted at the Electronic Defense Group is included in Fig. 40. In Figs. 41 and 42 the input resistance and reactance of the monopole are plotted for the following ground conditions:

- (a) ground plane of dry soil
- (b) ground plane of same soil thoroughly wetted
- (c) ground plane of muddy pool¹ surrounded by wet soil

¹

The pool was approximately elliptical in shape with semi-axes of 18 and 35 feet. The maximum depth of the pool was perhaps 2.5 feet. The conductivity of a sample of water taken from the pool was .023 mhos/m.

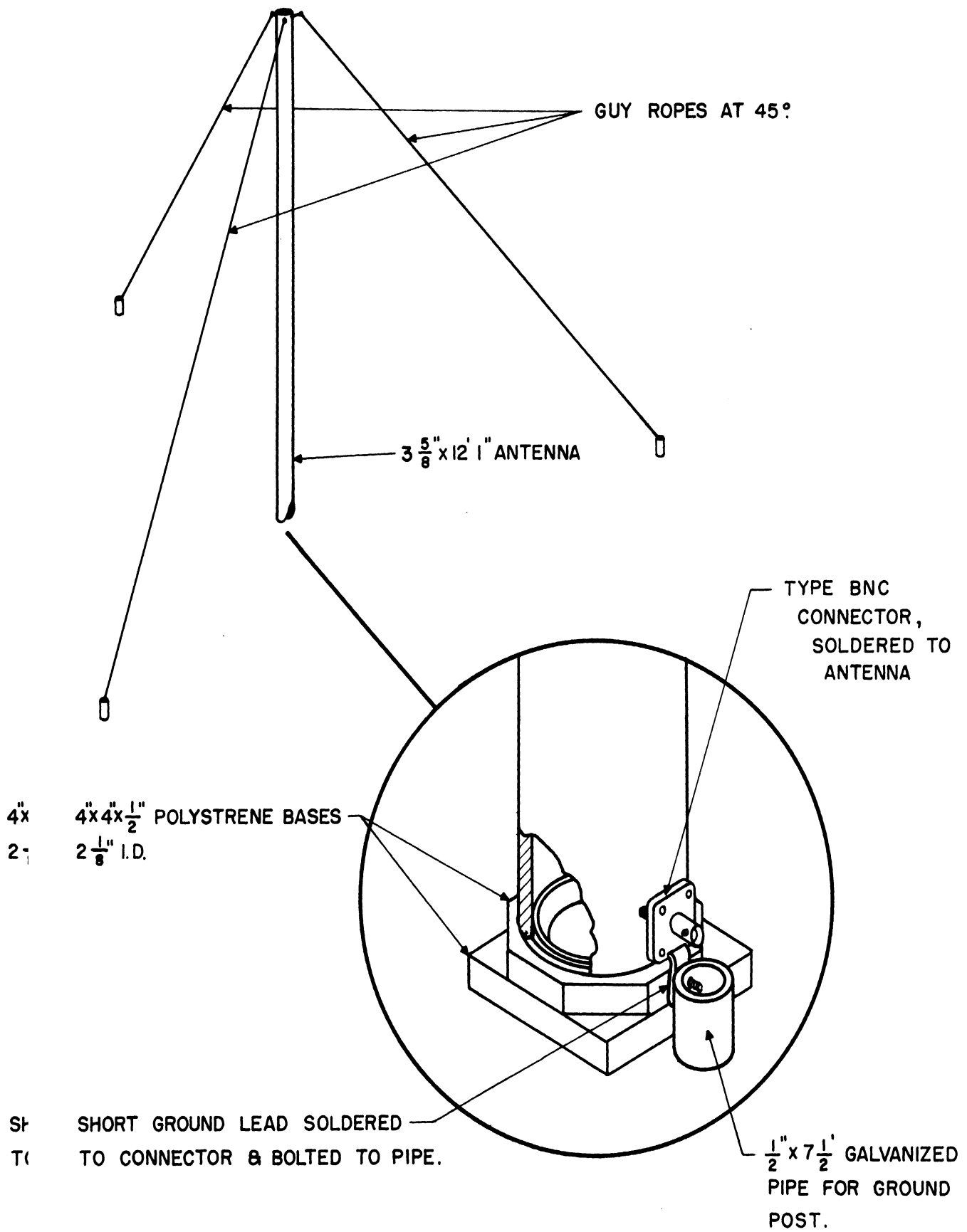
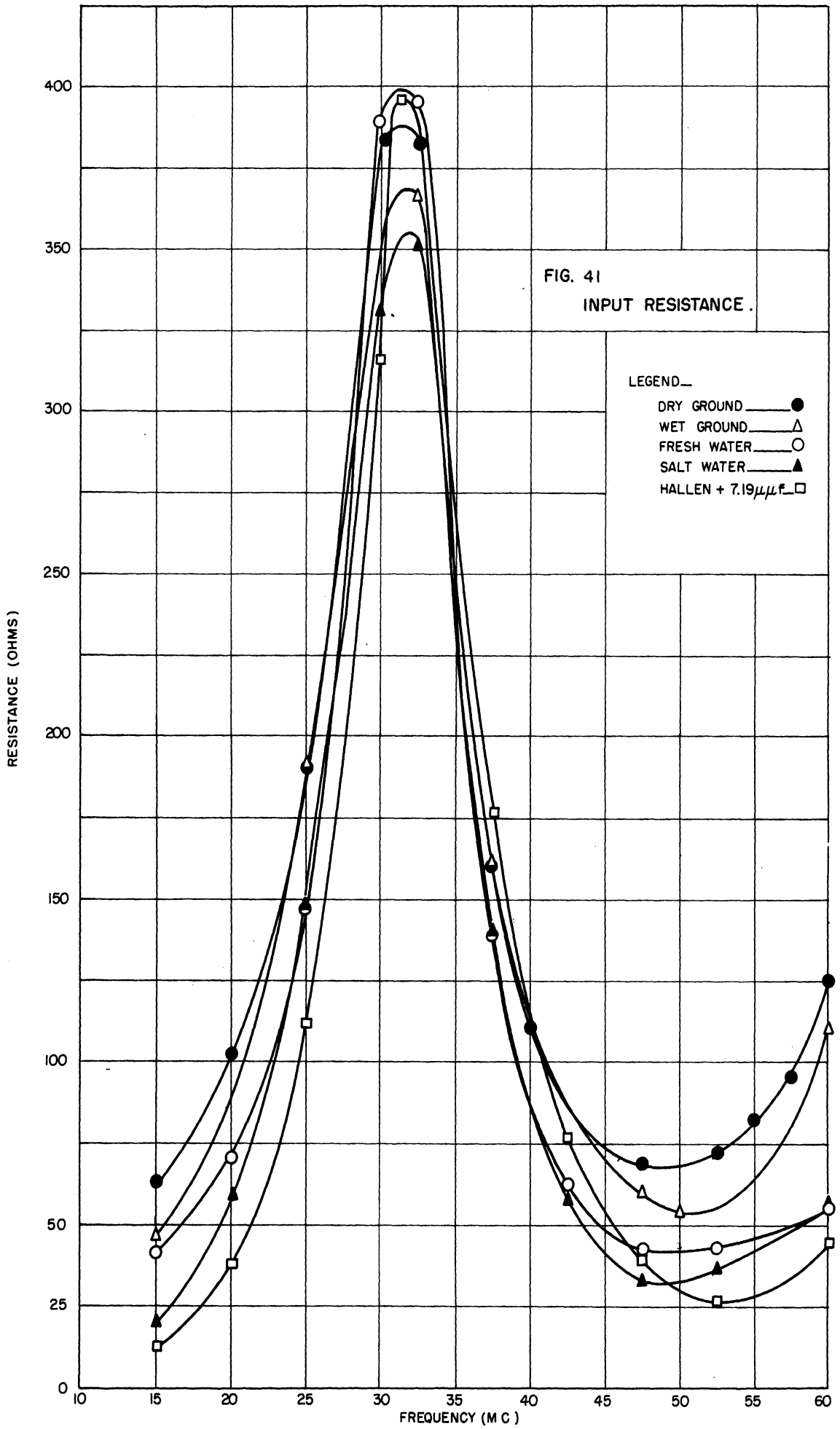
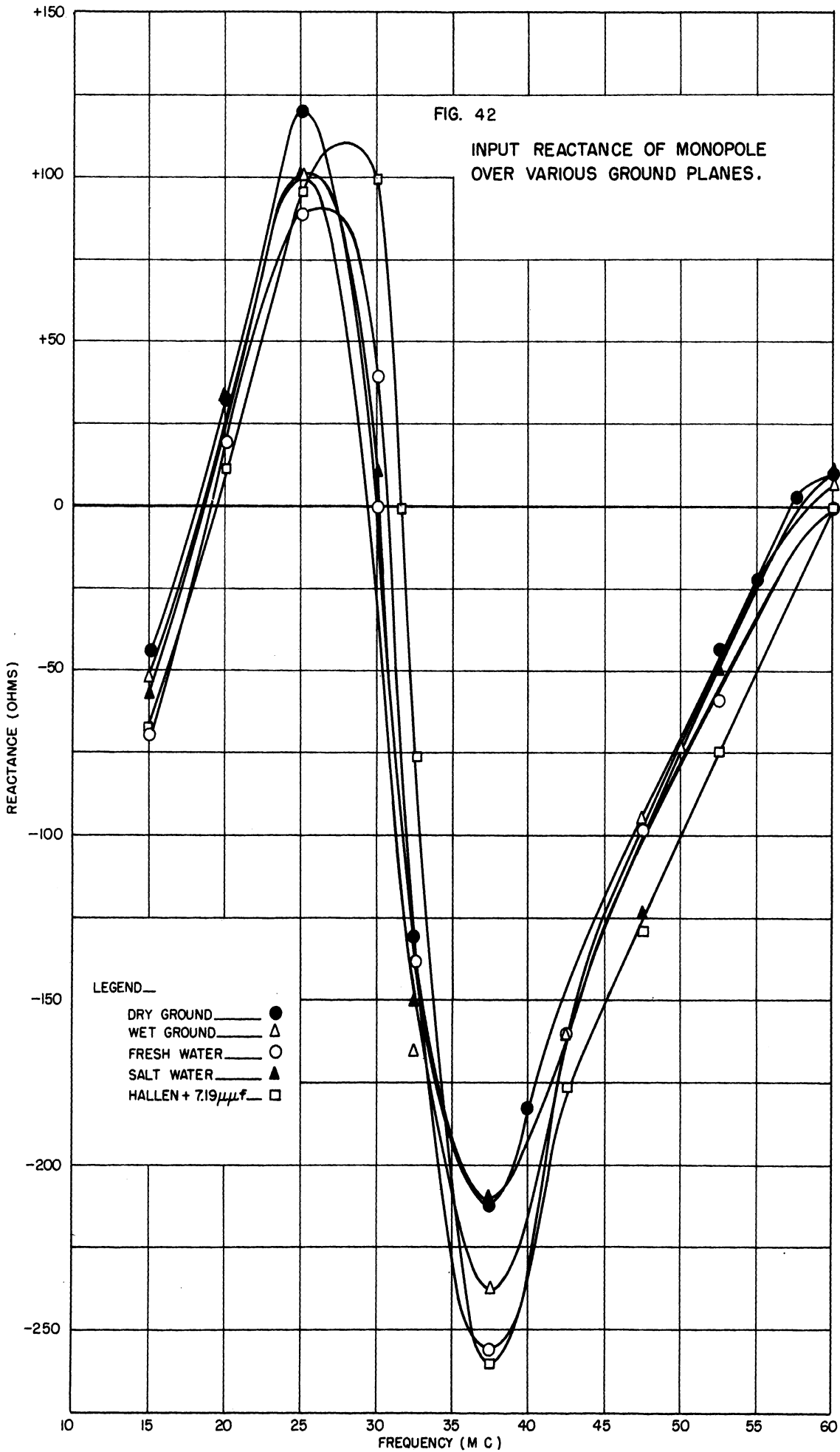


FIGURE 40.
EXPERIMENTAL ANTENNA
INSTALLATION





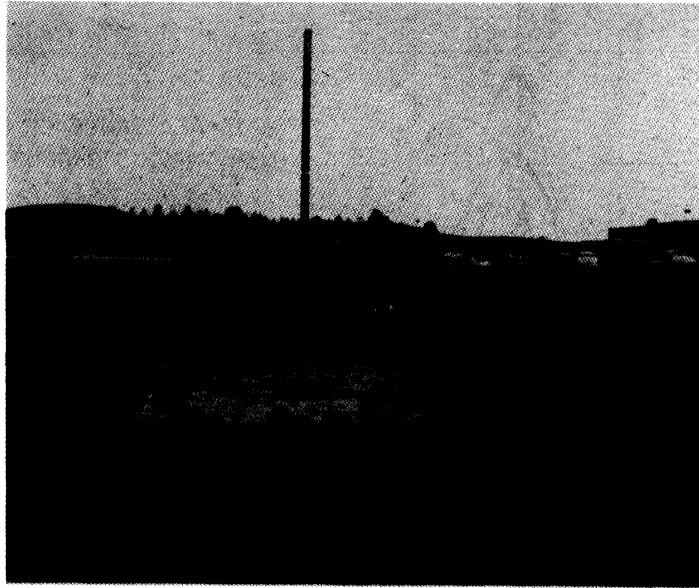


FIG. 43.

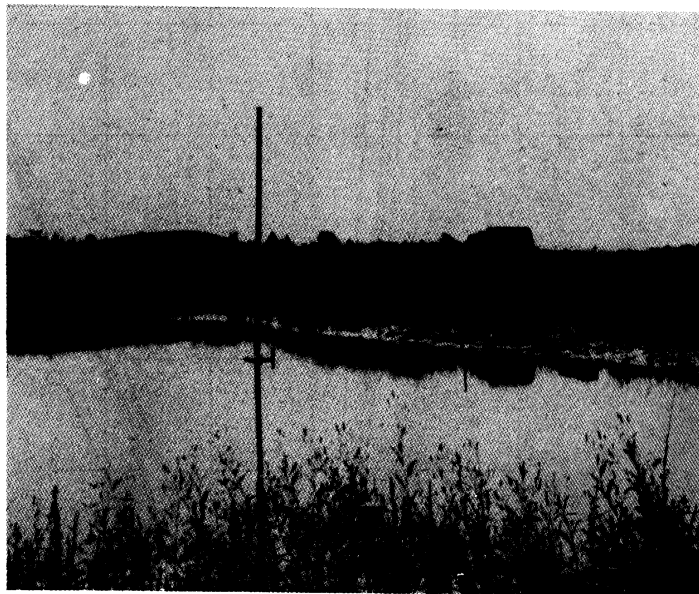


FIG. 44.

EXPERIMENTAL ANTENNA TEST SITES.

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(d) ground plane of muddy salt solution surrounded by wet soil.¹

The test area for conditions (a) and (b) is shown in Fig. 43 while the test area for conditions (c) and (d) is shown in Fig. 44. Also plotted in Fig. 41 and 42 are the curves of the parallel combination of Hallen's idealized monopole impedance for an antenna of this length (with length to radius ratio $l/a = 60$) and an assumed "base capacitance" of $7.2 \mu\text{f}$. A capacitance of this order actually exists, and addition of this capacitance makes the frequency of the resistive peak coincide with the peaks of the experimental curves. The input impedance of a copper monopole above a large copper ground plane might approach this curve rather closely.

It is noted that the experimental antenna of $l/a = 80$ was compared to the idealized characteristic of an antenna with $l/a = 60$. The curves suggest that the impedance characteristic of the ideal $l/a = 60$ antenna characteristic differs from that of an actual antenna of $l/a = 80$, over a wide range of ground conditions, by a factor which is approximately that which would result from the presence of an equivalent capacitance in parallel with the ideal characteristic. This equivalent capacitance has been referred to as "base capacitance".

An attempt was made by personnel of the Evans Signal Laboratory to obtain data from which this "base capacitance" could be estimated as a function of antenna base height. One approach was based on the fact that at a low enough frequency the monopole impedance would be almost purely capacitive. The variation of this capacitance with base height was interpreted by them as a variation in "base capacitance." The total capacitance of an antenna similar to the one used in the above tests was measured (at 100 kc) for a range of base

¹

The pool was modified by the introduction of 1000 lbs. of sodium chloride. The conductivity of a sample of the modified solution was 1.2 mhos/m. Sea water has a conductivity near 4 or 5 mhos/m.

heights, from which the estimated capacitances listed below were deduced. Whether these values are useful in the present analysis seems questionable. The ground

<u>Base Height (inches)</u>	<u>Estimated Base Capacitance ($\mu\mu\text{f}$)</u>
24	1
12	2
6	3
3	5
1	13

TABLE I

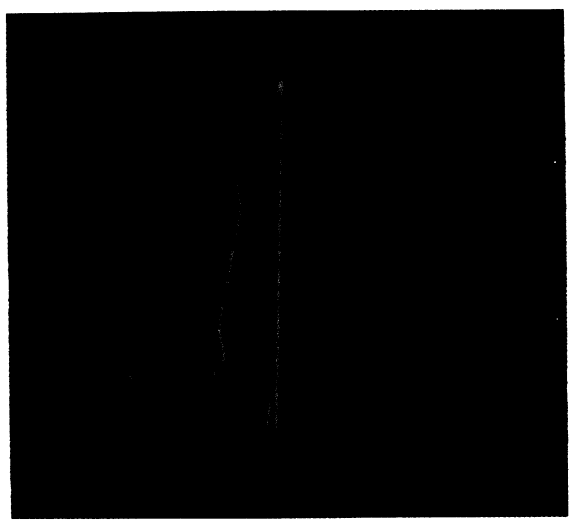
Base Height vs. Base Capacitance

parameters vary between 100 kc and 60 mc, and the variation of antenna impedance with geometry at a low frequency is not simply related to variation near resonance.

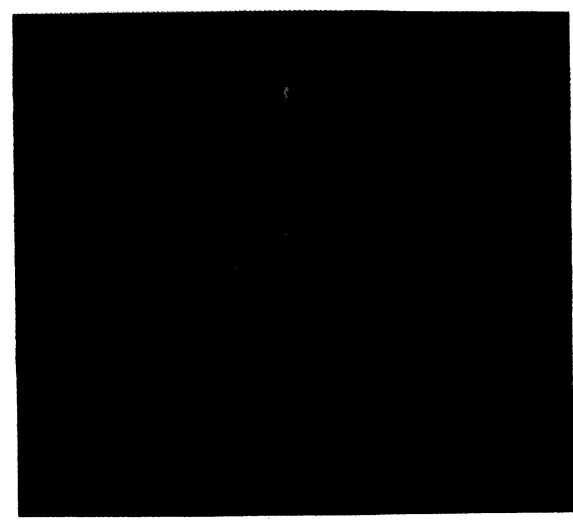
The effect of shunt capacitance at the antenna input is illustrated in Fig. 45. The photographs are power characteristics versus frequency for several values of C_b in the experimental circuit of Fig. 22. The antenna matching network and antenna approximating circuit are those of the 6-19 mc monopole. See Fig. 10. It is noted that the deterioration is moderate at $C_b = 10 \mu\mu\text{f}$. The same deterioration is expected with $3 \mu\mu\text{f}$ in the 19-60 mc band. The data of Table I suggests that a base spacing of six inches would lead to satisfactory results.

The Evans Signal Laboratory Report¹ also included plots of antenna impedance versus frequency on a 19-60 mc antenna. Substantial difference exists between their data with a 1" base spacing and the data of Figs. 41 and 42 for the same base spacing. It is felt that a thorough comparison of the experimental setup and methods is necessary to establish the significance of the differences. A reproduction of their curve obtained with a base spacing of six inches is given in Fig. 46. The plots of Fig. 5 and Fig. 46 are rather similar. A difference

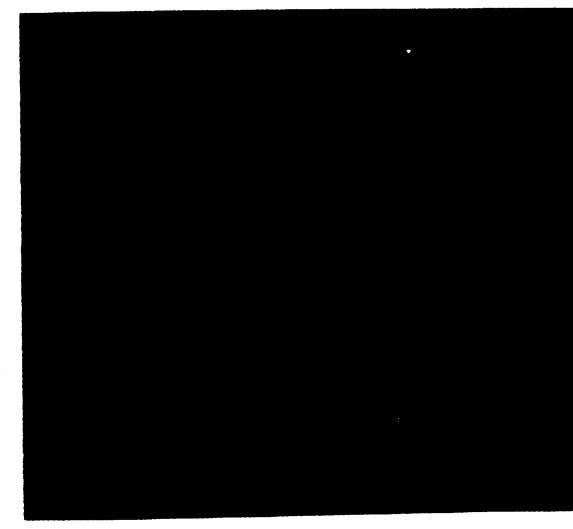
¹ See Reference 11.



(c)
 $C_b = 30\mu\mu f$



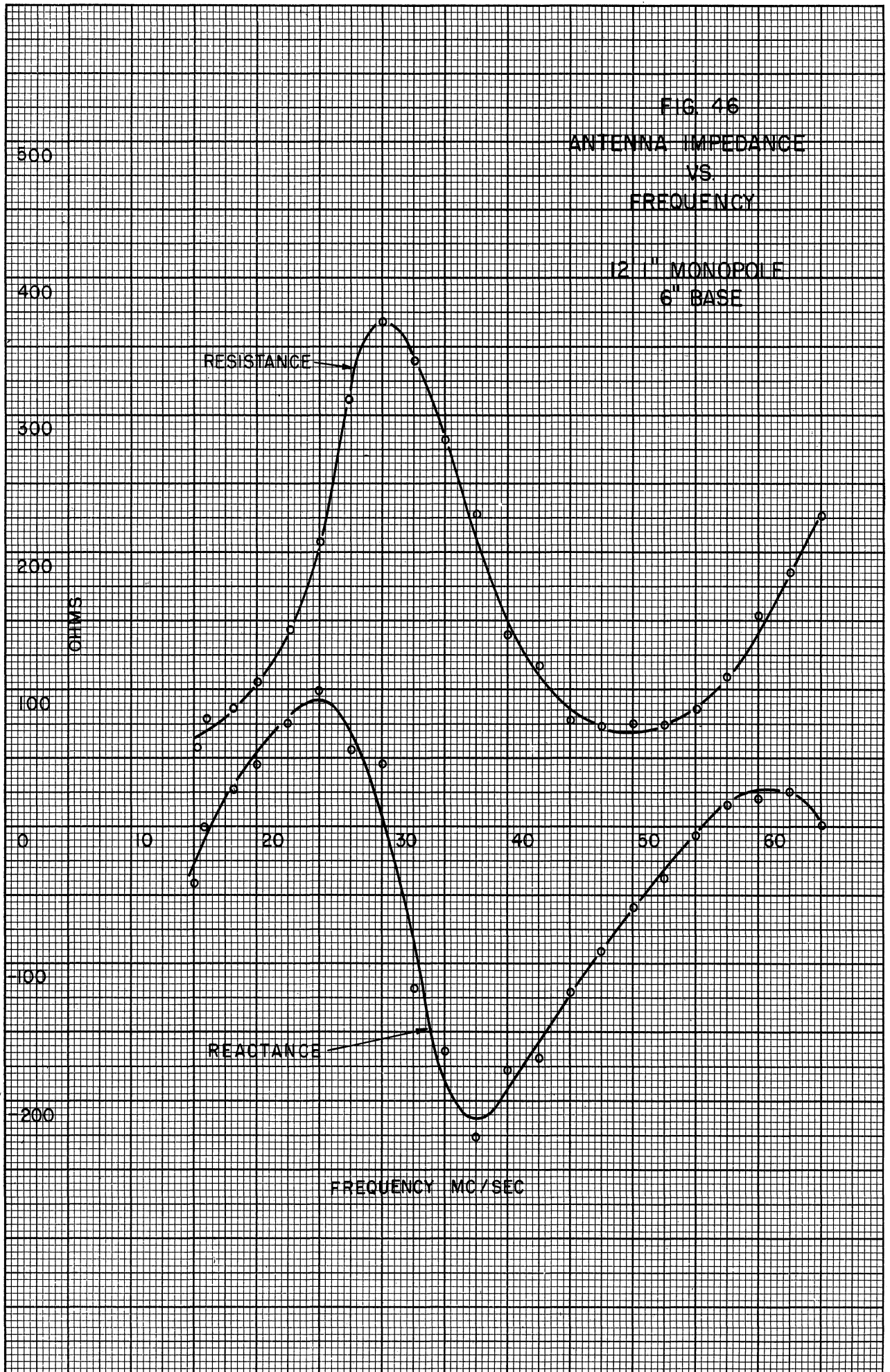
(b)
 $C_b \approx 10\mu\mu f$



(a)
 $C_b \approx 0$

NOTE: C_b IS THE SIMULATED BASE CAPACITANCE .

FIG. 45. EXPERIMENTAL STUDY OF EFFECT OF BASE CAPACITANCE .



is that peak resistance occurs at 34 mc for the ideal characteristics of a 12.08 foot monopole¹, while the experimental frequency is near 30 mc. This should be essentially corrected by shortening of the antenna. For this reason, it is suggested that the 19-60 mc monopole be

$$\frac{30}{34} \times 12.08 = 10.65 \text{ feet}$$

in length with a base spacing of six inches. For the 6-19 mc monopole, a length of 33.1 feet and a base spacing of 18 inches are indicated.

¹

This corresponds to $\beta l = 2.675$.

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