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THE SYNTHESIS OF MULTI-CHANNEL AMPLIFIERS

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by

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ABSTRACT

This dissertation presents new direct procedures for the design of wideband amplifiers. Wideband operation is achieved by partitioning, i.e., by parallel operation of a number of narrow amplifier chains. The only previous design procedure for such amplifiers involved iterative techniques.

It is shown that there is a simple criterion for determining the optimum number of channels, in the interests of maximizing the gain over a prescribed band with a fixed number of tubes of a given type and a given interstage complexity. The criterion is simply that a mean stage gain of ϵ ($\cong 2.718$) is achieved in the individual channels. The criterion is approximate, in that losses in the input and output coupling networks are neglected.

The design of input and output coupling networks providing equal transfer impedances to all channels is considered. It is shown that previously published studies by the present author and others determine optimum networks for this case.

The design of input and output coupling networks achieving selective transfer impedances (on a frequency basis) to the several channels (from the common source of load) is considered. Such "partitioning" networks are particularly desirable for power amplifiers, where the loss associated with an output coupling network providing equal transfer impedances to all channels has a detrimental effect on efficiency. Design principles for partitioning networks are presented, and a number of illustrative examples are worked out. It is shown that the resulting coupling networks are sub-

stantially superior to networks previously utilized, which provided equal transfer impedances to all channels.

A synthesis technique is introduced and illustrated, whereby partitioned amplifiers with prescribed overall gain characteristics can be obtained. The method involves the introduction of suitable all-pass factors into the overall gain function. The resulting function is then decomposed into a number of factors, each of which is associated with a particular portion of the circuit. Provisions for the type of input and output coupling networks sought (partitioning or non-partitioning) are made in straight forward manner, and a qualitative control of the shape of the channel responses is practical. The examples presented are confined to two-channel lowpass amplifiers. The principles discussed are useful in the design of more complex amplifiers.

A qualitative picture of the manner in which a gain-bandwidth advantage can accrue for a partitioned amplifier is presented. This discussion leads to a statement of the characteristics to be sought in a partitioned amplifier, in order that its gain approach the maximum feasible gain with the given tubes and interstage complexity. Certain conclusions are reached concerning the overall gain functions which should be studied in order to obtain exact prescribed gain characteristics and a maximum gain-bandwidth advantage.

An alternate design principle is also discussed. It is shown that a number of amplifier chains can be incorporated into an amplifier in such a way that a relatively good approximation to the rectangular gain characteristic is insured. The design method is based on the idea that, in a multichannel amplifier, a number of crossover regions (in frequency) occur, and that in each of these regions only two substantial output components need

exist at the load. The design procedure consists, then, of choosing the circuitry in such a way that the two substantial signals in each crossover region reinforce at the load at a single frequency. A region of satisfactory adjustment of the gain in the crossover region relative to the (equal) mid-channel gains of the amplifier is insured. Experimental results for amplifiers designed on this principle are presented which have amplification characteristics suitable for most applications. Examples are presented both with and without partitioning at the coupling networks.

ERRATA

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Page x	line 19	read "or" for "of"
Page 2	line 17	read "Distributed" for "Distributors"
Page 16	Eq. 26	should read $\frac{\epsilon}{2\sqrt{1 + \epsilon^2}}$
Page 18	line 2	read "trial" for "tiral"
Page 22	line 15	read $\frac{"m_a"}{M_a}$ for $\frac{"m_m"}{M_m}$
Page 23	Eq. 31	plus sign omitted between first two terms in brackets
Page 44	Eq. 76	first term of numerator read "p^3" for "p"
Page 45	line 3	read "p^3" for "p"
Page 57	line 5	read $\frac{"n_k"}{D_m}$ $\frac{"n_k"}{N_k}$
Page 63	line 11	read "stages" for "interstages"
Page 90	Fig. 43a	two parentheses missing
Page 109	line 16	sentence should begin "A midband gain of . . . . ."
Page 116	line 10	read "demonstrate" for "deomonstrate"
Page 117	line 4	read "having" for "have"

CHAPTER I

INTRODUCTION

The design of vacuum tube amplifiers is a broad subject because of the variety of specifications dictated by their diverse applications. Amplifier design problems usually contain, however, restrictions dealing with the magnitude and frequency dependence of the amplification. There is a limit to the amplification obtainable with a given amplifier over a given frequency band. Such a limitation results from the presence of the shunt capacitance of the tubes and interstages.

The problem of obtaining the largest feasible average amplification per tube in an amplifier chain; that is, a conventional cascade of tubes coupled by interstages; has been studied by many authors.[1-6†] Out of these studies have come such innovations as shunt-peaking,[3] two-terminal pair interstages, and stagger-tuning.[4-6] The character of the limitation imposed by shunt capacitance and a summary of present amplifier chain design capabilities are presented in Chapter II.

There is in fact a limit, due to the presence of shunt capacitance, to the bandwidth over which the gain of a single stage of amplification with any given conventional tube can be as great as unity. This restriction has given rise to the well-known figure of merit of a tube[1] (= $g_m/2\pi C$) which, for a value of C[‡] equal to the total estimated shunt capacitance between tubes, is the bandwidth over which a minimum gain of $1/\sqrt{2}$ can be

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† Superscript numbers identify individual references listed at the end of this study.

‡ At least two conventions exist in the choice of C. In one convention the C is the sum of the input and output capacitances of the tube. The resulting quantity has less physical significance than the figure of merit above.

realized with a simple tuned circuit interstage.

In most applications, it is necessary to employ more than one stage to obtain the desired level of amplification. The cascading of identical stages to obtain larger amplifications results in band shrinkage, ripple compounding, or a combination of the two effects. Even with the more refined technique of stagger tuning, the figure of merit of the tube is just half of the limiting bandwidth over which unity gain can be maintained with simple tuned interstages. Through the use of more complex interstages, it is possible to exceed this limiting bandwidth by modest factors. However, these more complicated circuits usually possess more parasitic shunt capacitance, which in part at least offsets their theoretical advantage.

A number of people have recognized that the bandwidth limitation of an amplifier chain can be circumvented.[7-12] This has been accomplished with distributed amplifiers and can be accomplished with multiple channel amplifiers, which will here be referred to as partitioned amplifiers.

Distributors amplifiers have been widely used to circumvene the single amplifier chain bandwidth limitations. Distributed amplifiers have, however, disadvantages which one might hope to avoid through the use of partitioned amplifiers. For example, power distributed amplifiers covering 250 mc with 4X150A tubes have a plate efficiency of about 10 per cent. Dr. P. H. Rogers showed[13] that one cannot obtain an efficiency higher than about 30 per cent with class A_1 operation and present circuits. With the conventional distributed amplifier circuit, one is restricted to class A_1 operation in most applications for one or more of the following reasons:

a) Larger outputs than obtained in conventional power distributed amplifiers are obtained only at large expense in driving power due to grid conduction which rapidly lowers the gain below a usable magnitude.

b) The requirement of even a moderate degree of fidelity may prohibit the use of greater grid signal voltages than presently utilized with the conventional circuit. Usually it is not practical to filter distortion products in subsequent circuits, since the output circuit of distributed amplifiers is broadband.

Such ideas as the application of push-pull techniques to distributed amplifier design must yet be brought to fruition.

The efficiency of the power distributed amplifier is basically limited by the impedance level of the output circuit. Presently, the output circuit is designed using constant-k and m-derived filter theory. Presumably superior circuits exist which would achieve higher impedance levels than those achieved at present under a prescribed limitation of shunt capacitance. Efforts to solve this problem have been made,[14,15] but much work remains to be done.

There are other disadvantages of the distributed amplifier circuit which in varying degree stimulate interest in the partitioned amplifier. For example, particular tube failures, such as by plate or grid to ground short circuits, can in the absence of inconvenient safeguards incapacitate the whole amplifier. Again, the r-f circuitry must approximate the idealized elements of the theoretical model over the entire band. Bypassing in distributed amplifiers is difficult, since this too must act over the entire band.

It is the opinion of the author that partitioned amplifiers are not in widespread use only because of the lack of satisfactory design procedures. As shown by Linvill,[11] the partitioned amplifier makes more economical use of tubes; that is, with a fixed number of tubes of a given type, the partitioned amplifier can give more amplification over any given band than can be obtained with a distributed amplifier. In certain applications, and for certain choices of channel bands, the filtering of distortion products generated in a channel can be achieved by the relatively narrow circuits of the channels themselves. Thus, operation without prohibitive distortion appears feasible at much higher efficiencies than is achieved in power distributed amplifiers.

The effect on gain of a tube failure can easily be confined to a limited portion of the total bandwidth of a partitioned amplifier. This advantage must be weighed against the fact that the shape of the amplification curve is affected by a variation, among the channels, of the aging characteristics of the tubes of a partitioned amplifier. Ideally, in the distributed amplifier, tube aging affects the amplification characteristic by a multiplying constant.

It is clear that, between the input and output tubes of a channel, the r-f circuitry and bypassing must be of such quality as to approach the ideal in the desired region of channel response, and in addition must prevent substantial amplification outside this region. This requirement is in general less stringent than the requirement for a distributed amplifier circuit of the same total bandwidth. The input and output circuits of the partitioned amplifier represent a substantial theoretical and practical problem to be considered extensively in this report.

A further potential advantage of a partitioned amplifier is that one can hope to progress from low level voltage amplification to power amplifier stages if desired. In fact, it will be illustrated that, if the principal objective is efficiency, this may permit a simplification of the design problem. This simplification results from the fact that, where power and efficiency are paramount, a simple amplifier input circuit with a consequent low level (voltage) loss may be unimportant. Similarly, the same type of loss may not be important in the output circuit of a low level amplifier, whereas such a loss in the input circuit would substantially reduce sensitivity. It may be worth noting that such freedom is lacking in distributed amplifiers. The author has, in fact, been forced to utilize a 4X150A distributed amplifier at about 1 per cent plate efficiency as a driver for the distributed amplifier with 10 per cent plate efficiency mentioned previously. This very undesirable procedure was necessitated by the unavailability of a satisfactory tube type for driver stages.

This paper is divided into several chapters, of which these introductory remarks comprise the first. Statements of the limitation imposed by shunt capacitance, of the capability of an amplifier chain, and of the distributed amplifier capability are given in the second section for purposes of later comparison. The results of a previous work dealing with partitioned amplifiers are also summarized. Chapter III contains a statement of the partitioned amplifier problem, and definitions of the symbols used in this report. An introduction to an exact design procedure is presented, and a discussion of the optimum gain of a partitioned amplifier containing a fixed number of tubes. Chapter IV is devoted to consideration of the input and output structures. In Chapter V, an exact design procedure is presented and illustrated. Chapter VI contains some general considerations dealing with partitioned amplifier design. Chapter VII is devoted to a discussion of an alternative approximate design procedure. Chapter VIII is devoted to a summary and conclusions.

CHAPTER II

THE STATE OF THE AMPLIFIER ART

As has been stated in the introduction, the presence of shunt capacitance imposes a limitation on the magnitude of the amplification which can be maintained over a given bandwidth with a given amplifier chain. A different sort of limitation, again related to the presence of shunt capacitance, prevails for distributed amplifiers. These limitations will now be discussed briefly. Previous literature dealing with partitioned amplifiers is also summarized.

2.1 <u>The Limitation Imposed on Amplifier Chains by Shunt Capacitance</u>

The basic building block of the amplifier chain is the amplifier stage, made up of a tube and an interstage. It will be assumed[†] that the amplification function of a cascade of these stages is in the form

$$\frac{E_o}{E_i} = (-g_m)^N \; Z_{kA} \; Z_{kB} \cdots Z_{kN} \quad , \qquad (1)$$

where the $Z_{kA}, \ldots Z_{kN}$ are the interstage characteristics. If two-terminal interstages are used, they are driving-point impedances, while with two-terminal pair interstages they are transfer impedances. In either case it is practically necessary that shunt capacitance appear at each terminal pair of the interstages.

Bode[1] has shown that for two-terminal networks, the maximum magni-

† This assumption clearly neglects coupling (aside from the tubes) that may exist between stages of a chain, and also between stages in different chains in the partitioned amplifier. The tubes are also idealized by the assumption of a constant g_m. These idealizations often lead to big gaps between paper work and an operative circuit.

tude of impedance that could be maintained over a band ω_o when shunted by a capacitance C is

$$|Z|_{max} = \frac{2}{\omega_o C} \quad . \tag{2}$$

Obviously, then, for a chain of N stages with two-terminal interstages, this implies an amplification over a bandwidth ω_o less than

$$\left|\frac{E_o}{E_i}\right|_{max} = (g_m)^N \left(\frac{2}{\omega_o C}\right)^N \quad . \tag{3}$$

It is useful to assess how close presently used techniques come to this limit. For a simple tuned interstage, the bandwidth ω_{3db} over which the gain is within 3 db of the maximum gain is given by $\omega_{3db}RC = 1$. Then over a bandwidth ω_{3db} the minimum gain of a stage with a simple tuned interstage is

$$\left|\frac{E_o}{E_i}\right|_{min} = \frac{(g_m)R}{\sqrt{2}} = \frac{(g_m)}{\sqrt{2}\,\omega_{3db}\,C} \quad . \tag{4}$$

For N such stages in an exact flat-staggered N-tuple,† the minimum gain over the bandwidth ω_{3db} is

$$\left|\frac{E_o}{E_i}\right|_{min} = \frac{(g_m)^N}{\sqrt{2}} \left(\frac{1}{\omega_{3db}\,C}\right)^N \quad . \tag{5}$$

For this case, the gain is lower than the theoretical limit by the factor $2^N/\sqrt{2}$. A 3 db variation in amplification occurs in this band.

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† That is, the poles are located in the lowpass case to give a maximally flat approximation to the rectangular lowpass characteristic;

$$\left|\frac{E_o}{E_i}\right|^2 = \frac{K^2}{1 + \left(\frac{\omega}{\omega_{3db}}\right)^{2N}} (g_m)^N \quad .$$

Bandpass characteristics are obtained with the same gain limitation by a lowpass to bandpass transformation. For design formulae, see Ref. 6.

The minimum gain, within a bandwidth ω_{3db}, of an amplifier employing N identical synchronous single-tuned stages is approximately[6]

$$\left|\frac{E_o}{E_i}\right|_{min} \simeq \frac{(g_m)^N}{\sqrt{2}} \left(\frac{1}{\omega_{3db}\, C}\right)^N \left(\frac{1}{1.2\sqrt{N}}\right)^N . \qquad (6)$$

The minimum gain, within a bandwidth ω_{3db}, of an amplifier employing N shunt-peaked stages with maximal flatness is approximately[23]

$$\left|\frac{E_o}{E_i}\right|_{min} \simeq \frac{(g_m)^N}{\sqrt{2}} \left(\frac{1.5}{\omega_{3db}\, C}\right)^N [1/N]^{N/4} . \qquad N < 10 \qquad (7)$$

Alternatively, the poles can be located to obtain a Tchebycheff approximation to the rectangular characteristic over a band ω_o centered at ω_c:

$$\left|\frac{E_o}{E_i}\right|^2 = \frac{M^2}{1 + \epsilon^2 T_N^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o \omega_c^2}{\omega}\right)} (g_m)^N . \qquad (8)$$

Here ϵ is a parameter determining ripple $= 10 \log_{10} 1+\epsilon^2$ and T_N is the Tchebycheff polynomial of degree N. The minimum gain over the band ω_o is

$$\left|\frac{E_o}{E_i}\right|_{min} = (g_m)^N \left(\frac{2}{\omega_o C}\right)^N \frac{\epsilon}{2\sqrt{1+\epsilon^2}} , \qquad (9)$$

whence for 3 db variations ($\epsilon = 1$),

$$\left|\frac{E_o}{E_i}\right|_{min} = (g_m)^N \left(\frac{2}{\omega_o C}\right)^N 1/2\sqrt{2}. \qquad (10)$$

This result, achieved with extremely simple circuits, may be regarded as remarkably close to the theoretical limit. The result is all the more remarkable when one considers that other simple driving point impedances such as shunt-peaked circuits are known which can maintain substantially greater impedance levels than the circuits used above over the same bandwidth and under the same capacitance limitation.

Bode[1] and Linvill[11] have shown that the maximum magnitude of transfer impedance of a two-terminal pair network Z_{12} which could be maintained over a band ω_o radians per second wide with shunt terminal capacitances C_1 and C_2 is

$$|Z_{12}|_{max} = \frac{\pi^2}{4\sqrt{C_1 C_2}\,\omega_o} , \tag{11}$$

giving a gain for N stages less than

$$\left|\frac{E_o}{E_i}\right|_{max} = (g_m)^N \left(\frac{\pi^2}{4\sqrt{C_1 C_2}\,\omega_o}\right)^N . \tag{12}$$

For $C_1 = C_2 = C/2$ this becomes

$$\left|\frac{E_o}{E_i}\right|_{max} = (g_m)^N \left(\frac{\pi^2}{2\omega_o C}\right)^N \cong (g_m)^N \left(\frac{4.94}{\omega_o C}\right)^N . \tag{13}$$

Let us consider an amplifier made up by cascading N stages with identical 3-pole Tchebycheff interstages. If the allowable ripple of N stages is x db, then the ripple per stage is x/N db. From

$$10N \log_{10} (1 + \epsilon^2) = x \tag{14}$$

one obtains

$$1 + \epsilon^2 = \log_{10}^{-1} \frac{x}{10N} . \tag{15}$$

Now, for a Z_{12} with a unit tolerance bandwidth, a ripple of x/N db and with a one ohm termination, there exists a uniquely defined 3-pole network of the form of Fig. 1.

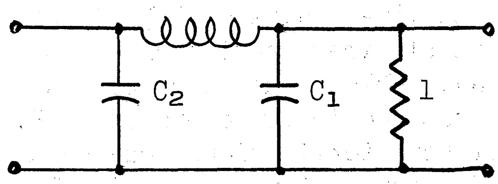


Fig. 1. A three-pole interstage.

These networks can be determined readily from the curves[16] of Fig. 2. One obtains from such a structure an interstage with the prescribed bandwidth ω_o and a termination R consistent with the capacitance limitations for the tube utilized.† Let us designate the total of the interstage terminal shunt capacitances by C, when

$$C = \frac{C_1 + C_2}{\omega_o R} \quad . \tag{16}$$

From this

$$R = \frac{C_1 + C_2}{\omega_o C} \quad . \tag{17}$$

Now, the gain of a stage at a passband minimum is

$$\left|\frac{E_o}{E_i}\right| = (g_m) \frac{R}{\sqrt{1 + \epsilon^2}} \quad . \tag{18}$$

Substituting, the gain of N such identical stages is

$$\left|\frac{E_o}{E_i}\right| = \left(\frac{g_m}{\omega_o C}\right)^N \left[\frac{C_1 + C_2}{\sqrt{\log^{-1}(x/10N)}}\right]^N \quad . \tag{19}$$

A study of the function

$$K = \frac{C_1 + C_2}{\sqrt{\log^{-1}(x/10N)}} \quad , \tag{20}$$

for the special case of $x = 3$db, shows that K is greater than 2 for $N < 7$. For $N = 10$, the function $(K)^N$ is less than $(2)^N$ by a factor of about 2.04. Thus, with these interstages, with less than about 10 stages

† The problem that the shunt capacitance may not be satisfactorily distributed between the terminal pairs will be ignored here. The networks defined are suitable where satisfactory approximations to ideal current sources are available. The networks can also be reversed in cases where grid conductance is low compared to R. Where substantial conductances exist at both terminations, one should use circuits based on transmission coefficient design. See Ref. 17.

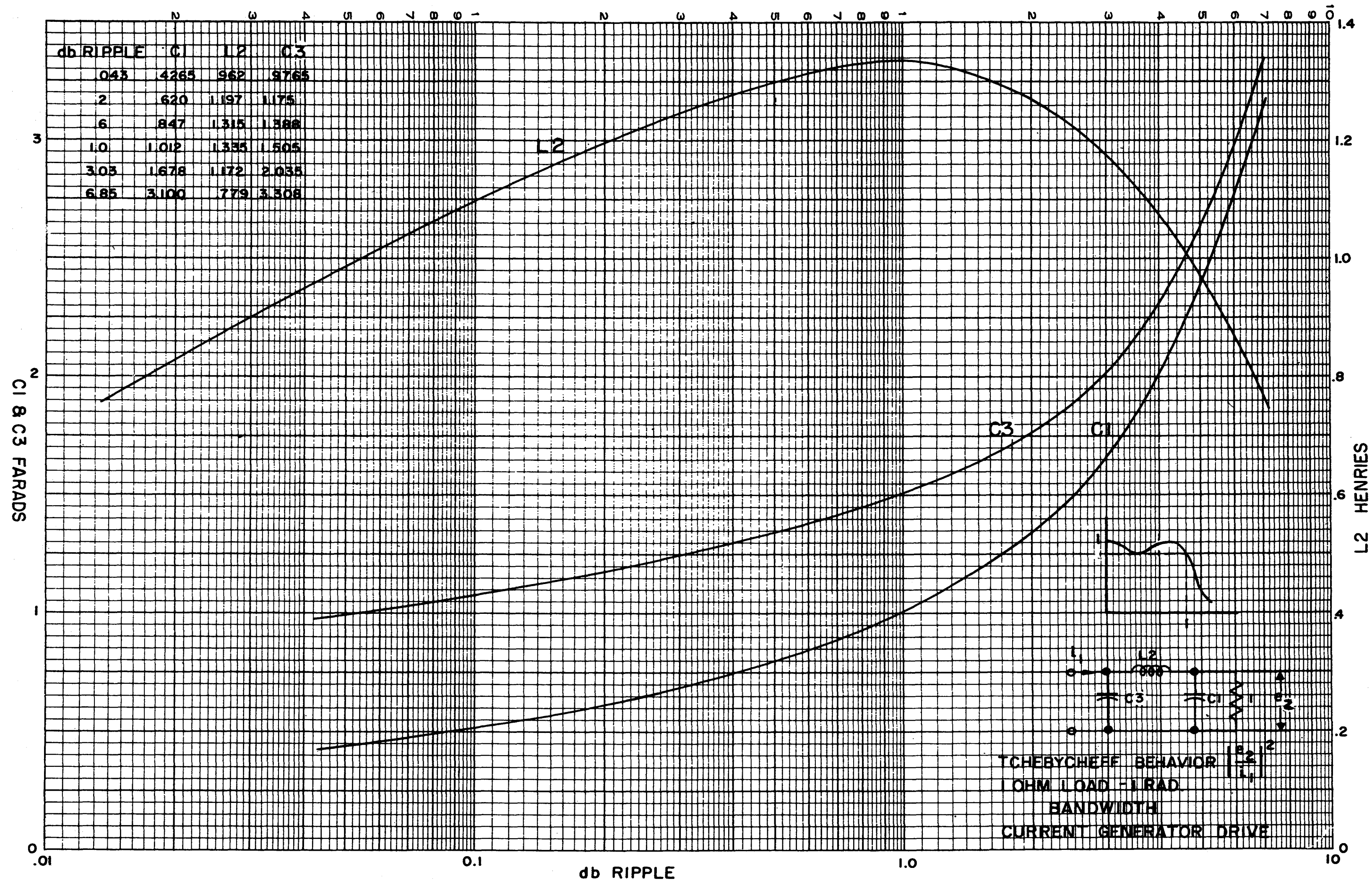


Fig. 2. Design curves for Tchebycheff three-pole interstages.

and with 3 db ripple in the overall response

$$\left|\frac{E_o}{E_i}\right| \simeq (g_m)^N \left(\frac{2}{\omega_o C}\right)^N \left(\frac{1}{.68 + .12\sqrt{N}}\right)^N \tag{21}$$

This circuit is therefore theoretically superior in many applications to the amplifiers previously discussed with simple stagger-tuned interstages. Two practical problems which often arise in using the interstages of Fig. 2 are concerned with the ratio of the terminal capacitances and the parasitic capacitance of the series inductance. In Ref. 16, it is shown that these problems can often be overcome through the use of circuits of the form of Fig. 3, at a modest cost in gain-bandwidth advantage.

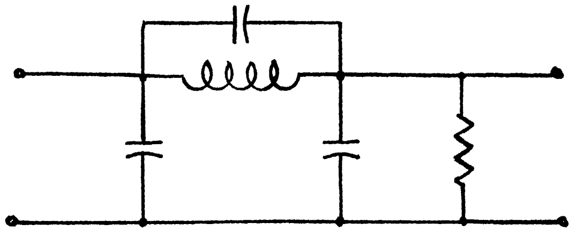


Fig. 3. A three-pole interstage.

2.2 The Distributed Amplifier

The distributed amplifier[7,8] can be considered as two artificial transmission lines joined by tubes. The tube input and output capacities contribute to the shunt capacities of the artificial lines.† The velocities of propagation on the two lines are made equal, but could both be functions of the displacement along the circuit. A signal propagating down the grid line causes plate current to be injected into the plate line by each tube in succession. These currents add constructively in

† This statement is true at low frequencies. At higher frequencies, due to transit time effects in the tube and lead inductances in particular, the tube impedances are more complex.

the forward plate line termination. The gain of a distributed amplifier of s stages with r tubes per stage is, from Linvill,[11]

$$\left|\frac{E_o}{E_i}\right| = \left(\frac{BA^{r-1}}{\sqrt{2}}\right)^s \cdot \left(\frac{rg_m\pi^2}{4\omega_o C}\right)^s . \qquad (22)$$

The symbols used in Eq. 22 are defined below.

B = a factor less than one indicating by how much the magnitude of the transfer impedance between adjacent terminal pairs differs from a theoretical limit

$$|Z_{p\ p+1}|_{max} = \frac{\pi^2}{\sqrt{2}\ 4\omega_o C} . \qquad (23)$$

A = the factor by which a signal traveling down a line is attenuated by a section of line. The attenuations of the two lines are assumed to be equal.

C = output capacitance of the tube. The limitation of shunt capacitance in the recurrent line structure results from the limit it places on line characteristic impedance, and the proportionality of the output signal to this characteristic impedance.

The factor $\sqrt{2}$ in the denominator of the first term of Eq. 22 arises from the fact that only 1/2 of the injected current of a tube flows toward the forward termination. The upper limit for B = A = 1 is

$$\left|\frac{E_o}{E_i}\right|_{max} = \left(\frac{1}{\sqrt{2}}\right)^s (rg_m)^s \left(\frac{\pi^2}{4\omega_o C}\right)^s . \qquad (24)$$

It is known[7] that, for a cascade of voltage distributed amplifier stages containing rs tubes, the maximum voltage gain is obtained if the stage gain is ϵ, the base of the Naperian logarithms ($\simeq 2.718$).

2.3 The Partitioned Amplifier

A partitioned amplifier is an amplifier made up of a number of conventional amplifier chains connected in parallel. The inputs of the individual chains, or channels, are coupled to a common source by a multi-terminal pair network; the channel outputs are similarly connected to a common load. The most significant prior treatment of partitioned amplifiers known to the author was made by Linvill.[11,12] Certain of his results are summarized below.

Linvill considered the circuit of Fig. 4, in which for any given input across the terminal pair shunted by C_i, the outputs at the remaining terminal pairs (each shunted by C_o) are equal.

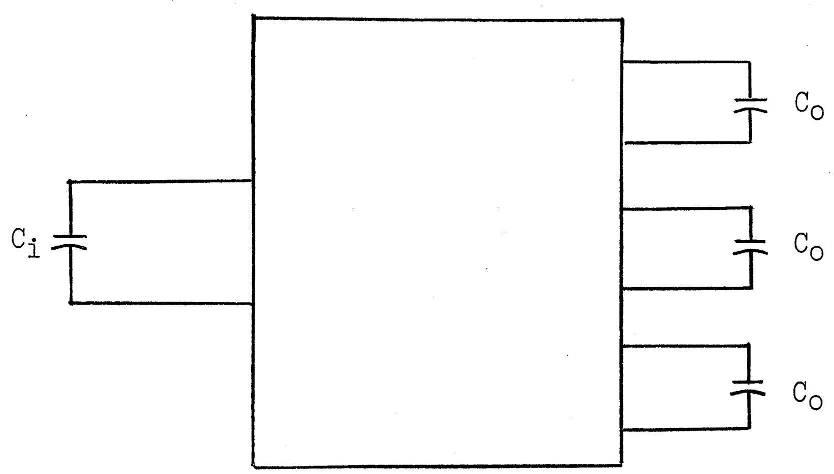


Fig. 4. A multi-terminal pair network.

Linvill stated that the voltage which could be uniformly maintained over a range of frequencies at these m output terminal pairs (m = 3 for the example drawn) for a unit input current can be no greater than could be maintained over the same band at a single output terminal pair with mC_o farads in shunt. This follows from an inspection of the two-terminal pair network in Fig. 5, constructed from the network of Fig. 4, and the results of Section 2.1. Thus, if one utilizes, for coupling the inputs

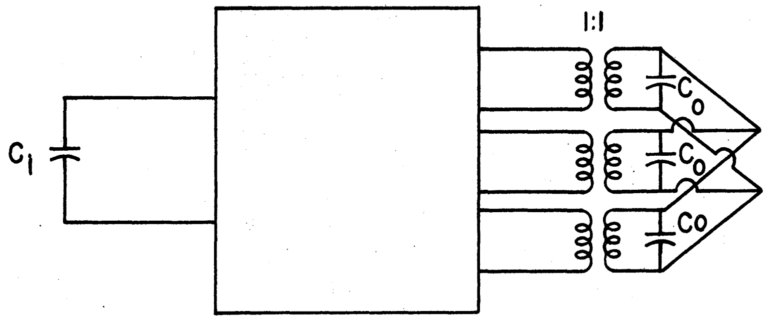


Fig. 5. Two-terminal pair network based on Fig. 4.

of the partitioned amplifier channels to a source, a multi-terminal set which causes identical signals to appear at all channel inputs, the magnitude of the transfer impedance from the source to any channel input is limited by the same quantity as when all the channel input capacitances are tied in parallel. From Eq. 11, it follows that the maximum transfer impedance from the source to any of m terminal pairs is, under these conditions, less than

$$\frac{\pi^2}{4\sqrt{mC_1C_2}\,\omega_0} .$$

This point will be shown subsequently to be extremely fundamental in partitioned amplifier theory.

Linvill also presented an empirical expression for the gain of partitioned amplifiers of a specific type. The formula obtained was

$$|\text{ampl}| = \frac{.2}{m^{3/2}} \left(\frac{\pi^2 g_m m}{4\omega_0 C}\right)^N , \tag{25}$$

where

N = number of stages per chain,

m = number of chains,

C = the magnitude of the capacitance at each terminal pair of the symmetrical two-terminal pair networks used.

The origin of the several factors will now be considered.

The N-th power of the quantity in parentheses is the theoretical limit of the uniform amplification of an N-stage amplifier chain covering the band ω_0/m radians per second wide with two-terminal pair networks having a shunt capacitance C at each terminal pair.

The factor .2 is an empirical number which is intended to describe the closeness with which the channels of his amplifiers approached the theoretical limit. It is analogous to the quantity

$$\frac{\epsilon}{\sqrt{2(1+\epsilon^2)}} \qquad (26)$$

associated with a chain utilizing Tchebycheff-staggered single-tuned interstages as indicated in Eq. 9.

The factor $m^{-3/2}$ results from the particular multi-terminal output coupling circuits used. These were in a form such as in Fig. 6, where a three-channel structure is illustrated. The structure is made up of four symmetrically positioned networks, each of these networks being identical. It will be noted that a signal entering this structure at any one of the input terminal pairs is transmitted equally well to the networks through

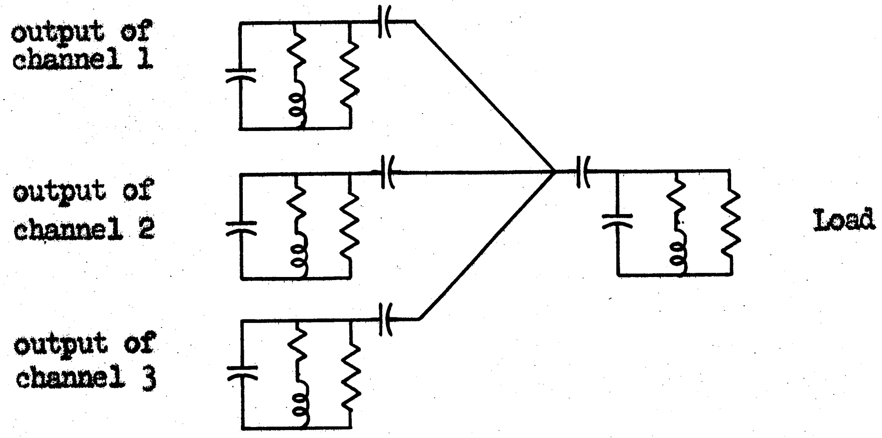


Fig. 6. A multi-terminal coupling network.

which the other channels are fed as well as the load. Thus, for an m channel amplifier, a power loss by a factor $1/m$ results. Now, one notes also that if only one chain were operated the output circuits of the remainder of the channels could be removed. The network could then be reduced in bandwidth by the factor m, when the impedance level could be increased by m under a fixed channel capacity limitation. An increase in load voltage by a factor m would result for a fixed channel output current. For this arrangement, then, the cost of tieing together the channel outputs is† given by the factor $m^{-3/2}$.

The gain given by Eq. 25 was maximized by Linvill under the restriction of a fixed total number of tubes $mN = t$. The results were expressed in terms of a parameter

$$b_n = \frac{\omega_o}{\frac{\pi^2 g_m}{4C}} . \qquad (26)$$

The resulting equation

$$\frac{3}{2} = N\,(1 + \ln b_n - \ln m) \qquad (27)$$

was utilized in obtaining optimum curves of N vs m with b_n as a parameter, which are represented in Fig. 7. One observes that the optimum number of chains depends little on N for N greater than 4.

In Linvill's design procedure m and N were determined from Fig. 7, for given bandwidth, tube type, and overall gain requirements. An output structure typified by Fig. 6 was then chosen consistent with the overall amplifier bandwidth, the number of channels, and the channel output capac-

† It should be noted that a loss in output power per unit channel current also results from the presence of the conductance across the terminal pair at which it is injected in Fig. 6. This point is discussed further in Chapter IV.

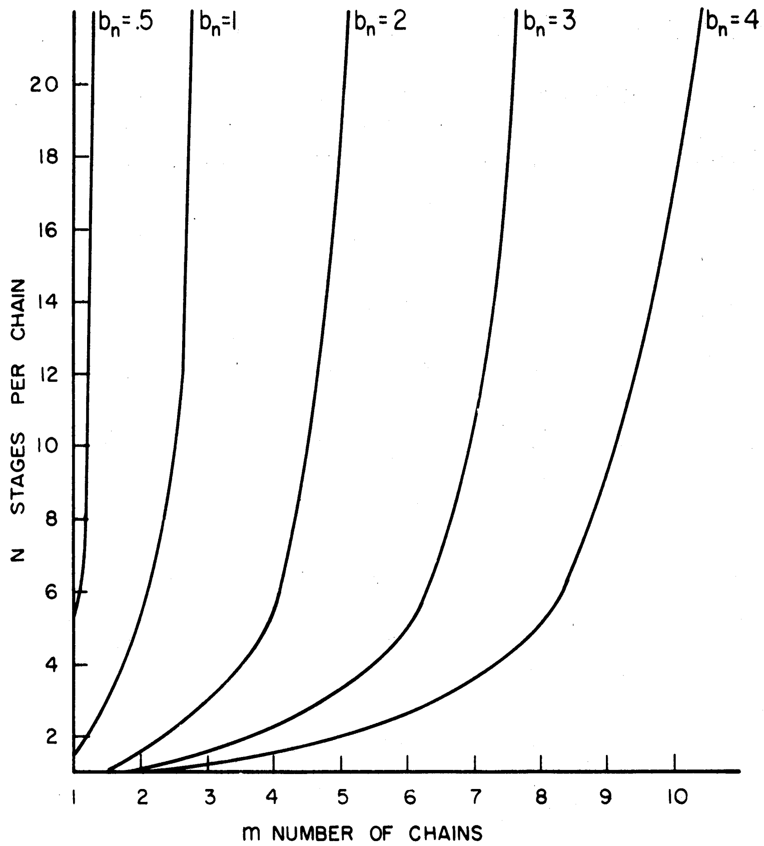


Fig. 7. Plot showing the optimum number of chains and stages per chain for a series of different normalized bandwidths.

itance limitations. A tentative choice of channel transfer functions was then made. The tiral positions of the poles and zeroes of each individual channel transfer function were qualitatively positioned in an attempt to maintain a high transfer impedance over the preassigned channel band. The gain was then evaluated, experimentally or analytically, and a more or less systematic iterative procedure was employed in shifting the poles and zeroes of the channel transfer functions to obtain a satisfactorily uniform response.

Linvill's results have not brought about a widespread use of partitioned amplifiers, even though he undoubtedly established their great capability. It is likely that more extensive use will follow with improved design procedures and a wider appreciation of their advantages.

CHAPTER III

INTRODUCTION TO THE PARTITIONED AMPLIFIER PROBLEM

3.1 Introduction

The term "partitioned amplifier," as used in this paper, refers to certain circuits made up of a number of conventional amplifier chains connected in parallel. By definition, a partitioned amplifier is a multi-channel amplifier in which there is a variation, among the channels, of the frequency dependence of the channel gains. Particular interest will be directed at such circuits in which the region of substantial amplification of each channel is confined to a unique portion of the total bandwidth covered. The reasons for this choice of attention will now be discussed briefly.

As an alternative to the idea of separating all the channels in frequency, it may be assumed that two particular channels provide nominally uniform amplification over the same portion of the total band. If, in order to achieve these channel amplifications, identical signals are developed at the two channel inputs, the transfer impedance from the common source to either channel input is limited by the same quantity as when the channel inputs are tied directly in parallel. See Section 2.3. Let us suppose, as again seems reasonable, that identical transfer functions are utilized from the two channel outputs to the common load. The transfer impedance from either channel output to the common load is then limited, similarly, by the same quantity as when the channel outputs are tied in parallel. Since the channels themselves were assumed to cover the same frequency range, one may conclude that no gain-bandwidth advantage accrues for the two coincident channels compared to a single channel circuit. In fact, if the two chains together with their input and output coupling arrangements were identical, the corresponding nodes in the two channels could be tied together without loss. A single channel circuit would result with the tubes paralleled and interstages differing only in impedance level from the interstages of the original individual channels. It is therefore concluded that confinement of the substantial gain of each channel of a partitioned amplifier to a unique portion of the total bandwidth of the amplifier is necessary, if maximum gain-bandwidth advantage is to be achieved.

3.2 Definition of Symbols

Before continuing the discussion of partitioned amplifiers, a system of notation will be defined. The individual channels will be designated respectively as channel a, channel b, etc. In generalized discussions, a typical channel will be referred to as the k-th channel, while m total channels will be presumed. The stages of the k-th channel will be designated, progressing from the input toward the output stages, by A_k, B_k... I_k...N_k. Thus, in generalized discussions, reference will be made to the I_k-th stage of channel k containing N_k stages.

Subsequent discussion will be restricted to circuits which can be represented in the form of Fig. 8. It will be assumed that the coupling from the common source to the several channel inputs, and from the channel outputs to the common load, is accomplished with multi-terminal networks made up of lumped passive elements.† These networks, as well as the

† This is a restriction to limit the scope of the problem, it being of course feasible to consider input and output circuitry with distributed or active character.

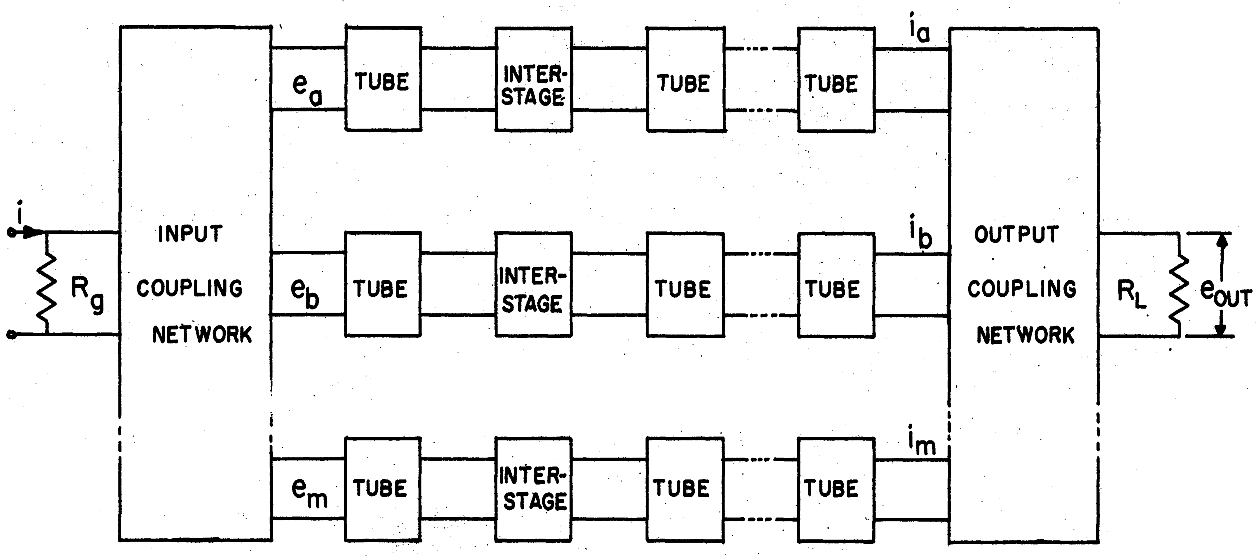


Fig. 8. The generalized partitioned amplifier.

channel interstages, may contain only lumped resistances, inductances, and capacitances.

It will be convenient to express the overall transfer function $\frac{e_{out}}{i}$ of the partitioned amplifier in terms of a set of transfer functions which can be associated with portions of the composite circuit. For example, it will be assumed that, as a result of injecting the source current i into the input structure, there appear at the respective channel inputs voltages e_a, e_b ... e_m (see Fig. 8). The presence of tube isolation at the channel inputs insures that the resulting transfer functions $\frac{e_a}{i}$, $\frac{e_b}{i}$ ... $\frac{e_m}{i}$ are dependent only on the input network. Similarly, define the respective channel output currents resulting from the injection of the source current i into the input structure as i_a, i_b ... i_m. The transfer admittance $\frac{i_a}{e_a}$ is then a characteristic of the a-th channel. Similar remarks hold for the transfer admittances $\frac{i_b}{e_b}$ ... $\frac{i_m}{e_m}$.

Finally, assume that when flowing individually, the channel output currents i_a, i_b ... i_m produce respectively voltages e'_a, e'_b ... e'_m at the common load. The transfer functions $\frac{e'_a}{i_a}$, $\frac{e'_b}{i_b}$ ... $\frac{e'_m}{i_m}$ then depend only on the multi-terminal output coupling circuit. The voltage at the common load resulting from the injection of the source current i when all channels are activated is then

$$e_{out} = e'_a + e'_b + \dots + e'_m \quad .$$

The overall transfer characteristic can be expressed in terms of the transfer functions defined above:

$$\frac{e_{out}}{i} = \frac{e_a}{i}\,\frac{i_a}{e_a}\,\frac{e'_a}{i_a} + \frac{e_b}{i}\,\frac{i_b}{e_b}\,\frac{e'_b}{i_b} + \dots + \frac{e_m}{i}\,\frac{i_m}{e_m}\,\frac{e'_m}{i_m} \quad . \tag{28}$$

The tube transconductances enter the channel transfer functions as multiplying constants. Let the product of the transconductances of the tubes of channel a be k_a, with analogous definitions for constants $k_b \dots k_m$. The channel transfer functions may then be written as

$$\begin{aligned} \frac{i_a}{e_a} &= k_a \frac{m_a}{M_a} \quad , \\ \frac{i_b}{e_b} &= k_b \frac{m_b}{M_b} \quad , \\ \frac{i_m}{e_m} &= k_m \frac{m_m}{M_m} \quad . \end{aligned} \tag{29}$$

where $\frac{m_m}{M_m}$ is the product of the transfer functions of the interstages of channel a, etc.

Certain properties of the transfer functions of the interstages, as well as the input and output coupling networks, will now be summarized. The functions $\frac{e_k}{i}$, $\frac{e'_k}{i}$, and the factors of $\frac{m_k}{M_k}$ can be expressed in general as rational functions of the complex frequency $p = \sigma + j\omega$. Their denominators are Hurwitz, while their numerators may contain roots anywhere in the complex plane. For the important special case of ladder structures without coupled coils, however, the transfer zeroes are also restricted to the left half plane.

The transfer functions, of which $\frac{e_k}{i}$ and $\frac{e'_k}{i_k}$ are typical, are characteristic of the input and output networks, respectively. These networks will be considered in detail in Chapter IV. It is known, however, that these transfer functions can always be expressed in the form

$$\frac{e_a}{i} = \frac{n_a}{D_{in}} , \frac{e_b}{i} = \frac{n_b}{D_{in}} \cdots \frac{e_m}{i} = \frac{n_m}{D_{in}} ;$$
$$\frac{e'_a}{i_a} = \frac{n'_a}{D_{out}} , \frac{e'_b}{i_b} = \frac{n'_b}{D_{out}} \cdots \frac{e'_m}{i_m} = \frac{n'_m}{D_{out}} . \tag{30}$$

The denominator D_{in} is in fact the denominator of the driving point impedance seen at any of the terminal pairs of the input network, for the circuit of Fig. 8. This follows from the fact that the presence of shunt elements at every terminal pair insures that the driving point impedances have no poles which are not found in the transfer impedances. A particular function $\frac{e_k}{i}$ may, however, have zeroes at the locations of some of these common poles.

If the definitions of Eqs. 29 and 30 are substituted in Eq. 28 the overall transfer function becomes

$$\frac{e_{out}}{i} = \frac{1}{D_{in}\,D_{out}} \left[\frac{k_a\,n_a\,m_a\,n'_a}{M_a} \quad \frac{k_b\,n_b\,m_b\,n'_b}{M_b} \cdots + \frac{k_m\,n_m\,m_m\,n'_m}{M_m} \right] . \tag{31}$$

Note that if identical tube complements are used in all channels,

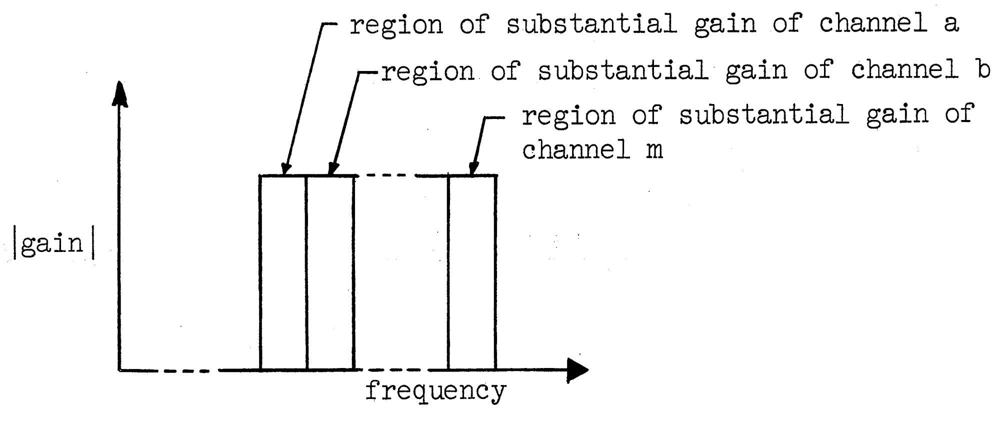
$$k_a = k_b = \cdots = k_m \overset{d}{=} k , \tag{32}$$

and a factor k is common to all channels. Then

$$\frac{e_{out}}{ki} = \frac{1}{D_{in}\, D_{out}} \left[\frac{n_a\, m_a\, n'_a}{M_a} + \frac{n_b\, m_b\, n'_b}{M_b} + \ldots + \frac{n_m\, m_m\, n'_m}{M_m} \right] . \qquad (33)$$

The tubes limit the composite functions $\frac{e_{out}}{i}$ which can usefully be identified as overall gain functions of a partitioned amplifier, in that they necessitate realizations in which certain minimum shunt capacitances (and perhaps conductances) are present at the various terminal pairs. These minimum terminal capacitances limit impedance levels throughout the amplifier and so the gain, at given bandwidths.

Emphasis in the following discussion will be placed on the problem of designing partitioned amplifiers with a magnitude of gain approximating the rectangular characteristic. The principles discussed are useful in approximating other ideal gain characteristics; the generalized subject of partitioned amplifiers is too large to permit coverage in this study. Certain considerations, some practical and some theoretical, seem to warrant the opinion that a desirable idealized distribution with frequency of channel gains to obtain an approximately rectangular overall gain characteristic is as sketched in Fig. 9.† Figure 9 suggests the use of a



g. 9. Desirable idealized distribution of channel gains.

† This point is discussed in Chapter VI.

number of adjacent channels of approximately equal gain and bandwidth. The gain of the channels would then approximate the desired overall gain. It is shown in Section 3.4 that economical use of the tubes dictates that a sufficient number of channels be utilized to realize a mean stage gain of ϵ in each channel. Thus it seems reasonable to assume identical channel tube complements at the outset of a design procedure. In subsequent design procedures departures from identical channel tube complements are treated as perturbations on this original assumption. One is thus led to consider Eq. 33, in which identical tube complements are assumed for the various channels. It is clear that the gains of the several channels of an amplifier defined by Eq. 33 can be made to approximate the ideal characteristics of Fig. 9 through control of the poles and zeroes of the additive terms in Eq. 33. The poles associated with D_{in} and D_{out} should presumably be so distributed as to achieve a reasonable compromise for all channels. The rest of the poles of say

$$\frac{n_a \; m_a \; n'_a}{D_{in} \; D_{out} \; M_a} \quad ,$$

which is associated with the gain from source to load through channel a alone, should be so located as to obtain the highest feasible gain over the portion of the total band assigned to channel a. The zeroes should in principle be as remote as is feasible from the region in which the gain of channel a is to be concentrated. Analogous remarks hold for the functions associated with the remaining channels.

3.3 Introduction to an Exact Design Procedure

An exact design procedure will now be introduced. It seems desirable to describe briefly the principles involved, and then to illustrate later many of the detailed problems which one can expect to encounter with specific examples. The basic starting point in a design problem would be a statement of the bandwidth to be covered, the tube type to be employed, and the required gain. Additional information such as the complexity of interstages to be employed may be given, or an assumption may be made. The realization of an amplifier with a given complexity of interstage using the principles of this report may tax the ingenuity of the designer, but can be achieved in principle. It will often prove convenient to obtain realizations with a mixture of simple and more complex circuits.

From the statement of the design problem one determines the number of channels necessary to permit the attainment of a mean stage gain of approximately ϵ in each channel, with the chosen tubes and interstage complexity. Practically, it may be advantageous to be somewhat conservative in the choice of the number of channels, since partitioning involves, in general, a loss which increases with the number of channels. Finally, one can estimate the number of stages necessary to achieve a prescribed overall gain, at a nominal gain of ϵ per stage.

Let us assume that the partitioned amplifier is to have a prescribed magnitude of overall gain given by

$$\left|\frac{e_{out}}{i}\right|^2 = \frac{K}{B(p)} \left.\frac{K}{B(-p)}\right|_{p^2 = -\omega^2} \tag{34}$$

where B(p) is the Hurwitz factor of the denominator of the prescribed overall gain function and **K** is a constant. Of course, the choice of the simple numerator is arbitrary, but it is a reasonable choice for a lowpass structure in that it locates all zeroes of transfer of the overall function at infinity. The choice of the numerator of $\frac{e_{out}}{i}$ is trivial in the example above due to the particular overall function chosen. The numerator of any realizable prescribed squared magnitude function determines a polynomial A(p) A(-p) in p. The numerator of $\frac{e_{out}}{i}$ must contain the factor A(p) which need not be Hurwitz. A difficulty arises for the methods of this paper for more general A(p) A(-p) in that a number of choices for the numerator of $\frac{e_{out}}{i}$ are possible and the best choice of A(p) A(-p) may depend on the particular application. This point is discussed further in Chapter V. The denominator of $\frac{e_{out}}{i}$ must be Hurwitz, and must contain B(p).

For the particular magnitude function chosen in Eq. 34, then, one considers the function

$$\frac{e_{out}}{iK} = \frac{P^*}{P} \frac{1}{B(p)} \tag{35}$$

where P is a Hurwitz polynomial in which all complex roots occur in conjugate pairs. P* is simply constrained so that

$$\left|\frac{P^*}{P}\right|_{p=j\omega} = 1 \quad . \tag{36}$$

The function $\frac{e_{out}}{i}$ then obviously has the prescribed magnitude.

The next step in the design is to choose input and output networks. This problem will be considered in detail later, but in principle amounts to choosing the sets of individual functions

$$\frac{e_a}{i}, \frac{e_b}{i} \cdots \frac{e_m}{i} \qquad \text{and} \qquad \frac{e'_a}{i_a}, \frac{e'_b}{i_b} \cdots \frac{e'_m}{i_m} \quad .$$

The basic problem in this choice is an approximation problem; ideally, one should choose these interrelated transfer functions in such a way that each transfer impedance is as high as practical over a pre-determined region in which the corresponding channel gain is to be concentrated. A result of selecting the input and output networks is to determine the sets of polynomials n_a, n_b ... n_m, D_{in} and n'_a, n'_b ... n'_m, D_{out}. It is clear that D_{in} and D_{out} must be factors of the denominator PB in Eq. 35. The polynomial P can be constrained to contain both D_{in} and D_{out}. Alternatively, one may be able to associate factors of the prescribed polynomial B with the input and output networks, when of course fewer constraints need be placed on P. The n_k and n'_k are also then determined in the equation

$$\frac{e_{out}}{i} = \frac{KP^*}{PB} = \frac{k}{D_{in} D_{out}} \left[\frac{n_a m_a n'_a}{M_a} + \frac{n_b m_b n'_b}{M_b} \cdots + \frac{n_m m_m n'_m}{M_m} \right] \quad (37)$$

which contains alternative expressions for the overall gain function previously discussed.

A relatively simple problem results if the poles of the channel functions are also common ($M_a = M_b = \ldots = M_m \overset{d}{=} M$). Then the gain functions from source to load through the several channels can differ only through variation of their zeroes. For this case

$$PB = D_{in} D_{out} M \quad , \quad (38)$$

$$KP^* = k \left[n_a m_a n'_a + n_b m_b n'_b + \ldots + n_m m_m n'_m \right] \quad . \quad (39)$$

It is certainly reasonable to suppose that, for given B, D_{in}, D_{out}, and n's, a number of P's and associated P*'s could be chosen satisfying the above equations. Numerical examples will be presented in Chapter V.

It would be expected that the use of channels with identical transfer poles would not lead in general to amplifiers achieving high gains for the number of tubes employed. A somewhat specific example will now be presented which suggests a method for obtaining channel functions in which all poles are not common. Consider, for example, Fig. 10 in which the poles of a prescribed overall response are indicated. Assume that one wishes

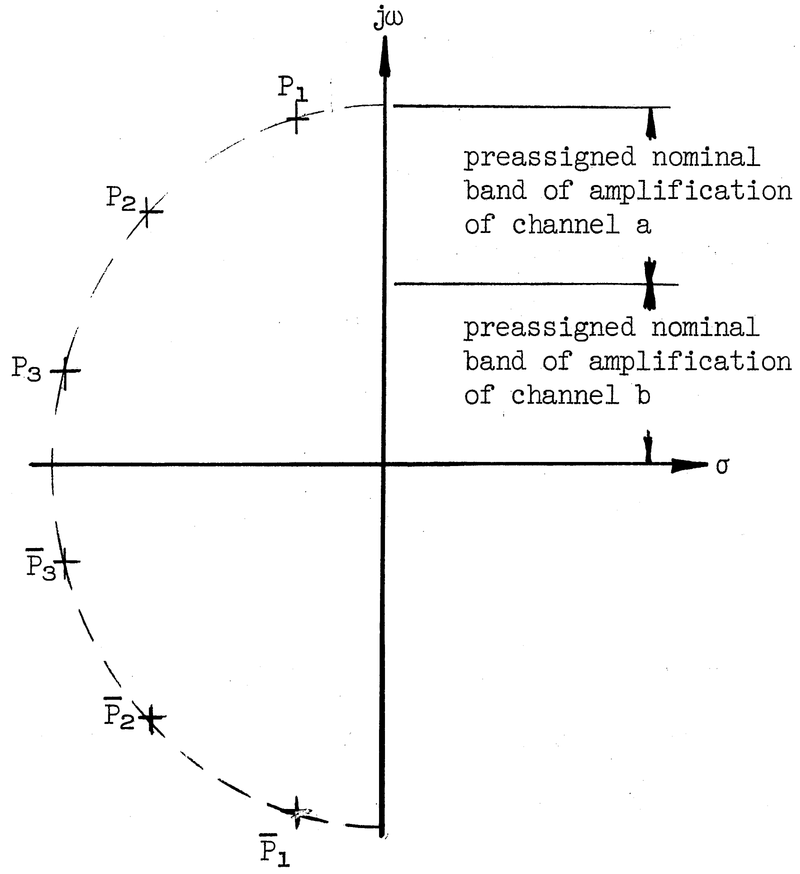


Fig. 10. Pole distribution for a prescribed overall response.

to realize this overall function with an amplifier made up of two channels which are to cover the bands indicated. It will further be assumed that the conjugate pair of poles associated with P_2 in Fig. 10 are to be associated with either the input or output network of the amplifier.

It would then appear that, in the interests of gain, it might be desirable to associate the conjugate pair of poles P_1 and $\overline{P}_1$, with the high frequency channel a. Similarly, the conjugate pair P_3 and $\overline{P}_3$ should be relatively desirable poles in the channel b characteristic, which is to cover the lower frequency band. In order to illustrate that this can be done, let us designate by p_1 the quadratic factor associated with the conjugate pair of poles P_1 and $\overline{P}_1$, with analogous definitions for p_2 and p_3. Further, let $P = P_\alpha P_\beta$ where P_α contains any additional factors other than p_2 to be associated with the input and output circuits. Now, if P contains a factor P_α, then P^* must contain a factor P_α^* where

$$\left|\frac{P_\alpha^*}{P_\alpha}\right|_{p=j\omega} = 1 \quad . \tag{40}$$

Note that P^* must also then contain an analogous factor P_β^*. We may constrain P^* to contain two additive terms, containing respectively p_1 and p_3, and write the overall transfer function in the forms

$$\frac{e_{out}}{i} = \frac{P_\alpha^* \, P_\beta^*}{P_\alpha P_\beta (p_1 p_2 p_3)} = \frac{\alpha p_1 + \beta p_3}{P_\alpha P_\beta (p_1 p_2 p_3)} = \frac{\alpha}{(P_\alpha p_2)(P_\beta p_3)} + \frac{\beta}{(P_\alpha p_2)(P_\beta p_1)} \, . \tag{41}$$

At this juncture in the discussion, several points are clear. First, the function α must contain all the transfer zeroes associated with transmission from source to load through channel a, including those associated with the input and output coupling networks. Similarly, the zeroes of the transfer from source to load through channel b are characterized by β. The functions $P_\alpha p_2 = D_{in} D_{out}$ by choice, while p_3 is associated with the lower frequency channel and p_1 with the higher frequency channel as desired. The poles P_β are common to the two channel transfer functions.

The procedure discussed would not appear to be justified unless more satisfactory distributions for the poles of the individual channels result than can be achieved with amplifiers having common channel transfer poles. This point is discussed in later numerical examples.

The factors n_a and n'_a in α, as well as n_b and n'_b in β, become known when the input and output networks of the amplifier are chosen. The regions in which the individual channel gains should be concentrated are then spelled out, and reasonable preliminary choices as to a desirable character of distribution for the transfer zeroes of the individual channels can be made. As will be illustrated later a number of arbitrary constants can be introduced at this point. The above choices fix the form of $\alpha p_1 + \beta p_3$ which was also constrained to contain a factor P_α^* as previously discussed. These constraints on the polynomial $\alpha p_1 + \beta p_3$ can be made to yield a set of simultaneous equations in the above-mentioned arbitrary constants. A preliminary function $\frac{e_{out}}{i}$ can then be obtained if a solution to the simultaneous equations exists. If more arbitrary constants than equations are present, an optimization procedure is feasible. Finally, obtaining a suitable realization often leads to interesting minor problems, some of which are considered in later numerical examples.

Before undertaking the illustration of design procedures for partitioned amplifiers having exact prescribed overall gain characteristics, two specific problems will be considered. The first of these problems is concerned with the optimum gain of a partitioned amplifier containing a fixed number of tubes. The second of these problems deals with the multi-terminal input and output networks of a partitioned amplifier.

3.4 The Optimum Gain of a Partitioned Amplifier of t Tubes

Referring to Linvill's equation (Eq. 25) for the gain of a partitioned amplifier, it will be noted that a considerable complication results from the inclusion of the factor $m^{-3/2}$ associated with losses in the multi-terminal pair output circuit. Subsequent material will show that circuits greatly superior to those utilized by Linvill can be obtained. It seems desirable, however, to have certain very simple results available immediately. In order to do this, it will be assumed temporarily that the losses associated with the input and output coupling networks are small compared to the overall gain. Now, let us further assume that over the total band which a partitioned amplifier is to cover, a stage gain of x can be achieved with some chosen quality of interstage. This gain may well be less than unity. If it is assumed that over $\frac{1}{m}^{th}$ of this band a stage gain of mx can be achieved, and that a channel contains N tubes, a channel gain of

$$|ampl| = k_o (mx)^N \qquad (42)$$

results, where k_o accounts for a cascading loss as in previous discussion. One then attempts to determine whether an optimum number of channels exists in the interest of maximizing the gain with a fixed number t(=mN) of total tubes. Setting

$$\frac{\partial |ampl|}{\partial m} = 0 \qquad (43)$$

results in the equation

$$\frac{t}{m^2_{opt}} (xm_{opt})^{\frac{t}{m_{opt}}} (1 - \ell n \; xm_{opt}) = 0 \; . \qquad (44)$$

The important solution occurs for

$$1 - \ell n \; xm_{opt} = 0 \qquad (45)$$

from which $xm_{opt} = \epsilon$. It is clear that the quantity xm_{opt} is the gain of a single stage in the final amplifier. The result dictates that for maximum gain with a fixed number of tubes one should always select a sufficient number of channels so that a mean stage gain of ϵ is obtained in each channel. An exception is where a stage gain greater than ϵ is obtainable over the desired band, when of course a single channel is optimum. It is worth repeating that no distinction has been made in this discussion as to whether two-terminal or two-terminal pair interstages are used. The effect of the quality of circuit used, in the sense of its gain-bandwidth advantage, lies simply in the value assigned to x in a particular problem. The above result is strikingly similar to one obtained in distributed amplifier theory, where however the term "stage" has a very different meaning. Reference to the curves of Fig. 7 by Linvill establishes that even with the large losses assumed there for the output circuit, m is approximately equal to $b_n \epsilon$ for $n > 4$. A comparison of Eqs. 25 and 42 shows that $b_n = 1/x$, from which the optimum relationship stated above would follow.

It is perhaps worth mentioning that the parameter k_o, which was assumed to be equal for all channels, and which was included to account for cascading loss, does not occur in the equation for the optimum number of channels. As an illustration of the optimum gain criterion, consider the 6AK5 which has a figure of merit of 75 mc. Then unity gain can be achieved over a 75 mc band with a flat-staggered cascade of these tubes and simple interstages. The above results indicate that, for a 50 mc bandwidth amplifier utilizing simple tuned interstages the optimum number of channels is $\epsilon/1.5 \simeq 2$. Similarly, for a 75 mc band and the same type interstages, the optimum number of channels is approximately three. Finally, the gain of a partitioned amplifier, optimum in the above sense, is $k_o(\epsilon)^N$.

CHAPTER IV

THE DESIGN OF MULTI-TERMINAL COUPLING CIRCUITS

4.1 Multi-Terminal Networks Providing Identical Channel Input Signals

As was stated in the introduction, the design of the multi-terminal coupling networks of a partitioned amplifier is a considerable theoretical and practical problem. In Chapter III, it was asserted that one can choose the characteristics of these coupling networks in order to insure efficient coupling to the channels, and then synthesize a set of channel transfer impedances which complement this output structure in such a way that an exact prescribed overall transfer function is obtained. Before further discussion of this technique, it is first necessary to consider the multi-terminal network design problem.

From Linvill's results (see Section 2.3) it is concluded that if a coupling set is utilized which causes identical signals to appear at all channel inputs, the magnitude of the transfer impedance from the source to any channel input is limited by the same quantity as when all the channel input capacitances are tied in parallel. This leads to consideration of the problem of Fig. 11 where all the channel inputs are tied together.

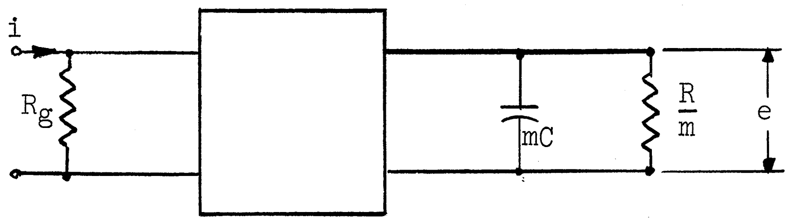


Fig. 11. Coupling network with all channel inputs tied together.

Each channel input impedance is represented approximately by a resistance R in parallel with a capacitance C. The source is represented by an ideal current generator in parallel with a resistance R_g. The maximum power which can be delivered from such a source is

$$P_{avail} = \frac{|i|^2 R_g}{4} \quad . \tag{46}$$

The efficiency of power transmission from a source is commonly studied in terms of a parameter t called the transmission coefficient; it is defined by

$$|t|^2 = \frac{P_{del}}{P_{avail}} \tag{47}$$

where P_{del} is the power delivered to the load (R/m in this case). The problem of maximizing the minimum transmission efficiency (that is, maximizing the minimum $|t|^2$) over a band ω_o for a prescribed product $\omega_o(R/m)\, mC = \omega_o RC$ has been studied.[17] Networks of given complexity optimum in the above sense are defined, and exhibit a Tchebycheff behavior of $|t|^2$.

The reciprocity theorem dictates that the ratio e/i in Fig. 12 is equal to that obtained in Fig. 11; it follows then that the $|t|^2$ is not dependent on the terminal pair from which the network is driven. Thus, the material above is also applicable in the output circuit of a partitioned amplifier, if the m outputs are connected in parallel.

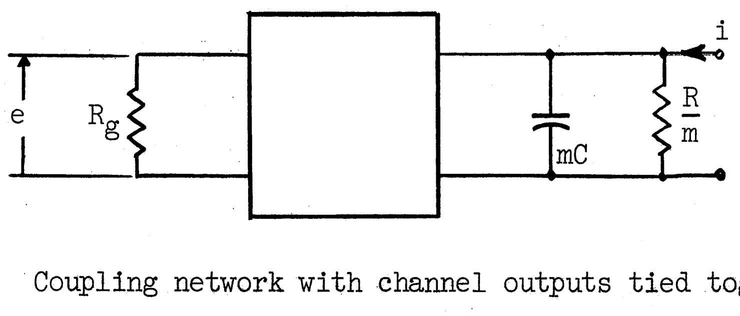


Fig. 12. Coupling network with channel outputs tied together.

A special problem arises often in the case where pentodes or tetrodes are used and the quantity $\omega_o RC$ associated with the source is very high. A study of the optimum networks described above indicates that as $\omega_o RC$ increases the ripple increases. For sufficiently high $\omega_o RC$ the networks optimum in the sense of transmission efficiency are undesirable due to passband nonuniformity. An alternative approach in this case is to approximate the tube as an ideal current source shunted by a capacity which leads for Tchebycheff behavior of $|e/i|^2$ to networks such as are defined by the curves of Fig. 2. One can then choose the tolerable ripple of the response and obtain suitable networks for the case where the actual $\omega_o RC$ is very high. After obtaining a network covering the bandwidth ω_o, an impedance level transformation is performed. The maximum impedance level consistent with the terminal capacitance limitation is chosen. These are optimum networks in the sense that the output voltage per unit input current is maximized for a given ω_o, mC, response tolerance and network complexity, with the current source.†

The material above was presented to show that optimum two-terminal pair structures can be defined for the case where all channel inputs or outputs are tied together. Such networks determine multi-terminal structures with the same capacitance limitations, in the manner illustrated in Fig. 13 where a three-pole structure was arbitrarily chosen. The impedances looking into the networks of Fig. 13 as indicated are obviously equal, and the delivered power is clearly divided equally among the m terminations on the right. The previous remarks indicate that the multi-terminal structure achieves individual transfer impedances as close to their theoretical maximum as is achieved in the network on the left, which has the same total terminal capacitance. Thus it is possible to design,

† These networks are very nearly those which would be obtained by also prescribing the passband response tolerance in the transmission efficiency problem with large $\omega_o RC$.

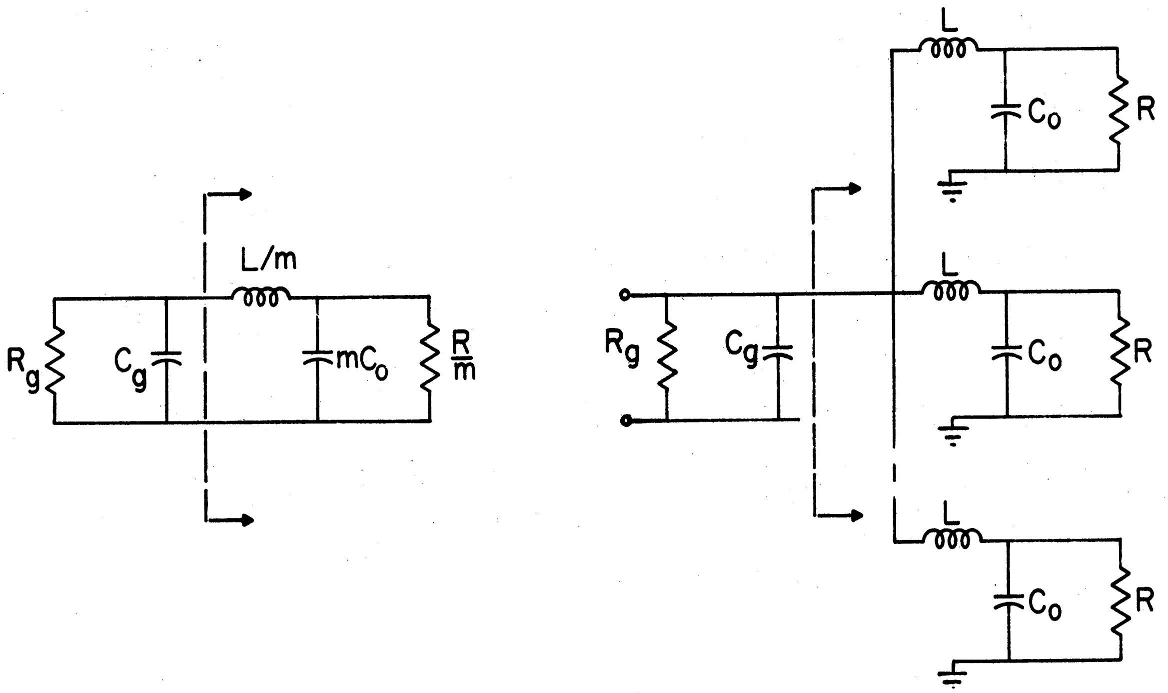


Fig. 13. Generation of a multi-terminal network providing equal transfer impedances to all channels.

in a straight forward way, a good set of networks providing equal signals at all channel inputs.

4.2 Multi-Terminal Coupling Networks with Partitioning

It is also possible to obtain structures which perform what will be referred to as partitioning; that is, providing different signals at several channel inputs. A simple example of such a principle can be found in complementary sets.[24] † These sets may be characterized as a group of networks which when tied together in a prescribed manner present a fixed resistive impedance R at all frequencies. The division of the power delivered from a source to such a set can be controlled. Three simple sets are given in Fig. 52, along with expressions describing the power division among the several terminations with frequency. The total power delivered

† These circuits are discussed further in Section 7.2.2.

to the lowpass and bandpass terminations in Fig. 52b is equal at all frequencies to the power delivered to the lowpass termination in Fig. 52a; for the networks shown this power is given by

$$\frac{P_{del}}{P_{avail}} = \frac{K}{1 + \omega^4} .$$

Thus the available power can be delivered efficiently (by using a source of internal impedance R when K = 1 above) over a given band (by suitable choice of crossover frequencies) while still performing efficiently the function of partitioning. The complementary sets shown in Fig. 52 have rather poor capacitance capability. Consider, for example, the minimum phase circuit for which

$$\frac{P_{del}}{P_{avail}} = \frac{1}{1 + \omega^4} \tag{48}$$

which is shown in Fig. 14. Here the terminal capacitance is $\sqrt{2}$ farads (one rad per sec bandwidth and one ohm reference impedance level) while the limiting terminal capacitances in Figs. 52a and 52b are $1/\sqrt{2}$ and $2/5\sqrt{2}$ respectively, at the same bandwidth and reference impedance level.

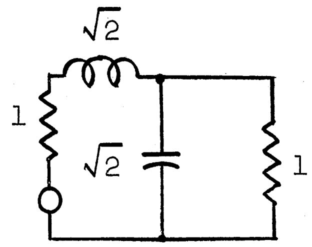


Fig. 14. A two-pole circuit with maximally flat transmission.

It is perhaps worth mentioning that the highpass structures, which can have no shunt capacitance, would presumably be "tacked on" as auxiliary networks if a complementary set were employed as a partitioned amplifier input or output network. Of course, nothing has been said about phase in this discussion; the complementary sets have been discussed here only to illustrate that partitioning can obviously be achieved.

One may question whether other partitioning systems do not exist which are superior to the complementary sets or the systems achieving identical channel input signals discussed above. This question can be answered in the affirmative. It will be convenient to consider the reflection coefficient ρ which is defined by

$$|\rho|^2 = 1 - |t|^2 \quad . \tag{49}$$

Let us arbitrarily choose a source with a one ohm internal impedance, when it is well-known that for the circuit of Fig. 15

$$|\rho|^2 = \left|\frac{Y'-1}{Y'+1}\right|^2 \quad . \tag{50}$$

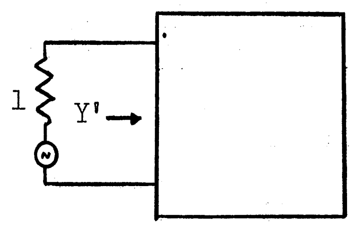


Fig. 15. Circuit for defining reflection coefficient.

It is then convenient to express ρ as a function of the complex frequency $p = \sigma + j\omega$. Further, since ρ is a rational function for lumped networks, polynomials m and n may be defined by

$$\rho(p)\,\rho(-p) \overset{d}{=} \frac{m(p)}{n(p)}\,\frac{m(-p)}{n(-p)} = \frac{Y'(p)-1}{Y'(p)+1}\,\frac{Y'(-p)-1}{Y'(-p)+1} \tag{51}$$

where

$$\frac{m(p)}{n(p)} \overset{d}{=} \frac{Y'(p)-1}{Y'(p)+1} \quad . \tag{52}$$

This leads to

$$Y'(p) = \frac{n(p) + m(p)}{n(p) - m(p)} \quad . \tag{53}$$

The use of the above procedure to obtain a Y' meeting certain restrictions on the character of ρ is well-known. It is reasonable to suppose that a number of networks of varying complexity exist which have the same character of $|\rho|^2$. Consider, for example, the function

$$\rho(p) = \frac{m(p)}{n(p)} \frac{a \pm pb}{a + pb} \stackrel{d}{=} \frac{Y(p)-1}{Y(p)+1} \tag{54}$$

where a and b are even polynomials and a + pb is defined to be Hurwitz.†
Note that the auxiliary rational function has a unit magnitude for $p = j\omega$ with either choice of the sign in its numerator. Choosing the minus sign results in

$$Y = \frac{a\ [n(p) + m(p)] + pb\,[n(p) - m(p)]}{pb\ [n(p) + m(p)] + a\,[n(p) - m(p)]} \stackrel{d}{=} \frac{N(p)}{D(p)} \quad . \tag{55}$$

It is observed that the bracketed terms are either the numerator or denominator of the function Y'(p) of Eq. 53. Further, for b = 0,

$$Y = Y' \qquad b = 0 \tag{56}$$

while for a = 0, one obtains the dual of Y'; i.e.,

$$Y = \frac{1}{Y'} \ . \qquad a = 0 \tag{57}$$

These admittances are known to give the required $|\rho|^2$

When the function Y of Eq. 55 is rationalized, the even part gives

$$\mathrm{Ev}\left\{Y(p)\right\}\Big|_{p^2=-\omega^2} = \frac{a^2 - p^2 b^2}{D(p)\,D(-p)}\Big|_{p^2=-\omega^2} = G(\omega^2) \quad . \tag{58}$$

† The poles of ρ must be in the left half plane, so that the choice of a Hurwitz denominator factor is necessary for realizability. The zeroes of ρ may lie anywhere, so that our choice is sufficient to insure realizability.

Thus, one has a measure of freedom in choosing the numerator of the $Ev\left\{Y(p)\right\}$, which becomes the numerator of $G(\omega^2)$ along the real frequency axis, independent of the function $\frac{m(p)}{n(p)}$ which determines the variation of $|\rho|^2$ with frequency. From the ρ of Eq. 54, and using

$$|\rho|^2 + |t|^2 = 1 \quad , \tag{59}$$

$$|t|^2 = \frac{P_{del}}{P_{avail}} = \frac{n(p)n(-p)-m(p)m(-p)}{n(p)\ n(-p)} \ \frac{a^2-p^2b^2}{a^2-p^2b^2}\Bigg|_{p^2=-\omega^2} \quad . \tag{60}$$

It thus becomes possible to control the division of the delivered power among a number of networks, while at the same time controlling the character of the individual networks <u>and</u> the quality of the match obtained. Some simple examples of this process will now be given.

The network of Fig. 14 has been described; with it one obtains a maximally flat approximation of $|t|^2$ to the rectangular lowpass characteristic;

$$|t|^2 = \frac{1}{1+\omega^4} \quad . \tag{61}$$

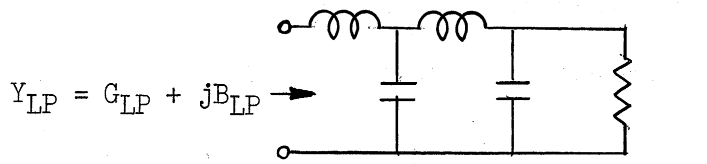
The midband loss ($\omega = 0$) for this network is zero, while 3 db loss occurs at one radian per sec. The admittance seen by the source is

$$Y' = \frac{\sqrt{2}p + 1}{2p^2 + \sqrt{2}p + 1} \tag{62}$$

from which is formed, using Eqs. 53 and 55,

$$Y = \frac{a\ [\sqrt{2}\ p+1] + pb\ [2p^2 + \sqrt{2}\ p+1]}{pb\ [\sqrt{2}\ p+1] + a\ [2p^2 + \sqrt{2}\ p+1]} \quad . \tag{63}$$

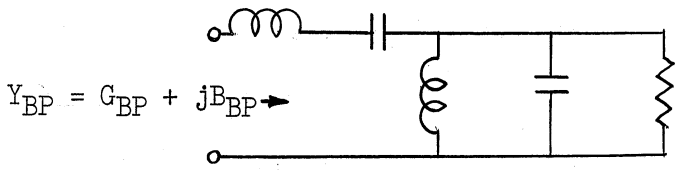
Let it be required that the above $|t|^2$ be obtained, and that the delivered power be divided between a lowpass ladder network and a bandpass ladder network so that a lowpass and a bandpass character of the individual transmission functions results. Now, for a lowpass ladder network such as in Fig. 16, the input conductance G_{LP} is in the form



$$G_{LP} = \frac{K'}{p^8 + \alpha_o p^6 + \beta_o p^4 + \gamma_o p^2 + \delta_o} \qquad (64)$$

Fig. 16. A lowpass ladder network.

while for the bandpass network



$$G_{BP} = \frac{K'p^4}{p^8 + \alpha' p^6 + \beta' p^4 + \gamma' p^2 + \delta'} \ . \qquad (65)$$

Fig. 17. A bandpass ladder network.

Consider then the choices for a and b in Eq. 63

$$a = p^2 + a_o^2 \qquad (66)$$

$$b = \sqrt{2}\, a_o \qquad (67)$$

when

$$a^2 - p^2b^2 = [p^4 + 2a_o^2\, p^2 + a_o^4] - p^2\, [2a_o^2] = p^4 + a_o^4 \ . \qquad (68)$$

For these choices of a and b, the total admittance of the two networks is

$$Y = \frac{(p^2 + a_o^2)\, [\sqrt{2}p+1] + \sqrt{2}\, a_o p\, [2p^2 + \sqrt{2}\, p+1]}{\sqrt{2}\, a_o p\, [\sqrt{2}p+1] + (p^2 + a_o^2)\, [2p^2 + \sqrt{2}p+1]} \ . \qquad (69)$$

The problem of finding the individual networks must now be undertaken.

Miyata[18] discussed the problem of finding the input admittance of a minimum phase network having a prescribed input conductance

$$G(\omega^2) = \frac{\lambda}{\lambda_1^2 + \omega^2\lambda_2^2} . \tag{70}$$

His technique involved obtaining, from the denominator of $G(\omega^2)$, the Hurwitz factor $\lambda_1 + p\lambda_2$. Use is then made of the fact that this polynomial determines two functions X_o and Y_o satisfying the relation

$$\lambda_1 X_o - p^2\lambda_2 Y_o = 1 . \tag{71}$$

The required admittance is then determined from the function

$$Q = \frac{(X_o+pY_o)\lambda}{\lambda_1+p\lambda_2} . \tag{72}$$

If the degree of the numerator and denominator differ by at most one, Q is the desired admittance. Otherwise, one removes an odd part of Q by dividing the polynomial of lower degree into the polynomial of higher degree. The process is repeated until a remainder is obtained with degrees of numerator and denominator differing by at most one. This remainder is then the required admittance.

Applying these techniques in the present problem, and using the Hurwitz denominator of Eq. 69, Eq. 71 becomes

$$[2p^4+p^2(2a_o^2+2a_o+1)+a_o^2](\alpha p^2+\beta) - \sqrt{2}(\delta p^2+\gamma)p^2[p^2+(a_o^2+a_o)] = 1 . \tag{73}$$

Here

$$X_o \stackrel{d}{=} \alpha p^2 + \beta ; \quad Y_o \stackrel{d}{=} \delta p^2 + \gamma . \tag{74}$$

Solving by equating coefficients

$$\alpha = \frac{1}{a_o^3} \qquad \beta = \frac{1}{a_o^2}$$

$$\delta = \frac{\sqrt{2}}{a_o^3} \qquad \gamma = \frac{1}{\sqrt{2a_o^2}}\,(2 + 1/a_o) \tag{75}$$

Now, for the lowpass network, we choose λ = constant, in the light of Eq. 64. The λ is chosen to obtain $G(0) = 1$ as required by Eq. 63. Substituting this λ and Eqs. 75 in Eq. 72,

$$Y_{LP} = \frac{a_o^4\left[\frac{\sqrt{2}}{a_o^3}\,p + \frac{1}{a_o^3}\,p^2 + \frac{1}{\sqrt{2}\,a_o^2}\,(2 + 1/a_o)\,p + \frac{1}{a_o^2}\right]}{2p^4 + \sqrt{2}p^3 + p^2\,(1 + 2a_o + 2a_o^2) + p\sqrt{2}\,a_o[1 + a_o] + a_o^2} \; . \tag{76}$$

The network of Fig. 18 is then easily obtained by a continued fraction expansion.

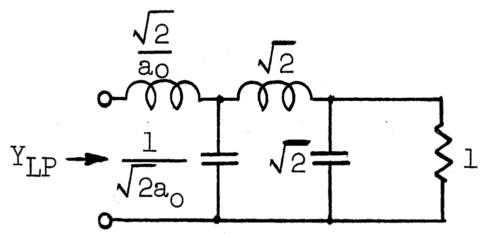


Fig. 18. The realization of Eq. 76.

The admittance of the bandpass structure Y_{BP} can be determined either by a reapplication of Miyata's technique using $\lambda = K'p^4$, or from the equation

$$Y_{BP} = Y - Y_{LP} \; . \tag{77}$$

Choosing Miyata's technique for illustration

$$Q_{BP} = \frac{K'p^4\left[\frac{\sqrt{2}}{a_o^3}\,p^3 + \frac{1}{a_o^3}\,p^2 + \frac{1}{\sqrt{2a_o^2}}\,(2 + 1/a_o) + \frac{1}{a_o^2}\right]}{2p^4 + \sqrt{2}p^3 + p^2\,(1 + 2\,a_o + 2a_o^2) + p\sqrt{2}\,a_o(1 + a_o) + a_o^2}$$

$$= \frac{K'}{\sqrt{2a_o^3}}\,p^3 - \frac{K'}{\sqrt{2a_o}}\,p + K'\,\frac{\sqrt{2}\,(1 + a_o)\,p^3 + (1 + a_o)\,p^2 + \frac{a_o}{\sqrt{2}}\,p}{2p^4 + \sqrt{2}p^3 + p^2(1+2a_o^2) + p\sqrt{2}\,a_o(1+a_o) + a_o^2} \; . \tag{78}$$

The K' is readily obtained from a comparison of the coefficients of a power of p common to the numerators of Eqs. 69 and 76 and the remainder of Q_{BP}. Using the coefficients of p

$$\sqrt{2}\, a_0 + K' \sqrt{2}\,(1 + a_0) = \sqrt{2} + 2\sqrt{2}\, a_0 \tag{79}$$

from which K' = 1. The bandpass network at the bottom of Fig. 19 results from a continued fraction expansion of the remainder of Q_{BP} with K' = 1. The required structure is then that indicated in Fig. 19. In Figs. 20a and 20b, an ideal transformer is inserted in the bandpass network and absorbed according to principles presented in Ref. 17.

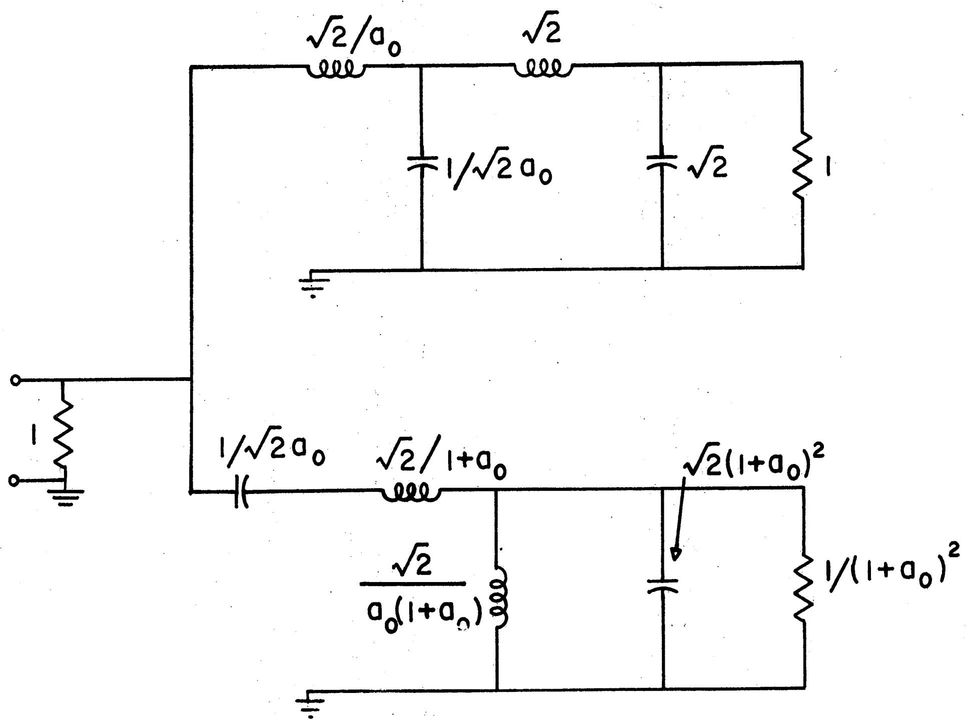
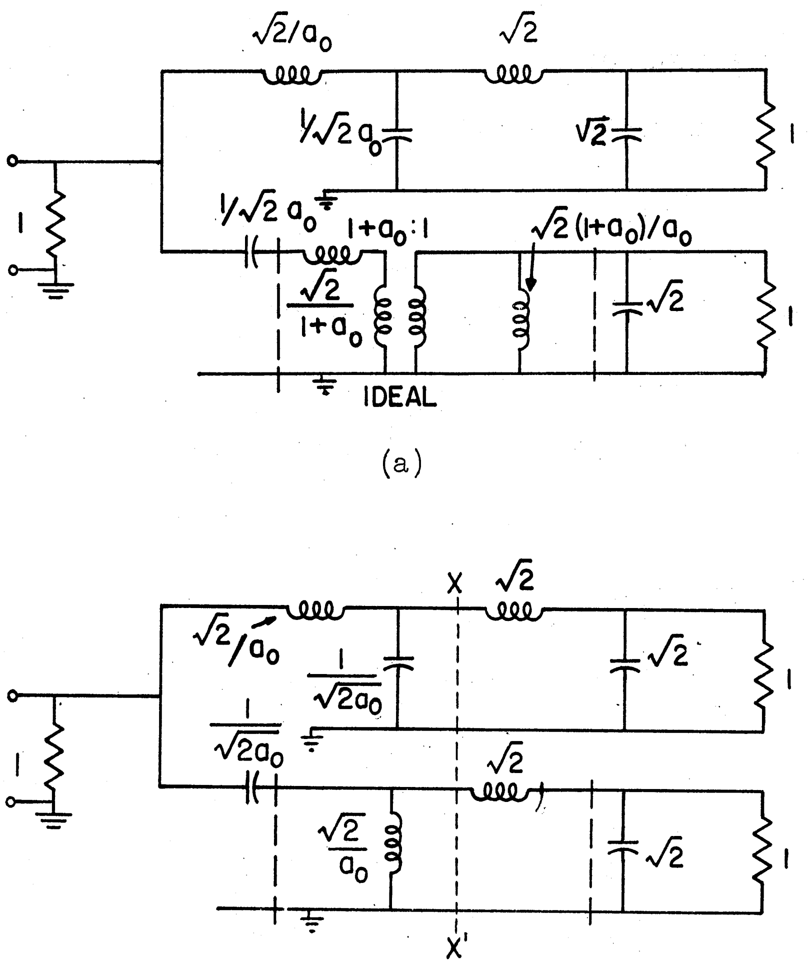


Fig. 19. A partitioning network.

The ideal transformer turns ratio chosen achieves the highest impedance level at the bandpass network termination which can be obtained without the use of coupled coils. The elements to the right of the line XX' in either network of Fig. 20b are identical with those of the original network of Fig. 14.



(b)

Fig. 20. Networks obtained by modification of Fig. 19.

It is now interesting to consider the character of the power transmission from the generator and the division of the delivered power between the individual terminations. The total power delivered by the generator was prescribed to be given by

$$\frac{P_{del}}{P_{avail}} = \frac{\omega^4 + a_o{}^4}{(\omega^4 + 1)\ (\omega^4 + a_o{}^4)} = \frac{1}{\omega^4 + 1} \tag{80}$$

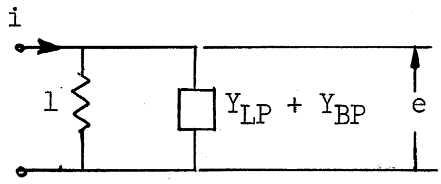
while the power delivered to the lowpass network termination $P_{del\ LP}$ and the power to the bandpass network termination $P_{del\ BP}$ are given by

$$\frac{P_{del\ LP}}{P_{avail}} = \frac{a_o{}^4}{(\omega^4 + 1)\ (\omega^4 + a_o{}^4)} \tag{81}$$

and

$$\frac{P_{del\ BP}}{P_{avail}} = \frac{\omega^4}{(\omega^4 + 1)\ (\omega^4 + a_o^4)} \quad . \tag{82}$$

To establish this, consider Fig. 21. One notes that



$$e = i \ \frac{1}{1 + Y_{LP} + Y_{BP}} \quad . \tag{83}$$

Fig. 21. A coupling circuit representation.

Then

$$P_{del\ LP} = |e|^2 \quad G_{LP} = \frac{G_{LP}\ |i|^2}{|1 + Y_{LP} + Y_{BP}|^2} \ , \tag{84}$$

$$P_{del\ BP} = |e|^2 \quad G_{BP} = \frac{G_{BP}\ |i|^2}{|1 + Y|^2} \quad . \tag{85}$$

The numerators of G_{LP} were prescribed to be a^4 and ω^4, while G_{LP}, G_{BP}, and $|1 + Y|^2$ were chosen to have the same denominator

$$D(p)\ D(-p)\big|_{p^2 = -\omega^2} \quad . \tag{86}$$

Substituting

$$Y = \frac{N}{D} \ , \tag{87}$$

$$\frac{P_{del\ LP}}{|i|^2/4} = \frac{P_{del\ LP}}{P_{avail}} = \frac{4a_o^{\ 4}}{|N + D|^2} = \frac{a_o^{\ 4}}{(\omega^4 + 1)\ (\omega^4 + a_o^4)} \tag{88}$$

and

$$\frac{P_{del\ BP}}{|i|^2/4} = \frac{P_{del\ BP}}{P_{avail}} = \frac{4\omega^4}{|N + D|^2} = \frac{\omega^4}{(\omega^4 + 1)\ (\omega^4 + a_o^4)} \ . \qquad (89)$$

Note that $P_{del\ LP}$ and $P_{del\ BP}$ per unit available power are equal at $\omega = a_o$, while $P_{del\ BP}$ per unit available power has a maximum at $\omega^2 = a_o$. The latter is readily established by setting

$$\frac{\partial \dfrac{(P_{del\ BP})}{P_{avail}}}{\partial \omega} = 0 \ . \qquad (90)$$

At $\omega^2 = a_o$

$$\left.\frac{P_{del\ BP}}{P_{avail}}\right|_{max} = \frac{a_o^2}{(a_o^2 + 1)\ (a_o^2 + a_o^4)} = \frac{1}{(1 + a_o^2)^2} \ . \qquad (91)$$

Summarizing the above results, the crossover frequency a_o can be varied while the total delivered power retains the prescribed form

$$\frac{1}{\omega^4 + 1} \ .$$

As a_o increases, the bandpass termination midband power (at $\omega^2 = a_o$) referred to unit available power decreases monotonely. For a crossover of 1/2 rad per sec, the bandpass midband power is 1.94 db below the generator available power. Of course, the lowpass midband power is equal to the generator available power for $a_o \neq 0$. For $a_o = 0$, the bandpass circuit degenerates into the lowpass structure of Fig. 14 while the response bandwidth of the lowpass circuit becomes zero.

It is worthwhile to depict the advantage which accrues from the use of the above technique, in terms of the capacitance capability and coupling network loss. The network of Fig. 14 could be used to obtain a coupling network producing equal voltages at two channel inputs, as in Fig. 22. The power to the individual terminations would be

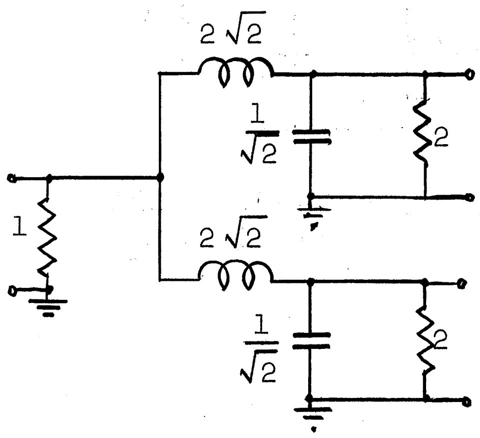


Fig. 22. Circuit providing equal transfer impedance to two channels and derived from Fig. 14.

$$\frac{P_{del}}{P_{avail}} = \frac{1/2}{1 + \omega^4} . \tag{92}$$

If this circuit were used to drive two equal bandwidth channels, the power to the lowpass channel input at $\omega = 0$ (midband) would be 3 db below the generator available power, while the bandpass midband termination power ($\omega^2 = 1/2$) would be 4 db below the generator available power. The channel capacitances in Fig. 22 are $1/\sqrt{2}$ farads per unit total bandwidth at a one ohm generator internal impedance level. By contrast, in the network of Fig. 20 with $a_o = 1/2$, losses of 0 db and 1.94 db are achieved with channel capacitances just twice as large. The same advantages accrue when the circuit of Fig. 20 is utilized as a partitioned amplifier output network, where of course the comparison is made for a one ohm common load.

Consider an alternative choice of a Y' corresponding to the structure

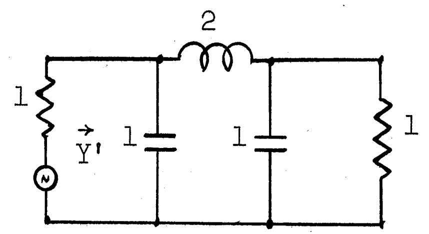


Fig. 23. A three-pole circuit with maximally flat transmission.

for which

$$|t|^2 = \frac{1}{1 + \omega^6} \tag{93}$$

and

$$Y' = \frac{2p^3 + 2p^2 + 2p + 1}{2p^2 + 2p + 1} \; . \tag{94}$$

From previous results, it is clear that this $|t|^2$ can be realized with networks having an input admittance

$$Y = \frac{a\ [2p^3 + 2p^2 + 2p + 1] + pb\ [2p^2 + 2p + 1]}{pb\ [2p^3 + 2p^2 + 2p + 1] + a\ [2p^2 + 2p + 1]} \; . \tag{95}$$

Again choosing

$$a = p^2 + a_o^2 \tag{96}$$

$$b = \sqrt{2}\, a_o \tag{97}$$

and by a method analogous to that utilized above, one can obtain the structure

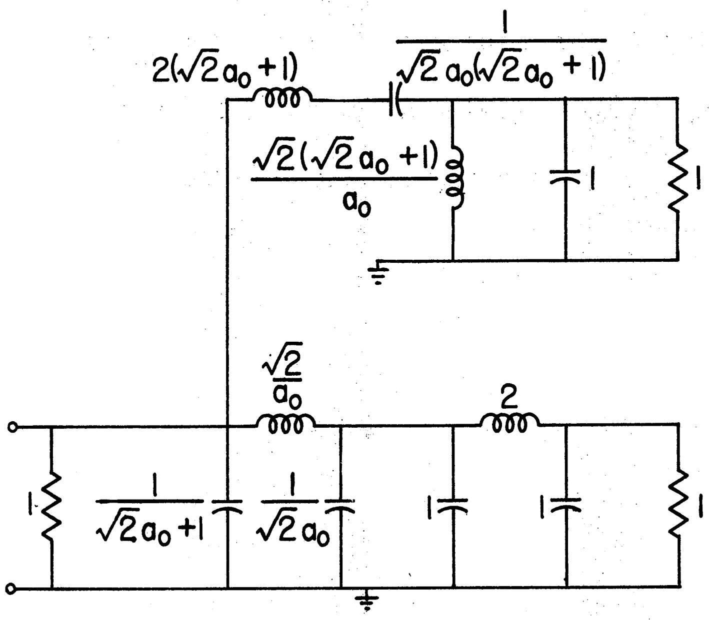


Fig. 24. A partitioning network derived from Fig. 23.

The power transmission from the generator and to the individual terminations are respectively

$$\frac{P_{del}}{P_{avail}} = \frac{1}{1+\omega^6} , \tag{98}$$

$$\frac{P_{del\ LP}}{P_{avail}} = \frac{a_o{}^4}{(\omega^6 + 1)\ (\omega^4 + a_o{}^4)} , \tag{99}$$

$$\frac{P_{del\ BP}}{P_{avail}} = \frac{\omega^4}{(\omega^6 + 1)\ (\omega^4 + a_o{}^4)} . \tag{100}$$

The crossover point again occurs at $\omega = a_o$ while the maximum power per unit generator available power delivered to the bandpass network termination occurs at a frequency ω_{max} determined by

$$-3\ \omega^{10}{}_{max} - \omega^6{}_{max}\ a_o{}^4 + 2a_o{}^4 = 0 . \tag{101}$$

Approximate solutions for ω_{max} are determined from the alternate form

$$a_o{}^4 = \frac{3\omega_{max}{}^{10}}{2 - \omega_{max}{}^6} \tag{102}$$

by plotting a_o for a range of choices of ω_{max} as in Fig. 25. For ω_{max} = $1/\sqrt{2}$, a_o^4 = 1/20, giving a crossover frequency of .472 radian per sec and a midband bandpass termination power 1.3 db below the generator available power.

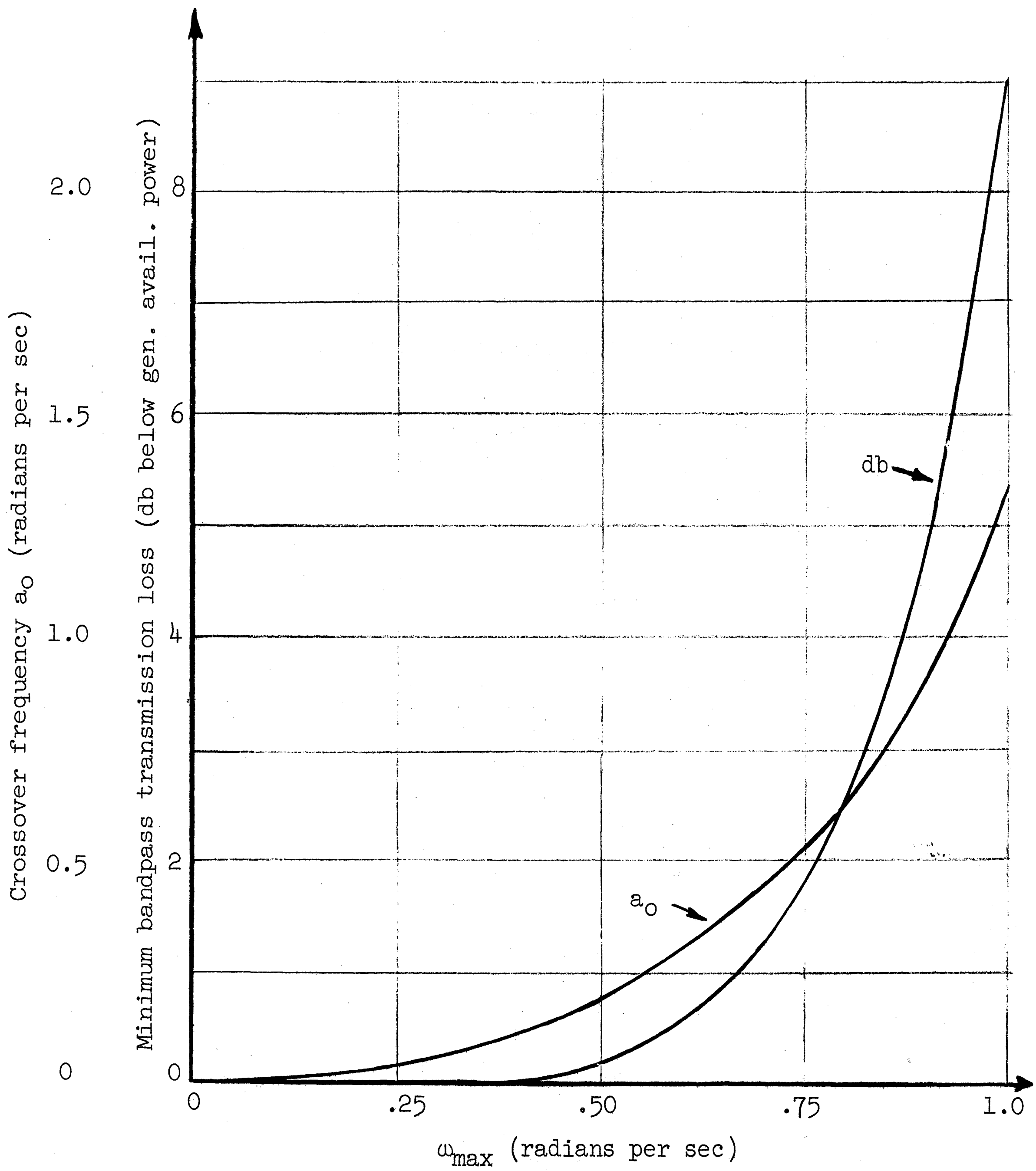


Fig. 25. Plot of crossover frequency a_o and minimum transmission loss to bandpass termination vs ω_{max} = frequency of minimum transmission loss to bandpass termination-circuit of Fig. 24.

In the above examples, the generator was matched at zero frequency. These circuits are desirable as partitioned amplifier input networks where substantial grid conductances are encountered, and for output networks where relatively high channel output conductances exist. In many practical cases, however, relatively low channel input and output conductances exist. The results of Chapter IV illustrate that, for example, with pentode channel output tubes, the problem is essentially to obtain maximum feasible load voltages per unit of current available at the channel output.† Consider, for application in these cases, a circuit which is derived by methods analogous to those used above, but based on the admittance Y' associated with Fig. 26, where unity source conductance has again been chosen.

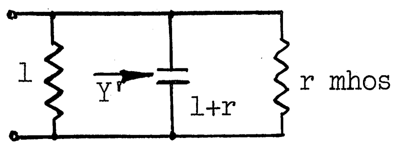


Fig. 26. A one-pole circuit with maximally flat transmission.

$$Y' = p(r + 1) + r \tag{103}$$

$$\frac{P_{del}}{P_{avail}} = \frac{K}{1 + \omega^2} \tag{104}$$

The parameter $K = \frac{4r}{(r + 1)^2}$ is a positive constant not greater than unity.

Taking

$$Y = \frac{a\ [p(r + 1) + r] + pb\ [1]}{pb\ [p(r + 1) + r] + a\ [1]}\ , \tag{105}$$

† This is true for sufficiently broad channels.

$$a = p^2 + a_o^2 \quad , \tag{106}$$

$$pb = \sqrt{2}\, a_o p \quad , \tag{107}$$

$$Y = \frac{(r + 1)p^3 + rp^2 + p\,[a_o^2\,(r + 1) + 2a_o] + a_o^2 r}{p^2\,[\sqrt{2}\,a_o\,(r + 1) + 1] + \sqrt{2}\,a_o rp + a_o^2} \quad , \tag{108}$$

the circuit of Fig. 27 is readily obtained.

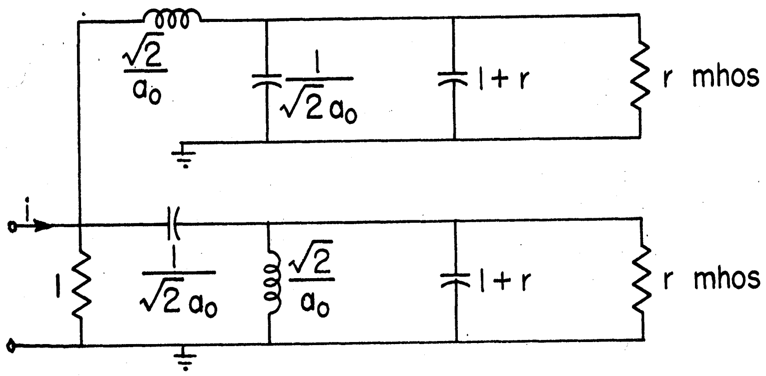


Fig. 27. A partitioning network derived from Fig. 26.

The power transmission from the source and to the individual terminations in Fig. 27 are given by:

$$\frac{P_{del}}{P_{avail}} = \frac{K}{1 + \omega^2} \quad , \tag{109}$$

$$\frac{P_{del\ LP}}{P_{avail}} = \frac{Ka_o^4}{(1 + \omega^2)\ (a_o^4 + \omega^4)} \quad , \tag{110}$$

$$\frac{P_{del\ BP}}{P_{avail}} = \frac{K\omega^4}{(1 + \omega^2)\ (a_o^4 + \omega^4)} \quad . \tag{111}$$

The product RC associated with the channel inputs at the prescribed nominal overall bandwidth of one radian per sec is $\frac{r + 1}{r}$, and may be made arbitrarily high at a cost in transmission efficiency. This result is comparable to that obtained in the circuit of Fig. 26.

By an analogous method, one may obtain the circuit of Fig. 28, of which the circuit of Fig. 20 is a special case ($K_0 = 0$). As in Fig. 20, an ideal transformer has been inserted and a realization obtained without coupled coils.

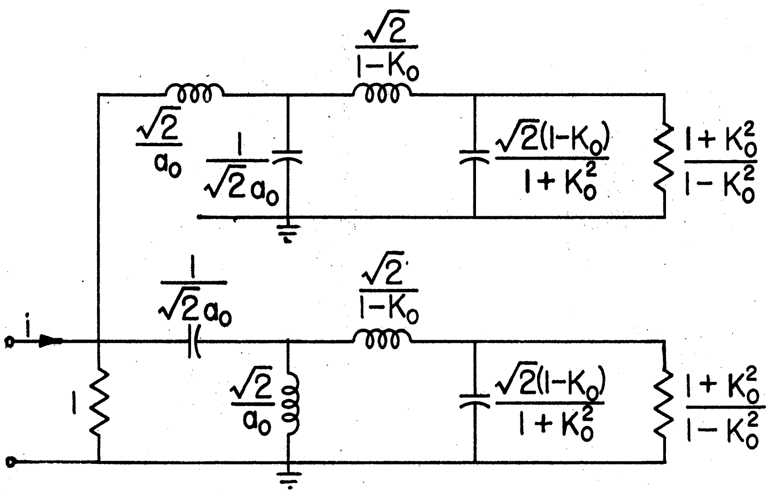


Fig. 28. A partitioning network.

For this circuit:

$$K_0^2 = \sqrt{1 - K^2} \, , \tag{112}$$

$$\frac{P_{del}}{P_{avail}} = \frac{K}{1 + \omega^4} \, , \tag{113}$$

$$\frac{P_{del\ LP}}{P_{avail}} = \frac{K a_0^4}{(1 + \omega^4)\ (a_0^4 + \omega^4)} \, , \tag{114}$$

$$\frac{P_{del\ BP}}{P_{avail}} = \frac{K\,\omega^4}{(1 + \omega^4)\ (a_0^4 + \omega^4)} \, . \tag{115}$$

The above examples establish that partitioning can be accomplished, and that in the particular examples considered it was advantageous to do so. What has been presented suggests several interesting areas for further consideration. However, it has not been possible to investigate in the course of this study a number of these areas which are therefore only mentioned here.

A number of problems which seem to the author to be suggested by the previous considerations are listed below.

1. The helping polynomial $a^2-p^2b^2$ was chosen in the form $p^4+a_o^4$ in the above examples. It is clear that, at the expense of increased network complexity, more rectangular transmission characteristics to the individual channels may be obtained, through the choice of, for example, $a^2-p^2b^2 = p^8+a_o^8$.
2. In the examples presented the generator match was arbitrarily prescribed to have a maximally flat character. The functions describing the transmission efficiency to the individual terminations are also relatively smooth. It is likely that through the use of Tchebycheff or other functions substantially superior results may be obtained.
3. It is possible to apply the above principles to generate coupling networks achieving partitioning among greater numbers of channels. It would seem that the work by Miyata together with the present results can readily be extended for this purpose. For example, the choices of helping polynomials $a^2-p^2b^2 = p^4+\alpha_o p^2+\beta_o$ and $a^2-p^2b^2 = p^8+\alpha_o p^4+\beta_o$ will lead to coupling networks potentially useful in three channel amplifiers.
4. The above coupling networks have transmission characteristics to the individual terminations differing only in their zeroes. It is particularly desirable, in the synthesis procedures to be discussed, to have simple transfer functions to the individual

terminations. It is also clearly desirable to obtain coupling networks with the minimum usable complexity. Improvement would seem to depend on obtaining transfer functions to the individual terminations with different poles. Note that this goal necessitates numerators and denominators of the functions $\frac{n_k}{N_k}$ of Section 3.2 with common factors.

5. The networks of Chapter IV may be thought of as achieving a distribution of substantially all the available power of the source among a number of networks with paralleled inputs. It appears feasible to achieve partitioning with circuits comparable to the artificial lines of a distributed amplifier. The relative merits of such circuits are uncertain.

6. Finally, there remains the task of illustrating the methods whereby the coupling networks of Chapter IV, or others subsequently chosen, may be incorporated into a partitioned amplifier. Since this problem is worthy of extensive consideration, and is the overlying objective of this paper, the remainder of the paper will be largely devoted to it.

CHAPTER V

THE DESIGN OF PARTITIONED AMPLIFIERS WITH A PRESCRIBED OVERALL GAIN FUNCTION

5.1 Introduction

The coupling networks of Chapter IV were prescribed in such a way as to insure a relatively uniform match over unity bandwidth between a source and a multi-terminal coupling network. In addition, it was possible to prescribe that what has been called partitioning was achieved efficiently; that is, a relatively high transfer impedance was maintained, for a given channel capacity and over a unique portion of the unit bandwidth, between the source (or load) and each of the individual terminal pairs of the multi-terminal coupling network associated with the channels. With such networks, it is possible to substantially reduce the loss associated with coupling from a common source or load to the channels of a partitioned amplifier, compared to the losses resulting with previously utilized circuits.† A principal objective in this chapter is to illustrate that such networks can be incorporated with a number of amplifier chains in such a way that the region of substantial gain of each channel is coincident with the region of substantial magnitude of the corresponding coupling network transfer impedance, while achieving a prescribed overall gain characteristic. The question of obtaining amplifiers with a maximum gain-bandwidth advantage will be considered in Chapter VI.

It is appropriate and convenient in the following examples to

† Linvill's coupling networks were discussed in Section 2.3.

consider the problem of an amplifier with a one radian per second bandwidth; it may then be converted to an optimum bandwidth amplifier for a given tube type by bandwidth and impedance level transformations. In a specific design problem, one is guided in the choice of the number of channels and the number of tubes per channel to be employed by the gain and bandwidth specifications, tube type, and interstage complexity to be used, as was discussed in Section 3.3.

In Section 3.2, a system of notation was defined which permitted the description of the overall gain of a partitioned amplifier in the form

$$\frac{e_{out}}{i} = \frac{e_a}{i}\frac{i_a}{e_a}\frac{e'_a}{i_a} + \frac{e_b}{i}\frac{i_b}{e_b}\frac{e'_b}{i_b} + \dots + \frac{e_m}{i}\frac{i_m}{e_m}\frac{e'_m}{i_m} \qquad (28)$$

It was assumed that multi-terminal coupling networks were present at both the input and output of the partitioned amplifier. The problem of the input coupling network could have been avoided, as was done in the previous treatment of the problem, by assuming that a voltage e_{in} was applied at all channel inputs. The function $\frac{e_{out}}{e_{in}}$ is then of interest, even though this voltage gain is not the transfer function from the Thevenin voltage of a practical source to the amplifier load. When $e_a = e_b = \dots = e_m = e_{in}$, Eq. 28 becomes

$$\frac{e_{out}}{e_{in}} = \frac{i_a}{e_{in}}\frac{e'_a}{i_a} + \frac{i_b}{e_{in}}\frac{e'_b}{i_b} + \dots + \frac{i_m}{e_{in}}\frac{e'_m}{i_{in}} \,. \qquad (116)$$

The functions $\frac{i_a}{e_{in}}, \frac{i_b}{e_{in}} \dots \frac{i_m}{e_{in}}$ are the gain characteristics of the channels, while the $\frac{e'_a}{i_a} \dots \frac{e'_b}{i_m}$ are determined by the output coupling network as before. An initial example will now be presented in which a prescribed gain function $\frac{e_{out}}{e_{in}}$ is realized; this will serve to illustrate a number of fundamental ideas.

5.2 Examples of the Synthesis of Partitioned Amplifiers with Prescribed Transfer Functions

5.2.1 An Example of the Synthesis of a Partitioned Amplifier with a Prescribed Voltage Gain When Driven by a Voltage Source.—Consider the problem of realizing the overall gain function

$$\left|\frac{e_{out}}{e_{in}}\right|^2 = \frac{K^2}{1 - p^6} \qquad (117)$$

with a two-channel partitioned amplifier. In particular, consider the choices of helping polynomials

$$P = (p^2 + \sqrt{2}\, a_o p + a_o^2)^2$$

and (118)

$$P^* = (p^2 + \sqrt{2}\, a_o p + a_o^2)\,(p^2 - \sqrt{2}\, a_o p + a_o^2) = (p^4 + a_o^4)$$

for which

$$\frac{e_{out}}{Ke_{in}} = \frac{P^*}{PB} = \frac{p^4 + a_o^4}{(p^2 + \sqrt{2}\, a_o p + a_o^2)^2\,(p + 1)\,(p^2 + p + 1)} \;. \qquad (119)$$

Here $B = (p + 1)\cdot(p^2 + p + 1)$ is the Hurwitz factor of the denominator of Eq. 117. This function $\frac{e_{out}}{e_{in}}$ may readily be shown to have the required magnitude, while the helping polynomials have been chosen in order to achieve certain objectives which will now be described. This can best be done by disassociating the function of Eq. 119 into two additive terms, each to be associated with the gain function from the common voltage source to load through a channel:

$$\frac{e_{out}}{e_{in}} = \frac{a_o^2}{(p^2 + p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} \frac{a_o^2}{(p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)}$$

$$+ \frac{p^2}{(p^2 + p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} \frac{p^2}{(p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)}$$

$$= \frac{i_a}{e_{in}} \frac{e'_a}{i_a} + \frac{i_b}{e_{in}} \frac{e'_b}{i_b} . \qquad (120)$$

Further, it is proposed that the individual transfer functions of Eq. 116 be identified as follows:

$$\frac{i_a}{e_{in}} = \frac{a_o^2}{(p^2 + p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} , \qquad (121)$$

$$\frac{e'_a}{i_a} = \frac{a_o^2}{(p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} , \qquad (122)$$

$$\frac{i_b}{e_{in}} = \frac{p^2}{(p^2 + p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} , \qquad (123)$$

and

$$\frac{e'_b}{i_b} = \frac{p^2}{(p + 1)(p^2 + \sqrt{2}\, a_o p + a_o^2)} . \qquad (124)$$

It will be observed in Eq. 120 that this particular choice of helping polynomials has enabled a disassociation of the overall function into two additive terms, one of which is a lowpass function while the other is a bandpass function. These are to be associated, respectively, with the transmission through a lowpass and a bandpass channel. In addition, the individual additive terms have certain merits in that each can be further disassociated into two multiplicative terms as in Eqs. 121 to 124 which also achieve certain objectives. Specifically, the functions $\frac{e'_a}{i_a}$

and $\frac{e'_b}{i_b}$ of Eqs. 122 and 124 are identical in form to the individual transfer functions of the multi-terminal network of Fig. 27. Finally, the regions of substantial channel transfer admittance clearly tend to be coincident with the regions of substantial magnitude of the associated individual transfer functions through the multi-terminal network. The magnitudes of the several transfer functions and their pole-zero plots are depicted in Fig. 29.

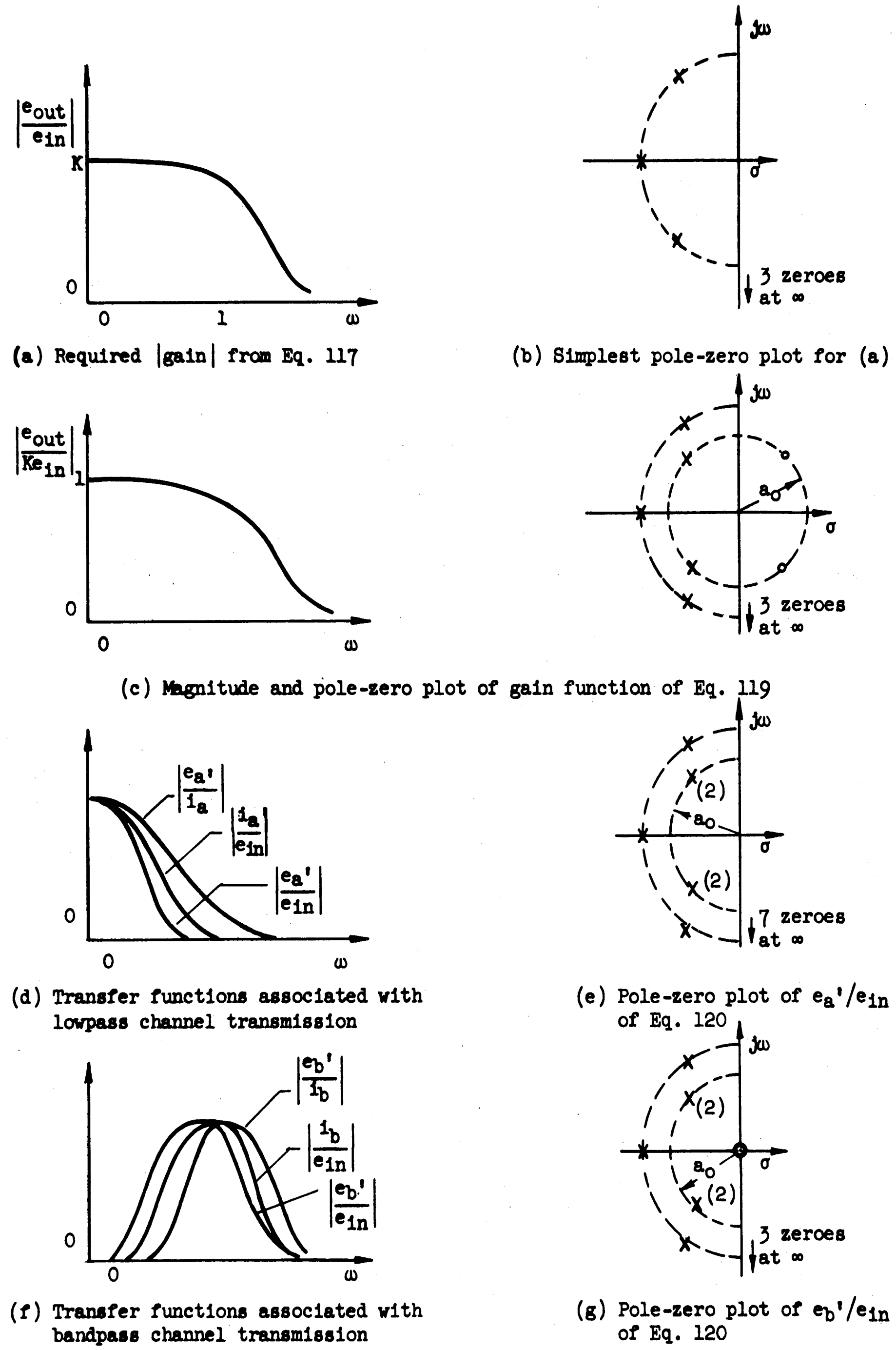


Fig. 29. Depictions of the magnitudes and pole-zero plots of the transfer functions of Eqs. 117, 119-124.

It is noted that the Eqs. 121 to 124 contain an as yet unspecified constant a_o, which is the crossover frequency of the individual transmission characteristics of the output network. This parameter can be chosen in such a way as to obtain a desirable capacitance distribution among the terminal pairs of the interstages. In order to illustrate this, it will be necessary to consider the actual realization of the channel transfer functions chosen above. The channel transfer functions could be realized in a number of ways; for the present example rather simple interstages will be selected.

The transfer function $\frac{i_b}{e_{in}}$ can be realized with a cascade of two simple-tuned interstages. The transfer function of such a circuit is

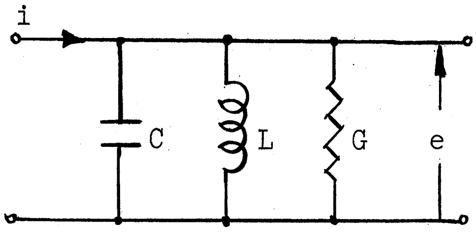
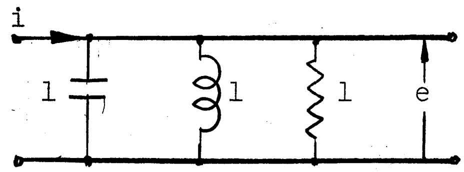


Fig. 30. The simple-tuned circuit.

$$\frac{e}{i} = \frac{p}{p^2C + pG + 1/L}. \tag{125}$$

Then $\frac{i_b}{e_{in}}$ can be disassociated into the following factors with the realizations indicated.

$$\frac{e}{i} = \frac{p}{p^2 + p + 1} \longrightarrow$$



$$\frac{e}{i} = \frac{p}{p^2 + \sqrt{2}\, a_o p + a_o^2} \longrightarrow$$

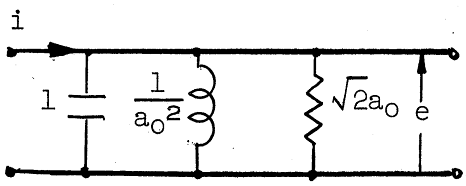


Fig. 31. Interstages in a realization of the $\frac{i_b}{e_{in}}$ of Eq. 123.

To realize the transfer function $\frac{i_a}{e_{in}}$, use is made of the network of Fig. 32. If $\frac{L}{R} = \frac{C_1}{G}$, the transfer impedance for this network is

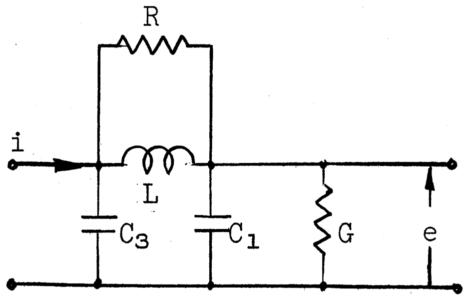


Fig. 32. A three-pole network.

$$\frac{e}{i} = \frac{1/G}{p^2 LC_3 + p\left(\frac{C_1 + C_3}{G}\right) + 1} \quad . \tag{126}$$

If the function $\frac{i_a}{e_{in}}$ is separated into the two factors below, the simultaneous equations indicated can be written down by comparison with Eq. 126.

$$\frac{1}{p^2 + p + 1} \quad \longrightarrow \quad \begin{aligned} \frac{L}{R} &= \frac{C_1}{G} \\ G &= 1 \\ LC_3 &= 1 \\ C_1 + C_3 &= 1 \end{aligned} \tag{127}$$

$$\frac{1}{\frac{1}{a_o^2} p^2 + \frac{\sqrt{2}}{a_o} p + 1} \quad \longrightarrow \quad \begin{aligned} \frac{L}{R} &= \frac{C_1}{G} \\ G &= 1 \\ LC_3 &= \frac{1}{a_o^2} \\ C_1 + C_3 &= \frac{\sqrt{2}}{a_o} \end{aligned} \tag{128}$$

Inspection of the simultaneous equations will disclose that a realizable network may be obtained in the first case with a total capacitance $C_1 + C_3$

of one farad distributed between the input and output terminal pairs as desired. In the second case, a total capacitance of $\frac{\sqrt{2}}{a_o}$ farads may be distributed between the terminal pairs as desired. Thus, in both cases it is possible to divide the total interstage capacitance between the terminal pairs in accordance with the input to output capacitance ratio of the tube utilized. It will also be noted that for $a_o = \sqrt{2}$ a total interstage terminal capacitance of one farad results at each interstage. The choice $C_3 = \frac{5}{6} C_1$ is desirable for the 6AK5 tube, which with $a_o = \sqrt{2}$ results in the element values of Fig. 33. The parameter r in Fig. 27 was chosen to be zero on the assumption of pentode output tubes, in selecting the output circuit illustrated below. The impedance level of the output circuit was then chosen to meet the requirement of tube output capacities of $\frac{5}{11}$ f.

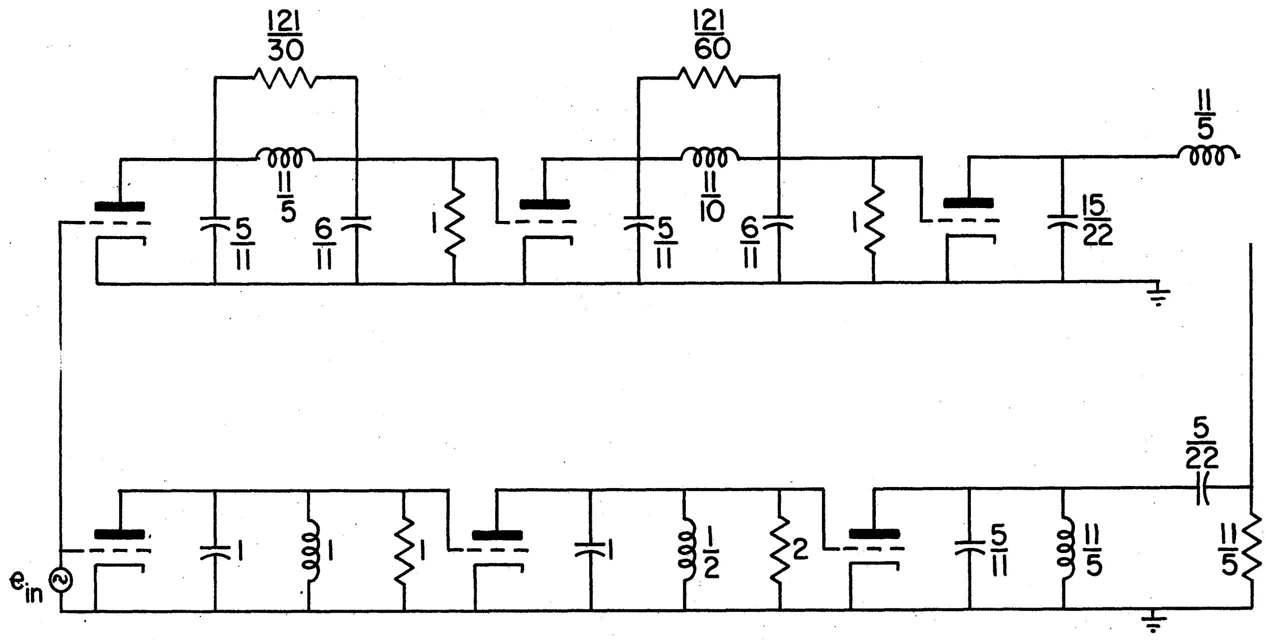


Fig. 33. A realization of the gain function of Eq. 120 with $a_o = \sqrt{2}$.

It will be noted that a unit nominal gain[†] can be achieved with the amplifier of Fig. 33 over a unit bandwidth with a total interstage terminal capacitance of one farad and $g_m = 1$.

Reference to Eqs. 7 and 8 of Section 2.1 will substantiate that with a cascade of six simple RC stages with $\omega_{3db} = 1$, $C = 1$, and $g_m = 1$, a midband gain of .00156 results, while with a cascade of six shunt-peaked stages with maximal flat characteristics, a midband gain of approximately .775 results.[‡] Clearly, for the tube constants chosen, the partitioned amplifier is substantially superior. In the light of the results of Section 3.4, it would be expected that if the bandwidth were decreased at a constant g_m and interstage capacitance, the single channel circuits would eventually become superior in gain capability to the two-channel circuit. That this effect will be observed for the circuits compared above is obvious.[‡‡]

5.2.2 Examples of the Synthesis of Partitioned Amplifiers with Prescribed Transfer Characteristics Between a Resistive Source and Load.— A second design problem will now be considered which will lead to an amplifier in which the poles of the transfer functions from source to load through the individual channels are again common. However, it will be

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† The gain of the amplifier is stated here for channel output capacities equal to the total interstage terminal capacitance. This convention facilitates a direct comparison of the amplification with gains given by the equations of Section 2.1, where the same assumption was made. The actual gain may differ from this gain by a factor which depends on the input to output capacitance ratio of the tube employed.

‡ The amplifier employing shunt-peaking is markedly superior to the amplifier utilizing simple RC stages due to its having a gain bandwidth advantage of approximately 1.5, while also possessing a substantially superior bandwidth shrinkage factor.

‡‡ A further consideration of the example of Section 5.2.1 will be presented in Section 5.3.2.

assumed for this example that the gain is to be prescribed between a practical source and the amplifier load.

Let it be prescribed that the overall response of a two-channel amplifier is to be in the form

$$\left|\frac{e_{out}}{i}\right|^2 = \frac{K^2}{1 + \omega^{12}} . \tag{129}$$

One considers then the function

$$\frac{e_{out}}{Ki} = \frac{P^*}{P(p^2 + \sqrt{2 + \sqrt{3}}\, p + 1)(p^2 + \sqrt{2 - \sqrt{3}}\, p + 1)(p^2 + \sqrt{2}\, p + 1)} . \tag{130}$$

It is again convenient to choose

$$P = (p^2 + \sqrt{2}\, ap + a^2)(p^2 + \sqrt{2}\, ap + a^2 ,$$

$$P^* = (p^2 + \sqrt{2}\, ap + a^2)(p^2 - \sqrt{2}\, ap + a^2) = p^4 + a^4 . \tag{118}$$

The resulting function can be broken into the additive terms shown below.

$$\frac{e_{out}}{Ki} = \frac{1}{(p^2+\sqrt{2+\sqrt{3}}p+1)} \frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2-\sqrt{3}}p+1)} \frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)}$$

$$+ \frac{1}{(p^2+\sqrt{2+\sqrt{3}}\,p+1)} \frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2-\sqrt{3}}p+1)} \frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)}$$

$$= \frac{e_a}{i} \frac{i_a}{e_a} \frac{e'_a}{i_a} + \frac{e_b}{i} \frac{i_b}{e_b} \frac{e'_b}{i_b} . \tag{131}$$

These terms have been disassociated into multiplicative factors which can be associated with the transfer functions previously defined as indicated.†
As before, the factors associated with the gain through one channel are lowpass, while the factor associated with the other channel is bandpass.

† The choice of the quadratic factor for the denominator of $e_a/i = e_b/i$ appears arbitrary at this juncture. The choice is related to the problem of adjusting the relative channel gains with given channel tube complements, as discussed in Section 5.3.

The channel transfer functions $\frac{i_a}{e_b}$ and $\frac{i_b}{e_b}$ are similar in form to those discussed in the previous example. Further, the indicated functions $\frac{e'_a}{i_a}$ and $\frac{e'_b}{i_b}$ can be realized with the multi-terminal coupling network of Fig. 28.

The functions $\frac{e_a}{i}$ and $\frac{e_b}{i}$ have been chosen identical, so that the input networks develop equal signals at the channel inputs. The particular function chosen could be realized with input coupling networks such as in either Figs. 34a or 34b, the choice presumably depending on whether significant source capacitance were present.

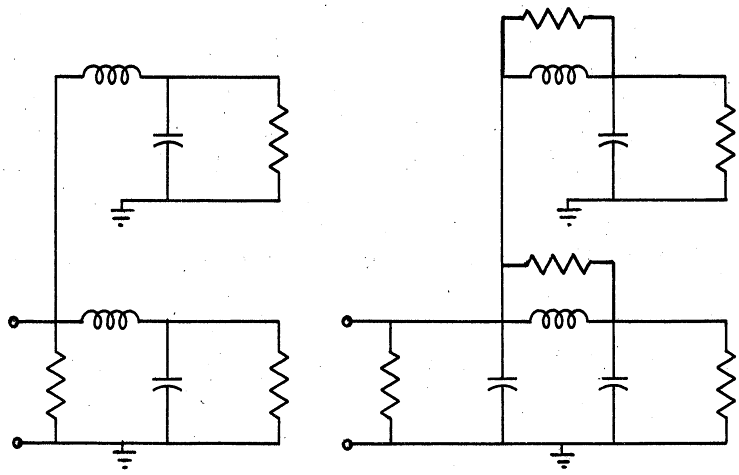


Fig. 34. Feasible input coupling network configurations for an amplifier based on Eq. 131.

If one chooses as the prescribed overall gain characteristic

$$\left|\frac{e_{out}}{i}\right|^2 = \frac{K^2}{(1 + \omega^{12})^2} \qquad (132)$$

one can choose

$$P* = p^{12} + a^{12} \qquad (133)$$

$$P = (p^2 + \sqrt{2}\,ap + a^2)^2\,(p^2 + \sqrt{2 + \sqrt{3}}\,ap + a^2)^2\,(p^2 + \sqrt{2 - \sqrt{3}}\,ap + a^2)^2 \;. \qquad (134)$$

This permits the disassociation

$$\frac{e_{out}}{Ki} =$$

$$\frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)}\,\frac{a^8}{(p^2+\sqrt{2+\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2-\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2+\sqrt{3}}p+1)^2(p^2+\sqrt{2-\sqrt{3}}p+1)^2}\,\frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)} +$$

$$\frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)}\,\frac{p^8}{(p^2+\sqrt{2+\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2-\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2+\sqrt{3}}p+1)^2(p^2+\sqrt{2-\sqrt{3}}p+1)^2}\,\frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p^2+\sqrt{2}p+1)} \tag{135}$$

This function enables the use of the partitioning networks of Fig. 28 at both the input and output of the partitioned amplifier.

Again, it may occur to the reader that the prescribed gain function

$$\left|\frac{e_{out}}{i}\right|^2 = \frac{K^2}{(1 + \omega^{10})^2} \tag{136}$$

will, for the same choice of P* and P as in the previous example, lead to the function

$$\frac{e_{out}}{Ki} =$$

$$\frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p+1)}\,\frac{a^8}{(p^2+\sqrt{2+\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2-\sqrt{3}}ap+a^2)^2(p^2+\frac{\sqrt{5}+1}{2}p+1)^2(p^2+\frac{\sqrt{5}-1}{2}p+1)^2}\,\frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p+1)} +$$

$$\frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p+1)}\,\frac{p^8}{(p^2+\sqrt{2+\sqrt{3}}ap+a^2)^2(p^2+\sqrt{2-\sqrt{3}}ap+a^2)^2(p^2+\frac{\sqrt{5}+1}{2}p+1)^2(p^2+\frac{\sqrt{5}-1}{2}p+1)^2}\,\frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p+1)} \tag{137}$$

having a realization containing the networks of Fig. 27 at both the input and output of the amplifier.

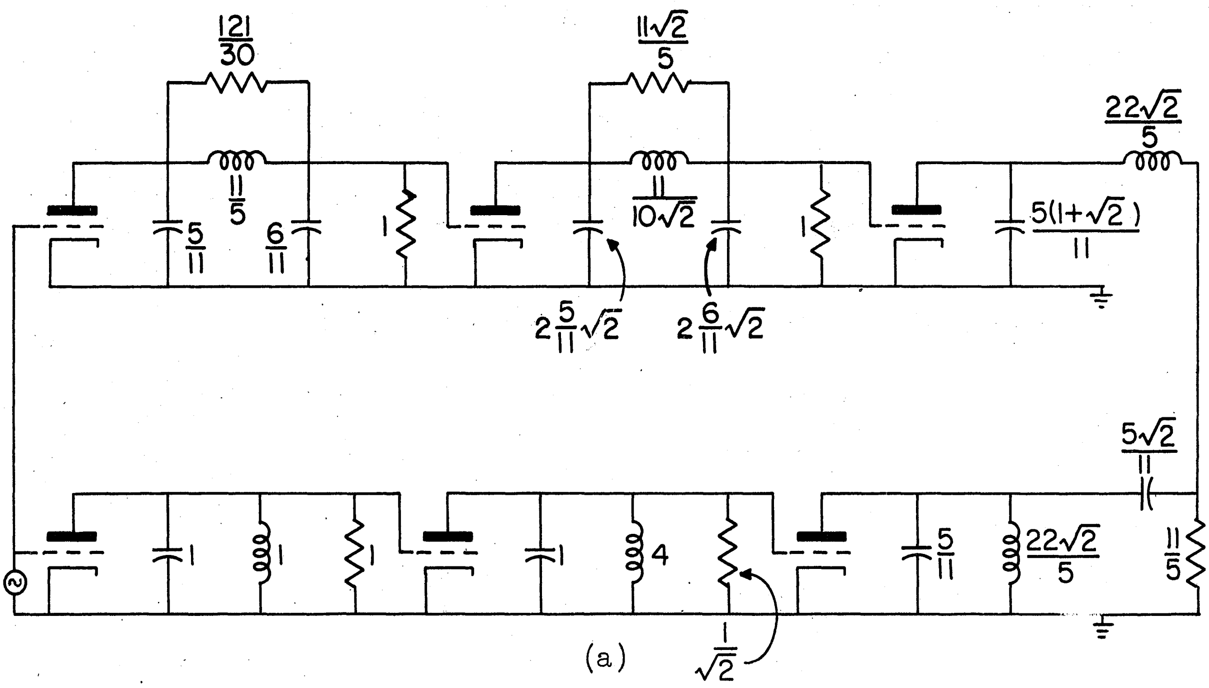
5.3 Some General Considerations Arising from the Examples of Sections 5.1 and 5.2

5.3.1 Factors Influencing the Choice of Coupling Networks.—It has been shown in the above examples that a number of possible input and output coupling arrangements can be provided for in designing a partitioned amplifier. The relative merits of the various arrangements are of interest. If the transfer function from a practical source to an amplifier load is to be prescribed, the design technique based on Eq. 117 is not applicable.† Amplifiers utilizing partitioning networks at both input and output have a higher ultimate gain capability than amplifiers without two partitioning networks. However, an advantage of a coupling arrangement providing equal transfer impedances to all the channels is simplicity, both of the circuit and the design problem.‡ It should be noted that the input and output coupling arrangements can be interchanged without affecting any of the various transfer functions from source to load. Thus, the loss associated with the use of one inferior coupling arrangement can be sustained at either the input or output of the amplifier, as desired. In a power amplifier where efficiency is paramount, such a loss could be sustained in a low level input circuit at negligible cost in efficiency. In a low level circuit, on the other hand, a coupling network with partitioning would be desirable at the input in the interests of sensitivity, while the loss resulting from the use of an inferior output coupling arrangement might be relatively unimportant.

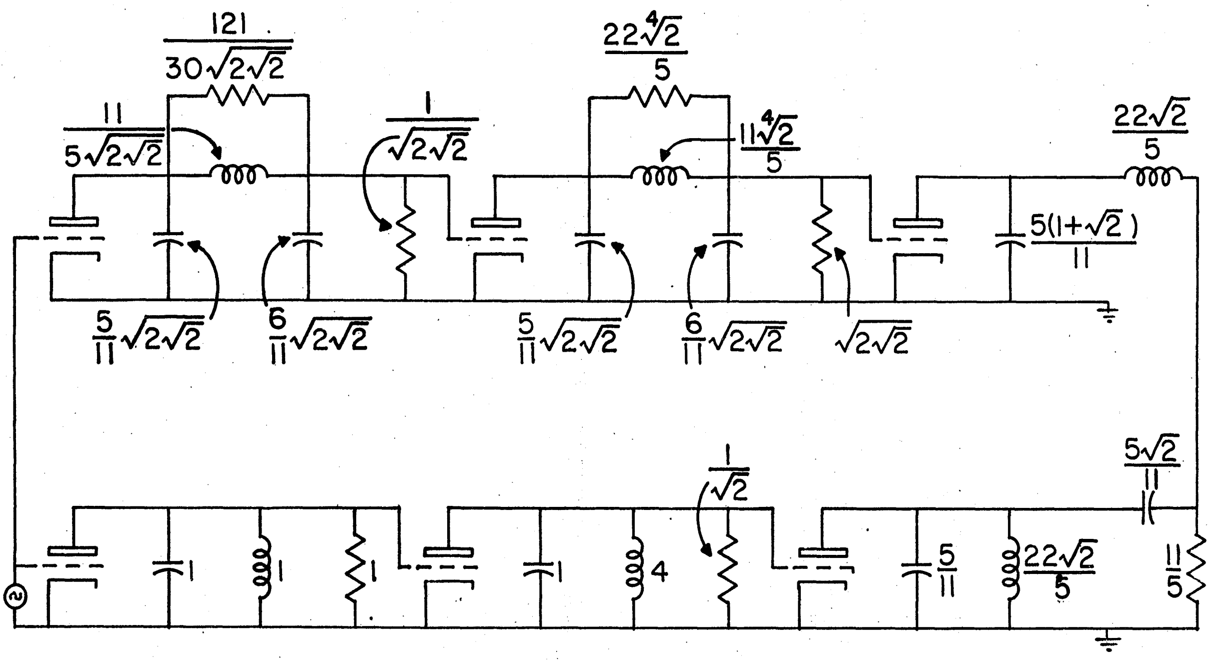
† A broadband (input) coupling network can be used which realizes an essentially uniform magnitude of transfer impedance to all channels over the amplifier passband. With such a network the transfer function from a resistive source to the amplifier load can be made to approximate as closely as desired a prescribed function by control of e_{out}/e_{in}. This is discussed further in Section 6.3.

‡ Note that if, for example, $e_a/i = e_b/i = e_m/i = 1/(ap+b)$, it is possible to simply tie all the channel inputs together and drive the amplifier with a source having the proper internal resistance. This would be extremely easy to construct provided the inductance necessary to join the inputs did not become appreciable. In this event a circuit such as Fig. 34a would be desirable.

5.3.2 The Conflict Between Relative Channel Gain Adjustment and Prescribed Channel Tube Complements.—In obtaining realizations of any of the overall transfer functions considered above, as well as in subsequent examples, a problem is generally encountered. The problem is how to obtain a distribution of capacitance at the various terminal pairs of the amplifier consistent with the tubes chosen, while maintaining the relative gains of the channels as required to realize a prescribed overall response. It will be remembered that an equalization of the capacitance capability of the several interstages was achieved in the amplifier of Fig. 33 by choosing $a_o = \sqrt{2}$. Consider the problem of obtaining an amplifier based on Eq. 120 and with equal bandwidth channels ($a_o = 1/2$). In this particular case, equal bandwidth channels can be obtained while still utilizing the tubes chosen at full capability by using tubes with a lower figure of merit in the lowpass channel. To illustrate this, take $a_o = 1/2$ in Eq. 120 and assume $C_3 = 5/6\ C_1$ for the lowpass channel interstages in the form of Fig. 32. The circuit of Fig. 35a results. The relative channel gains are correct if all transconductances are equal. In Fig. 35b, the impedance level of the first interstage of the lowpass channel of Fig. 35a has been lowered by a factor $\sqrt{2\sqrt{2}}$, while the second lowpass channel interstage impedance level has been raised by the factor $\sqrt{2\sqrt{2}}$ compared to Fig. 35a. In this way the capacitance capabilities of the lowpass channel interstages have been equalized while maintaining the required channel transfer admittance. This particular choice is desirable if the tubes of the lowpass channel are to be identical. Note that for uniform transconductances, tubes with a mean figure of merit of $\frac{1}{\sqrt{2\sqrt{2}}}$ = 59.5% of those in the bandpass channel may be utilized in the lowpass channel. At a fixed tube capacitance, the mean figure of merit of the lowpass



(a)



(b)

Fig. 35. An alternate realization of the transfer function of Eq. 120.

channel tubes may be $\frac{1}{\sqrt[3]{2\sqrt{2}}}$ = 70.7% of those in the bandpass channel. Continuous intermediate compromise is possible.† The circuit of Fig. 35b is well suited to the utilization of 6AU6's in the lowpass channel and 6AK5's in the bandpass channel.

Unity midband nominal gain is achieved with the circuit of Fig. 35 with a 3 db bandwidth of one radian per sec. The circuit employs three tubes with $\frac{g_m}{C} = \frac{1}{\sqrt{2\sqrt{2}}}$ and three tubes with $\frac{g_m}{C} = 1$. It is interesting to compare this gain with the gain which can be achieved using the same tubes in an exact flat-staggered cascade of simple stages. Reference to Eq. 5 will show that a midband gain of .21 results. Unity gain can be achieved with the cascade over a bandwidth of $\frac{1}{\sqrt[4]{2\sqrt{2}}} \simeq .771$ radians per sec. Further, the circuit of Fig. 35 has greater gain than can be achieved with a cascade of simple stages employing the same tubes for bandwidths greater than .595 radians per sec.‡ The difficulty in building bandpass circuits achieving extreme fractional bandwidths are well known. The circuit of Fig. 35 is in fact a staggered bandpass channel which need not achieve great fractional bandwidths since it is augmented by a lowpass channel.

5.4 Examples of the Synthesis of Partitioned Amplifiers with Prescribed Responses and Channel Transfer Functions Differing in Both Their Poles and Zeroes

In the examples of Section 5.2 the amplifier channels have had common transfer poles. Superior circuits should result if the transfer poles of

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† Other input to output capacitance ratios for the lowpass channel tubes than the one illustrated can be accommodated, as discussed in connection with Eqs. 127.

‡ At this bandwidth the gain, at a load impedance corresponding to 5/11 farad bandpass channel output capacity, is 20.4 db. Identical transconductances were assumed for all tubes.

the individual channels can be positioned in the interests of obtaining high individual channel gains. In Section 3.3, an approach to this problem was discussed. In order to illustrate the principle, let it be prescribed that the overall response of a two-channel partitioned amplifier be in the form

$$\left|\frac{e_{out}}{i}\right|^2 = \frac{K^2}{1 + \omega^{12}} \tag{138}$$

leading as before to the function

$$\frac{e_{out}}{Ki} = \frac{P^*}{P(p^2 + \sqrt{2-\sqrt{3}}\,p+1)(p^2 + \sqrt{2}\,p+1)(p^2 + \sqrt{2+\sqrt{3}}\,p+1)} \;. \tag{139}$$

Consider then choosing

$$P^* = p^4\,(p^2 + \sqrt{2+\sqrt{3}}\,p+1) + (\alpha p + k)\,(p^2 + \sqrt{2-\sqrt{3}}\,p+1). \tag{140}$$

Subsequently a Hurwitz polynomial P will be formed with the same magnitude as P*. The overall function can be disassociated into two additive terms, one being a bandpass and the other essentially a lowpass function. Neither of the factors $p^2 + \sqrt{2+\sqrt{3}}\,p+1$ and $p^2 + \sqrt{2-\sqrt{3}}\,p+1$ are common to the channel functions. In the interests of high gain, the "high frequency" conjugate pair of poles corresponding to the factor $p^2 + \sqrt{2-\sqrt{3}}\,p+1$ is to be retained in the bandpass channel function, while the "low frequency" pole pair corresponding to the factor $p^2 + \sqrt{2+\sqrt{3}}\,p+1$ is to be retained in the lowpass channel function. The factor $p^2 + \sqrt{2}\,p+1$ is common to the transfer functions from source to load through the two channels and may be associated with one of the partitioning networks of Chapter IV. A number of disassociations of the resulting overall function are possible. To illustrate this, let us assume that P* contains a factor $p^2 \pm \sqrt{2}\,ap + a^2$. Since P* is sixth degree, it may then be assumed that P is in the form

$$P = (p^2 + \sqrt{2}\, ap + a^2)\, P_1 P_2 \tag{141}$$

where a is positive and P_1 and P_2 are quadratic factors. Two possible disassociations of the overall function are given below.

$$\frac{e_{out}}{Ki} = \frac{1}{P_1} \frac{\frac{1}{a^2}(\alpha p+k)}{(p^2 + \sqrt{2+\sqrt{3}}\, p+1)P_2} \frac{a^2}{(p^2+\sqrt{2}\, ap+a^2)(p^2+\sqrt{2}\, p+1)}$$

$$+ \frac{1}{P_1} \frac{p^2}{(p^2 + \sqrt{2-\sqrt{3}}\, p+1)P_2} \frac{p^2}{(p^2+\sqrt{2}\, ap+a^2)(p^2+\sqrt{2}\, p+1)} \; . \tag{142}$$

This would lead to a realization with an input coupling network providing equal signals to all channels, and an output coupling network of the form of Fig. 28. The lowpass channel might be realized with a shunt-peaked stage and a stage employing a circuit such as in Fig. 32, if α and k have suitable values. The bandpass channel might be realized with a pair of stages employing simple-tuned interstages.

Alternatively, one might seek a realization based on the disassociation

$$\frac{e_{out}}{Ki} = \frac{1}{(p^2+\sqrt{2}p+1)(p+\alpha_1)} \frac{\frac{\alpha}{a^2}(p+\alpha_1)(p+\beta_1)\left(p+\frac{k}{\alpha}\right)}{P_1P_2(p^2+\sqrt{2+\sqrt{3}}\, p+1)} \frac{a^2}{(p^2+\sqrt{2}ap+a^2)(p+\beta_1)}$$

$$+ \frac{p^2}{(p^2+\sqrt{2}p+1)(p+\alpha_1)} \frac{(p+\alpha_1)\,(p+\beta_1)}{P_1P_2(p^2+\sqrt{2-\sqrt{3}}\, p+1)} \frac{p^2}{(p^2+\sqrt{2}ap+a^2)(p+\beta_1)} \; . \tag{143}$$

Partitioning networks in the form of Fig. 27 would be used at both the input and output of the amplifier. In Eq. 143 helping polynomials have been employed in the functions associated with transfer through the individual channels to enable the use of the partitioning networks of Fig. 27.

In order to achieve the prescribed magnitude while still obtaining realizable transfer functions, it is necessary that the auxiliary numerator and denominator factors be identical and Hurwitz. Whether this is a sufficient condition presumably depends on the realization sought. This is suggested by the following example. Let us suppose that one wished to realize the channel transfer function of Eq. 143

$$\frac{i_a}{e_a} = \frac{\frac{\alpha}{a^2}\,(p+\alpha_1)\,(p+\beta_1)\left(p+\frac{k}{\alpha}\right)}{P_1P_2\,\left(p^2+\sqrt{2+\sqrt{3}}\,p+1\right)} \qquad (144)$$

as a cascade of three shunt-peaked circuits. Note that the network of Fig. 36 has the transfer function indicated in Eq. 145. For the special case G = 0, the transfer function is given by Eq. 146.

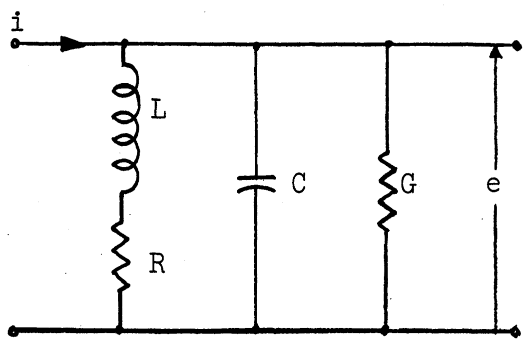


Fig. 36. A shunt-peaked circuit.

$$\frac{e}{i} = \frac{p + R/L}{p^2C + p\left(G + \frac{RC}{L}\right) + \left(\frac{1+RG}{L}\right)} \qquad (145)$$

$$\frac{e}{i} = \frac{p + R/L}{pC\left(p + \frac{R}{L}\right) + \frac{1}{L}} \;. \qquad G = 0 \qquad (146)$$

Inspection of these functions reveals that as G approaches zero, the magnitude of the real part of the pole position approaches one-half that of the zero position. For $G > 0$, the magnitude of the real part of the pole position is greater than half that of the zero position. Then, the function

$$\frac{e}{i} = \frac{r_1 p + s_1}{p^2 + r_2 p + s_2} \tag{147}$$

may be realized with a circuit of the type of Fig. 36 only for $\frac{s_1}{r_1} \leq r_2$. The restrictions on the relative positions of the poles and zeroes would presumably be different if a realization were sought containing a more complex interstage, such as one for which

$$\frac{e}{i} = \frac{(p+\alpha_1)\ (p+\beta_1)}{P_1 P_2} \quad . \tag{148}$$

It will be noted that the capacitance C of Fig. 36 is determined by the ratio of the coefficients of the highest powers of p in the numerator and denominator in the Eqs. 145 and 146. Thus, for the transfer function of Eq. 147, an interstage capacitance of $\frac{1}{r_1}$ farads is indicated. Similarly, the interstage capacitance of the realizations of the factors below, in the forms indicated, is easily determined by referring to Eqs. 125 and 126.

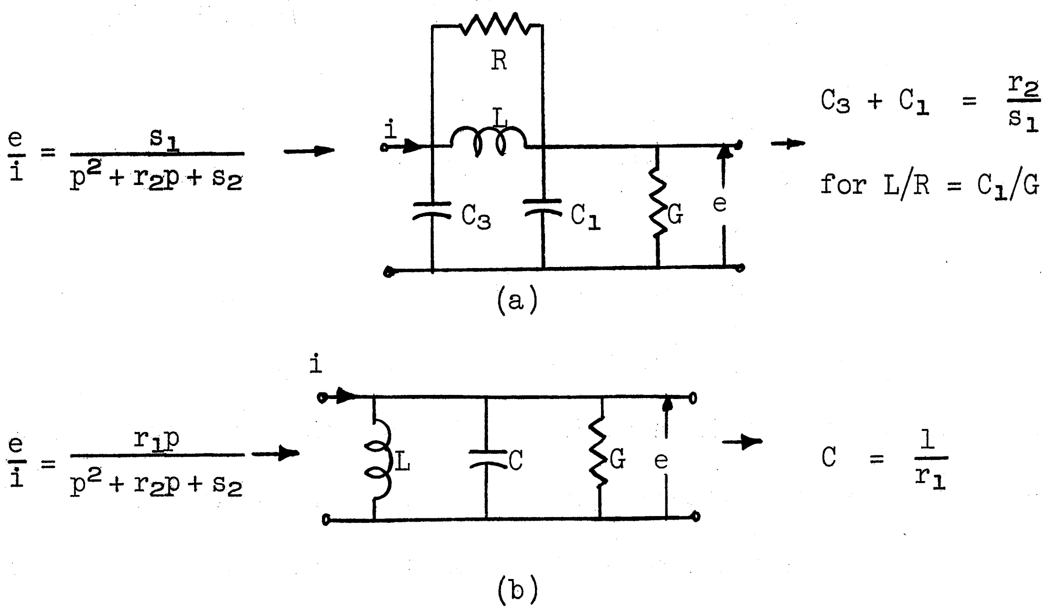


Fig. 37. Total terminal capacitance of two useful interstages.

Let us return to a consideration of Eq. 139 with a P* given by Eq. 140 and containing a factor $p^2 \pm \sqrt{2}\,ap + a^2$. If $p^2 + \sqrt{2}\,ap + a^2$ is divided into the polynomial P* of Eq. 140, two useful results are obtained. The quotient is

$$p^4 + [\sqrt{2+\sqrt{3}} - \sqrt{2}a]p^3 + [1 + a^2 - \sqrt{2}a\sqrt{2+\sqrt{3}}]p^2$$

$$+ \quad [\alpha - \sqrt{2}a + a^2\sqrt{2+\sqrt{3}}]p + [-a^4 + k + a^2 - \sqrt{2}a\alpha + \alpha\sqrt{2-\sqrt{3}}] \qquad (149)$$

while the remainder is

$$[\alpha + \alpha a^2 - \sqrt{2}\,ak + k\sqrt{2-\sqrt{3}} - \sqrt{2}\,a\alpha\sqrt{2-\sqrt{3}} - a^4\sqrt{2+\sqrt{3}} + \sqrt{2}a^5]p$$

$$+ \quad [a^6 - a^2k - a^4 + \sqrt{2}\,a^3\,\alpha - a^2\alpha\sqrt{2-\sqrt{3}} + k] \qquad (150)$$

If $p^2 \pm \sqrt{2}\,ap + a^2$ is to be an exact factor, the two cases being covered by allowing a to be positive or negative, each of the coefficients of p in the remainder must be zero; collecting terms in α and k results in the equations

$$\alpha[1 + a^2 - \sqrt{2}\,a\sqrt{2-\sqrt{3}}] + k[-\sqrt{2}a + \sqrt{2-\sqrt{3}}] - a^4\sqrt{2+\sqrt{3}} + \sqrt{2}a^5 = 0 \qquad (151)$$

$$\alpha[\sqrt{2}\,a^3 - a^2\sqrt{2-\sqrt{3}}] + k[1 - a^2] + a^6 - a^4 = 0\,. \qquad (152)$$

It is now clear that the choice of a P* containing the two arbitrary constants α and k has resulted in two simultaneous equations in the three variables a, α, and k. It is easily established that for many choices of "a," two linear independent equations in α and k result. Thus, a measure of freedom in the choice of "a" exists. For a = 1/2, the approximate solution is

$$\alpha = .0987$$
$$k = .056\,. \qquad (153)$$

Using the above values, the polynomial 149 containing the factors of P* other than $p^2 + \sqrt{2}a\, p + a^2$ may be evaluated. The factors of this polynomial are (approximately)

$$(p^2 + 1.925\, p + .9516)(p^2 - .7005\, p + .2812)$$

and

$$P = [p^2 + \sqrt{2}\, 1/2\, p + (1/2)^2](p^2 + 1.925\, p + .9516)(p^2 + .7005\, p + .2812)\ . \tag{154}$$

The overall function then has a disassociation (in the form of Eq. 142)

$$\frac{e_{out}}{Ki} =$$

$$\frac{1}{(p^2+1.925p+.9516)}\,\frac{p}{(p^2+.518p+1)}\,\frac{p}{(p^2+.7005p+.2812)}\,\frac{p^2}{\left(p^2+\frac{\sqrt{2}}{2}\,p+\frac{1}{2^2}\right)(p^2+\sqrt{2}p+1)} +$$

$$\frac{1}{(p^2+1.925p+.9516)}\,\frac{p+.5675}{(p^2+1.932p+1)}\,\frac{.0987(2)^2}{(p^2+.7005p+.2812)}\,\frac{(1/2)^2}{\left(p^2+\frac{\sqrt{2}}{2}+\frac{1}{2^2}\right)(p^2+\sqrt{2}p+1)}\ . \tag{155}$$

It will be observed that interstage capacitances of one farad result in the bandpass channel, for realizations in the form of Fig. 37b. A shunt-peaked stage with one farad results for the lowpass channel transfer function

$$\frac{p + .5675}{p^2 + 1.932\, p + 1} \tag{156}$$

while a capacitance of 1.773 farads results for a realization of

$$\frac{.0987(2)^2}{p^2 + .7005\, p + 1} \tag{157}$$

with the form of Fig. 37a. The association of the constant $.0987(2)^2$ with the polynomial $p^2 + .7005\,p + 1$ was arbitrary. The several interstages of a channel may be modified by impedance level transformations in the interest of obtaining a desired capacitance distribution, it being only necessary that the overall channel transfer admittance remain fixed. The mean interstage capacitance of the two channels above differ by the factor $\sqrt{1.773}$. With identical tube complements, the lowpass channel has 5 db of gain capability in excess of that required to obtain the desired overall response. By using tubes with different figures of merit, a relative gain adjustment can be effected while still utilizing all tubes at full capability. In the above example, employing a 6AK5 output tube and two 6AU6 tubes in the lowpass channel, with three 6AK5's in the bandpass channel, the relative channel gains are within 1.1 db. An error of 1.1 db in relative gains results for a mean variation of 4 per cent in the transconductances of the tubes of one channel from those of the other channel.

The association of the factor $p^2 + 1.925\,p + .9516$ with the coupling network rather than the factor $p^2 + .7005\,p + .2812$ was arbitrary. Note also that the lowpass channel transfer function could be obtained with interstage realizations of

$$\frac{e}{i} = \frac{p + .5675}{p^2 + .7005 + .2812}$$

$$\frac{e}{i} = \frac{.0987(2)^2}{p^2 + 1.932\,p + 1} \quad . \tag{158}$$

For realizations in the form of a shunt-peaked circuit and a network of the form of Fig. 37a, a mean interstage capacitance of $\sqrt{\frac{1.932}{.3948}} = 2.21$ farads results in the lowpass channel. Relative channel gain adjustment while still using all tubes at full capability could be achieved with tubes in the lowpass channel having a figure of merit approximately half of those in the bandpass channel for this case.

A plot of the poles and zeroes of the transfer functions from source to load through the two channels for Eq. 155 is presented in Fig. 38. The merits of the above technique can best be assessed by comparison with the previously discussed amplifier with transfer functions from source to load through the two channels having identical transfer poles and with the same prescribed overall magnitude of gain function (see Eq. 131). A plot, analogous to Fig. 38, is presented in Fig. 39 for this function with $a = 1/2$. It appears, qualitatively, that the pole and zero configuration of Fig. 38 is not substantially superior to that of Fig. 39.

A second approximate solution of Eqs. 151 and 152 occurs for

$$\begin{aligned} a &= 1 \\ \alpha &= 0 \\ k &= -.5786 \end{aligned} \qquad (159)$$

when $P^* = p^4(p^2 + \sqrt{2 + \sqrt{3}}\,p+1) - .5786\,(p^2 + \sqrt{2 - \sqrt{3}}\,p+1)$. This suggests the possibility of using different numbers of tubes for the two channels, the minus sign resulting from the 180° phase shift through an extra tube in one channel. There appears to be some merit in the idea of using different numbers of tubes in the two channels; the partitioning loss of several of the coupling networks discussed in Chapter IV is confined to the bandpass channel.

An alternate choice for P^* is

$$P^* = p^5(p^2 + \sqrt{2 + \sqrt{3}}\,p+1) + (\alpha p + k)\,(p^2 + \sqrt{2 - \sqrt{3}}\,p+1) \quad . \qquad (160)$$

If $p^2 + \sqrt{2}\,ap + a^2$ is divided into this P^*, the quotient is

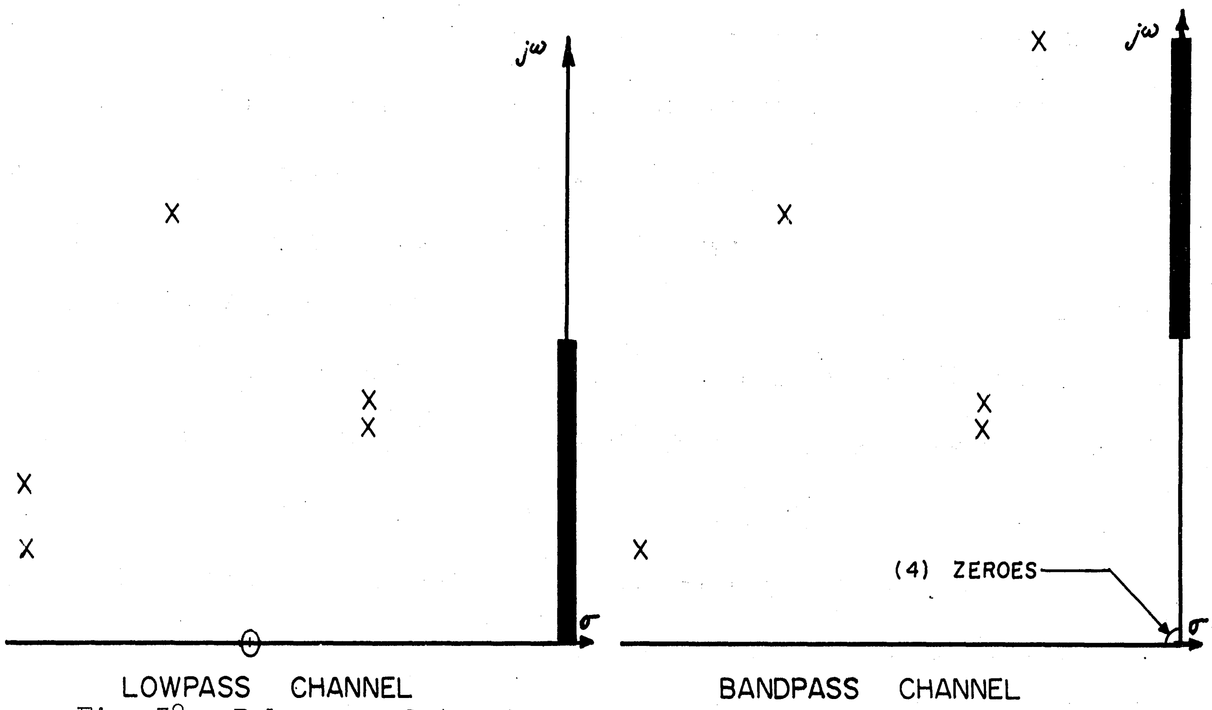


Fig. 38. Pole-zero plots of source to load transfer functions of Eq. 155 (crossover frequency a = 1/2 radian per second).

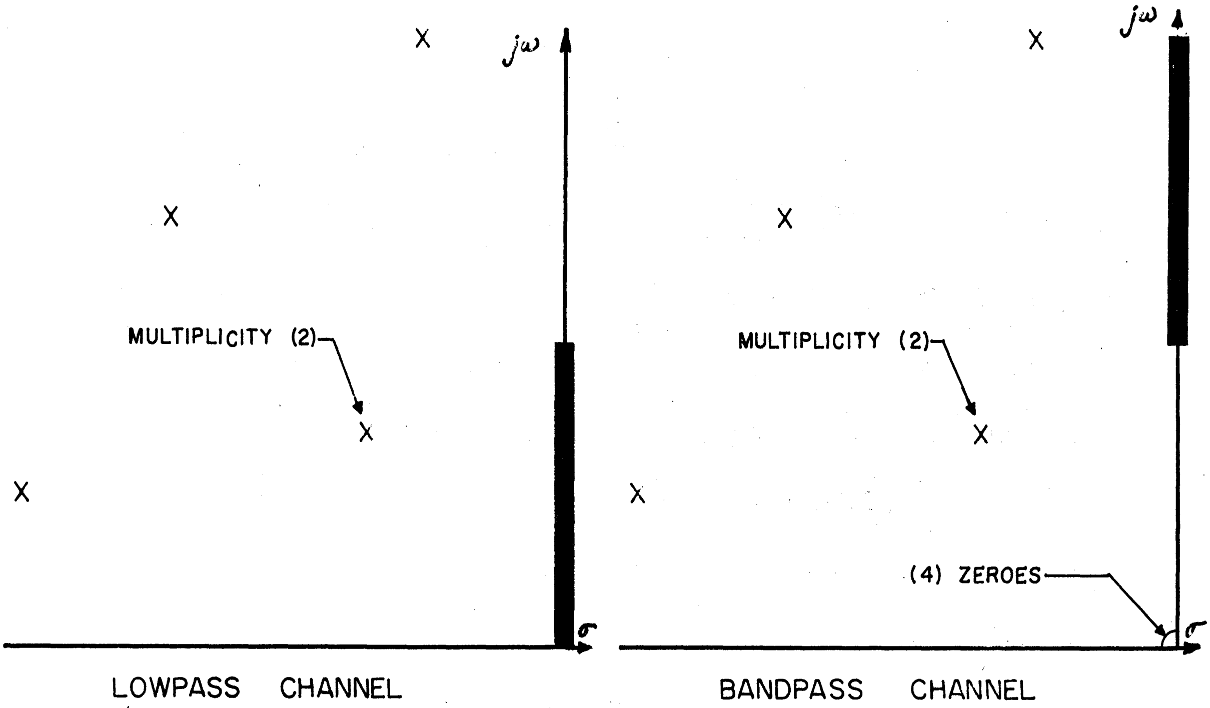


Fig. 39. Pole-zero plots of source to load transfer functions of Eq. 131 (crossover frequency a = 1/2 radian per second).

$$p^5 + (\sqrt{2+\sqrt{3}} - \sqrt{2}a)p^4 + [1 + a^2 - \sqrt{2(2+\sqrt{3})}a]p^3 + [\sqrt{2+\sqrt{3}}a^2 - \sqrt{2}a]p^2$$

$$+ (\alpha + a^2 - a^4)p + (k + \sqrt{2-\sqrt{3}}\,\alpha - \sqrt{2}a\alpha - \sqrt{2+\sqrt{3}}a^4 + \sqrt{2}a^5 \;, \qquad (161)$$

while the remainder determines the two simultaneous equations:

$$\alpha[1 - \sqrt{2(2-\sqrt{3})}a + a^2] + k[\sqrt{2-\sqrt{3}} - \sqrt{2}a] - a^4 + \sqrt{2(2+\sqrt{3})}a^5 - a^6 = 0 \;, \qquad (162)$$

$$\alpha a^2[\sqrt{2}a - \sqrt{2-\sqrt{3}}] + k[1 - a^2] + \sqrt{2+\sqrt{3}}\,a^6 - \sqrt{2}a^7 = 0 \;. \qquad (163)$$

For $a = 1/2$, the approximate solution of Eqs. 162 and 163 is

$$\alpha = -.01345 \;, \qquad k = -.0247 \;. \qquad (164)$$

The factors of the polynomial 161 are, then, approximately

$$(p - .463)(p^2 + 1.987p + 1.059)(p^2 - .299p + .201) \;, \qquad (165)$$

and a possible disassociation

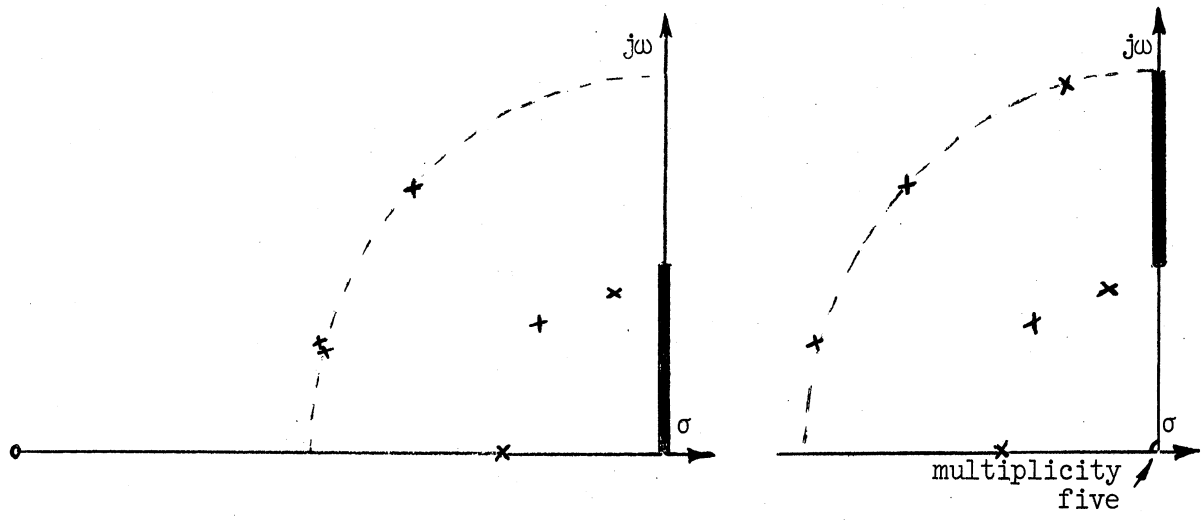
$$\frac{e_{out}}{K1} = \frac{P*}{PE} =$$

$$\frac{1}{(p^2+\sqrt{2}p+1)(p+A')}\,\frac{-.01345(2)^2(p+1.837)(p+A')}{(p^2+1.987p+1.059)(p^2+1.932p+1)}\,\frac{(p+B')}{(p+.463)(p^2+.299p+.201)}\,\frac{(1/2)^2}{\left[p^2+\sqrt{2}\,\frac{1}{2}p+\left(\frac{1}{2}\right)^2\right](p+B')} +$$

$$\frac{p^2}{(p^2+\sqrt{2}p+1)(p+A')}\,\frac{p+A'}{(p^2+.518p+1)(p+.463)}\,\frac{p+B'}{(p^2+1.987p+1.059)}\,\frac{p}{(p^2+.299p+.201)}\,\frac{p^2}{\left[p^2+\sqrt{2}\,\frac{1}{2}p+\left(\frac{1}{2}\right)^2\right](p+B')} \;. \qquad (166)$$

The pole-zero plots of the transfer functions associated with the two channels are presented in Fig. 40. The results are superior to those depicted in Figs. 38 and 39.

Partitioning the networks of the form of Fig. 27 are proposed at both the input and output of the amplifier in Eq. 166. This has been



(a) Lowpass channel (b) Bandpass channel

Fig. 40. Pole-zero plot for transfer function of Eq. 166.

facilitated by the insertion of the factors $p + A'$ and $p + B'$ in the numerator and denominator of the transfer function associated with each channel. Desirable choices of A' and B' depend on the following considerations related to the partitioning networks. The power transmitted from the source of Fig. 27 is described by

$$\frac{P_{del}}{P_{avail}} = \frac{a^4 + \omega^4}{(1 + \omega^2)\ (a^4 + \omega^4)} \tag{167}$$

If a bandwidth transformation by a factor ω_{3db} is made on the circuit of Fig. 27, the resulting transmission is

$$\frac{P_{del}}{P_{avail}} = \frac{a^4 + \left(\frac{\omega}{\omega_{3db}}\right)^4}{\left[1 + \left(\frac{\omega}{\omega_{3db}}\right)^2\right]\left[a^4 + \left(\frac{\omega}{\omega_{3db}}\right)^4\right]} \ . \tag{168}$$

and the individual termination powers become:

$$\frac{P_{del\ LP}}{P_{avail}} = \frac{\omega_{3db}{}^2[\omega_{3db}a]^4}{[(\omega_{3db})^2 + \omega^2][(\omega_{3db}a)^4 + \omega^4]} \ , \tag{169}$$

$$\frac{P_{del\ BP}}{P_{avail}} = \frac{\omega_{3db}{}^2\omega^4}{[(\omega_{3db})^2 + \omega^2]\ [(\omega_{3db}a)^4 + \omega^4]} \tag{170}$$

The resulting transfer impedances to the lowpass and bandpass network terminations, respectively, are

$$\frac{e_a}{i} = \frac{\omega_{3db}{}^3 a^2}{(p + \omega_{3db})\ [p^2 + \sqrt{2}\ (\omega_{3db}a)p + (\omega_{3db}a)^2]}$$

and (171)

$$\frac{e_b}{i} = \frac{\omega_{3db}p^2}{(p + \omega_{3db})\ [p^2 + \sqrt{2}\ (\omega_{3db}a)p + (\omega_{3db}a)^2]} .$$

Consider the transfer functions associated with the output network in Eq. 166:

$$\frac{e'_a}{i} = \frac{(1/2)^2}{\left[p^2 + \sqrt{2}\,\frac{1}{2}p + \left(\frac{1}{2}\right)^2\right]\ (p + B')} ,$$

(172)

$$\frac{e'_b}{i} = \frac{p^2}{\left[p^2 + \sqrt{2}\,\frac{1}{2}p + \left(\frac{1}{2}\right)^2\right]\ (p + B')} .$$

A comparison of the denominators of Eqs. 171 and 172 reveals that for $B' = 1$, the source is matched over a nominal bandwidth $\omega_{3db} = 1$ radian per sec, while the crossover of the transmission characteristics occurs at $a = 1/2$ radian per sec. Reference to Fig. 27 establishes that with $B' = 1$, the transfer functions of Eq. 172 can be achieved with a minimum capacitance at the terminal pairs associated with the channels of one farad.

The indicated transfer functions in Eq. 166 for the input network are

$$\frac{e_a}{i} = \frac{1}{[p^2+\sqrt{2}p+1]\,[p+A']}$$

$$\frac{e_b}{i} = \frac{p^2}{[p^2+\sqrt{2}p+1]\,[p+A']} \tag{173}$$

A comparison of the denominators of Eqs. 171 and 173 establishes

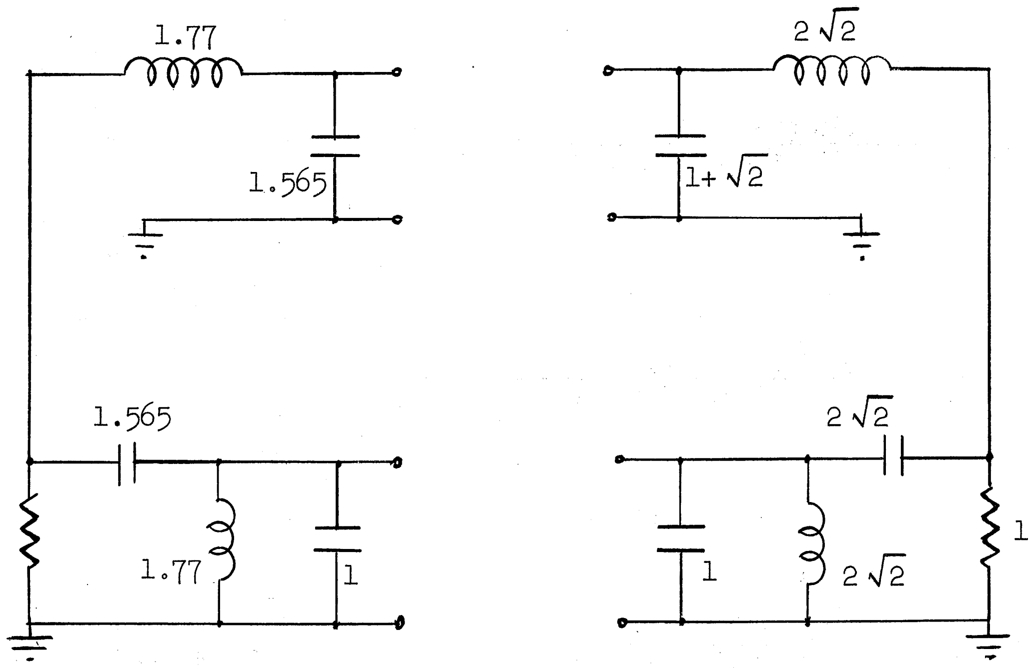
$$\omega_{3db} = A'$$

$$\omega_{3db}a = 1\ . \tag{174}$$

Note that for $A' = 2$, the crossover of the transmission characteristics occurs at $a = 1/2$ radian per sec. Thus, to obtain a crossover of the transmission characteristics of the input network at a frequency of 1/2 radian per sec, a generator match over 2 radians per sec is necessary for the functions of Eq. 173. In the realization of Eqs. 173 to be presented in Fig. 41, a value of $A' = .798$ was chosen. This value resulted from considerations associated with the realization of the lowpass channel transfer function, as will be discussed, and results in relatively inefficient coupling to the bandpass channel at the input circuit.

The input and output networks realizing the transfer functions of Eqs. 172 and 173, with $A' = .798$ and $B' = 1$, are shown in Fig. 41. The parameter r of Fig. 27 was made zero in these realizations. The output network is obtained directly for $r = 0$, and $a_o = 1/2$. The input network is obtained by setting $a_o = 1/.798$, making a bandwidth transformation by a factor $\omega_{3db} = .798$, and an impedance level transformation by a factor of 1/.798.† This results in a network with a minimum channel input capacitance of one farad. Such a minimum channel capacitance was insured by the choice of the constants in the numerators of Eqs. 172 and 173.

† See Eqs. 174 with $A' = .798$ as previously discussed.



(a) Input network

(b) Output network

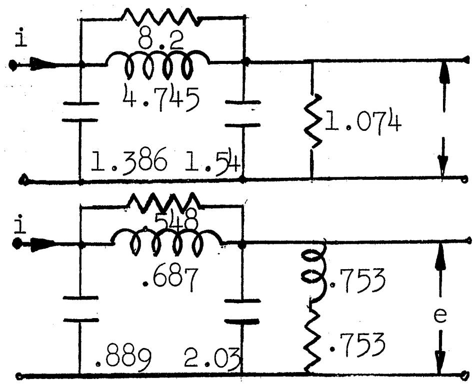
Fig. 41. Realizations of partitioning networks of Eq. 166 with B' = 1, A' = .798.

The minus sign of Eq. 166 can be associated with the 180° phase shift through an extra tube in one channel. For the Eq. 166, a realization of the channel transfer functions was therefore sought enabling the use of three tubes in the lowpass channel and four tubes in the bandpass channel. The channel transfer functions of Eq. 166 (with A'= .798, B' = 1) can be disassociated into the factors in Fig. 42, which have the realizations indicated. The determination of the first three interstages is outlined in Appendix A. The last two realizations are obtained using the results of Eqs. 145 and 125, which refer to Figs. 36 and 30, respectively.

If the networks of Figs. 41 and 42 are interconnected with tubes with $g_m = 1$, a nominal midband gain of one is obtained for a 3 db bandwidth of one radian per sec. This is achieved with three tubes with $g_m/C \simeq 1/3$ in the lowpass channel and with three tubes with $g_m/C = 1$ and

$$\frac{e}{i} = \frac{.0538(p+1.837)}{(p+.463)(p^2+.299p+.201)}$$

$$\frac{e}{i} = \frac{(p+1)\ (p+.798)}{(p^2+1.987p+1.059)(p^2+1.932p+1)}$$

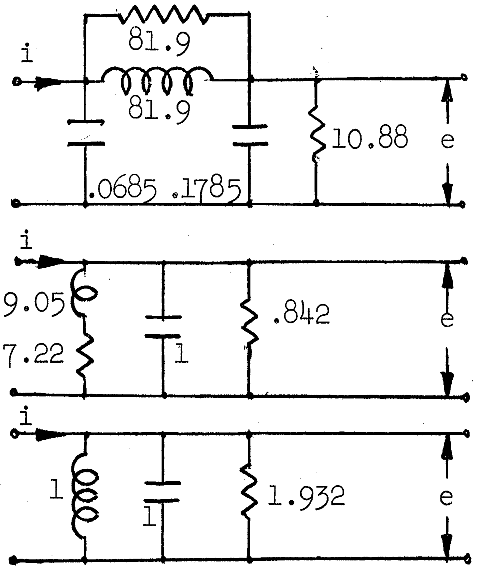


(a) Interstages of lowpass channel

$$\frac{e}{i} = \frac{p+1}{(p+.463)(p^2+.299p+.201)}$$

$$\frac{e}{i} = \frac{p+.798}{p^2+1.987p+1.059}$$

$$\frac{e}{i} = \frac{p}{p^2+.518p+1}$$



(b) Interstages of bandpass channel

Fig. 42. Realizations of channel transfer functions of Eqs. 166 with B'= 1, A'= .798.

one having $g_m/C \simeq 4$ in the bandpass channel.† In obtaining a practical amplifier from such a circuit, three types of modifications would normally be effected. These are a bandwidth transformation, impedance level transformations, and a variation of the tube transconductances from $g_m = 1$.

† The bandpass channel interstage capacities could be equalized by impedance level adjustments, such as was illustrated in connection with Fig. 35. The gain-bandwidth advantage of this circuit results both from the use of two-terminal pair interstages, and from the use of a multi-channel circuit.

The latter two of these factors influence the requirements on the tubes in obtaining a satisfactory relative gain adjustment of the two channels with different tube complements. In order to illustrate this, let us assume $g_m = .005$ for the extra tube in the bandpass channel, with the remainder of the channel tube complements identical. The relative channel gains are then correct if the product of the bandpass channel interstage impedance levels is increased by 200, to compensate for the reduction of the transconductance of the extra bandpass channel tube by 200. If the interstages of each channel are modified by factors with the product y, the relative channel gains are maintained in correct adjustment. Equating the resulting mean interstage capacitances for the lowpass and bandpass channels, respectively,

$$\frac{2.92}{\sqrt{y}} = \sqrt[3]{\frac{.247}{200y}} . \qquad (175)$$

The mean channel capacitances are equal for

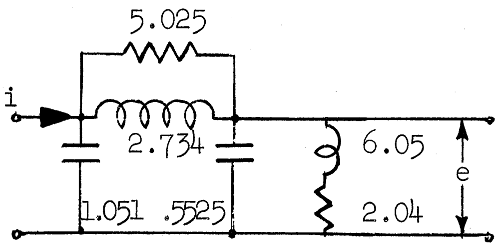
$$\sqrt{y} = (2.92)^3 \, (810)$$

when with $g_m = .005$ for all tubes, a mean stage gain of $(2.93)^3(810)(.005) \simeq 100$ results. Using tubes in the lowpass channel with $g_m = .0039$ and an interstage capacitance of 1.36 times those in the bandpass channel, equalization is achieved at a mean lowpass channel stage gain of eight.

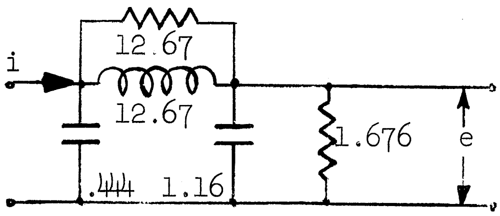
The difficulty of equalizing channel gain capability can be alleviated by a different choice of A' in Eq. 166, at a cost in gain-bandwidth advantage. A satisfactory realization of the lowpass channel transfer function can also be obtained with $A' = 3.36$. Referring to Eqs. 174, with the accompanying discussion, a generator match over a bandwidth of 3.36 radians per sec results, with a crossover frequency a = .3 radian per sec.

The cost of this excessive input circuit bandwidth can be assessed from an inspection of Fig. 43, where the corresponding channel transfer function realizations are indicated.

$$\frac{e}{i} = \frac{.349(p + 1.837)\ (p + 3.36)}{p^2+1.987p+1.059)(p^2+1.932p+1)}$$

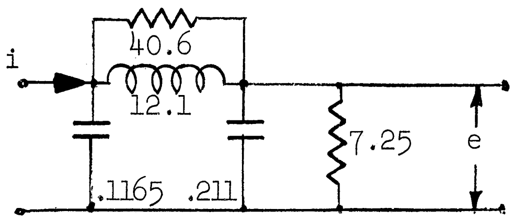


$$\frac{e}{i} = \frac{.1539(p+1)}{(p+.463)(p^2+.299p+.201}$$

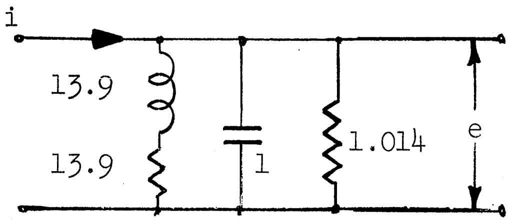


(a) Interstages of lowpass channel

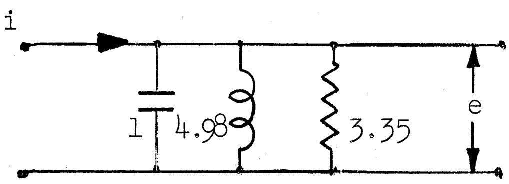
$$\frac{e}{i} = \frac{p + 3.36}{(p + .463)\ (p^2 + .518\ p + 1)}$$



$$\frac{e}{i} = \frac{p + 1}{p^2 + 1.987\ P + 1.059}$$



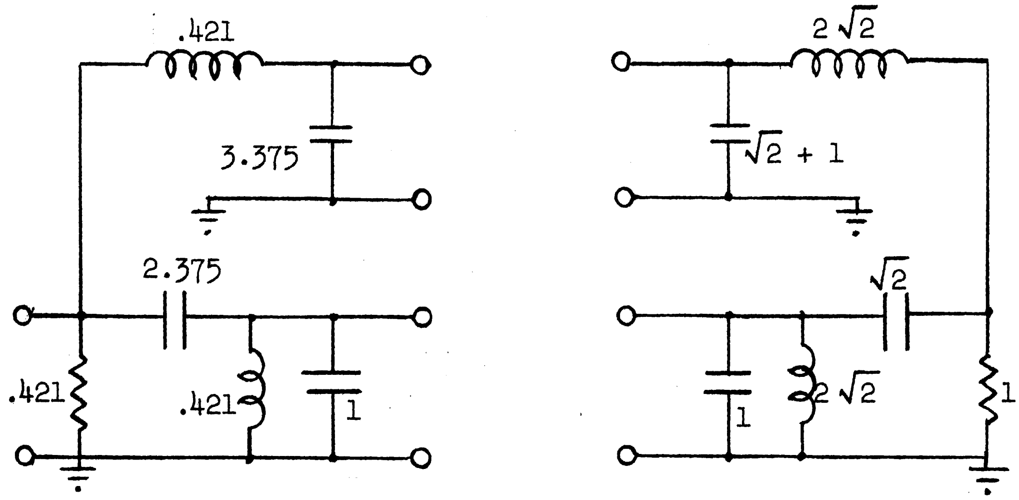
$$\frac{e}{i} = \frac{p}{p^2 + .299\ p + .201}$$



(b) Interstages of bandpass channel

Fig. 43. Realizations of channel transfer functions of Eq. 166 with $B' = 1$, $A' = 3.36$.

These interstages are obtained in the same manner as the interstages of Fig. 42. The input and output coupling networks of Fig. 44, for $B' = 1$ and $A' = 3.36$, are obtained using the same methods as in obtaining Fig. 41.



(a) Input network

(b) Output network

Fig. 44. Realizations of partitioning networks of Eq. 166 with $B' = 1$, $A' = 3.36$.

Again assuming 5000 µmhos transconductance for the extra tube in the bandpass channel, the mean interstage capacitances can be equated:

$$\frac{1.604}{\sqrt{y}} = \sqrt[3]{\frac{.3275}{200\,y}} \qquad \therefore \sqrt{y} = (1.604)^3(610) \ .$$

With $g_m = .005$ for all tubes, channel gain equalization occurs at a mean lowpass channel stage gain = $(1.605)^3(610)(.005) \simeq 12.6$. With interstage capacitances in the lowpass channel 1.361 times those in the bandpass channel, for example, satisfactory channel gain adjustment occurs at a mean lowpass channel stage gain of

$$\left(\frac{1.605}{1.361}\right)^3 (610)\ (.005) \simeq 5 \ .$$

CHAPTER VI

GENERAL CONSIDERATIONS IN THE DESIGN OF PARTITIONED AMPLIFIERS WITH CONTROLLED RESPONSE CHARACTERISTICS

6.1 Factors Affecting the Gain-Bandwidth Advantage of Partitioned Amplifiers

The principle underlying the fact that a gain-bandwidth advantage can accrue for a multi-channel amplifier is depicted in the following example. Consider an amplifier made up of two bandpass channels, and with input and output coupling circuits providing equal transfer impedances to (and from) the two channels. For such a circuit, only the magnitude of the transfer impedances of the coupling networks influences the magnitude of the amplifier gain. Let us assume that broadband input and output networks are used, so as to provide nearly constant magnitude of transfer impedances to the channels over the entire amplifier passband. The magnitude of the overall gain is then given, to as close an approximation as is desired, by

$$\left|\frac{e_{out}}{K'i}\right| \simeq \left|\frac{i_a}{e_a} + \frac{i_b}{e_b}\right| = \left|\frac{k_a p^{\alpha}}{M_a} + \frac{k_b p^{\beta}}{M_b}\right| = \left|\frac{k_a p^{\alpha} M_b + k_b p^{\beta} M_a}{M_a M_b}\right| . \quad (176)$$

K' approximates the product of the magnitudes of input and output coupling network transfer impedances within the passband, while for the bandpass channels $m_a = p^{\alpha}$ and $m_b = p^{\beta}$. Otherwise, the notation conforms with that of Section 3.2. It was shown in Section 3.1 that in order to maximize the gain-bandwidth advantage of a partitioned amplifier it was necessary for the substantial gain of each channel to be confined to a unique portion of the total band. The gain of the amplifier within each channel band is then essentially determined by the gain of the channel covering this band. It is therefore concluded that an optimum partitioned amplifier should contain chains with the highest feasible nominally uniform gain over their prescribed bands, for the tubes and interstage complexity employed. The limitations on amplifier chains have been widely studied, and were summarized in Section 2.1. The best presently available amplifier chains achieving an approximately uniform gain over a prescribed band have distributions of their transfer poles such as in Figs. 45a or 45b. Assume, then, for the two-channel amplifier discussed in connection with Eq. 176, that the poles of the transfer functions of the individual channels are distributed with respect to their prescribed individual bands in a manner analogous to Fig. 45. The result is illustrated in Fig. 46a.[†] The distribution of poles of the individual channel transfer functions under the same conditions, but for a three-channel amplifier, is shown in Fig. 46b. The same total number of poles are illustrated in the several

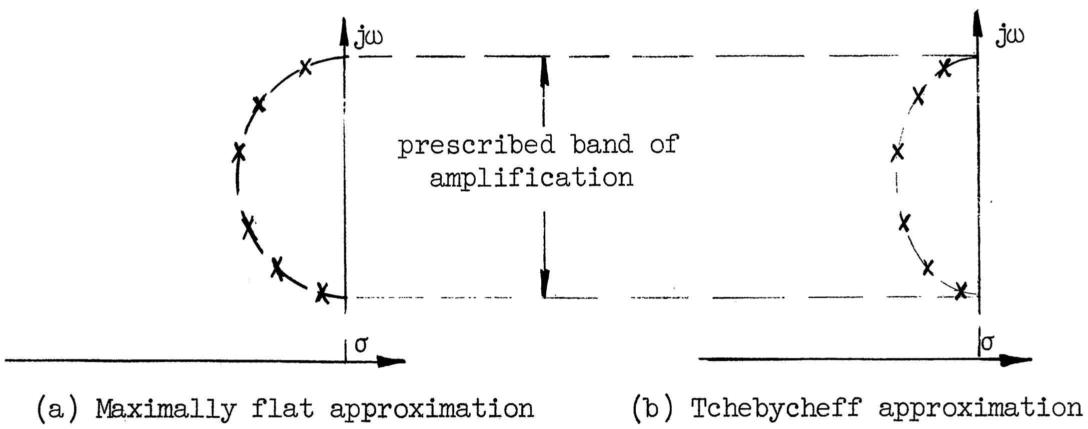
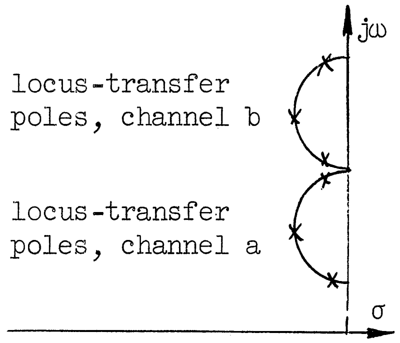
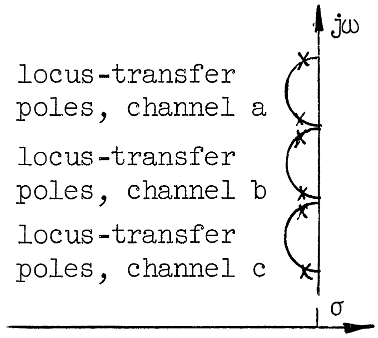


Fig. 45. Conventional distributions of transfer poles for bandpass channels with nominally uniform gain.

† The bandwidths of the individual channels were arbitrarily chosen equal for this illustration. The pole distributions are consistent with a maximally flat approximation to the rectangular gain characteristic, which is also arbitrary.



(a) Two-channel amplifier



(b) Three-channel amplifier

Fig. 46. Idealized distributions of poles of channel transfer functions.

sketches, to suggest the restriction to a fixed total number of tubes and interstage complexity. It will be noted that, because the poles in Figs. 46a and 46b are distributed with respect to individual portions of the total band, their proximity to the jω-axis is greater than in a single channel amplifier covering the same total band (see Fig. 45a). Such a decrease in the mean pole displacement from the jω-axis is necessary if a gain-bandwidth advantage is to be realized by an increase in the number of channels.

In Section 3.4 it was shown that there is an optimum number of channels for a prescribed total bandwidth, tube type, and interstage complexity. It follows that a decrease in the mean pole displacement from the jω-axis is not a sufficient condition for a gain-bandwidth advantage to accrue from an increase in the number of channels. In order to illustrate the principle underlying this fact, it is convenient to consider again the two-channel amplifier discussed in connection with Eq. 176. The factors M_a and M_b are Hurwitz. Further, in order for a gain-bandwidth advantage to be realized, M_a and M_b must be different, and there will be zeroes of the overall gain function in the finite region of the p-plane associated with the factor $k_a p^{\alpha} M_b + k_b p^{\beta} M_a$ in Eq. 176. The gain of a three-channel amplifier, analogous to Eq. 176, is

$$\left|\frac{e_{out}}{K'i}\right| \cong \left|\frac{k_a p^{\alpha} M_b M_c + k_b p^{\beta} M_a M_c + k_c p^{\gamma} M_a M_b}{M_a M_b M_c}\right| \quad . \qquad (177)$$

It will be noted that for distributions of poles as in Fig. 46, the factor $k_a p^{\alpha} M_b M_c + k_b p^{\beta} M_a M_c + k_c p^{\gamma} M_a M_b$ of the three-channel amplifier tends to be of higher degree than the factor $k_a p^{\alpha} M_b + k_b p^{\beta} M_a$ of the two-channel amplifier. There are, therefore, generally more zeroes of the overall transfer function in the finite p-plane, for a three-channel amplifier than for a two-channel amplifier, at a fixed number of tubes and interstage complexity. The number of these zeroes tends to increase, as the number of channels is increased, while for a fixed number of tubes and interstage complexity the number of poles of the overall function ($M_a M_b$ in Eq. 176 and $M_a M_b M_c$ in Eq. 177) is fixed. The potential analogy (Refs. 21,22) would lead to the conclusion that an optimum number of channels exists, as was proved in Section 3.4.

The zeroes associated with a factor such as $k_a p^{\alpha} M_b + k_b p^{\beta} M_a$ in Eq. 176 can serve the useful purpose of flattening the overall response of a multi-channel amplifier. Note that such zeroes could be "cancelled out" by introducing properly positioned poles in the coupling network transfer impedances. The resulting overall responses for pole distributions such as in Fig. 46 would then be very non-uniform. This follows from the potential analogy also, or may be seen by plotting such a function.

A single-channel amplifier could be constructed with poles of the transfer function distributed as in Figs. 46a and 46b. It is also possible to distribute the transfer zeroes of such an amplifier so as to achieve a relatively uniform gain. However, in this case the zeroes would be characteristic of the channel itself, i.e., in a simple cascade, interstages with transfer zeroes in the finite region of the p-plane would be necessary to realize such a transfer function. It follows from the results of Fano[25] and Sharpe[26] that the gain of such a channel is less than gain of a channel with the same number of poles of transfer but distributed in a manner such as Fig. 45, and with the transfer zeroes remotely located from the region of desired amplification. In a multi-channel amplifier, as has been pointed out, the zeroes associated with a factor such as $k_a p^{\alpha} M_b + k_b p^{\beta} M_a$ in Eq. 176 are not immediate attributes of the individual channels, but are characteristic of the several channels when operated in parallel. These zeroes can, with proper design, flatten the overall gain characteristic while still permitting a gain-bandwidth advantage to accrue for a restricted number of channels in a particular application.

6.2 A Realistic Design Objective for Partitioned Amplifiers Without Partitioning Networks

The preceding discussion would seem to justify the following description of a partitioned amplifier with coupling networks providing uniform and equal transfer impedances to all channels, the gain of which approaches the maximum feasible uniform gain over a prescribed band for a given number of tubes of a given type and a given interstage complexity:

1. It contains approximately an optimum number of channels, in the sense of Section 3.4, for the case of high gain channels. For gains less than perhaps 20 db, the optimum number of channels is less than that of Section 3.4 due to the lack of partitioning. Figure 7 usually provides a satisfactory guide in this case.

2. Each of the channels achieves the maximum feasible nominally uniform gain (consistent with 3. below) over its prescribed unique portion of the total band. This is accomplished if the individual channels have a high gain in terms of present cascade design capability, with the given tubes and interstage complexity.

3. The channels are interrelated in such a way that a satisfactory overall response results, and no appreciable cancellation of load components occurs due to the relative phase shifts through the several channels.

The above description is useful in that it gives a design objective as well as a criterion whereby any particular circuit may be evaluated. Viewed in terms of these results, the circuits of Chapter V are not optimum. However, they have some gain-bandwidth merit, and in addition illustrate many detailed problems which can be expected in any exact design procedure. In Chapter VII, an alternative design procedure is discussed. It is shown that a number of amplifier chains can be incorporated into an amplifier in such a way that a relatively good approximation to the rectangular gain characteristic can be insured. The amplifiers described in Chapter VII satisfy the above requirements very well. In particular, the individual chains are designed in a conventional manner. Little or no compromise in the gain capability of these channels was made in the interests of achieving uniform overall amplifier gain characteristics.

6.3 A Statement of the Design Problem for Partitioned Amplifiers Without Partitioning Networks

The material in Sections 6.1 and 6.2 permits certain conclusions relating to an extension of the exact design procedures of Chapter V for amplifiers without partitioning. The generalized equation for such amplifiers is written (from Section 3.2)

$$\frac{e_{out}}{i} = \frac{n_a n_a'}{D_{in} D_{out}} \sum_{j=a}^{m} k_j \frac{m_j}{M_j} \quad . \tag{178}$$

If coupling networks providing nominally uniform magnitudes of transfer impedances are assumed,† attention can be directed to the simplified expression

$$\left|\frac{e_{out}}{K'i}\right| \simeq \left|\sum_{j=a}^{m} k_j \frac{m_j}{M_j}\right| = \left|\frac{\sum_{k=a}^{m} k_k \frac{m_k}{M_k} \prod_{j=a}^{m} M_j}{\prod_{j=a}^{m} M_j}\right| \tag{179}$$

where K' approximates the product of the magnitudes of input and output coupling network transfer impedances within the passband. For an important class of bandpass channels (transfer zeroes at 0 or 00), this reduces to

$$\left|\frac{e_{out}}{K'i}\right| = \left|\sum_{j=a}^{m} \frac{k_j p^j}{M_j}\right| . \tag{180}$$

For the special case of topologically similar channels with identical tube complements, Eq. 180 becomes

$$\left|\frac{e_{out}}{Ki}\right| = \left|p^{\alpha} \sum_{j=a}^{m} \frac{1}{M_j}\right| \tag{181}$$

where $K = kK'$, and $k_a = k_b = \ldots = k_m \stackrel{d}{=} k$.

In any of the above cases, it is necessary to determine a rational function approximating the desired gain characteristic, and which can be disassociated into factors meeting practical requirements, as previously discussed. The discussion in connection with Eqs. 176 and 177 indicates that, for optimum gain-bandwidth circuits, rational amplifier gain functions with zeroes in the finite region of the p-plane will result. The problem of finding rational functions suitable for identification with the above equations is complicated by the fact that previous studies have

† The restriction to "flat" input and output coupling networks is not necessary. It is pointed out shortly that certain resulting difficulties can be overcome by making alternative assumptions.

been concerned largely with the squared magnitudes of approximating functions, rather than with the functions themselves.

A difficulty in Eqs. 178-181 is that the factors M_j occur in both numerator and denominator of the overall expression (see, for example, Eq. 179). An interrelationship of the pole and zero loci of the approximating function is therefore necessary to enable a decomposition into the forms of Eqs. 178-181. It is known that Tchebycheff rational functions can be formed with arbitrary poles.[19,20,26,27] The zeroes are determined, however, when the poles are chosen. Alternatively, the zeroes may be chosen arbitrarily, when the poles are determined. The above difficulty can be circumvented, as can be seen by rewriting Eq. 178 in the form

$$\frac{e_{out}}{i} = \frac{\sum_{k=a}^{m} k_k \frac{m_k}{M_k} \prod_{j=a}^{m} M_j}{D_{in} D_{out}} \cdot \frac{n_a n'_a}{\prod_{j=a}^{m} M_j} . \qquad (182)$$

It is possible to identify

$$\frac{n_a n'_a}{\prod_{j=a}^{m} M_j}$$

with a Tchebycheff rational approximating function F_1, when the M_j can be chosen independently. For the given M_j, a Tchebycheff rational approximating function F_2 is obtained to be identified with

$$\frac{\sum_{k=a}^{m} k_k \frac{m_k}{M_k} \prod_{j=a}^{m} M_j}{D_{in} D_{out}} .$$

This can be done, since it can be done with both the $k_k m_k$ and M_j prescribed, which amounts to picking the zeroes of F_2. The magnitude of the overall gain is then given by

$$\left|\frac{e_{out}}{i}\right| = |F_1| \cdot |F_2| \quad . \tag{183}$$

While the product of the magnitudes of two Tchebycheff approximating functions will not in general be Tchebycheff, the maximum possible pass-band variation is known, and can be prescribed.

6.4 Further Considerations in the Design of Partitioned Amplifiers with Partitioning Networks

The overall response of a partitioned amplifier with partitioning networks has been characterized in Section 3.2 by the equation

$$\frac{e_{out}}{i} = \frac{1}{D_{in}D_{out}}\left[n_a n'_a \frac{m_a}{M_a} + n_b n'_b \frac{m_b}{M_b} + \ldots + n_m n'_m \frac{m_m}{M_m}\right] \quad . \tag{184}$$

It is possible to reduce the complexity of the function to be considered in a design problem, without appreciably compromising the usefulness of the results. An important potential application for partitioning networks is at the output of power amplifiers. The loss resulting from the use of an input coupling network without partitioning can easily be offset with extra low-level tubes in each channel. For example, in a two-channel amplifier a loss in gain by a factor not greater than two will result if an input network without partitioning is used, as compared to a single-channel amplifier with the same total bandwidth.† This follows since the input coupling network impedance level can simply be reduced by two to provide for the doubling of the total capacitance at the channel inputs. This loss by a factor of two is more than offset by a single tube per channel in an optimum amplifier, where a stage gain of ϵ is achieved. If partitioning is not employed at one coupling network, the simplified form of Eq. 185 may be considered:

† It was shown in Section 4.1 that the loss need not be even this great.

$$\left|\frac{e_{out}}{i}\right| \simeq \left|\frac{n_k}{D_{in}}\right| \cdot \left|\frac{1}{D_{out}}\left[n'_a \frac{m_a}{M_a} + n'_b \frac{m_b}{M_b} + \dots + n'_m \frac{m_m}{M_m}\right]\right|$$

$$\simeq \left|\frac{n_k}{D_{in}}\right| \cdot \left|\frac{1}{D_{out}} \frac{\sum_{k=a}^{m} \frac{n'_k m_k}{M_k} \prod_{j=a}^{m} M_j}{\prod_{j=a}^{m} M_j}\right|$$

$$\simeq \left|\frac{\sum_{k=a}^{m} \frac{n'_k m_k}{M_k} \prod_{j=a}^{m} M_j}{D_{in}}\right| \cdot \left|\frac{n_k}{D_{out} \prod_{j=a}^{m} M_j}\right| \stackrel{d}{=} |F_1| \cdot |F_2| \quad . \qquad (185)$$

The final form in Eq. 185 suggests a direct design procedure analogous to that discussed in connection with Eq. 182. In principle, F_2 is identified with a suitable approximating function. A measure of freedom exists in the choice of the function F_2, and a D_{out} suitable for identification with a partitioning network and desirable M_j are chosen. Next, the partitioning network is chosen. Both the n'_k and M_j are then known, when for desirable m_k the D_{in} can be determined by identification of F_1 with a suitable approximating function. As in Eq. 182, the maximum passband variations can be constrained if, for example, Tchebycheff approximating functions are used. In either Eq. 182 or 185, the M_j could be chosen in accordance with the principles discussed in Section 6.1.

CHAPTER VII

AN APPROXIMATE PARTITIONED AMPLIFIER DESIGN PROCEDURE

7.1 Design Philosophy

The material previously presented represents an introductory treatment of the problem of designing partitioned amplifiers with exact prescribed gain characteristics. It is to be expected that an initial brief study of the problem cannot be comprehensive, since decades of effort by many individuals have been expended in bringing the single channel amplifier, which is the special case of a partitioned amplifier with m = 1, to the present state of development. Extension of the above exact design techniques to provide for greater numbers of channels and more efficient amplifiers is possible, and is of practical importance. In spite of this, the author feels justified in undertaking now a discussion of a second design principle which is less exact than the previous approach. This approximate design procedure has the compelling advantage of great simplicity. The following discussion will be couched in terms of the problem of realizing approximately a rectangular overall gain characteristic, although it may be applicable to other problems.

One must be able to construct amplifier chains in order to be able to construct a partitioned amplifier. Let us assume that it is possible to construct a number of individual amplifier chains with transfer functions $\frac{i_k}{e_k}$ (see Section 3.2) as suggested by Fig. 47. It seems reasonable that if these chains are paralleled, with nominally uniform magnitude of transfer impedances in the input and output circuits to the individual channels (at least within their individual regions of substantial gain), the major potential difficulties will be associated with the several cross-

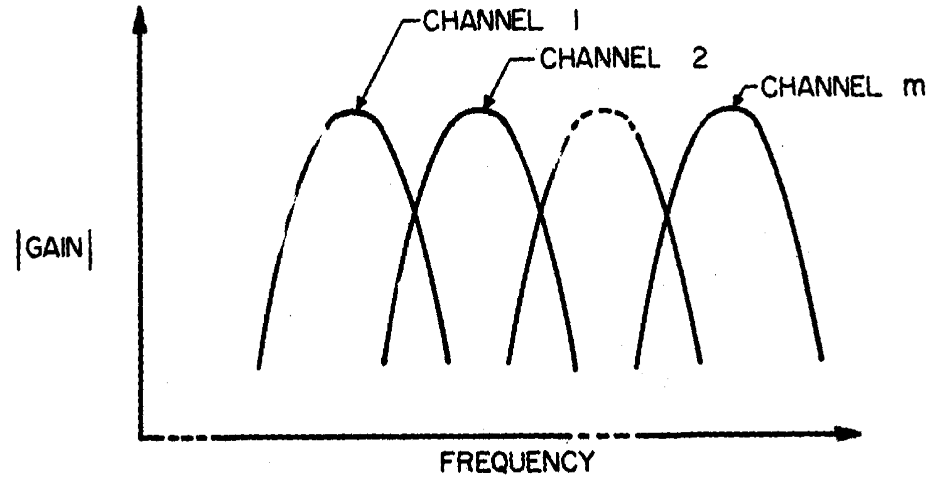


Fig. 47. Assumed distribution of channel gains.

over regions at each of which just two channels need have substantial gain. It would appear that, by proper control of the relative magnitude and phase of the two important signal components at the load in each crossover region, the resultant gain could be made equal to the channel midband gains at a single frequency. If the magnitude and phase functions are then assumed to be continuous, a region of relatively satisfactory adjustment would result. Whether such a region of satisfactory adjustment is sufficiently wide to result in a satisfactory overall response is dependent on the degree of pass-band uniformity required in a particular application. The experimental results in Sections 7.2.1 and 7.2.2 would be acceptable for most applications.

If the coupling circuits achieving nominally uniform and equal transfer impedances to all channels are utilized, the phase shifts through the coupling networks are the same for every channel at a given frequency, and the relative phase of the two substantial output signal components at each crossover frequency† depends only on the relative phase shift through the two channels with substantial gain. One design possibility is then, to make the 6 db points of the two important channel outputs coincide at each

† The frequency at which the gain of the two channels having substantial gain in a given crossover region are equal will be referred to as the crossover frequency for that region.

crossover frequency, with relative channel phase shifts of 360 n degrees.

Alternatively, networks achieving partitioning could be utilized at either the input or output of the amplifier, or in both places. In these cases, different phase shifts may be expected through the coupling networks to the channels of interest at a given crossover frequency. If the individual coupling network transfer impedances are also in the form of Fig. 47, the individual output components are reduced in magnitude at the crossover frequency compared to the mid-channel values due to variation with frequency of both the channel and coupling network transfer function magnitudes. If such depreciation of magnitudes at a crossover frequency becomes too great, a dip in the overall characteristic will result.

A particularly desirable situation (from the design standpoint) results if one partitioning network and one coupling network achieving equal transfer impedances to all channels are utilized in a partitioned amplifier. It is then possible to make the nominal 3 db points of the partitioning network individual transfer impedances and the channel responses for adjacent channels all coincide. This can be done while still insuring that the relative phases of the two substantial output signal components are suitably controlled at the crossover frequency, as illustrated later by an example.

7.2 Examples of the Approximate Design Procedure

7.2.1 Example 1 - Bandpass Partitioned Amplifiers.—The phase angle of $\frac{i_k}{e_k}$ for an amplifier chain containing $N_k + 1$ tubes separated by N_k simple tuned interstages in an exact flat-stagger is easily evaluated at the 3 db points. The phase shifts ϕ_1 and ϕ_2 associated with the N_k interstages at the lower and upper 3 db frequencies, respectively, are

$$\phi_1 = 45\ N_k \text{ degrees} \quad ,$$
$$\phi_2 = -45\ N_k \text{ degrees} \quad . \tag{186}$$

Including the idealized shift through the $N_k + 1$ tubes, the total channel phase shifts θ_1 and θ_2 at the lower and upper 3 db frequencies, respectively, are

$$\theta_1 = 45\ N_k + 180(N_k + 1) \text{ degrees} \quad ,$$
$$\theta_2 = -45\ N_k + 180(N_k + 1) \text{ degrees} \quad . \tag{187}$$

If the tube complements of the channels are identical, the relative phase shift ($= \theta_1 - \theta_2$) through two adjacent channels with coincident 3 db response frequencies is then 90 N_k degrees at the crossover frequency. If pairs of N-tuples are cascaded, with the 6 db response frequencies of adjacent channels coincident, the relative phase shift at the crossover frequency is 2(90 N) degrees. This is, of course, again 90 N_k degrees where N_k is the total number of interstages of the channel.

Consider, then, an amplifier employing input and output networks with equal magnitudes of transfer impedances to all channels. Assume that the magnitudes of these transfer impedances are made essentially uniform over the total bandwidth of the amplifier. As an example, the channel inputs could be tied together, resonated at the center frequency of the total amplifier band, and a sufficiently low source impedance utilized to achieve nominally uniform applied voltages to all channels. A similar technique, or other circuits based on the concepts of Section 4.1, could be employed in the output circuit. Assume further that the channels are made up of pairs of exact flat-staggered N-tuples of equal bandwidth and with identical tube complements. It is evident that, if the crossover frequency of adjacent channels occurs at the 6 db response frequencies, both the magnitude and phase of the resultant load signal at the crossover frequency will be satisfactory for

$$90\ N_k = 360\ n \text{ degrees} \quad . \tag{188}$$

A solution of this equation occurs for $N_k = 4$, when each channel consists of a pair of identical exact flat-staggered doublets. Other solutions suggest channels containing pairs of exact flat-staggered 4-tuples, 6-tuples, etc.

A three-channel amplifier, designed on the basis of the above discussion, was constructed. Type 6AK5 tubes were used throughout, which have a figure of merit of 73.8 mc for assumed interstage capacitances of 11 μμf. Each channel was made up of two exact flat-staggered doublets. **The design 3 db bandwidth of each pair was 31 mc, so that the design** 6 db channel bandwidths were 31 mc. The experimentally measured channel bandwidths were found to be approximately 27 mc.† In order to obtain a relatively smooth overall response, the resonant frequencies of the high and low frequency channel interstages were shifted slightly from the design values to bring the channel 6 db response frequencies approximately into coincidence. The output circuit was derived from the prototype lowpass circuit of Fig. 48. Design curves for this circuit are given in Fig.

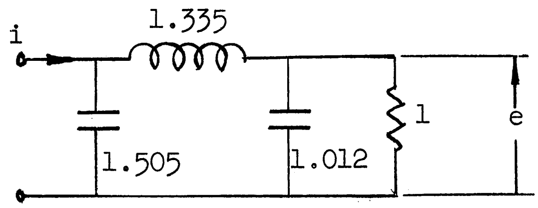
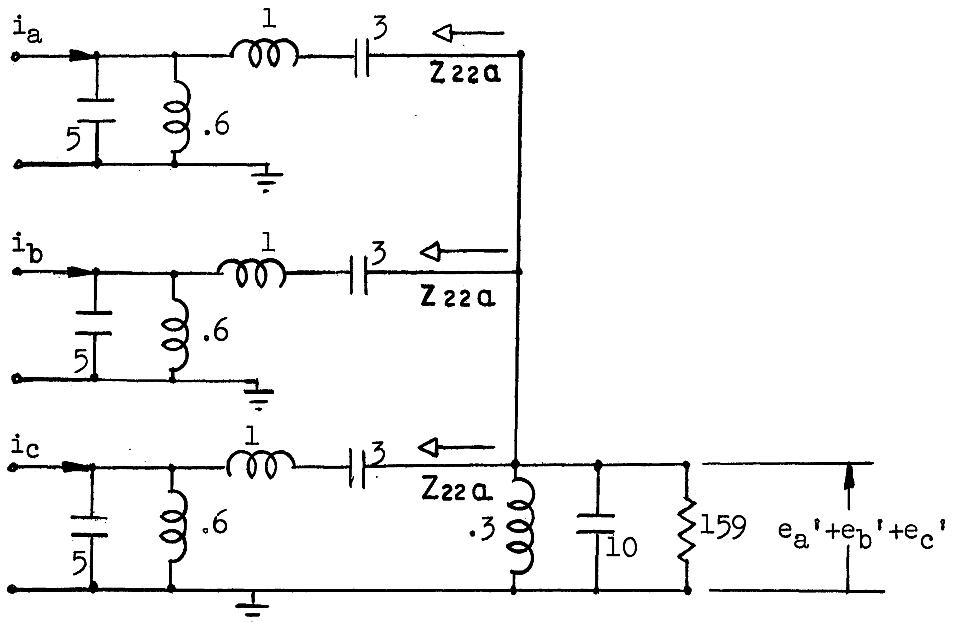


Fig. 48. Tchebycheff three-pole lowpass network.

† The experimental channel bandwidths were presumably lower than the design bandwidths due to the presence of interstage capacitance greater than the design value of 11 μμf.

2. The function $\left|\frac{e}{i}\right|^2$ of this circuit is Tchebycheff, with a tolerance bandwidth of one radian per sec. and a ripple of 1 db. A bandpass equivalent of this circuit centered at 90 mc and with a tolerance bandwidth of 100 mc was obtained. The maximum usable impedance level is determined by the channel output capacities. The resulting output network is shown in the amplifier circuit diagram of Fig. 50. The alternate form of this output circuit shown in Fig. 49 was inferior in practice. Note that in this network the

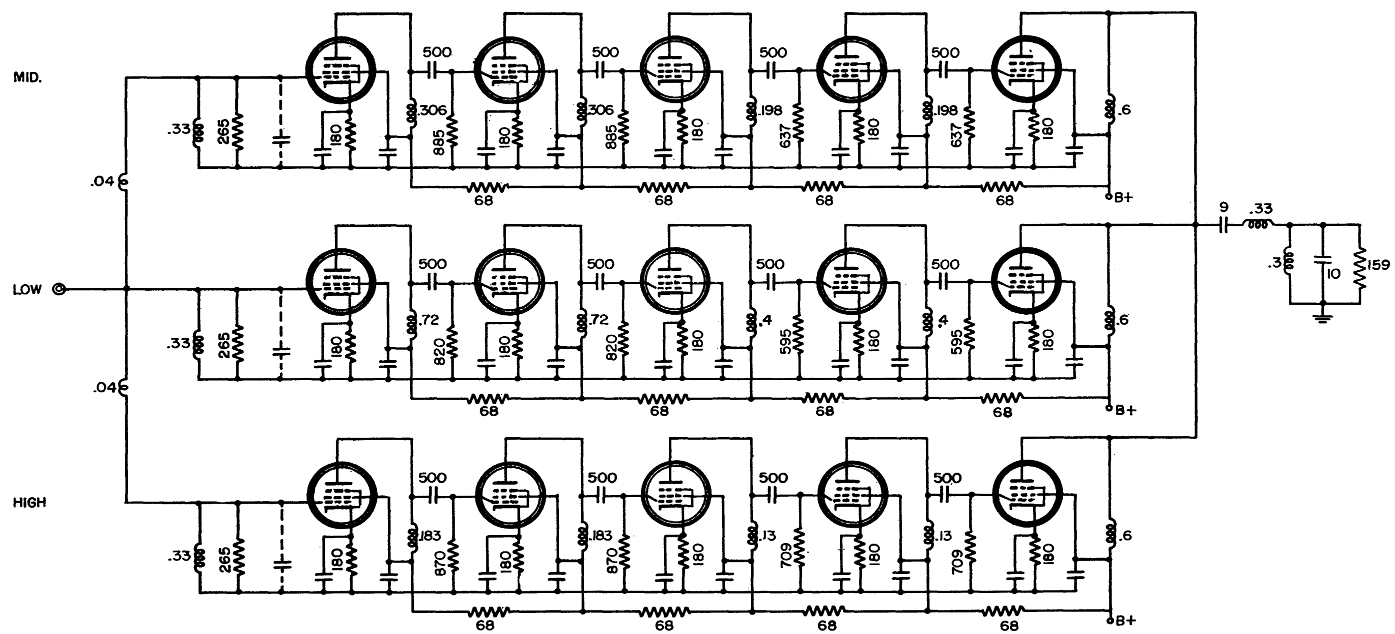


(values in ohms, μh, and μμf)

Fig. 49. Alternate form of output coupling network of Fig. 50.

transfer impedance $\frac{e_a'}{i_a}$, for example, has zeroes at the zeroes of the driving point impedances z_{22b} and z_{22c}, unless z_{22a} has a coincident zero. Practically, this exact symmetry among the three input branches is difficult to obtain.

In the experimental amplifier, the output tubes were spaced about 2-1/2 inches apart (center to center). It had been decided initially to locate the lowest frequency channel between the two upper frequency channels, as a precaution against parasitic oscillations. As a result, approximately



NOTE: All tubes 6AK5; heater pin 3 grounded; heater pin 4 bypassed with 1000 μμf and joined with .47 μh to form heater supply line for each channel; screen-plate bypass capacitors 1000 μμf; element values in ohms, microhenrys, and micro-microfarads.

Fig. 50. Three-channel partitioned amplifier.

.04 μh of parasitic series inductance was introduced in both the middle and high frequency channel outputs in connecting to the centrally located low frequency channel output, where the common output connection was made. This symmetry was intended to prevent a relative phase shift at the higher crossover frequency. The performance of this network was satisfactory.

The channel inputs were simply tied together and resonated at 90 mc in the experimental circuit. The amplifier was driven with a 50 ohm source, so that the input circuit bandwidth (approximately 130 mc) was adequate. A circuit comparable to that used at the output of the amplifier could have been incorporated advantageously. Such a circuit would be particularly advantageous if a source with substantial shunt capacitance were employed.

Plots of measured gain vs. frequency, with the channels activated individually and with all channels activated, are shown in Fig. 51.† The uniformity of the overall curve is sufficient for most applications, and could presumably be improved by further adjustment. A midband gain the approximately 24 db was obtained, and a 6 db bandwidth of 79 mc. This was accomplished with fifteen tubes having a figure of merit of 73.8 mc, and with simple tuned interstages.

The mean transconductances of the tubes in the three channels of the amplifier of Fig. 50 were nearly equalized by tube selection. The transconductances of approximately thirty new 6AK5 tubes checked varied from 3750 μmhos to 6500 μmhos, with fixed supply voltages and a 180 ohm cathode resistor. If such variations of tube parameters are encountered, a certain amount of tube selection or individual channel gain controls are necessary. Alternatively, special techniques can be employed to more

† These gains were obtained with an average cathode current of 10 ma.

nearly equalize tube characteristics.†

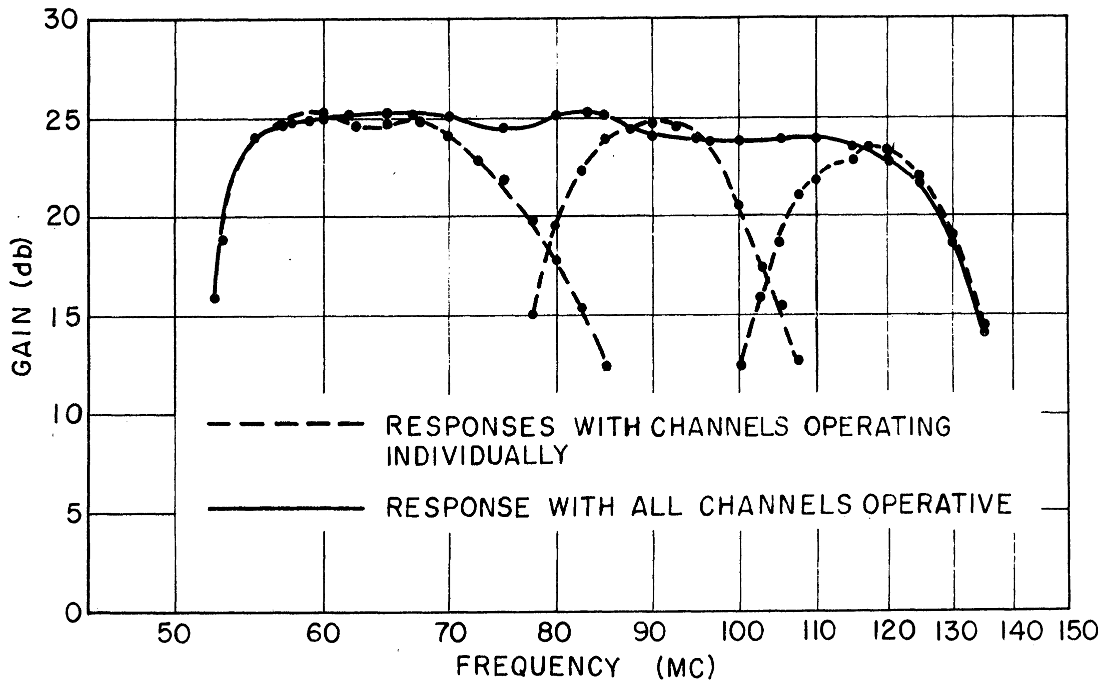


Fig. 51. Experimental gain curve-amplifier of Fig. 50.

The combination of gain and bandwidth obtained with the circuit of Fig. 50 can be achieved or surpassed using the same tubes in Tchebycheff-staggered cascades of simple-tuned circuits, or with circuits employing more complex interstages. However, both the design and the construction of such circuits would be more difficult than for the circuit of Fig. 50. Further, such techniques can presumably be utilized to advantage in the construction of partitioned amplifiers. It seems, therefore, that a comparison with an exact flat-staggered cascade of simple-tuned 6AK5 stages is fair. No gain can be obtained with such a circuit over the 79 mc meas-

† The transconductance varies much less at a fixed total tube current than at fixed supply voltages. Professor Macnee has pointed out that this could be more nearly achieved by using a fixed positive cathode bias supply in conjunction with cathode resistors larger than the conventional value.

ured bandwidth of the amplifier of Fig. 50.

7.2.2 Example 2 - A Bandpass Partitioned Amplifier with a Partitioning Network.—The partitioning network used in an experimental amplifier which will now be discussed was derived from a complementary filter set.[28] A common application of complementary sets has been in the woofer-tweeter circuits for splitting audio power efficiently among several speakers of a system on a frequency basis. A complementary set may be characterized as a group of networks which when tied together in a prescribed manner present a fixed resistive impedance R at all frequencies. When driven with a generator made up of an ideal voltage source in series with a resistance R, the available power of the generator is delivered to the set at all frequencies. If the internal resistance of the generator differs from R, a fixed mismatch (independent of frequency) results.

A certain degree of freedom exists in the distribution of the power delivered by the generator among the networks of a complementary set. This is illustrated in Fig. 52, where three simple complementary sets are shown, along with the functions describing the distribution of the available power of the matched generator among the network terminations. In Fig. 52b, the total power delivered to the lowpass and bandpass network terminations is

$$\frac{P_{del\ LP} + P_{del\ BP}}{P_{avail}} = \frac{1}{1 + \omega^4} . \tag{189}$$

The transmission to the lowpass network is over a nominal bandwidth of 1/2 radian per sec; while the transmission to the bandpass network occurs, nominally, between 1/2 and 1 radian per sec. In Fig. 52c, the same total power ($\frac{1}{1+\omega^4}$ per unit available power) is divided among the lowpass and two bandpass networks, each of the three networks covering nominally one-third of the unit bandwidth. The choice of the relative bandwidths of the individual characteristics was arbitrary. In Fig. 52b, a midband loss of 2.2 db (referred to generator available power) occurs for the bandpass network. The midband losses of the bandpass networks in Fig. 52c are 2.2 db and 5.5 db. That such losses occur is obvious since neither the lowpass nor highpass termination powers are zero at the bandpass midband frequencies. However, for the matched generator case, one-half of the Thevenin voltage of the generator appears across each termination at the midband frequency of the corresponding network. If any of the above networks were utilized as coupling networks for a partitioned amplifier, the highpass network would presumably be incorporated to achieve a purely resistive input impedance. Its presence is necessary to achieve the transfer functions indicated in Fig. 52.

Two important points should be made. While nominally the available power of the generator can be divided among as many networks as desired within the unit bandwidth, the midband losses tend to increase for each succeeding added network of a given complexity; and the limiting capacitance at the terminal pairs associated with the channels decreases for the sets of Fig. 52 as the number of channels covering the unit bandwidth is increased. The networks of Fig. 52 are inferior to the partitioning networks of Section 4.2, measured in terms of the midband losses as well as in capacitance capability.† However, the network of Fig. 52b was used for designing the partitioning network of an experimental amplifier further illustrating the principles discussed in Section 7.2.1.

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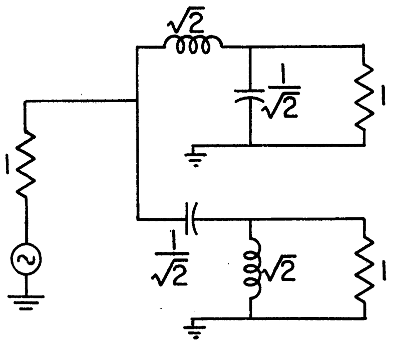
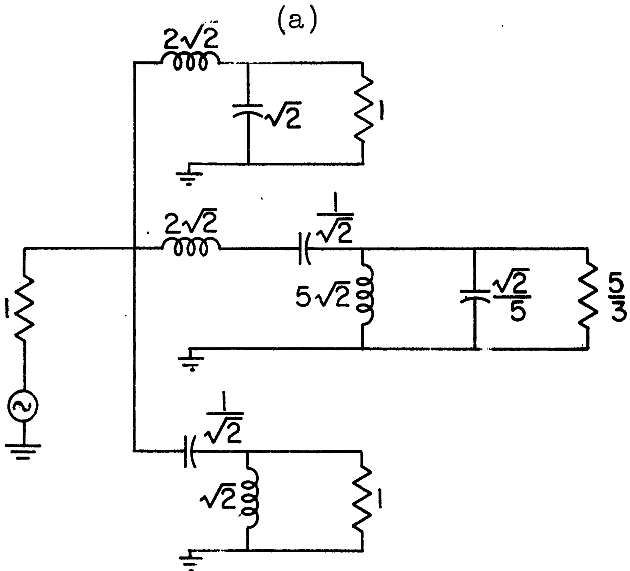
† A study of complementary sets would be necessary to establish what theoretical limitations exist.



(a)

(A) $\frac{P_{del}}{P_{avail}} = \frac{1}{1 + \omega^4}$

(B) $\frac{P_{del}}{P_{avail}} = \frac{\omega^4}{1 + \omega^4}$

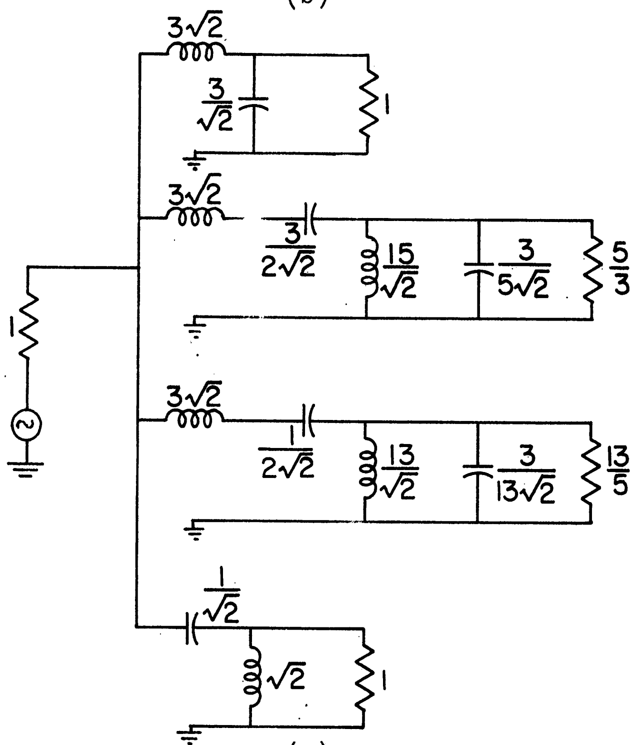


(b)

(A) $\frac{P_{del}}{P_{avail}} = \frac{1}{1 + 16\omega^4}$

(B) $\frac{P_{del}}{P_{avail}} = \frac{15\omega^4}{(1 + \omega^4)(1 + 16\omega^4)}$

(C) $\frac{P_{del}}{P_{avail}} = \frac{\omega^4}{1 + \omega^4}$



(c)

(A) $\frac{P_{del}}{P_{avail}} = \frac{1}{1 + 81\omega^4}$

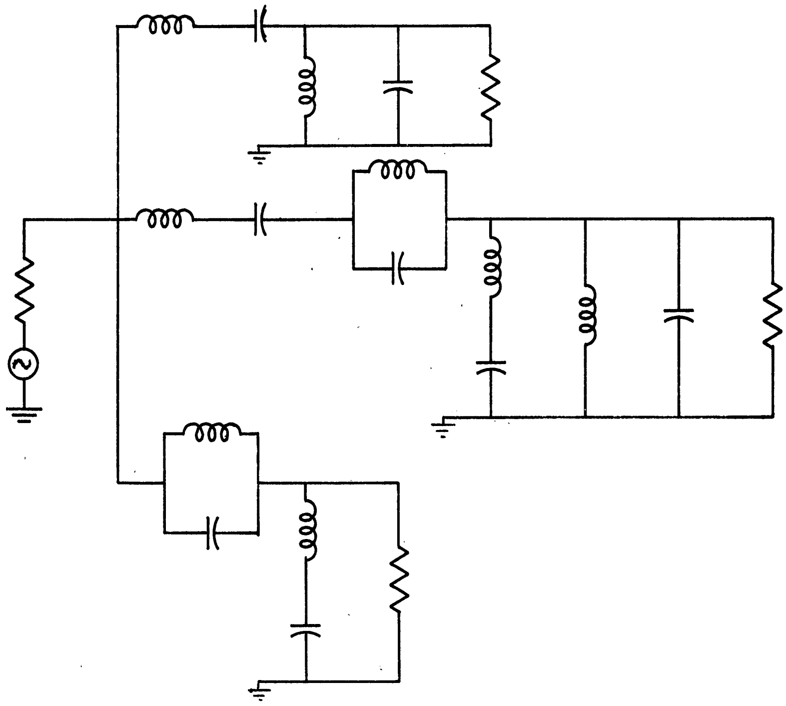
(B) $\frac{P_{del}}{P_{avail}} = \frac{15(81)\omega^4}{(1+81\omega^4)(16+81\omega^4)}$

(C) $\frac{P_{del}}{P_{avail}} = \frac{65\omega^4}{(16+81\omega^4)(1+\omega^4)}$

(D) $\frac{P_{del}}{P_{avail}} = \frac{\omega^4}{1 + \omega^4}$

Fig. 52. Complementary filter sets.

The tolerable capacity at the terminal pairs associated with the channels, in the sets of Figs. 52b and 52c, decreases with increasing center frequency of the individual network transfer characteristic. For this reason, it seems highly undesirable to use, for example, the two bandpass networks of Fig. 52c as coupling networks to a bandpass partitioned amplifier. Consider, an an alternative, the network of Fig. 53 obtained by application of a lowpass-to-bandpass transformation to each element of Fig. 52b. This structure is also a complementary set.



(A')

(B')

(C')

Fig. 53. A complementary set derived from circuit of Fig. 52b.

The network B' in Fig. 53 has an alternative realization (from Foster's canonical forms[29]) in the form of Fig. 54, which has an approximate realization in the form of the two bandpass networks of Fig. 55.†

† The transmission to the network B' of Fig. 53 is "double-humped" with frequency. In the region of one of these humps, the admittance Y_1 (see Fig. 54) is high due to the high admittance of the branch containing L_a and C_a, which is near series resonance. In this same region, the impedance Z_2 is high due to the high impedance of the circuit containing L_A and C_A, which is nearly anti-resonant. In this frequency region, then, Y_{in} is approximated with the circuit containing L_a, C_a, L_A, C_A, and R in Fig. 55.

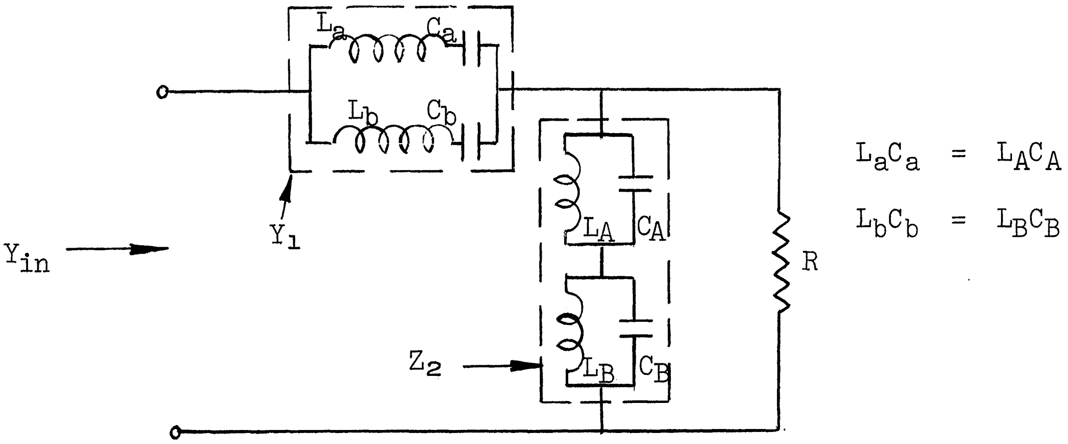


Fig. 54. Form of alternate realization of network B' of Fig. 53.

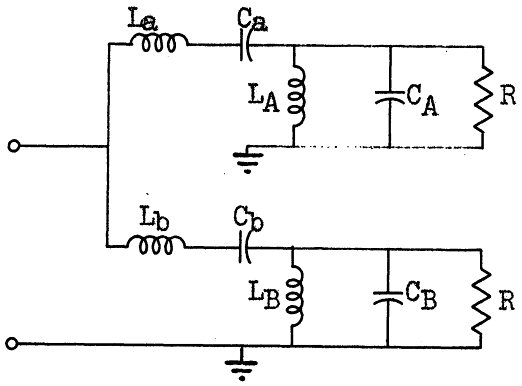
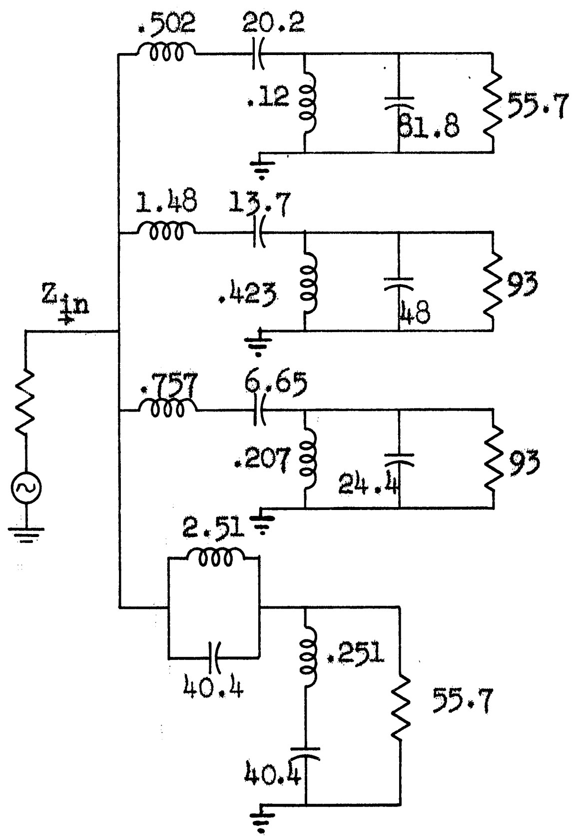


Fig. 55. Approximate equivalent of Fig. 54.

An experimental circuit, Fig. 56, derived from Fig. 53 but with the network B' replaced by the approximate equivalent circuit of Fig. 56, was constructed. The three bandpass circuits were designed to cover a nominal band of 50 mc centered at 50 mc. An impedance level R of 55.7 ohms was chosen for the experimental circuit.† The measured input impedance of this set, presented in Fig. 57, was obtained with a Hewlett-Packard 803A UHF bridge. When this

† For a previous circuit with a design impedance level of 100 ohms, input impedance measurements indicated that a minimum mismatch occurred for a source impedance of approximately 93 ohms. The objective in choosing a network with a design impedance level of 55.7 ohms was to obtain a network with a minimum mismatch with a source impedance of 53 ohms. This simplified measurements with 53 ohm coaxial lines and a 53 ohm signal generator.



(values in ohm, μh, μμf)

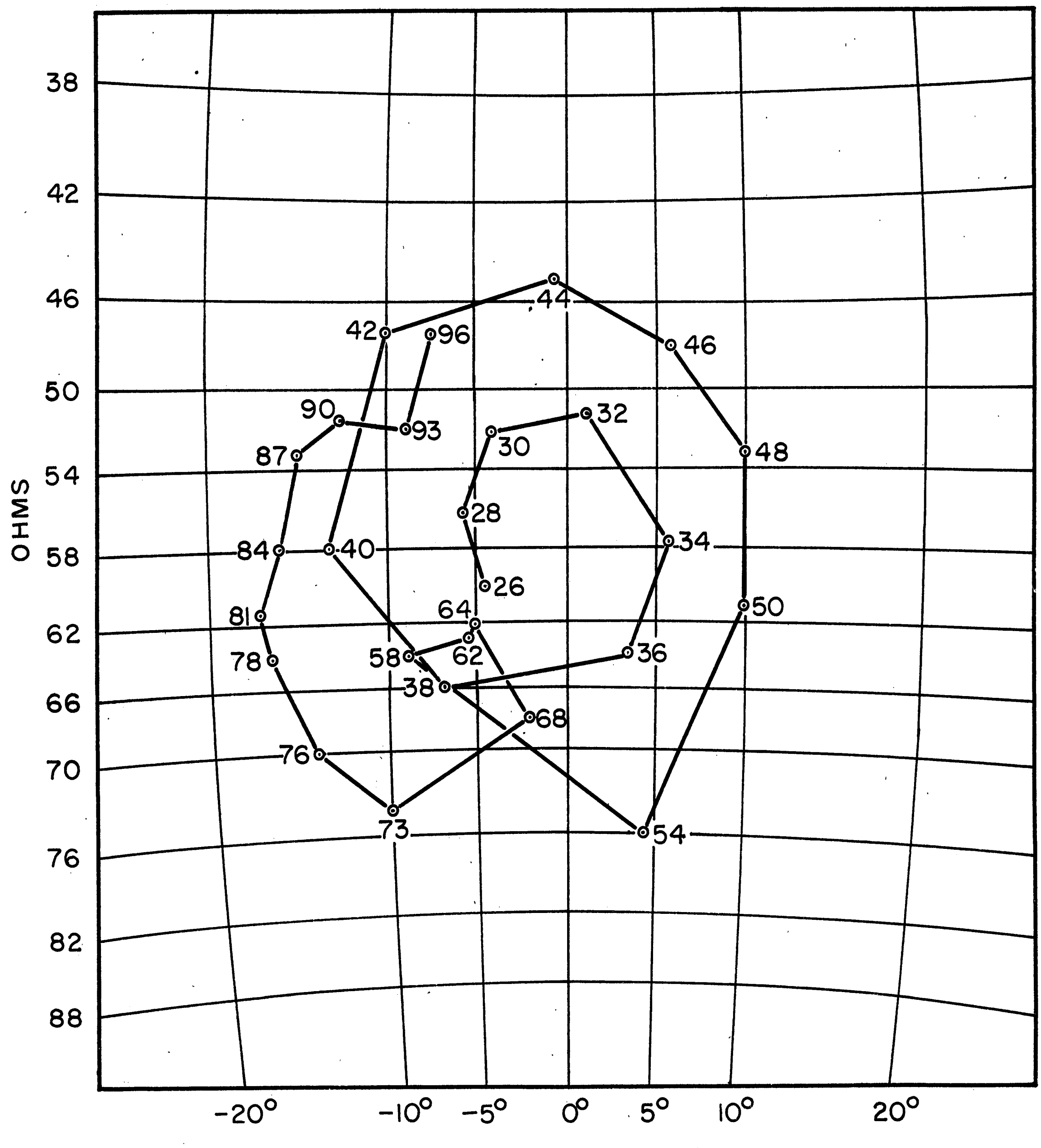
Fig. 56. Partitioning network for amplifier of Fig. 60.

structure was driven with a generator having an internal impedance of 53 ohms, the power to the several bandpass network terminations, in db below generator available power, varied as in Fig. 58. The midband losses approach the theoretical values of 0 db for the network centered at 50 mc and 2.2 db for the bandpass networks centered at 34.7 mc and 72 mc. These networks were assembled with only moderate care, and the above results were obtained without further trimming of the element values. These characteristics, as well as subsequent amplifier response curves, are presented to indicate that <u>good</u> results can be readily obtained, rather than to deomonstrate the best results obtainable with the method.

The relative phase shifts through the above coupling network at the frequencies at which the individual transfer functions cross can be estimated using the transfer functions of the derivative set of Fig. 52b. These crossings occur nominally at 3 db response frequencies, where relative phase shifts of approximately 180° exist. Consider, then, the use of three adjacent channels consisting of exact flat-staggered doubles have 3 db response frequencies coincident with the frequencies of crossings of the coupling network transfer impedances. The notes on exact flat-staggered N-tuples in Section 7.1.2 indicate that for doublet channels with coincident 3 db response frequencies, the relative phase shift through the channels at the crossover frequencies is 180°. Thus, a satisfactory adjustment of relative phase and magnitude is achieved at each crossover frequency, if the other coupling network provides a uniform coupling to all channels over the amplifier passband. The outputs of the several channels of the experimental amplifier were tied together, resonated at 50 mc, and a load resistance consistent with the overall amplifier bandwidth requirement was used.

It should be noted that no attempt was made in the experimental circuit to obtain equal bandwidth channels. This could be closely approached by more judicious choice of the relative bandwidths of the individual transfer functions in Fig. 52b. Another recourse is to choose tubes with different figures of merit for the several channels. In this way it is possible to equalize the channel midband gains, while still utilizing the various tubes at full capability. For the experimental circuit, three 6AK5's were used in each channel, and the channel gains were equalized by controlling the transconductances of the tubes.

The experimental amplifier circuit diagram is given in Fig. 60. In Fig. 59, the measured gains with the individual channels activated individually, and with all three channels operative, are presented. The technique of obtaining smooth partitioned amplifier response curves by insuring satisfactory responses at discrete frequencies in the crossover regions has been almost completely successful.



(parameter = frequency in mc)

Fig. 57. Measured input impedance Z_{in}-network of Fig. 56.

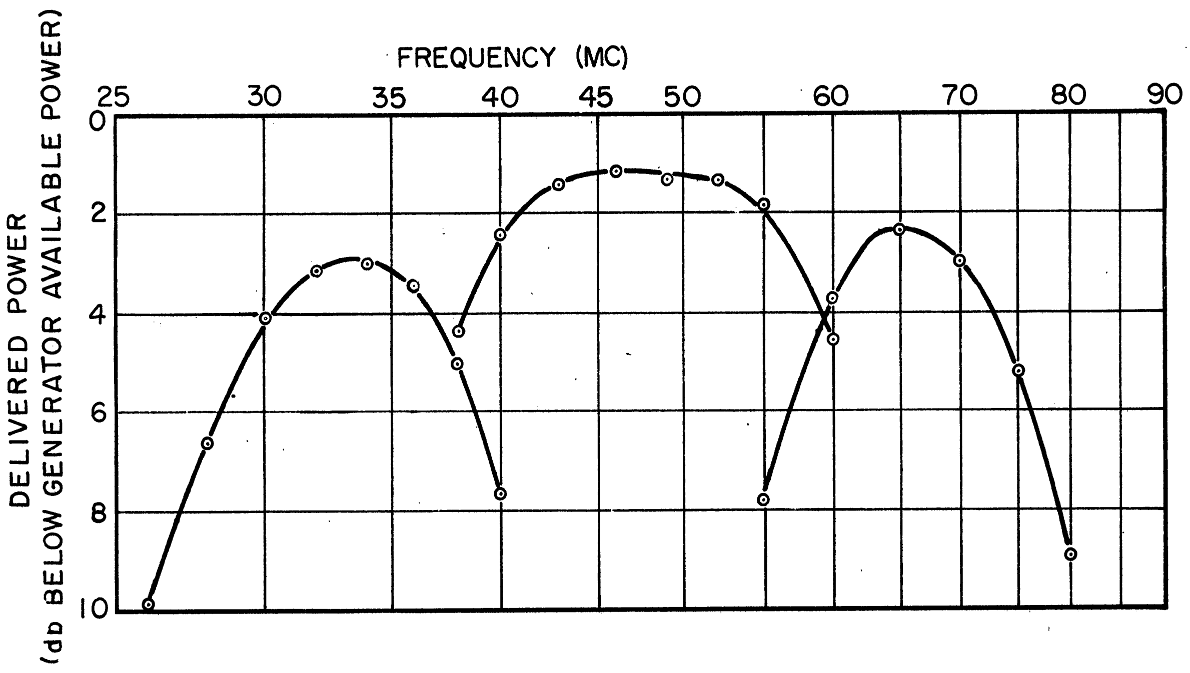


Fig. 58. Transmission efficiency-network of Fig. 56.

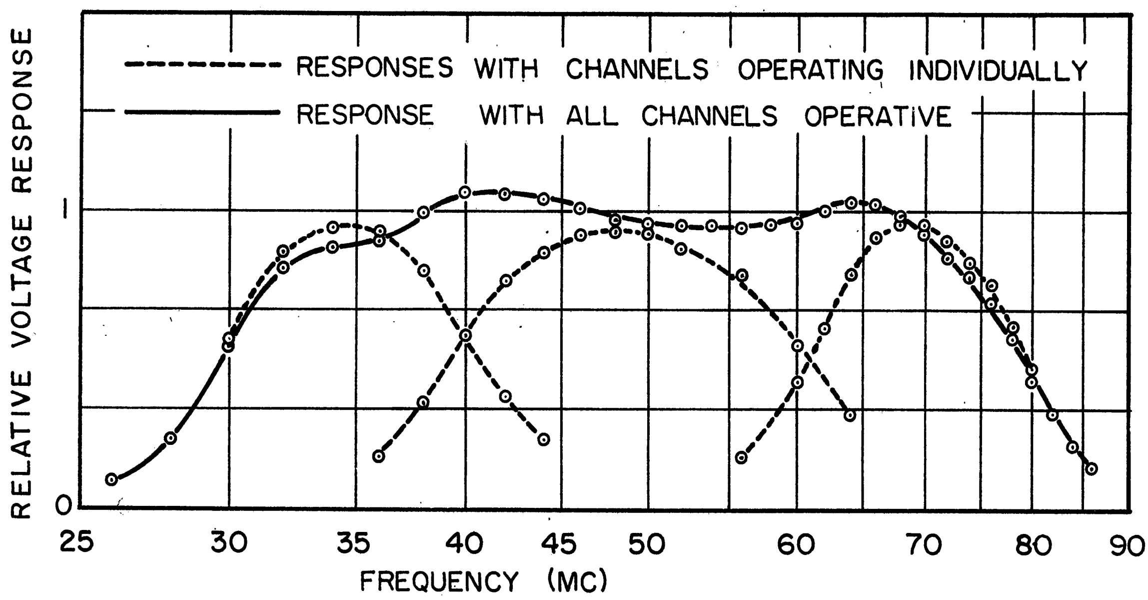


Fig. 59. Experimental response-amplifier of Fig. 60.

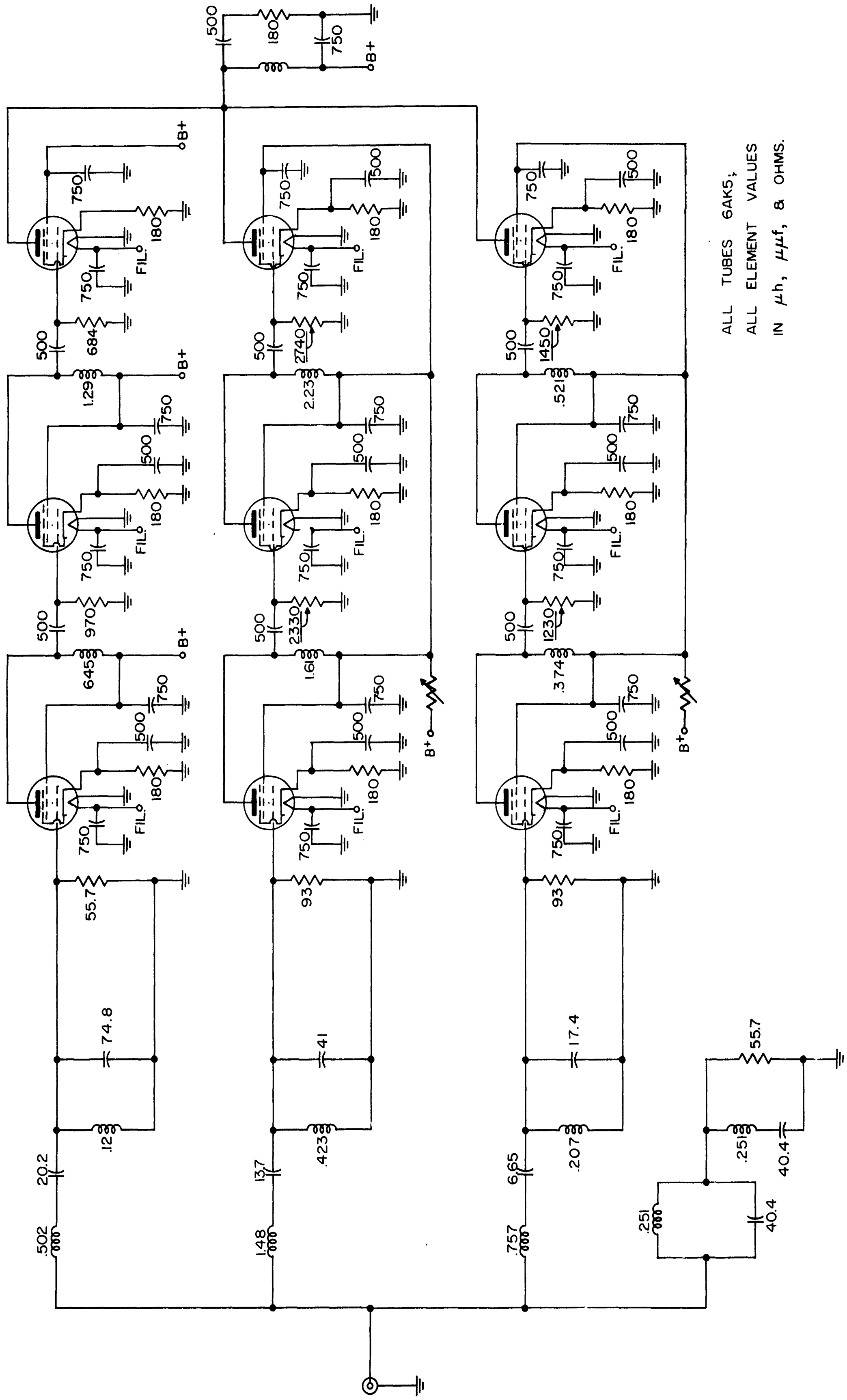


Fig. 60. An experimental three-channel amplifier.

CHAPTER VIII

SUMMARY AND CONCLUSIONS

8.1 Introduction

In the preceding chapters two direct design procedures for partitioned (multi-channel) amplifiers with controlled gain characteristics have been presented. The first of these procedures leads to amplifiers with exact prescribed gain characteristics. The second technique was used successfully to obtain amplifiers with approximately uniform gain over a prescribed band.

8.2 Summary of Results

1. It was shown in Section 3.4 that there is a simple criterion for determining the optimum number of channels, in the interests of maximizing the gain over a prescribed bandwidth with a fixed number of tubes of a given type and a given interstage complexity. The criterion is that a mean stage gain of ϵ ($\simeq 2.718$) is achieved in the individual channels. The criterion is approximate, in that losses in the input and output network are neglected.

2. It is pointed out in Chapter IV that previously published studies determine optimum coupling networks of the type which provide equal transfer impedances to all channels from the common source or load. Design techniques for coupling networks achieving selective transfer impedances (on a frequency basis) to the several channels are also presented in Chapter IV. Illustrative examples of these "partitioning networks" for the two-channel lowpass case establish their superiority over networks which provide equal transfer impedances to all channels.

3. A synthesis technique is introduced and illustrated in Chapter V, whereby partitioned amplifiers with prescribed overall gain characteristics can be obtained. The method involves the introduction of suitable all-pass factors into the overall gain function. The resulting function is then disassociated into a number of additive factors, each of which is associated with a particular portion of the circuit. Provisions for the type of input and output coupling network sought can be made in a straightforward manner, and a qualitative control of the shape of the channel responses is practical. The examples presented are confined to two-channel lowpass amplifiers.

4. It is concluded in Chapter VI that optimum gain-bandwidth amplifiers should contain optimum amplifier chains, in terms of present chain design techniques, operated at bandwidths such that mean stage gains of ϵ are obtained in each chain. Design techniques are proposed for such amplifiers with controlled gain characteristics, which represents an extension of the concepts of (3) above to obtain optimum gain-bandwidth structures.

5. An alternate design principle is introduced and illustrated in Chapter VII. It is shown that a number of conventional amplifier chains can be employed to construct an amplifier so that a relatively good approximation to the rectangular gain characteristic can be insured. The design method is based on the idea that, in a multi-channel amplifier, a number of crossover regions (in frequency) occur, and that in each of these only two substantial signal components need exist at the load. The design procedure consists, then, of choosing the circuitry in such a way that the two substantial signals in each crossover region reinforce at the load at a single frequency. A region of satisfactory adjustment of the gain in

the crossover region relative to the (equal) mid-channel gains of the amplifier is insured by the continuity of the gain and phase functions. Experimental results for amplifiers designed on this principle (both with and without partitioning networks) are presented which have amplification characteristics suitable for most applications.

8.3 Suggestions for Further Research

The results of this investigation suggest several problems which seem worthy of study.

1. A number of interesting problems associated with the design of coupling networks were enumerated at the end of Chapter IV.

2. In the design procedures presented in this study (as well as in Linvill's procedure) the basic approach is to determine suitable channel transfer functions for some original choice of coupling network type. In attempting to use or extend any of the presently available design procedures, a basic problem is the choice of desirable coupling networks.

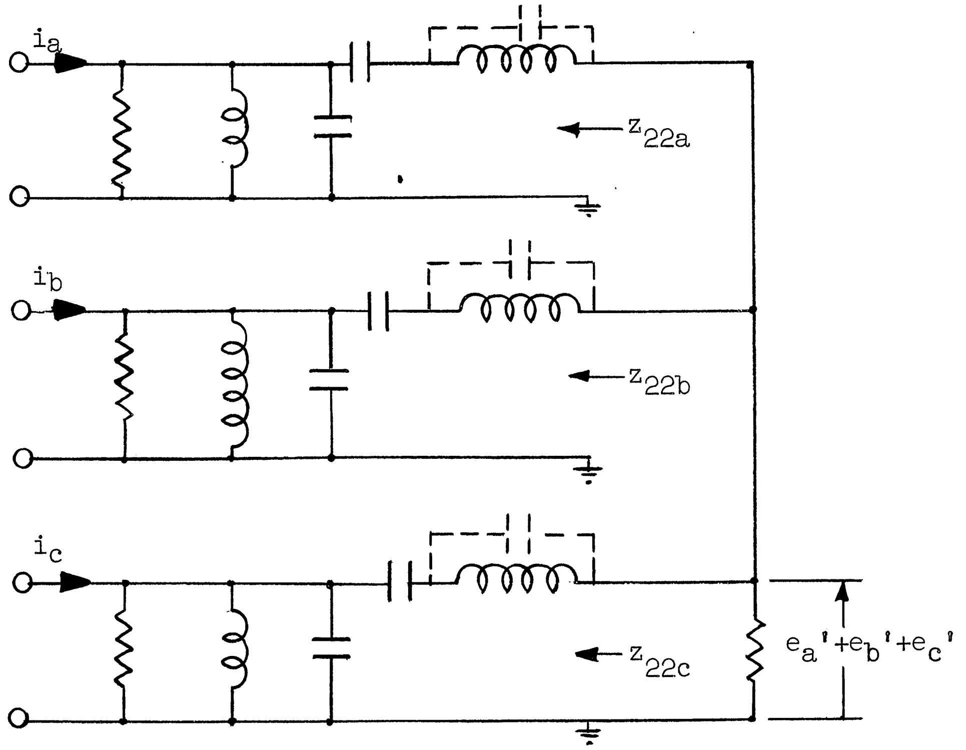


Fig. 61. Proposed partitioning network.

For the particularly important case of bandpass partitioned power amplifiers at center frequencies of hundreds of megacycles, which frequencies are only reached by (lowpass) distributed amplifiers at great cost, extreme structural simplicity is essential. Partitioning networks of the form of Fig. 61, where a three-channel structure is illustrated, would seem to be worthy of study for this application. Conceptually, the series resonant circuit associated with each channel may be regarded as a switch which is closed at the resonant frequency and open at frequencies remote from the resonant frequency. The transfer function $\frac{e'_a}{i_a}$, for example, has zeroes at the zeroes of the driving point impedances z_{22b} and z_{22c}, unless z_{22a} also has a driving point zero. The zeroes of the several transfer functions can be restricted in number, and removed from the region of substantial magnitude, by locating the zeroes of the driving point impedances in the manner suggested in Fig. 62. Note that $\frac{e'_a}{i_a}$ would

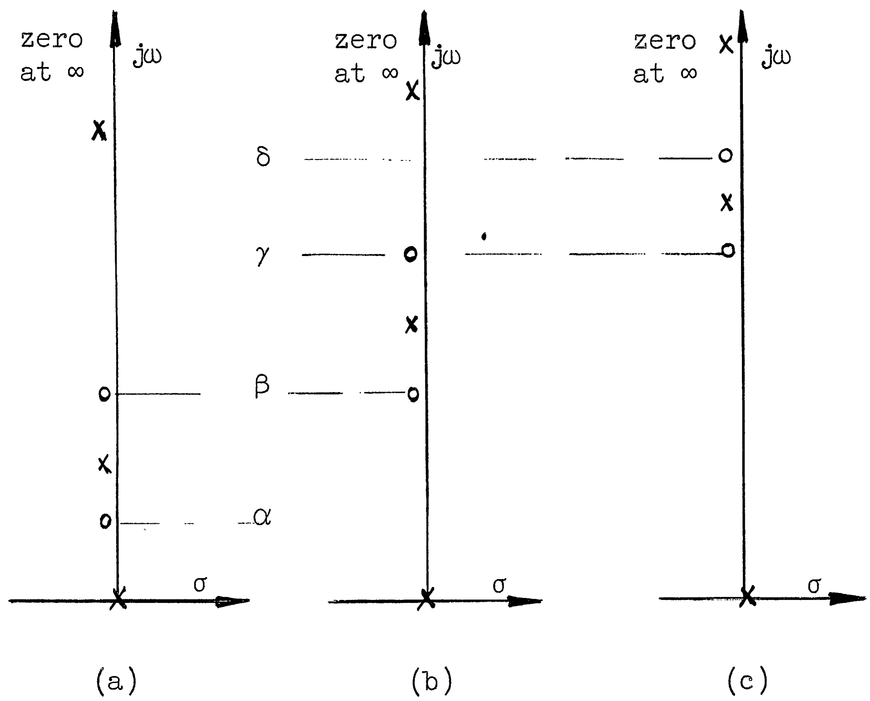
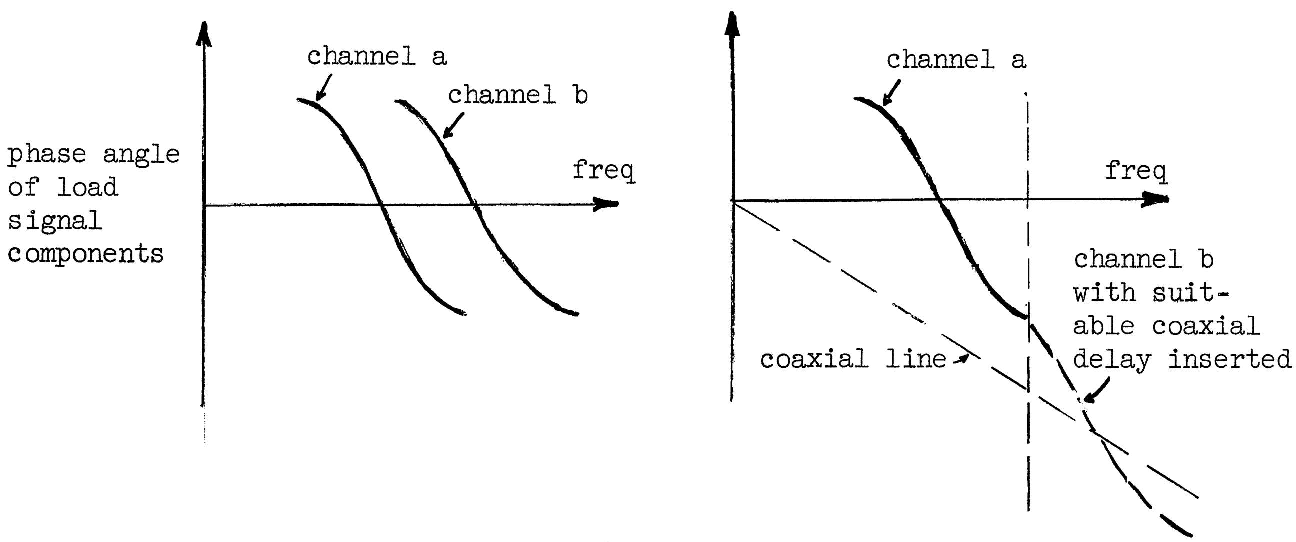


Fig. 62. Proposed pole-zero loci of (a) z_{22a}, (b) z_{22b}, and (c) z_{22c} for the partitioning network of Fig. 61.

have relative magnitude minimums at frequencies γ and δ, while smooth transmission is possible over a frequency range encompassing α and β. Similarly $\frac{e'_b}{i_b}$ would have relative magnitude minimums at frequencies α and δ, with smooth transmission in the region of β and γ, etc. Preliminary numerical studies indicate that such a circuit has substantial merit.

3. The results of further study of partitioning networks could be utilized in amplifiers designed on the basis of the principles of Chapter VII. The realization of increased gain-bandwidth advantage through the use of Tchebycheff-staggered chains in amplifiers designed on the approximate basis appears feasible.

4. The techniques of Chapter VII should be extended to obtain approximately linear-phase amplifiers. The insertion of coaxial lines in (m-1) channels could effect such an approximation in the frequency range of hundreds of megacycles, as illustrated in Fig. 63 for a two-channel bandpass amplifier. The problem of finding a suitable place for inserting



(a) Uncorrected phase angle of load signal components

(b) Phase angle of load signal components corrected with suitable coaxial delay

Fig. 63. Proposed phase correction technique.

the coaxial lines in bandpass amplifiers could perhaps be solved by using a symmetrical interstage in the necessary channels as suggested in Fig. 64.

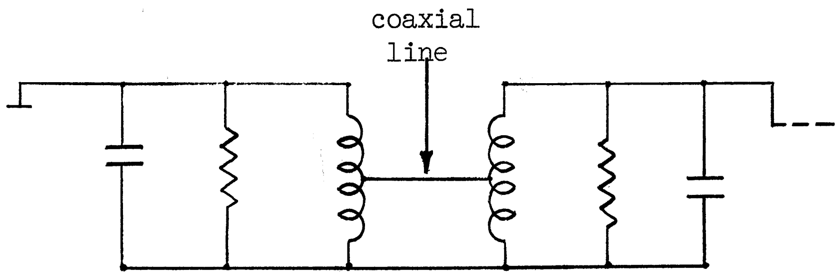


Fig. 64. Proposed interstage to facilitate phase correction.

The use of all-pass networks for phase correction is also feasible. No evaluation of the transient response of the experimental circuits of Chapter VII was made.

5. The proposed design procedures of Chapter VI should be brought to fruition. The further study of the coupling network problem proposed above could profitably precede this undertaking, particularly if the objective is to synthesize high-frequency power partitioned amplifiers.

APPENDIX A

THE REALIZATION OF THE INTERSTAGES OF FIG. 42

The transfer function of the network of Fig. A.1 is

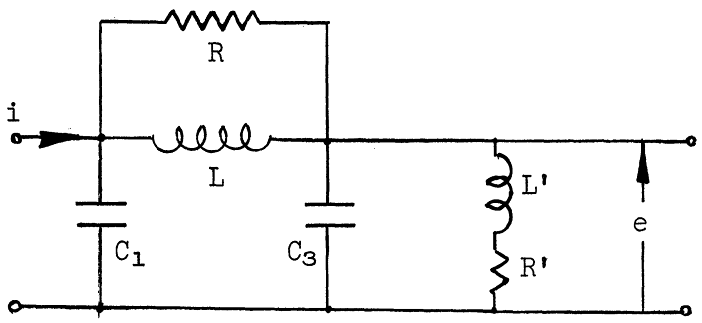


Fig. A.1. A series-shunt-peaked interstage.

$$\frac{e}{i} = \frac{(p+u)\ (p+v)}{p^4RC_1C_3+p^3[C_1+C_3+vRC_1C_3]+p^2\left[(u+v)(C_1+C_3)+\frac{RC_1}{L'}\right]+p\left[uv(C_1+C_3)+\frac{1}{L'}\right]+\frac{u}{L'}} \tag{A.1}$$

where $u = R/L$ and $v = R'/L'$. To realize the transfer function of Fig. 42

$$\frac{e}{i} = \frac{(p+1)\ (p+A')}{(p^2 + 1.987p + 1.059)\ (p^2 + 1.932p + 1)}$$

$$= \frac{(p+1)\ (p+A')}{(p^4 + 3.919p^3 + 5.899p^2 + 4.034p + 1.059)}\ , \tag{A.2}$$

one may equate coefficients in Eqs. A.1 and A.2. The results are:

$$RC_1C_3 = 1\ , \tag{A.3}$$

$$C_1 + C_3 + v = 3.919\ , \tag{A.4}$$

$$(u+v)(C_1+C_3) + \frac{RC_1}{L'} = 5.899\ , \tag{A.5}$$

$$uv(C_1+C_3) + \frac{1}{L'} = 4.034\ , \tag{A.6}$$

$$\frac{u}{L'} = 1.059 \quad . \qquad \text{(A.7)}$$

It is further necessary to associate the zero loci u and v of Eq. A.1 with the A' and 1 of Eq. A.2. Equations A.4, A.6, and A.7 yield

$$uv(v - 3.919) + \frac{1.059}{u} = 4.034 \quad . \qquad \text{(A.8)}$$

For v = 1, a useful network results for u = A' = .798 from Eq. A.8. The resulting network is readily determined from Eqs. A.3 to A.7, and is given in Fig. 42.†

For the network of Fig. A.2, the transfer function is:

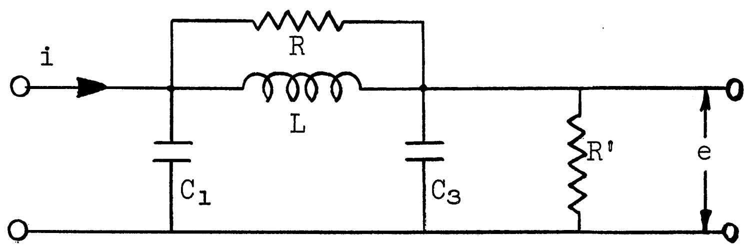


Fig. A.2. A series-peaked interstage.

$$\frac{e}{i} = \frac{p+u}{p^3RC_1C_3+p^2\left[(C_1+C_3) + \frac{RC_1}{R'}\right] + p\left[u(C_1+C_3) + \frac{1}{R'}\right] + \frac{u}{R'}} \qquad \text{(A.9)}$$

where u = R/L. To realize the transfer function of Fig. 42

$$\frac{e}{i} = \frac{.0538(p + 1.837)}{(p + .463)\ (p^2 + .299p + .201)} \qquad \text{(A.10)}$$

one may consider

$$\frac{e}{i} = \frac{p + 1.837}{p^3 + .762p^2 + .339p + .092} \quad . \qquad \text{(A.11)}$$

† For u = 1, a solution v = A' = 3.36 was used in obtaining the realization of Fig. 43.

Equating coefficients in Eqs. A.9 and A.11, the network of Fig. A.3 is obtained. The network realizing the transfer function of Eq. A.10 is obtained by an impedance level transformation. The result is given in Fig. 42.

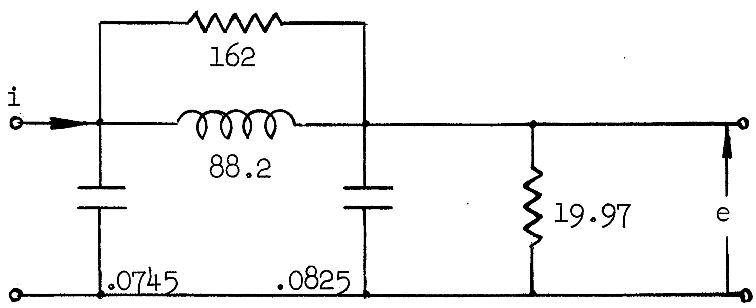


Fig. A.3. Network realizing transfer function of Eq. A.11.

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