THE UNIVERSITY OF MICHIGAN

INDUSTRY PROGRAM OF THE COLLEGE OF ENGINEERING

TURBULENT PROPANE-AIR FLAMES
STABILIZED IN SMOOTH CERAMIC TUBES

(Thomas David)

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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan
Department of Chemical and Metallurgical Engineering
1962

December, 1962

IP-598

UNION

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ACKNOWLEDGEMENTS

TABLE OF CONTENTS

TABLE OF CONTENTS (CONT'D)

LIST OF TABLES

v

LIST OF FIGURES

LIST OF FIGURES (CONT'D)

LIST OF APPENDICES

NOMENCLATURE

A   =   area, ft.^{2}

A(x,θ), A(I)   =   oxygen concentration, lb./lb. gas mixture.

AER(I)   =   annular area defined in Equation (32).

B(x,θ), B(I)   =   propane concentration, lb./lb. gas mixture.

C   =   constant defined in Equation (34)

C_p =   heat capacity, Btu/lb.°F

D   =   diameter, ft.

DR   =   wall thickness and/or radial increment thickness, ft.

DT   =   time increment, hr.

DX   =   size of axial length increment, ft.

E   =   energy of activation, Btu/lb.mol.

$\mathcal{F}_{xy}$ =   compensated view factor as defined in Equation (15).

F_{xy} =   view factor

F_1, F_2 =   heat flux associated with flow, Btu/hr.

G   =   mass flow rate, lb./ft.^{2}hr.

h_g =   heat transfer coefficient between bulk gas and tube wall, Btu/ft.^{2}°F.

I   =   radial subscript

J   =   axial subscript

k_g =   thermal conductivity of gas, Btu/hr.ft.°F.

KRX   =   reaction rate constant, ft.^{3}/lb.hr.

k_w, KCER   =   thermal conductivity of tube wall, Btu/hr.ft.°F.

N_{Re} =   Reynolds number, dimensionless

$Q_1 \ldots Q_7$ = heat fluxes as defined in Figure 13, Btu/hr.

R_O = gas constant, $Btu/lb.mol.^{\circ}R$.

R_W = wall radius, ft.

$R(I)$ = radius of Ith increment, ft.

t = time, hr.

T_S = surface temperature, $^{\circ}R$.

$T(x,\theta)$, $T(I)$ = gas stream temperature, $^{\circ}R$.

U_O = bulk velocity, ft./hr.

V = volume, $ft.^3$

$V(I)$ = axial velocity in the Ith radial increment, ft./hr.

$VR(I)$ = radial velocity at the Ith increment, ft./hr.

w = mass flow, lb./min.

$X(J)$ = wall temperature, $^{\circ}R$.

x = axial location from top of tube, ft.; flame stabilization point, Equation (1) ft.

Y = subscript in the axial direction

β = constant defined in Equation (1)

ϵ = radiant emissivity, dimensionless

θ = time subscript

π = ratio of circumference to diameter of a circle, dimensionless

ρ = density, $lb./ft.^3$

σ = constant in Stefan-Boltzmann law, $.1714 \times 10^{-8}$, $Btu/ft.^2 hr.^{\circ}R^4$

φ = fuel-air ratio, lb. propane/lb. air

Δ = heat of reaction, Btu/lb. propane

ABBREVIATIONS

Psia = absolute pressure, $lb._f/in.^2$

Psig = gauge pressure, $lb._f/in.^2$

SCFM = standard cubic feet per minute (at $60^{\circ}F$ and 1 atm)

TURBULENT PROPANE-AIR FLAMES
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ABSTRACT

Turbulent, propane-air flames were stabilized in smooth, insulated ceramic tubes without the use of flameholders in the flow stream. The tubes were insulated so that the radial heat flux was negligible compared to the heat generated in the reaction. Some difficulty was encountered in fabricating an experimental system which would contain the flame and still maintain a smooth surface. A high purity Al_2O_3 tube 1-inch in inside diameter, 1 1/4-inch in outside diameter, and 24 inches long, surrounded by a shell of phosphate bonded Zr_2O_3 cement 7 inches in diameter, provided a satisfactory solution to the problem. Data were taken on axial tube-wall temperature profiles and flame stability limits for lean flames as a function of mass flow rate and fuel-air ratio. Stability is defined experimentally in this study by the wall temperature profiles. If the wall temperature profile changed less than $10^\circ F$ at each measuring point over a period of half an hour the operating conditions were termed stable. The stable fuel-air ratio ranged up from .049 lb. propane/lb. air and the range of stable mass flow ranged from .45 to 1.35 lb. gas mixture/minute with the highest flow rates being observed at the stoichiometric fuel-air ratio. These flow rates correspond to a range of 4,950 to 14,850 lb. gas mixture/ft.^{2}hr. and to Reynolds Numbers of 9,700 to 29,100 based on the cold, entering gas stream.

xi

A mathematical model was proposed in which the importance of back-radiation from the tube wall downstream from the reaction zone to the tube wall upstream of the reaction zone was stressed. The actual mechanism must be one in which flame stabilization is promoted by heating the gases in the boundary layer ahead of the reaction zone to the combustion point by the hot tube wall. The flame will then spread to the rest of the gas stream by turbulent mixing. It became obvious that the mathematical description of a model based on the above ideas was much too complex to solve with the algorithm presently formulated. Various limiting approximations were investigated and it was found that a "plug flow" model assuming no radial effects in the gas stream predicted velocities much lower than the observed. Another approach was to assume that the reaction went to completion across the tube when the tube wall reached a certain temperature. However, in order to avoid instability of the partial differential-difference equations involved it was necessary to make the time increment so small that convergence required a prohibitive amount of 7090 time.

The observed flame was at no time a smooth burning flame such as one observes in a bunsen flame. It is best characterized by its randomness, sometimes flashing back up into the tube, sometimes 3 to 4 inches out of the end of the tube but never in any periodic fashion. The noise levels were never any higher than that of a loud truck horn. It was recommended that the study be extended to consider the effect of the variation of diameter at constant length-diameter ratio.

I. INTRODUCTION

Considering the fact that man has known and used fire for the past 50,000 years[6], there is still remarkably little known about it. Only since about 500 BC has man begun to regard fire with interest rather than suspicion. The past fifty years have produced a great deal of scientific research, both theoretical and applied on the subject of combustion.

Flames are exothermic, continuous chemical reactions usually characterized by some luminescence caused by incandescent particles in the reaction zone. The most important single thing about flames is their stabilization, i.e., keeping them burning. In gaseous flames, with which we are concerned in this study, this implies supplying enough heat to the unburned reaction mixture to cause it to become a part of the flame. There are several different mechanisms by which gaseous flames may be stabilized. In stagnant gases the flame is propagated by conduction and free convection where it applies. In flow systems the flame is usually stabilized by conduction and diffusion of "active radicals" or by some forced mixing of the burned and unburned gases. This mixing is quite often promoted by mechanical means. This involves creating eddies in the flow profile of the stream by causing it to flow around sharp-edged bodies. This type of stabilization is usually referred to as "bluff-body" stabilization. Flames can also be stabilized by causing them to flow over or around heated surfaces. In this case the flame occurs first in the boundary layer close to the hot surface, spreading to the rest of the reaction mixture by mixing and conduction. This phenomenon is known as boundary-layer stabilization. Various authors[3, 45] have observed stabilization

-1-

of this type in systems in which it was unnecessary to supply heat to the

hot surface. This study is of this type in that it uses the products of

the combustion reaction indirectly to furnish the heat required to warm

the hot surface. The probable mechanism is as follows: (1) The boundary

layer gases are heated to the ignition point by convection from the hot

surface and then, (2) this reaction spreads to the rest of the gas stream

by mixing, raising it to some fairly high temperature; (3) the hot gas

stream heats by convection the boundary surface downstream from the reac-

tion zone and (4) heat is transferred from it both by conduction and

thermal radiation to that part of the surface which initiated the reaction.

It has long been observed that when a combustion reaction is

carried out in a region which is bordered by ceramic materials, the com-

bustion reaction goes farther toward completion and has a greater stabil-

ity both with respect to flow rate and fuel-oxidizer ratio than any flame

not stabilized by artificial means. At the turn of the century, making

use of this property, Bone and M'Court[3] designed and patented ceramic

burners in which the combustion mixture flowed through a porous ceramic

plate. They attributed the desirable properties of these burners to

the "catalytic" action of the ceramic surface on the combustion reaction.

Although they offered no proof of any catalytic reaction, this concept

has been accepted and carried through to the present time[1]. The work

of Khitrin and Solovyeva[21] indicates that the mechanism may not be

catalytic in all cases. Their data indicate that the combustion surface

has little effect on the reaction for tube diameters greater than 5 mm.

It should be noted here that the interstices in Bone's burner were much

smaller than 5 mm circles. It was originally intended that this study should be an extension of the work of Khitrin and Solovyeva confirming their results and extending their ideas to include heat transfer along the tube by thermal radiation.

The experimental program which was initially planned was to determine the stability limits of propane-air flames with respect to both flow rate and fuel-air ratio for various insulated ceramic tubes. The tube sizes and materials were to be varied to show the effects of diameter and surface emmissivity on the flame stability. This was to be tied in with a mathematical model, an extension of earlier seminar work by Winer[48] and McCarty[31]. The extended model assumed: (1) That there was no radial heat flux, (2) that the turbulent flow could be treated as "plug" flow with no wall effects, and (3) that back mixing was unimportant both for heat transfer and in its effect on the second order Arrhenius-type reaction rate equation which was assumed. The mathematical model in finite difference form was to be run on the IBM 709 in an attempt to duplicate the experiments.

As the investigation progressed, it became evident that the experimental program outlined above was overly ambitious and further that the mathematical model as originally proposed was naive in certain very important factors concerning the stabilization process. A major difficulty encountered in executing the experimental program was in finding a ceramic material which could contain the reaction without losing the smoothness of interior surface which was necessary in order that there by no eddies in the boundary layer flow at the tube wall. This same

boundary layer flow was the neglected factor in the "plug flow" approximation which prevented it from giving reasonable results. It thus became necessary to consider a two-dimensional problem with a variable boundary condition instead of a one-dimensional one. A closer look at the model showed that the eddy diffusivities for heat and mass transfer in a reacting system must also be considered (see Mathematical Models). A satisfactory solution to the rather involved heat and material balance equations which result has not yet been obtained. The materials problem was finally circumvented after nearly a year of full time effort. The resulting experimental system was such that it would be expensive and time consuming to vary tube diameters and materials. Also, there was no guarantee that data could be taken with the tube wall materials other than the one used. Thus, the purpose of the research has been changed to developing a system for containing turbulent, propane-air flames in a smooth refractory tube and measuring the effect of fuel-air ratio and mass flow rate on the stability of these flames.

This thesis presents the results of the experimental and mathematical work done with the above objects in mind. There are data on the effects of fuel-air ratio and mass flow rates on the stability limits and temperature profiles observed in a one-inch, insulated alumina tube. The phenomenon of stabilization which occurs on the smooth tube walls with no external heat addition is discussed extensively, and the results of the two attempts at mathematical analysis of the system are set forth. The data and observed phenomena are compared with those found in several sources in the literature and analyzed.

II. LITERATURE SURVEY

In spite of, or perhaps due to, the fact that the field of
heat transfer from flames and the stabilization of flames is one of the
oldest "arts" known to man, relatively little had been done in the way
of basic scientific studies until about 1938. Since that time, the
science of the study of flames and their properties has grown rapidly.
As an example of this growth we can consider the frequency of interna-
tional technical meetings devoted entirely to combustion phenomena.
The first two large symposia on combustion were held in 1928 and 1937
under the auspices of the American Chemical Society. Since 1948, how-
ever, some seven International Symposia on Combustion have been held.
Thus, because of the great amount of recent work which has been done in
this field, a cursory survey of the general "state of the art" of gase-
ous combustion will be made and tied in with the principles of chemical
reaction, heat and mass transfer, and fluid flow. A more detailed sur-
vey of the combustion literature directly pertinent to this work will
then follow. The topics covered will be laminar stability, turbulent
stability without flame holders, turbulent stability with flame holders,
general related topics, and the stabilization of flames by boundary
layer effects.

1. Laminar Stability

Since most applications of flames require them to take place
essentially instantaneously or in a controlled manner at a certain point
in a flow system, the question of whether or not the flame (combustion

reaction) will occur as desired is of paramount importance. When the flow rate in question is laminar, the problem is known as one of laminar stability or calculation of the laminar flame speed. Since laminar flow is more amenable to mathematical description than turbulent flow, there has been more basic theoretical work done on it. One of the earliest analytic considerations of this problem was made by Mallard and LeChatelier [28] for one-dimensional adiabatic, laminar flames, assuming that heat conduction was the only mechanism important in stabilizing the flame. Their theory depends on the existence of an "ignition temperature" beyond which the heat given off by the reaction is no longer negligible. This so-called "thermal theory" has been studied in a more sophisticated manner by many authors including von Kármán and Millan[43], who calculated the range of ignition temperatures which may be assumed without affecting the value of the flame speed. Tanford and Pease[40] assumed that the diffusion of active radicals was the controlling factor in stabilizing the flame and that the concentration of activated complexes in the mixture determined the reaction rate. The true mechanism must naturally encompass both of these concepts. Attempts have been made to predict more accurately the values observed experimentally both by numerical techniques[15] and by making various simplifying assumptions either about the thermal or mass transfer properties of the system[25, 44. 49]. None of these techniques gives good agreement with experimental results. However, most of the approximate techniques lead to agreement as good as or better than the laborious numerical methods of Hirschfelder[15]. Fristrom[11] has reviewed the data available on temperature and concentration gradients. There

is also a considerable amount of more recent work in the 7th, 8th, and 9th

Combustion Symposia. In general, combustion velocities are measured by

measuring the volume of gas encompassed by a flame front over a measured

length of time and then dividing it by the area of the flame front[14].

The values recorded for propane-air mixtures[14] are around 35 cm/sec. It

is much more difficult to measure these parameters when dealing with a

laminar flame stabilized within a tube. Flame velocities have been meas-

ured by Coward and Hartwell[7], however, and their investigations showed

that the gas velocities for the unburned gas were higher than the average

stream velocity close to the tube wall (except for the boundary layer)

while for the burned gas they were higher at the center. Data by Lewis and

von Elbe[23] and by Mentser and von Elbe[30] show that the upper (blowoff)

and lower (flashback) stability limits for bunsen burner type flames go

to smaller flow rates as the diameter of the burner tube is decreased.

2. Turbulent Flame Stabilization

The material covered here refers to flames stabilized in turbulent

flow without the use of flame holders. Such data as are available show very

little correlation with the mathematical model (the so-called "wrinkled

laminar flame model") which has been set forth by Damköhler[8] and

Shchelkin[35]. This model predicts that the turbulent flame velocity at

high Reynolds numbers will be independent of the fuel and linearly dependent

on the Reynolds number. Experimental data taken by Bollinger and Williams

[47] show very little agreement with the above model. The data also show

a decrease in burning velocity with decreasing burner diameter and range up

to as high as 80 cm/sec for propane-air mixtures in a 1-1/8 inch diameter

tube.

Since the data seemed to disagree markedly with the wrinkled-flame model, several investigations were initiated to attempt to get at the heart of the matter. One of the first of these was conducted by Summerfield who, in 1955[39], presented data taken in an attempt to test the fluctuating laminar flame model. The results obtained clearly indicate that the wrinkled flame model is a failure. According to Summerfield, they indicate that the turbulent flame is really a zone of reaction distributed in depth having smooth spatial variations in the time average values of temperature and composition. A mathematical model based on this assumption seems to fit the data fairly well, particularly at high intensity turbulence. However, the approach turbulent diffusivity and the thickness of the turbulent reaction zone which must be evaluated seem to be somewhat arbitrary.

Toong[41] studied ignition and combustion in the laminar boundary layer of flow over a hot plate. He developed, by the numerical integration of a somewhat idealized model, equations for the velocity, temperature and concentration profiles in such a boundary layer. His predicted values seem to fit his data well.

Khitrin and Gol'denberg[19] assumed that the flame would stabilize at the point where the heat generation by the reaction exceeded that removed from the gas stream by radiation and convection. This so-called "thermal" theory has been the predecessor of a number of papers by the above and associated authors which are considered to be quite significant by most reviewers. The development given allows the prediction of allowable concentration limits, burning velocities and flame front stabilization criteria with the evaluation of certain constants.

In the next paper in the series[20] Khitrin considered the effect of non-uniform heat transfer rates in various parts of a body bounding a flame as applied to the thermal model.

Shetinkov[37] came up with a new approach in a paper in the 7th Combustion Symposium. He agrees with Summerfield that there is little likelihood of there being laminar flame fronts in strongly developed turbulence. He assumes a homogeneous burning volume "micro-model" which gives results in agreement with experiment. He feels that at very high turbulence distributed homogeneous burning exists but that the amount of fuel which may burn up in "wrinkled" laminar flame fronts increases with decreasing degree of turbulence. Shchelkin, who was one of the first proponents of the wrinkled flame model[35], seems to agree completely with Summerfield and Shetinkov in a paper published in 1960[36]. In a more recent paper applying Khitrin's work on the thermal theory in the 7th Symposium (International) on Combustion to the ignition of a stream flowing past a hot, flat plate, Gol'denberg [12] applies mathematics to gain some interesting results. He notes that since Toong[41] has established by numerical integration of the mass, energy and momentum equations of the system, that chemical reaction has little initial effect on the velocity and temperature profiles in the boundary layer, one must assume a sharp variation in the temperature profile in a narrow zone along the plate. This leads to the result that the flame stabilization point is directly proportional to velocity at constant fuel/air ratio and surface temperature, and further that

$$\frac{X}{U_o} \cong \beta \, e^{\left(E/R_o T_s\right)} \tag{1}$$

3. Bluff Body Stabilized Turbulent Flames

The majority of interest in the applied use of flames requires the stabilization of flames with high sensible heat outputs and therefore at correspondingly high flow rates. These necessarily turbulent flames are usually stabilized by so-called "bluff body" flame holders. These bodies promote the stability of the flame by mixing the hot, reacted gas with some of the unburned inflammable mixture in the wake of an object which partially blocks the stream of flow. This continual recirculation creates a flame front in the wake of the baffle which may gradually extend the combustion zone to the rest of the stream. Flame stabilization by small scale bluff bodies of various geometries has been studied by many investigators, including Longwell[26] on parallel cylinders, Scurlock[34] on normal cylinders, deZubay[10] on discs, and Weir et al.[45] on spheres.

Several workers have attempted to analyze this phenomenon. Since the flow rates which can be obtained with bluff body flame holders are several orders of magnitude greater than those reached by other techniques, there is much interest in being able to fit the observed data to a mathematical model for design purposes.

Some of the best data in the literature on bluff body stabilized flames are those of Zukoski and Marble[51, 52]. They indicate stream velocities of the order of 700 ft/sec as compared to the laminar burning velocity of about 2 ft/sec.

Longwell[26] felt that the recirculation zone could be treated as a homogeneous reactor in the manner of Avery and Hart[2]. Spalding[38]

felt that the stability was the result of a combination of three effects:
the mixing of hot and cold jets of gas, stabilization by two standing
vortices, and a recirculation of part of the reaction mixture. Cheng and
Kovitz[5] fully develop a theory of stabilization of a flame on a gutter
(v-type) flame holder, going to great lengths to consider chemical reaction
and fluid flow (including heat transfer) as equally important in the flame
stabilization mechanism. This work is based on Adamson's initial paper
[29] and an extension of it by Cheng and Kovitz in the Sixth Symposium[6].

4. General Related Topics

As can be seen from glancing through the previous pages of this
section, flame research has become a very broad field. Because it requires
knowledge of almost all of the basic engineering phenomena (fluid flow,
heat transfer, mass transfer, and chemical reaction), there are men of
almost all of the various engineering disciplines doing some sort of work
in the field. This has led to a great deal of confusion in nomenclature.
The fact that there is such a broad gap between experiment and theoretical
understanding has further complicated the situation. One of the best
source books in the field is the 1961 edition of Lewis and von Elbe[24].
Its major headings are: chemical kinetics, flame stability, considerations
of the burned gas, and problems in technical combustion processes. Through-
out the book they try to clarify the ideas of various workers by present-
ing the assumptions made in terms of fundamental scientific concepts. In
this new work they give very little attention to their older theory of
excess enthalpy which they now say has been demonstrated to be unprovable.

5. The Effects of Boundary Layers on Flame Stability

It has always been accepted as fact that combustion processes cannot be considered independently of the wall effects of their boundaries. This has led to much discussion in the field as to whether or not surface reactions or boundary layer concepts should be used to describe the effect of these surfaces. In some fairly recent studies, Gross[13] studied flame stability in the laminar boundary layer behind a flat plate, noting that the plate itself acted as a heat sink, forcing the flame to stabilize in back of the thin plate. In a 1957 paper Hottel, Toong, and Martin[16] studied the stability of a lean propane-air flame in the boundary layer along a water-cooled, slender rod. As expected, they observed that the distance from rod tip to flame decreased as velocity decreased, as coolant temperature increased, and as the propane-air ratio increased toward stoichiometry. In this study we are really interested in the effect of heating the walls which bound the flame zone. There has been a good deal of conjecture as to whether or not the hot surface catalyses the chemical reaction of the flame or just causes stability by a boundary layer heating mechanism. Before World War I, Bone and M'Court[3] patented a device which they called a diaphragm heater. This heater consisted of a porous ceramic plate in which the flame was stabilized within the plate. The flame produced had almost no incandescent carbon entrained in it and was thus known as a "flameless flame". Bone and M'Court indicated that the flame was stabilized due to the catalytic action of the ceramic surface. Because much higher efficiency was obtained using burners of this type (97% of theoretical heat used), there was a great deal of interest in industry for

the use of the burners. As the burners were widely accepted, so was the catalytic hypothesis. As late as 1960[1] a "news report" in an industrial magazine described a ceramic burner under a patent by Mr. Gunther Schwank of Germany saying that: "Catalytic action of the ceramic material gives surface combustion on the ceramic material's outer surface." There is no evidence for any catalytic action of ceramic surfaces in any oxidation reactions except at relatively low temperatures. In an apparent attempt to resolve this problem once and for all, Khitrin and Solovyeva [21] considered the possibility of both homogeneous and heterogeneous reactions in an oxidation process. Their data and theory were in good agreement and gave the results that for tube diameters greater than 1 cm and wall temperatures of 1100°C surface processes were negligible. They were even less important at 1400°C. As tube diameters went below 1 cm surface effects increased rapidly becoming equal to homogeneous effects at about 0.5 cm.

Ziemer and Cambel[50] have conducted an interesting study on "Flame Stabilization in the Boundary Layer of Heated Plates". They indicated that they could explain their data with a bunsen-burner-type mechanism. Although there is some question about some of the empirical extrapolations used in the comparison, fair agreement was obtained, and therefore, their choice of stabilization mechanism seems valid, at least for lower temperatures. It should be noted that they studied a laminar boundary layer. Turcotte[42] made a similar study but was concerned with the "Stable Combustion of a High Velocity Gas in a Heated Boundary Layer". He felt that the hot surface (above 1700°F) produced stability

by continuous ignition of the gas flowing across it. The characteristic
length required for ignition was a systematic and reproducible variable.
Turcotte says that because of this the stabilization observed can be
explained in terms of the continuous ignition of a laminar sublayer of
the turbulent boundary layer. In confirmation of his results he shows
that he is able to correlate the heat flux required for stabilization on
a semi-log plot vs reciprocal wall temperature, the result being a minimum
activation energy in an Arrhenius-type relationship with a value within
30% of the best estimates available.

In addition to the above, which are rather basic studies, there
has been a good deal of data taken on the stabilization of flames in hot
tubes, which is basically the topic of this thesis. Weir[46] studied the
stability and combustion products (spectroscopically) of a very high
velocity burner. After noting that flame holders in ram jets do not
necessarily prevent blow-off at high mass velocities, he sets forth data
taken on the propane-air system in a ceramic-lined 5-inch steel pipe.
Combustion efficiencies of up to 92% at the extremely high mass flow rates
of 50 lb/(sq.ft)(sec) were obtained. Wall temperatures were as high as
3000°F. Weir explained his results in terms of some spectroscopic data
which he took on the concentration of C_2 ion, but there is some feeling
that the really important thing in causing the high stability which he
observed was that the walls of his burner tube were quite rough, providing
eddies in the boundary layer which could promote flame stability. In
another interesting paper, Howland and Simmonds[18] observed methane-air
flames stabilized in refractory tubes (usually silica). They believe that

the combustion occurs in a series of explosions ignited by both convection and radiant heating from the hot walls and turbulent mixing with the products of the previous explosion. They defined instability as that flow rate at which the flame blew out of the tube. There is some argument about several of their ideas. The flow rates which they observed ranged up to 4.3 lb/ft² sec.

Another pertinent study is that of Lee, Martin, and Moore[22]. They studied the combustion of propane-air mixtures in a heated stainless steel tube which was held at a constant temperature. The combustible gases entered the constant temperature tube through a smaller pipe. For most purposes the flame was considered unstable if it drifted away from the mouth of the samller tube where it existed in bunsen-type stability. However, it was noted that if wall temperatures were high enough (above 950°C), the flame front could stabilize on the large tube wall or oscillate back and forth from the mouth of the small tube to the large tube's wall. Note the agreement with the above mentioned temperature and that cited by Turcotte[42]. The stabilization of the flame front on the tube wall at high wall temperatures and flows was thought to be due to "surface effects".

III. DESCRIPTION OF EXPERIMENTAL APPARATUS

This apparatus was designed in order to determine the stability limits with respect to flow and fuel-air ratio of a flame contained in an insulated, ceramic tube. It is conceptually very simple apparatus and has the following basic components: (1) Gas supply and pressure regulating equipment, (2) gas flow metering equipment, (3) mixing and reaction chambers, and (4) temperature measuring apparatus. Drawings indicating the location and function of the various components are presented in Figures 1 and 2. After a short functional description of the relationship of the components, they will be described in detail in the order given above.

Compressed air from the 95 psig building supply is filtered and then passed through a pressure regulator and a rotameter which determines the flow rate at its calibration pressure. At the same time, the vapor from a tank of commercial liquified propane is filtered, passed through a surge tank, a pressure regulator, and another rotameter. The two calibrated gas streams are then mixed in a chamber packed with ceramic spheres and passed into the reaction tube where they burst into a self-sustained flame and flow out the end of the tube, passing over a triangular baffle which acts as a source of radiant energy to the end of the reaction tube. The tube wall temperatures are measured every few inches by platinum, 10% platinum-rhodium thermocouples. The individual components of the apparatus will now be described.
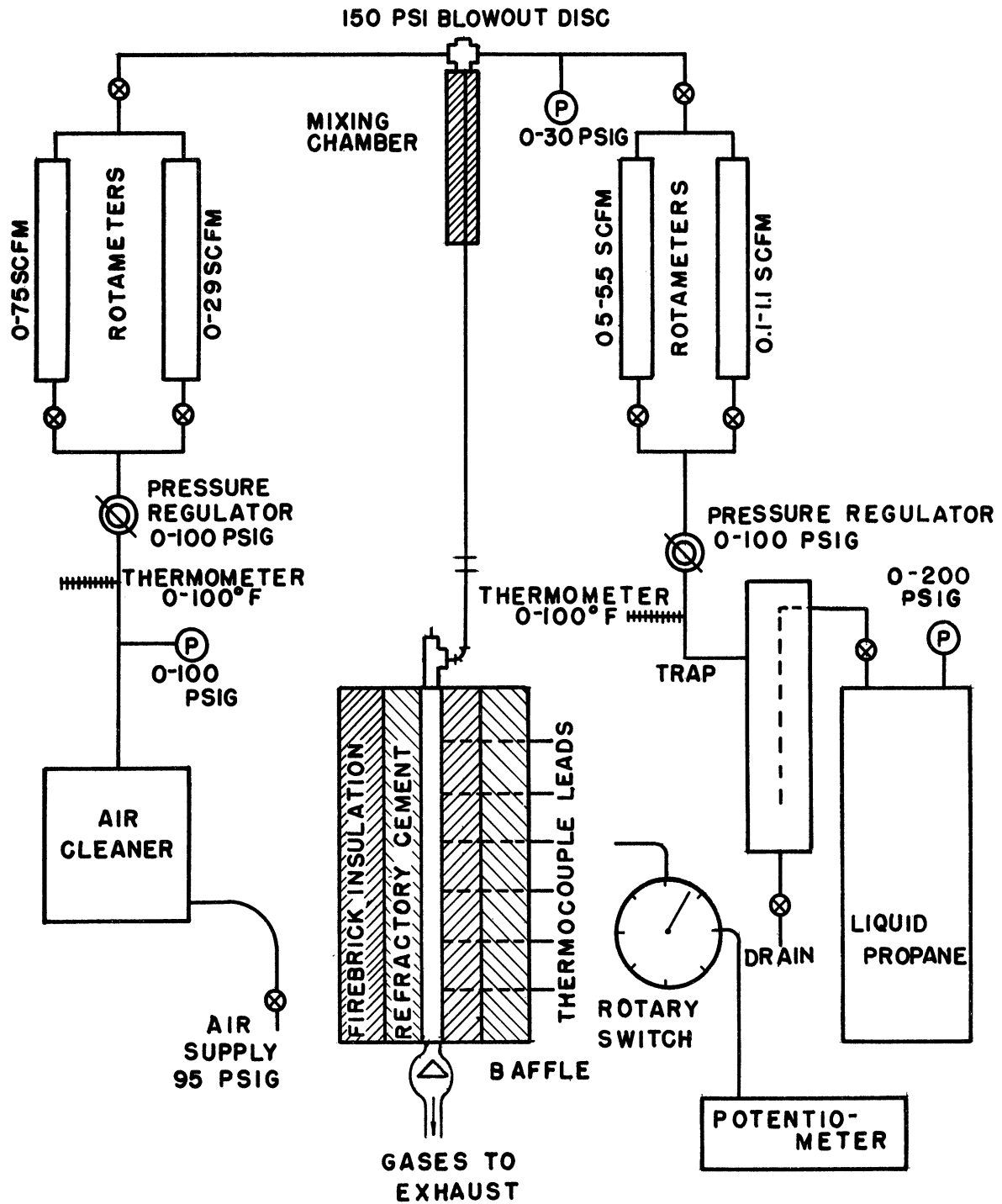
Figure 1. Schematic Diagram of Experimental System.

START-UP PLUG

PROPANE-AIR INPUT

98% Al$_2$ O$_3$ TUBE (I"IDxI-1/4"ODx 24")

PHOSPHATE BONDED Zr$_2$ O$_3$ (7"OD x 24")

THERMOCOUPLE WELLS

IO GAUGE SHEET STEEL

SILICA FIREBRICK 9" #3 ARCH 2-1/2" SERIES (9"x4-1/2" x 2-1/2"-1)

COMBUSTION PRODUCTS
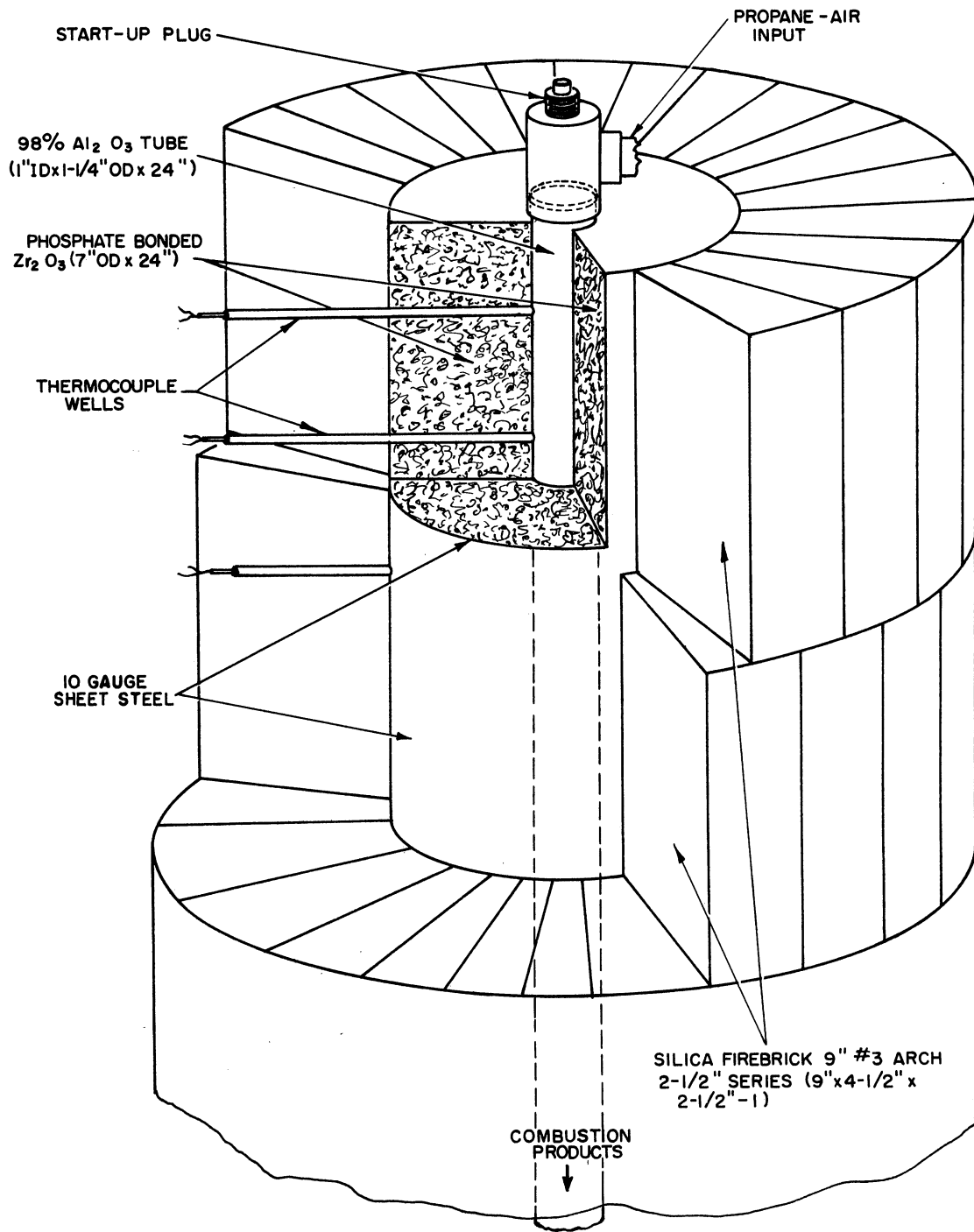
Figure 2. Detailed Diagram of Reaction Chamber.

1. The Propane-Air Supply Systems

        Compressed air is available in the Automotive Engineering

Building from two industrial sized (40 HP) compressors which furnish air

at 90-94 psig. This building air must be filtered to remove entrained

oil and dirt. This is accomplished by passing the air through a 1-1/2 foot

high box, 1 foot square, containing 1/4 inch ceramic balls covered with

a thick layer of glass wool. The propane was purchased from a local

distributor and is of the order of 97% $C_3H_8 \pm 1\%$ with the remainder being

propylene (C_3H_6). The propane is stored in a 400 pound tank and pressure

is supplied by its own vapor pressure. The tank is equipped with three

Chromelox heating elements each rated at .75 kw at 115 v and having 100

square inches of surface area. The function of these heaters was to add

heat to the propane in case the pressure should fall too low due to

external cooling or evaporation. It was found that it was unnecessary

to use these heaters at the flow rates encountered in this work except

in very cold weather or when the tank was less than 20% full. The propane

passes through first a surge tank 12 inches high and then a flash vessel

9 inches in diameter and 15 inches high which cleans it and makes sure

that there is no liquid propane entrained in the system. Both gas streams

are then led to Moore "nullmatic" pressure regulators, model 42 H,

0 - 100 psig, the propane line on 1/4 inch pipe and the air stream on

1 inch pipe. The flow capacities of the regulators are some 123 SCFM

at the rated pressure (50 psia).

2. Flow Meters

There are four flow meters incorporated in the system, two each for the propane and air streams. The calibration curves (Figures 17 and 18) for the two air rotameters give the pertinent data as to floats and flow tubes. The low range flow meter has a range of from 1 to 29 SCFM at 60°F and 1 atmosphere while the high range flow meter has a range of from 7.5 to 75 SCFM. These flow meters were calibrated at 50 psig by use of a orifice critical flow prover and a wet test meter at low flow rates. Similarly, the propane rotameters were calibrated with a wet test meter and a critical flow prover at 40 psig. The low capacity flow meter has a range of from .05 to 1.05 SCFM at 60°F and 1 atmosphere while the high capacity flow meter has a range of 0.5 to 6 SCFM. The tubes and floats used are shown with the calibration curves, respectively Figures 19 and 20. In both cases the operation pressure was set on the pressure regulator and the rotameter desired selected by opening the valve between it and the pressure regulator and closing off the valve to the other rotameter. The flow rate was controlled by opening a needle valve downstream of the rotameters. The temperature of both feed streams usually ranged between 70 and 75°F.

3. Mixing and Reaction Chambers

The fuel and air streams were mixed by running the two lines together at a 1 inch cross fitting. One of the fittings of the cross had a 150 psi blowout disc attached to it. The 4th fitting was attached to a 6 inch length of 1-1/2 inch pipe packed with 1/4 inch ceramic balls. The gas stream then passed through a 30 inch length of 1 inch pipe and then through an elbow and into the reaction chamber.

The reaction chamber, which is the heart of the apparatus, presented more difficulty than all of the rest of the equipment combined. The materials problem which was presented by the experimental requirements of the system proved almost insurmountable. Basically, all that was required was a smooth tube to contain the reaction. Since qualitative theoretical examination of the process leads one to expect extremely sharp temperature gradients on the tube wall, the first type of material attempted was one having very good thermal shock properties which could stand high temperatures. A material which seemed to meet these requirements was zircon $(ZrO_2 \cdot SiO_2)$, a refractory material having extremely good thermal shock properties and good mechanical strength up to temperatures around 2500°F. Several tubes were secured and the apparatus was set up. In the first attempt to take data the tube was attached to the pipe above it with some "Sauereisen" type liquid ceramic cement. The tube was then surrounded by "Fiberfrax" insulation, which is a cotton-like substance, and it was in turn covered by magnesium oxide fiberboard. The tube was thus, effectively, a vertical cantilever beam, supported at the top and free to vibrate except at the top. After runs totaling approximately 1-1/2 hours, the reactor was disassembled and inspected. It appeared that the tube had either cracked and then softened and deformed or else the cracking had been brought about by stress hardening due to prior deformation. The "Fiberfrax" packed around the tube had fused and in some cases melted and much of the MgO board had its binder destroyed. It seemed that there were probably two things necessary here. First, the reaction tube needed better support, and, second, higher temperature insulation

materials were needed. In an attempt to attain both of these objectives

at once a second tube was insulated by fitting it inside holes drilled in

commercial refractory brick which were stacked on top of each other. The

spaces between the hole edges and the tube wall were filled with tightly

packed Al_2O_3 insulation. The bricks were assumed to be self supporting

and the area around them filled with more "fiberfrax" insulation. Un-

fortunately, the results of this attempt proved to be much the same as

the first time except that the tube seemed to be more deformed, some of

the fire-brick had fused and two platinum thermocouples melted, but none

of the "fiberfrax" was fused. This seemed to indicate that it might be

necessary to consider another higher temperature material for the combus-

tion tube itself. After a good deal of inquiry and search it was decided

to use a high purity aluminum oxide refractory. There are several

suppliers of this product. The tubes used in this project were ordered

from Morganite, Inc. because of their ready availability. The grade used

was their "Triangle RR" recrystallized impervious alumina. The first

attempt with this kind of tube was made with the same insulation and

support set up as is described above for the last zircon tube. After

three hours of running the tube was examined and it was found to be badly

cracked. Although there were no serious deformations as were observed

with the zircon tube, there were some dislocations in the tube wall which

provided stabilization points for the flame in the form of back eddies.

Because of this it was impossible to take meaningful data. In an attempt

to eliminate the cracking the tube was wrapped in asbestos sheet and

stuffed into the holes in the firebrick and then cemented at each of the

joints with Sauereisen cement. The bricks around the column were then

supported by other bricks so as to give added support. The tube wall

cracked this time in between the cemented joints. Another attempt was

made, replacing the asbestos with very tightly packed anhydrous magnesium

oxide powder. This, too, failed completely. In an attempt to provide

over-all support without the rigidity of the silica brick, the central

brick fitting around the tube was removed leaving a hole approximately

three inches square around the central tube. This was packed with

anhydrous MgO under fairly high pressure by ramming with a steel rod 1/2

inch in diameter. This tube operated for some ten hours before examina-

tion of its interior with a flashlight revealed serious dislocations.

Further consultation led to the suggestion that perhaps it was the

rigidity of the tube itself which was at fault since the tube would be

subject to mechanical shocks due to detonation and deflagration waves in

addition to the anticipated thermal shock. It was suggested that a lower

density Al_2O_3 tube would probably have more flexibility. Unfortunately,

this tube behaved worse than any tried until then.

A letter to Morganite, Inc. in England asking for recommenda-

tions produced the suggestions of trying thorium oxide or rhenium oxide

tubes. They indicated that the cracks observed might be caused by work

hardening of the Al_2O_3 at temperatures close to its plastic temperature.

On the basis of this and the other data gleaned from previous failures it

was decided to try to stiffen up the support of the tube wall once more

before getting into a great deal of expense and delay by trying to use

exotic tube materials. Correspondence with the Zirconium Corporation

of America had pointed out that their Zr_2O_3 compounds could withstand temperatures in excess of 3500°F and so it was decided to cast a high density "Triangle RR" alumina tube in a 7 inch outside diameter cylinder of phosphate bonded Zr_2O_3. The zircronia is sold in a ramming mix with phosphoric acid. It was tamped down with an iron bar inside a sheet of 10 gauge steel rolled in a 7 inch cylinder 24 inches high. Provision was made for inserting thermocouples next to the tube wall by welding short lengths of 1/4 inch pipe to the inside of the form and inserting 8 mm OD x 5 mm ID Al_2O_3 tubes into the pipe butting up against the central tube. The whole apparatus (Figure 2) was then cured according to the instructions accompanying the ramming mix and put into operation. The system is further insulated by silica refractory brick 4-1/2 inches thick. All of the experimental data taken with alumina tubes have been taken using the set up described above with an alumina tube surrounded by a phosphate-bonded zircronia backing. Hairline cracks are still observed in the alumina tube but due to the support given by the zircronia there are no dislocations.

4. Temperature Measuring Apparatus

The temperature measurements recorded were made by observing the emf generated across the junction of a platinum, platinum-10% rhodium thermocouple. The emf generated was measured by a Leeds and Northrup portable potentiometer, No. 8662. This type of potentiometer has a built-in reference junction which eliminates the need for the use of an ice bath for a reference junction. The thermocouple beads, set at the end of 10 inch, ceramic thermocouple guides, butted up against the outside wall of

the reaction tube at approximately 3 inch intervals. There were six of

the thermocouples and their exact distances from the end of the tube were

3, 6, 9, 12, 15-1/2, and 18 inches, respectively. The thermocouples were

made from 28 gauge wire and thus had to be handled carefully.

5. Location of Experimental Work

  The above described apparatus was located in cell 255 of the

Automotive Engineering Building at the North Campus of the University of

Michigan. The room in question was originally designed as a rocket test

cell and has double, explosion-proof doors and concrete walls 1 foot thick.

The room and the one next to it are equipped with high capacity ventilat-

ing and exhaust systems which facilitated the removal of such noxious

fumes as were created and thus allowed the experimentation to be carried

out with a minimum of disturbance to others.

IV. EXPERIMENTAL PROCEDURE

The following discusses preliminary investigations, preparations for a run, operating procedures, and the form in which the data were recorded.

1. Preliminary Investigations

Preliminary runs were made to determine, in general, what the limits of operation of the system were, what start up conditions were necessary, and what materials would be required for a reliable system. The first of these three in turn dictated what range of rotameters would be needed and, most important of all, demonstrated that a flame actually could be stabilized in an insulated, non-heated ceramic tube. There was no great certainty about this at the initial point of the investigations. It was found, secondly, that it was necessary to preheat the tube walls to temperatures of the order of 1500-2000°F in order to start up the system. Otherwise, the flame front moved on out of the tube and the flame stabilized on the end of the tube just as does a bunsen flame. The many trials necessitated by the objectives of the third phase of the preliminary investigations are set forth in detail in that portion of this work devoted to the description of apparatus. Suffice it to say that after several months of trial and error a successful system was fabricated, consisting of a high-purity, high-density Al_2O_3 tube surrounded by 3 inches of Zr_2O_3 cement.

2. Preparations for a Run

In order to prepare for a run the plug at the top of the reaction tube was removed (see Figure 2) so that there could be a free flow of air from top to bottom. The tube was then heated to the range of 1500-2000°F. This was done slowly so as to minimize the thermal shock. The actual heating took from two to three hours and was done by first placing a bunsen burner under the vertical tube at very low flame. After a short while the flame was increased and air was jetted into the flame stream through a piece of copper tubing, providing the additional oxygen needed for efficient combustion. The air and gas flow rates were increased until the desired temperature range was reached. The air and propane were then shut off, the plug was replaced and the triangular baffle placed under the outlet of the tube. Then a bunsen burner was lighted and placed at right angles to the exit so that the flame would blow across it. This was done as a safety measure and was not really necessary if the preheat was sufficient. This last part was all done as rapidly as possible so that the wall temperature would not drop significantly. The apparatus was then ready to operate by simply turning on the main gas streams which had already been brought up to pressure.

3. Operating Procedure and Data Format

In general, the only limitations on the number of data points which could be obtained after start-up were the time available for experimentation and whether a given set of conditions was unstable. Due to the fact that if the apparatus were operated under unstable conditions

it was necessary to shut down and go through the start-up procedure again,

data runs were always made by first bringing the apparatus to thermal

steady state under a set of operating conditions which was known to be

stable. This operation usually required another two to three hours due

to the long time transient response of the tube wall temperatures caused

by the high heat capacity of the surrounding cement. The criterion for

steady state in this case was that all six of the temperatures being

observed remain the same ($\pm$ 10°F) for a period of one half hour. Having

reached steady-state at a set of conditions which was known to be stable,

the experimental conditions were then altered to the region of interest

at that particular time. It was then necessary to determine whether or

not this new set of conditions gave rise to a stable flame. This was

done by essentially the same techniques as mentioned above. If the wall

temperatures remained constant for a long period of time after the

transition period brought about by changing the operating conditions,

then the operating conditions were considered "stable". If the observed

wall temperatures continued to drop and showed no sign of leveling off,

then the operating conditions were considered unstable. An indication

of the length of time required for an instability to show up can be seen

by examining the following raw data for wall temperatures taken on July 24,

1962. This is an example of a creeping instability. If data had been

taken for a longer period of time, the 9 inch temperatures would have

dropped far enough to cause the 12 inch wall wall temperature to decline,

and so on until the flame blew out the tube.

TABLE I

WALL TEMPERATURE DATA SHOWING CREEPING INSTABILITY

Distance from Tube Top (inches)	Temperature at Time (PM):						
	2:30	3:30	4:00	4:15	4:30	4:45	5:00
3	250	not recorded					100
6	1250	1000	800	700	600	500	400
9	2300	2440	2310	2050	1950	1810	1600
12	2330	2440	2470	2490	2520	2530	2530
15	2130	not recorded					
18	1770	not recorded					

Generally, successive runs were made in the direction of increasing flow as less time was required for steady state to be reached when the "flame front" moved down the tube, as it usually did when the flow rate was increased, than for it to move upstream as it did in most cases where the flow rate was decreased. When changing from one set of conditions to another the propane flow rate was always changed first because it had relatively little effect on the over-all flow rate.

The raw data which were taken on the process were the rotameter readings and the wall temperatures if the conditions were stable, otherwise only the rotameter readings and the notation "unstable" were recorded.

Before the start-up for a run was begun, the interior of the the tube was examined carefully with a flashlight and a dental mirror

in order to be certain that there were no dislocations in the smooth line of the tube wall and that such cracks as existed were hairline cracks. Only once did the apparatus fail this inspection. At that time the flaw was corrected by use of some of the refractory cement.

V. EXPERIMENTAL RESULTS

The results of the experimental work performed in this study can perhaps best be presented by grouping them under the separate headings of quantitative results and qualitative results. Quantitatively, data were obtained for the stability limits or propane-air flames burning within the length of a 24 inch alumina tube, 1 inch in inside diameter. Most of these data were taken in the lean region (i.e., the oxygen present was in excess of stoichiometric requirements). By the stability limit is meant that flow rate beyond which the flame will not perpetuate its existence in the tube indefinitely as previously defined. At the same time data were taken with platinum, platinum-rhodium thermocouples on the temperature profiles of the outer tube wall as a function of gas flow rate and fuel-air ratio for stable flow rates. The stability data are listed in Appendix B, and are presented in graphical form in Figure 3, a plot of mass flow rate (lbs gas/min) vs fuel-air ratio (lb propane/lb air). All of the data taken are shown in Figure 3, the unstable points being represented by circles and the stable points by squares. The data on temperature profiles are also in the Appendix B and are presented here in Figure 4, 5, 6, 7, 8, 9, and 10 as plots of external tube surface temperature vs length at different mass flow rates. The average peak positions for various flow rates were determined from the above and correlated in Figure 11.

The qualitative results of the experimental work are perhaps even more important than the quantitative ones. First and foremost, it was determined that turbulent propane-air flames could be stabilized in smooth,
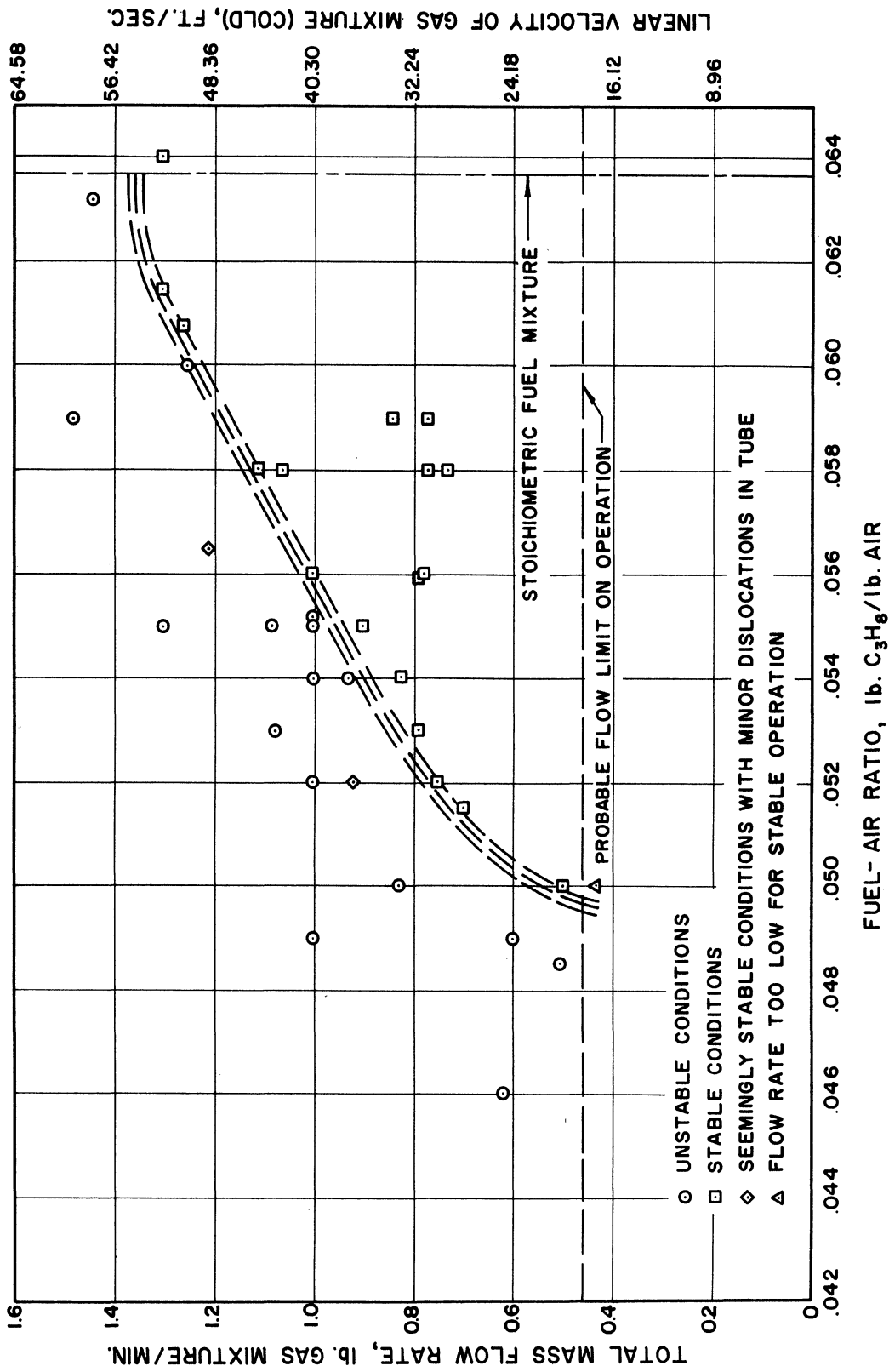
Figure 3. A Plot of Total Mass Flow Rate (lb.mixture/min.) vs. Fuel-Air Ratio (lb.C_3H_8/lb. Air).
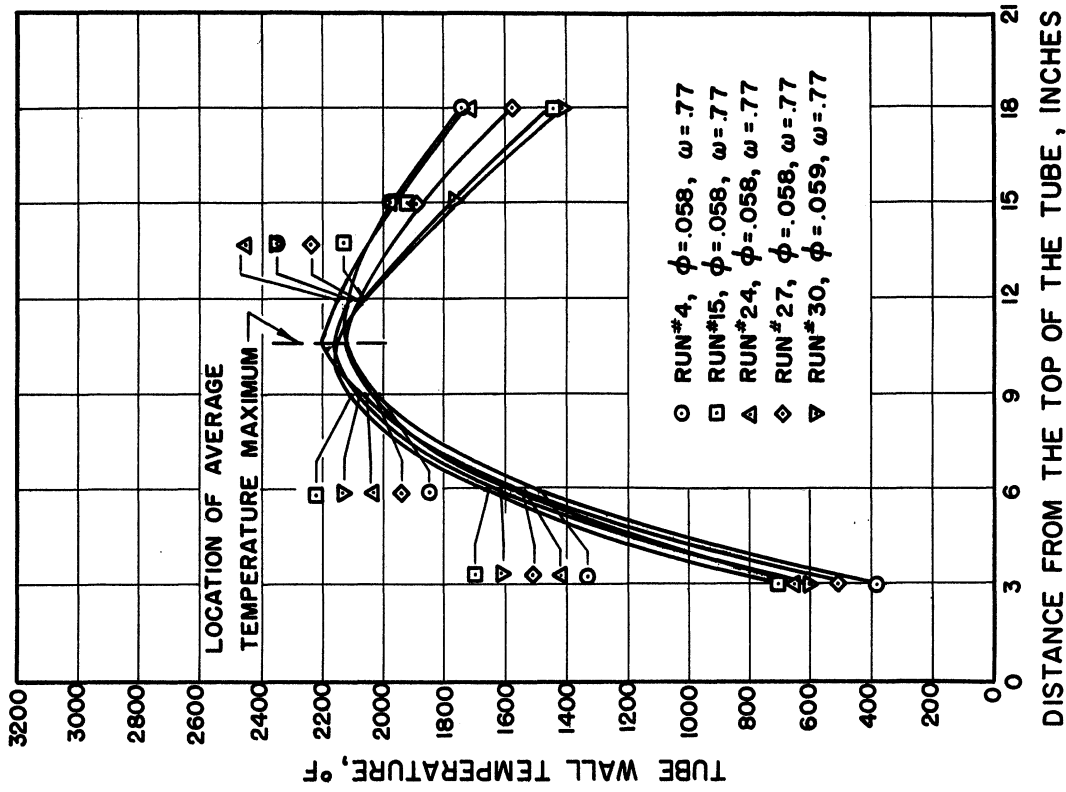
Figure 5. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .058, Mass Flow Rate ≈ 0.77.
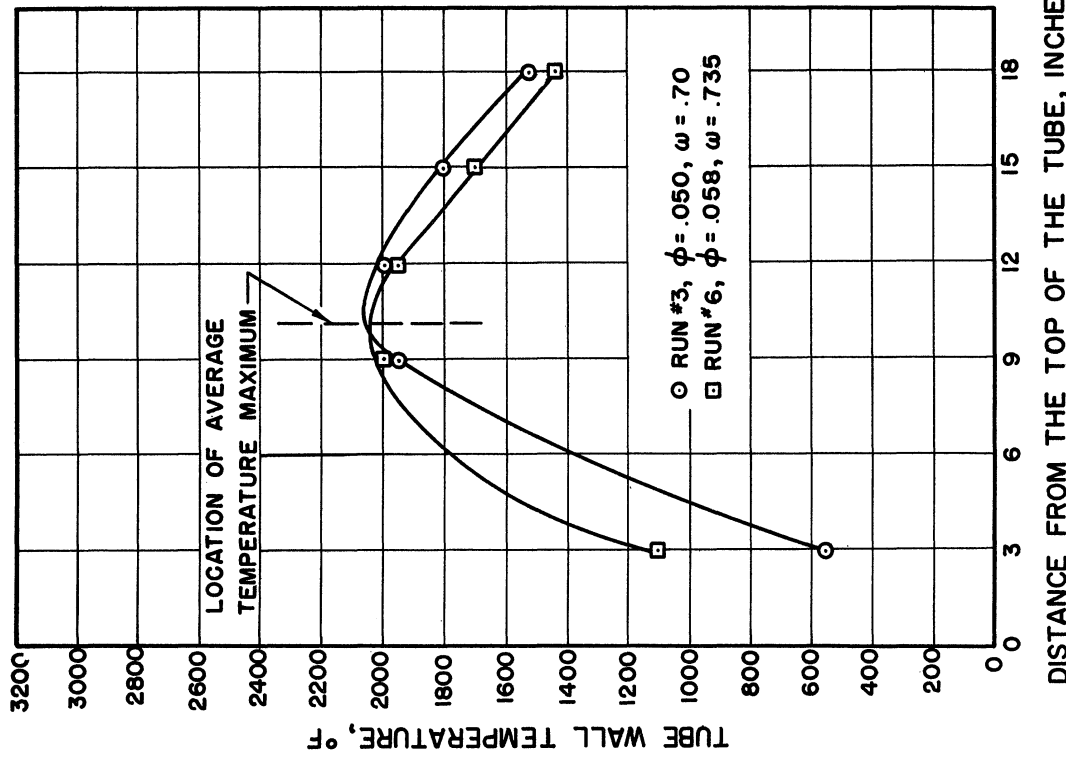
Figure 4. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .054, Mass Flow Rate ≈ .715.
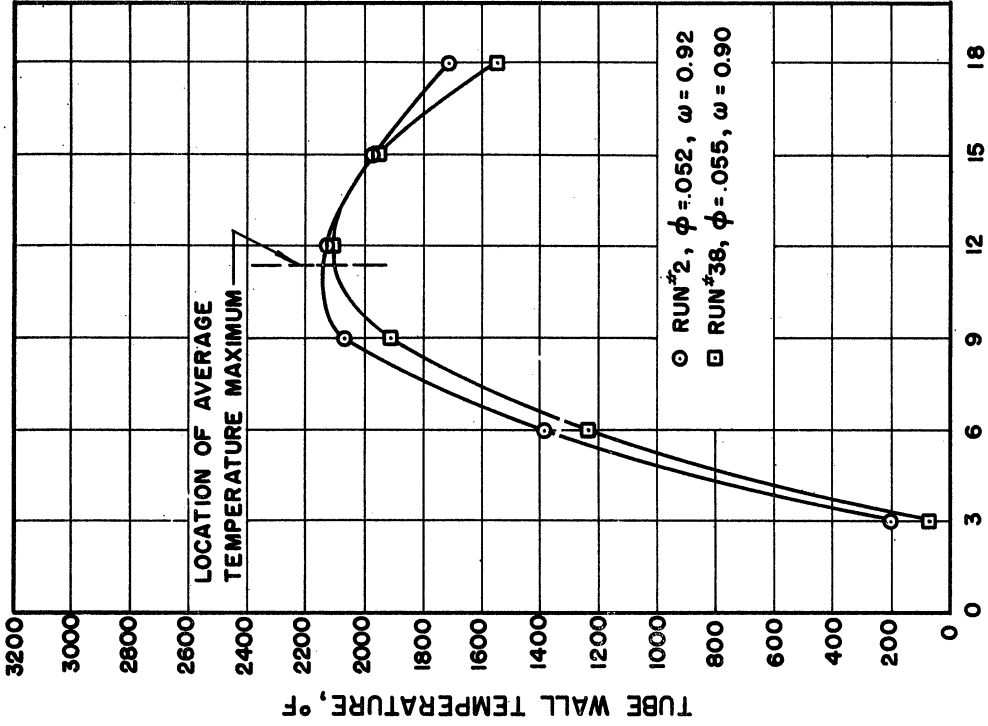
Figure 7. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .0535, Mass Flow Rate ≈ 0.91.
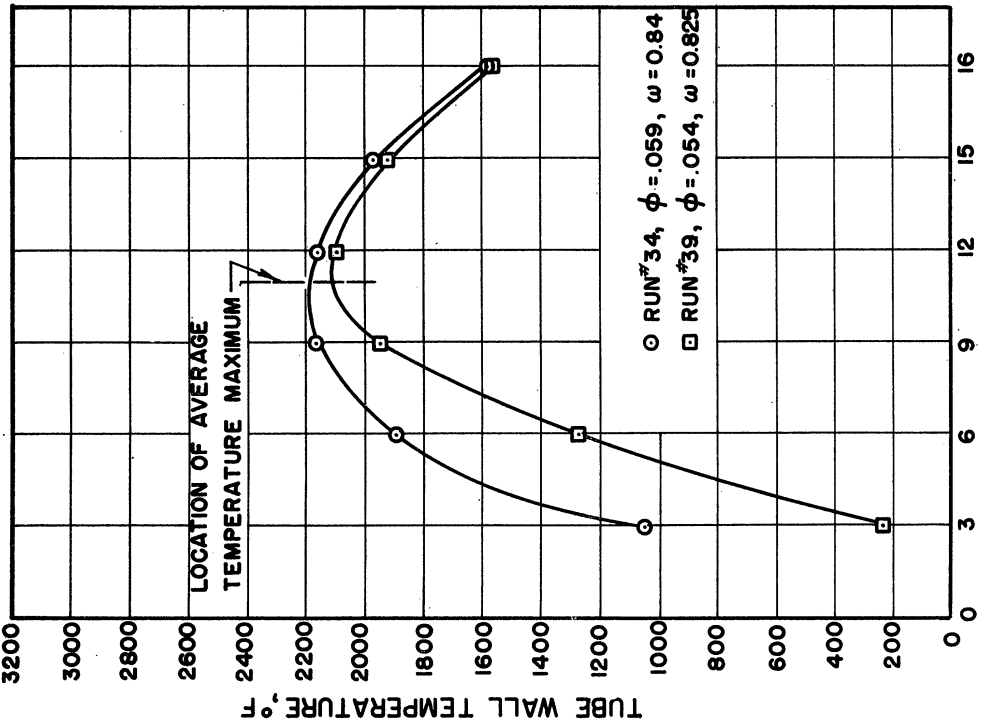
Figure 6. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .057, Mass Flow Rate ≈ 0.83.

Figure 9. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .058, Mass Flow Rate ≈ 1.08 .

Figure 8. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .056 to .066, Mass Flow Rate ≈ 1.0 .
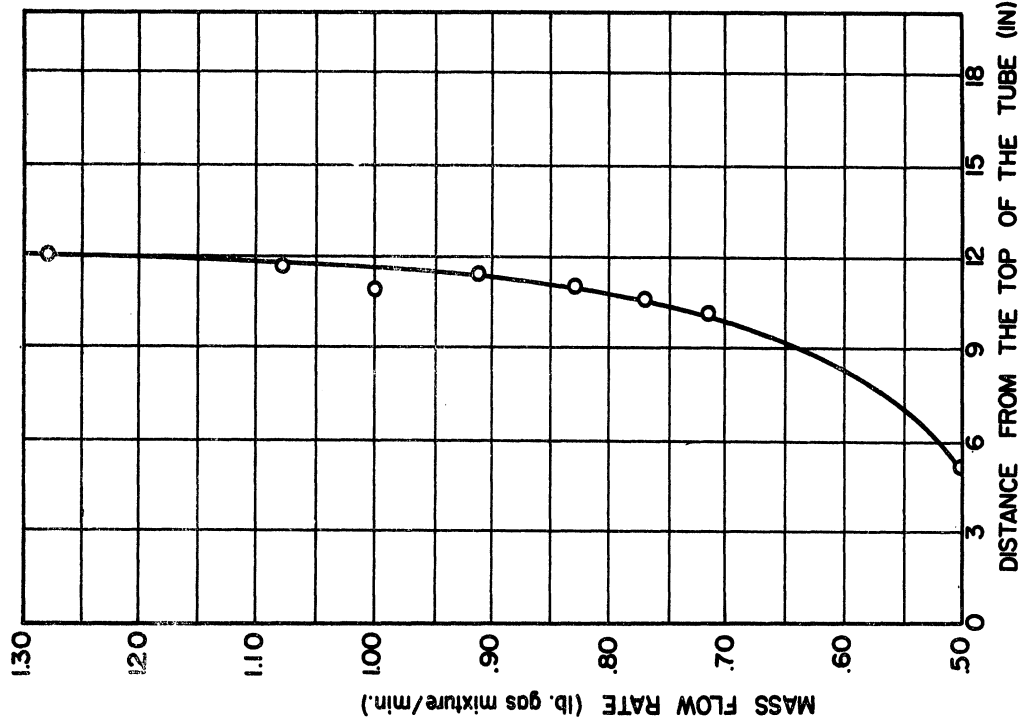
Figure 11. A Plot of Mass Flow Rate (lb/min) vs Maximum Temperature Location (inches).
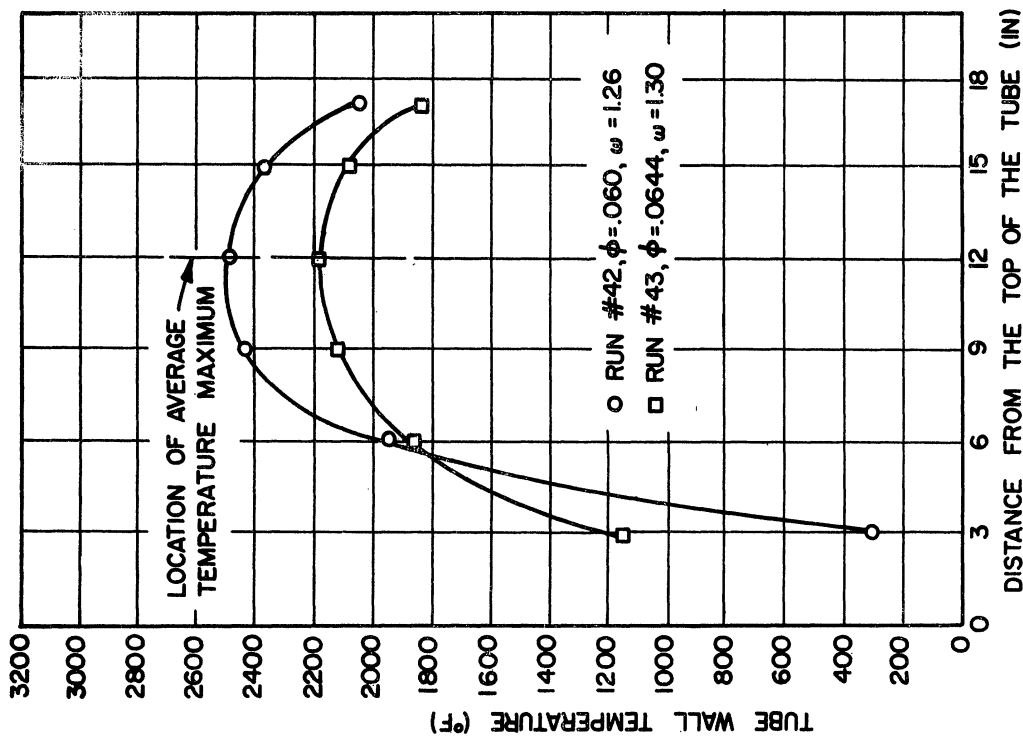
Figure 10. A Plot of Tube Wall Temperature (°F) vs Tube Position (inches), Fuel-Air Ratio ≈ .062, Mass Flow Rate ≈ 1.28.

insulated ceramic tubes without the need for any external heat addition.
The flames observed were, in general, of low luminosity (except for rich
mixtures) and had only a short "tongue" sticking out the end of the tube.
In order to contain the flame inside of a smooth ceramic surface, it was
found to be necessary to use a high quality, high temperature oxide ceramic
backed up by a massive refractory cement casting laid around the refractory
tube. This did not completely prevent cracking, but it did prevent any
roughness or dislocations from occurring in the tube wall. It was neces-
sary to preheat the tube wall to 1500-2000°F in order to start up the
operation. After this, however the flame was self-perpetuating or else
moved out of the tube very slowly (over several hours), indicating that
long term heat transfer transients were controlling. If flow rates were
changed from a stable point to another one at a lower flow rate, the flame
would back up the tube, but much more slowly than it would slide out. It
was observed during the search for a suitable reactor material that if
there were dislocations or jagged edges along the tube wall the flame
would stabilize at much higher flow rates than were observed with the
smooth-walled tube. These flow rates were as much as ten times higher
than the highest stable flow rate in the system observed. Finally, it
was shown that the data could not be reconciled with a "plug flow" type
of mathematical model and that it must be necessary, therefore, to con-
sider in some way the boundary layer effects acting on the system.

VI. DISCUSSION OF EXPERIMENTAL RESULTS

Before discussing the experimental results mentioned in the previous section it would be well to give some attention to a complete description of the phenomenon itself as it was observed in taking the aforementioned data. Perhaps the best way to describe it is by its randomness. The flame creates some noise within the tube, but this is not of any great volume nor is it uniform or of constant frequency. Sometimes there are organ type sounds, sometimes a gentle rumbling, and sometimes a rather loud klaxon horn-type sound. At the exit from the tube the external tongue of flame would waver back and forth, sometimes disappearing, sometimes thrusting far out of the tube in a fashion similar to the photographs in Howland and Simmonds' paper[18]. When making temperature measurements, the high sensitivity circuit of the 8662 potentiometer could not be used because the local wall temperature varied so rapidly and widely that it was impossible to get a representative temperature reading using the high sensitivity circuit. All of this leads one to attempt to visualize flame fronts eddying back and forth both radially and longitudinally with the random flow patterns of the turbulent gas. These flame fronts would give rise to radial and longitudinal detonation and deflagration waves which would explain the difficulty encountered in finding a mechanical system capable of containing the flame.

1. Stability Data

The accuracy of the stability data presented in the preceding section is estimated as +1.5% with respect to fuel-air ratio, as +2% with respect to mass flow rate. Therefore, the stability curve may be drawn in between the stable and unstable data points with the confidence limits shown (Figure 3). This curve is redrawn (Figure 12) in this section and compared with the data of Harris, Grumer, von Elbe, and Lewis (14) on laminar flames, Williams and Bollinger[47] on turbulent flames, Turcotte[42] on heated flat plates, Howland and Simmonds[18] on ceramic ram jet burner. The qualitative relationship of the data of this study to that of the above studies can easily be explained on the basis of the conceptual two-dimensional model as outlined in the section entitled Mathematical Models. The data on laminar and turbulent flame velocities are naturally lower than the data presented here because they were measured by igniting cold gases in laminar and turbulent flow and measuring the value of $(dV/dt)(1/A)$. That they are almost of two orders of magnitude lower is of great practical significance. There were no heated boundary layers involved. The values of Turcotte[42] are of the same order of magnitude as those in this experiment but slightly higher. The probable reason for this is that his boundary layers were kept at various constant temperatures by heat supplied to them. They were not, therefore, self sustaining in the sense of this study and should give higher flow values. The data of Howland and Simmonds[18] cannot be applied as a comparison to this study since their method of evaluating stability limits was, "to increase the air flow rate at constant methane flow rate until
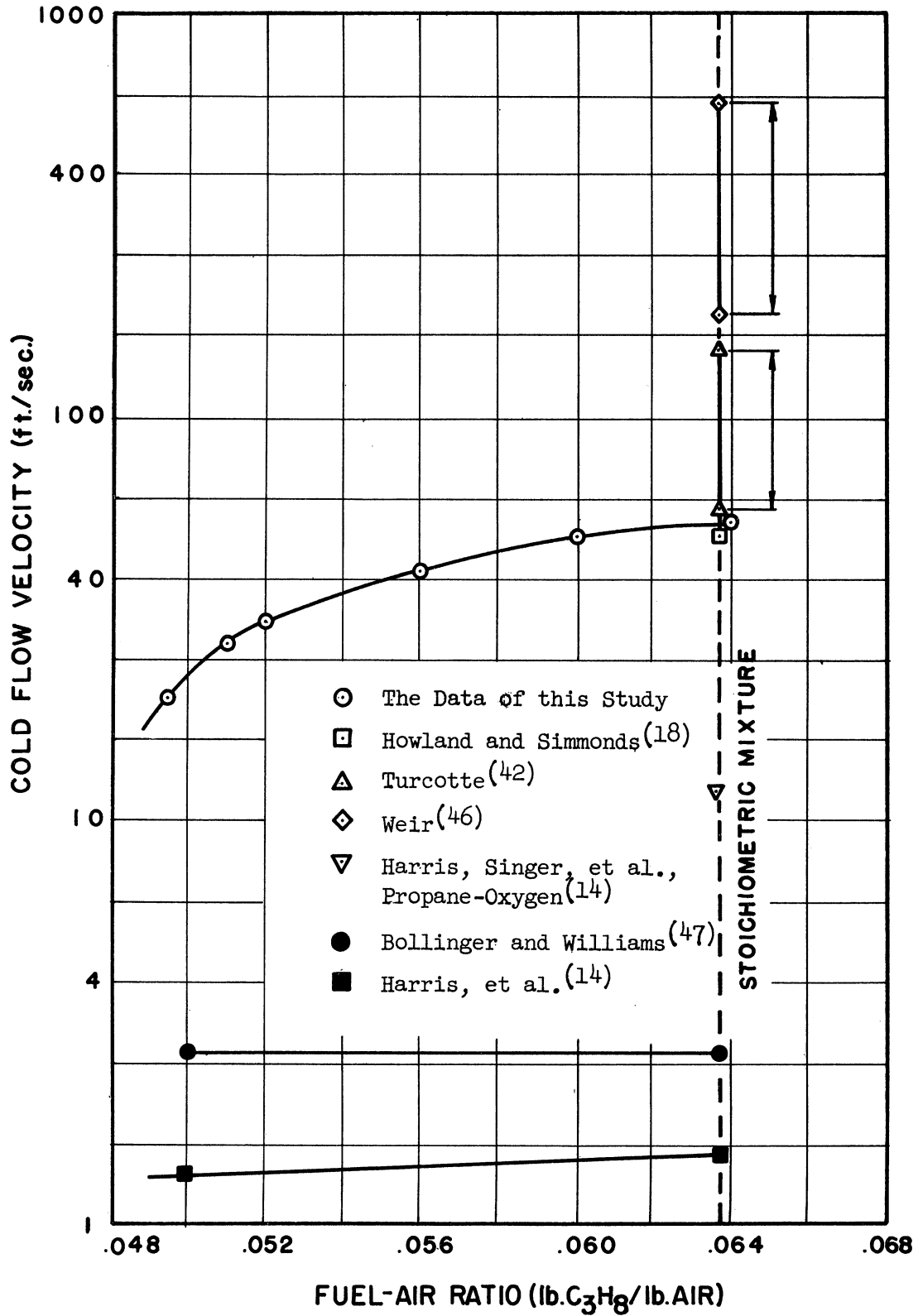
Figure 12. A Plot of Flow Velocity for Stable Flames Based on Gas
Temperatures of 70°F vs Fuel-Air Ratio. (If Data on the
Effect of Fuel-Air Ratio Were Not Available, the Available
Data are Shown as an Experimental Range on the
Stoichiometric Line.)

the flame blew out the end of the tube." It is possible that they thus actually obtained values for which the flow rapidly overwhelmed heat transfer from the wall and thus gave flow values considerably higher than those observed in this study where the long-time heat transfer transients were taken into account. The data of Weir[46] show the highest flame velocities of all. It is obvious from a photograph of Weir's apparatus in a University of Michigan report of 1950 (UMM73) that the inside of the ceramic ram jet burner is extremely rough. Professor J.L. York of the Department of Chemical and Metallurgical Engineering at the University of Michigan has indicated that when the apparatus was in operation the rough spots glowed white hot and flames were trailing from their edges. This would lead one to expect higher velocities than those obtained in this study because of eddies formed in the boundary layers due to roughness in the ceramic walls. This, of course, leads to the question as to whether or not the high velocities obtained in this experiment might not be due to this same roughness effect but in a lesser degree. The answer to this lies in the reproducibility of the data. The tube walls were smooth initially and developed hairline cracks only after several hours of operation. The fact that the data taken when the tube was first in use could be reproduced after 100 hours of tube use should show that the hairline cracks which formed in the tube wall have an insignificant effect on the boundary layer streamlines.

2. Temperature Profile Data

The data on temperature profiles are presented in the preceeding section. It should be noted that there may be as much as $\pm 100°F$ variation in actual recorded temperatures on runs made with identical flow rates. This is partly due to the short term transient effect due to the turbulent flow eddies. Note, however, that the shapes of the temperature profiles for identical runs are quite similar and all have their temperature maximum at essentially the same place. These temperature profiles are similar to the predictions of Winer[48] and McCarty[31] modified for Arrhenius reaction rates except that the temperature peaks are not as high. This would lead one to the conclusion that not as much of the gas is preheated before reacting which in turn supports the boundary layer model. On the basis of the similarity of temperature profiles determined in this study, the distance from the top of the tube to the maximum temperature point can be plotted as a function of flow rate (Figure 12). In the two-dimensional model one would expect this distance to increase as a function of the mixing length (less than a linear relationship) while the one-dimensional model would predict a straight line relationship if the wall temperatures were essentially the same. The observed temperatures are higher for fuel-air ratios increasing toward the stoichiometric value. This is to be expected due to the maximum in the theoretical flame temperature at the stoichiometric fuel mixture.

3. Qualitative Results

In general the qualitative results lend confirmation to the concept of the two-dimensional, boundary layer stabilized model. The fact that the experimental results cannot be predicted in terms of the one-dimensional model and that dislocations in the tube wall allow flame stabilization at flow rates an order of magnitude greater than any observed with smooth tubes both seem to support the more complex model and to indicate that Weir's data may be the result of the extreme roughness of his burner walls.

In summary then, it can be said that none of the experimental data give any cause to doubt the two-dimensional model and that there is a good deal of evidence which seems to support it.

VII. MATHEMATICAL MODELS

There will be presented in this section two mathematical models by which an attempt was made to predict the behavior of the experimental apparatus. The first model is a "plug flow" type system, assuming radial symmetry. The equations pertaining to it are thus one-dimensional with variable boundary conditions (the tube wall temperatures). The second and more complex model takes some, but not all, of the radial effects into account and must therefore be two-dimensional. It was impossible to take all of the radial effects into account as several of them would require separate and extensive studies to evaluate them well enough to fit them into the computer program. They are all taken into account in the conceptual description of the two-dimensional model.

One-Dimensional Model

Consider a section of the reaction tube of length DX (see Figure 13). The various sources of heat flux are indicated on the diagram and defined below: (all quantities are in Btu/hr.)
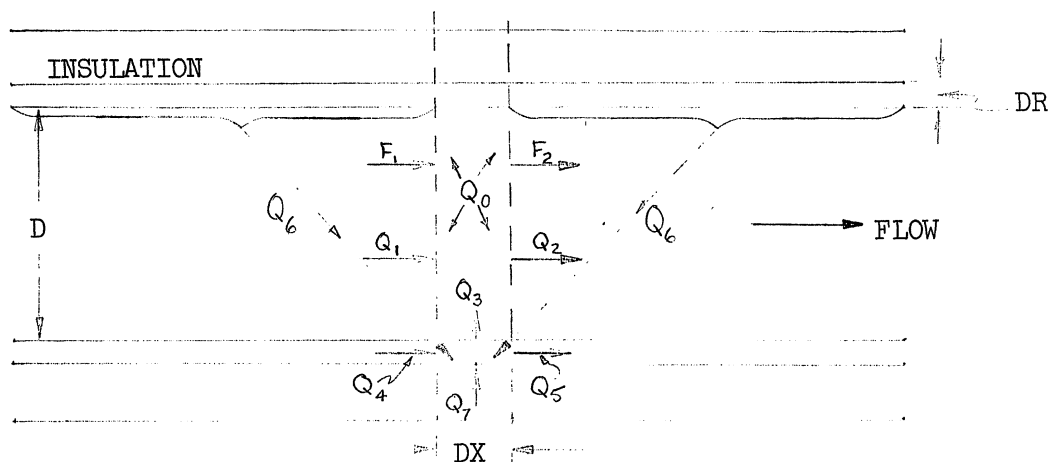
Figure 13. Reaction Tube Cross Section.

Q_0 = The heat released in the zone by chemical reaction

$$= \frac{\pi D^2}{4} \cdot DX \left[2 KRX \cdot A(x,\theta) \cdot B(x,\theta) \cdot \Delta H \cdot \rho^2 \, e^{-E/R_o \cdot T(x,\theta)} \right] \tag{2}$$

Q_1, Q_2 = The conduction of heat in the gas phase

$$= -k_g \frac{\partial T(x,\theta)}{\partial x} \left(\frac{\pi D^2}{4} \right) \tag{3}$$

Q_3 = The transfer of heat from the wall to the gas stream by

convection $= h_g \cdot \pi D (DX) \left[X(x,\theta) - T(x,\theta) \right]$ (4)

Q_4, Q_5 = The conduction of heat in the tube wall

$$= -\frac{k_w \pi}{4} \left[(D + 2 DR)^2 - D^2 \right] \cdot \frac{\partial X(x,\theta)}{\partial x} \tag{5}$$

Q_6 = The transfer of heat to the wall at DX by radiation

$$= \sigma \left\{ \int_{-\infty}^{\infty} \mathcal{F}_{x \to y} \left[X^4(y,\theta) - X^4(x,\theta) \right] dx \right\} DX \cdot \pi \cdot D \tag{6}$$

Q_7 = The heat transfer through the tube.

F_1, F_2 = Heat input and output associated with the heat

capacity of the gas $= \frac{\pi D^2}{4} G C_p \cdot T(x,\theta)$ (7)

We now assume that Q_1, Q_2, and Q_7 are negligible and also that back mixing and axial diffusion may be neglected. This leads to the following heat balance equations:

(1) Energy Balance on Gas Stream Element

Input = Flow In + Heat Transfer + Chemical Reaction

$= F_1 + Q_3 + Q_0$

$$= \frac{\pi D^2}{4} G C_p T(x,\theta) + \pi D (DX) \cdot h_g \left[X(x,\theta) - T(x,\theta) \right] \tag{8}$$

$$+ \frac{\pi D^2}{4} (DX) \left[2 KRX \cdot A(x,\theta) \cdot B(x,\theta) \cdot \Delta H \cdot \rho^2 \, e^{-E/R_o T(x,\theta)} \right]$$

Output = Flow Out = F_2

$$= \frac{\pi D^2}{4} G C_p T(x+DX, \theta) \tag{9}$$

Accumulation $= \frac{\pi D^2}{4}(DX)(\rho C_p)\left[\frac{T(x, \theta+DT) - T(x, \theta)}{DT}\right]$ (10)

Now, by definition,

Input - Output = Accumulation.

Therefore, we may write in differential form -

$$Q_0 + 2\pi D\left[X(x,\theta) - T(x,\theta)\right] - \frac{\pi D^2}{4} G C_p \frac{\partial T(x,\theta)}{\partial x} = \frac{\pi D^2}{4} \rho \cdot C_p \frac{\partial T(x,\theta)}{\partial \theta} \tag{11}$$

or

$$\frac{8}{D\rho C_p}\left[X(x,\theta) - T(x,\theta)\right] - \frac{G}{\rho}\frac{\partial T(x,\theta)}{\partial x} + 2 KRX \cdot A(x,\theta) \cdot B(x,\theta) \cdot \Delta H\left(\frac{\rho}{C_p}\right) e^{-E/R_0 T(x,\theta)}$$
$$= \frac{\partial T(x,\theta)}{\partial \theta} \tag{12}$$

This may be written as an explicit, averaged difference equation for

T (x, θ + dθ) in the following manner (see Appendix A, Section 1):

$$Z(J) = T(J) + \frac{DT}{2 \cdot DX \cdot \rho \cdot C_p}\left[G C_p\left[T(J-1) - T(J+1)\right] + \frac{4 DX \cdot U}{R}\left[X(J) - T(J)\right] + 2 KRX \cdot A(J) \cdot B(J) \cdot \Delta H \rho^2 \cdot DX \, e^{-E/R_0 \cdot T(J)}\right] \tag{13}$$

where, J = incremental value of x

   Z(J) = the value of T(J) at the next time step (°R)

   T(J) = the temperature at the Jth increment in tube length (°R)

   DX = the size of the tube length increment (ft.)

   DT = the size of the time increment (hr.)

   X(J) = the tube wall temperature at the Jth increment (°R)

   U = the overall heat transfer coefficient at the wall (Btu/hr.·ft.^{2} F)

   KRX = the reaction rate constant (cu. ft./lb. hr.)

$A(J)$ = the concentration of oxygen in the Jth increment (lb.O_2/lb. mixture)

$B(J)$ = the concentration of propane in the Jth increment (lb.C_3H_8/lb. mixture).

The other variables are as defined in the Nomenclature.

(2) Energy Balance on Tube Wall Element -

Input = Conduction in + Radiation in = $Q_4 + Q_6$

$$= -k_w \frac{\partial X(x,\theta)}{\partial x}\Big|_x \left[\pi((R+DR)^2 - R^2)\right] + \sigma \pi D \cdot DX \int_{-\infty}^{\infty} \mathcal{F}_{y \to x}\left[X^4(y,\theta) - X^4(x,\theta)\right] \qquad (14)$$

where

$$\mathcal{F}_{y \to x} = \frac{1}{\dfrac{1}{F_{y \to x}} + \dfrac{1-\epsilon_x}{\epsilon_x} + \dfrac{1-\epsilon_y}{\epsilon_y}} \qquad (15)$$

(assuming that the areas of all increments are equal [33]).

Output = convection heat transfer to gas stream + conduction out

$$= Q_3 + Q_5$$

$$= -k_w\left(\frac{\pi}{4}\right)\left[(D+2DR)^2 - D^2\right] \cdot \frac{\partial X(x,\theta)}{\partial x}\Big|_{x+DX} + \pi D h_g DX\left[X(x,\theta) - T(x,\theta)\right] \qquad (16)$$

$$\text{Accumulation} = \frac{\pi}{4}\left[(D+2DR)^2 - D^2\right] \cdot DX \, (\rho C_p)_{cer}\left[\frac{X(x,\theta+DT) - X(x,\theta)}{DT}\right] \qquad (17)$$

We thus have the integro-differential equation,

$$k_w \frac{\partial^2 X(x,\theta)}{\partial x^2} + \frac{4\sigma D}{\left[(D+2DR)^2 - D^2\right]} \int_{-\infty}^{\infty} \mathcal{F}_{y \to x}\left[X^4(y,\theta) - X^4(x,\theta)\right] - \frac{4Dh_g}{\left[(D+2DR)^2 - D^2\right]}\left[X(x,\theta) - T(x,\theta)\right]$$

$$= (\rho C_p)_{cer} \frac{\partial X(x,\theta)}{\partial \theta} \qquad (18)$$

This, too, can be expressed in a finite difference form with the integral being treated as a summation over the elements (see Appendix A, Section 1). Once the programming of the simultaneous solution of the above equations

had progressed to the point where the required execution time on the
IBM 709 could be estimated, it was deemed wise to test the feasibility
of the model by assuming a reasonable temperature distribution for the
wall temperatures (from experimental data). These temperatures were
assumed invariant and the gas stream energy balance equation was
solved for the resultant gas temperatures at various values of the
mass flow rate, G. The computer flow diagram and program may be found
in Appendix A, Section 2. This procedure gave an indication of the flow
rates which would allow stability (given the assumed wall temperature
profile) in a region fairly close to that which should give rise to the
assumed temperature profile. The results (see Figure 14) obtained from
the "plug flow" model did not come close to approximating the already
observed flame stability data. This model was therefore discarded with-
out ever completing work on the program for the solution of the two
simultaneous equations. The intended technique for solution of the
equations was to first assume a wall temperature profile and from this
calculate a gas temperature profile which could in turn be used to
calculate a new wall temperature profile. Some combination of this
new profile and the old one would then be used to calculate a new
series of gas stream temperatures, and so on ad infinitum until the
wall and gas stream temperature profiles were compatible for the flow
rate given. There was some fear that this method of solution might
give rise to stability problems, but it was never explored.

A solution of a simplified form of the above problem was
undertaken in the Fall Term of 1959 by two students in course CM 363 at
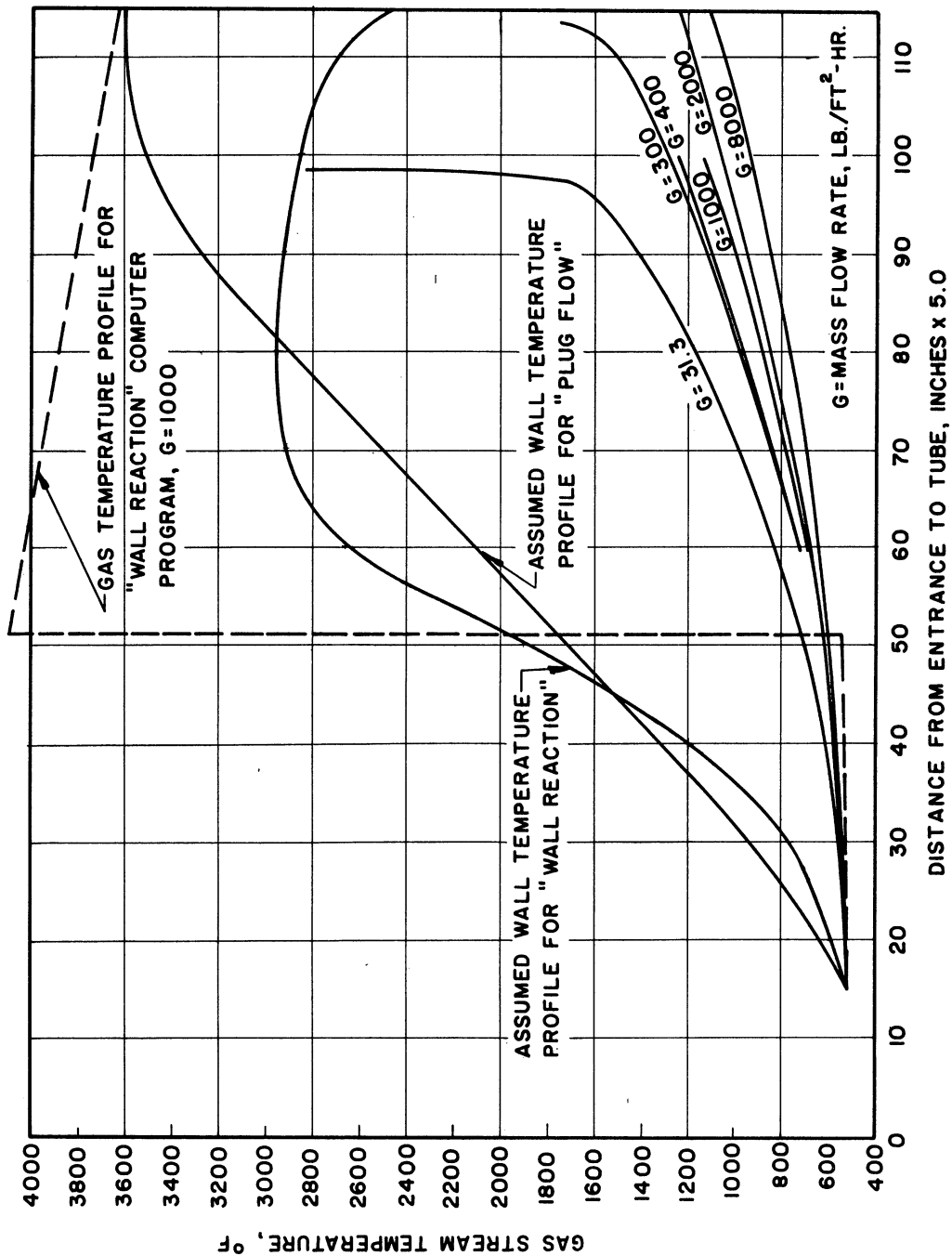
Figure 14. A Plot of Gas Stream Temperature vs Distance as Calculated from "Plug Flow" and "Wall Reaction" Computer Programs.

the University of Michigan. The students, McCarty[31] and Winer[48]

assumed in addition to the above assumptions that conduction in the

tube wall was negligible and that the chemical reaction took place

instantaneously and completely at the point x=0 in an infinite tube

(no end effects). They then tried to solve the energy balance equa-

tions indicated above by hand calculation. Using different methods,

they arrived at essentially the same qualitative results for the

temperature profiles in the infinite tube. These results are shown

schematically below:
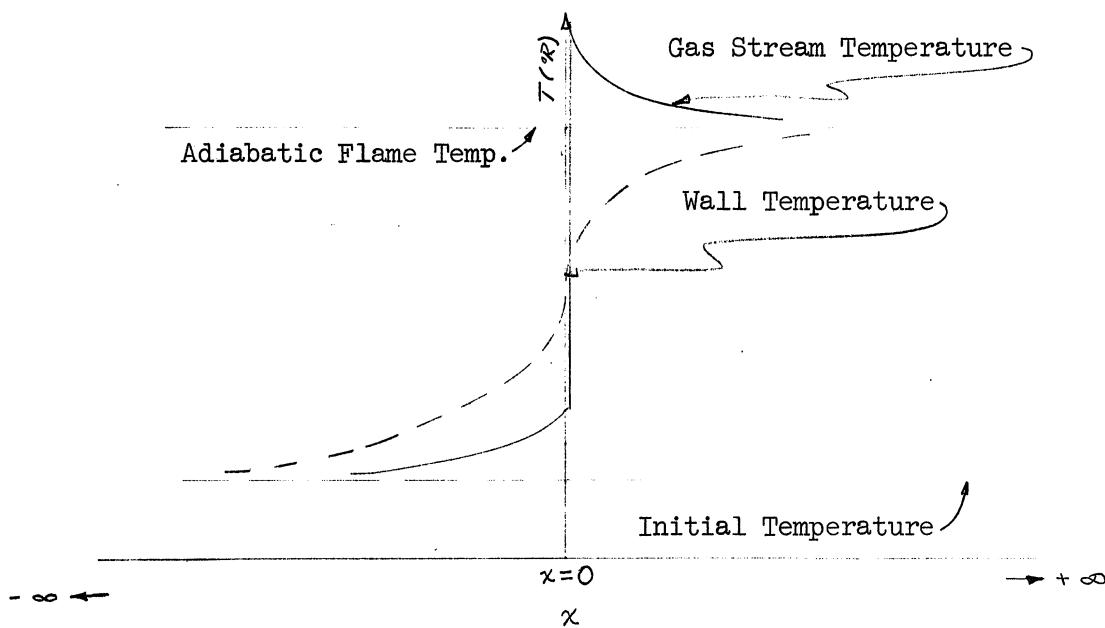
Figure 15. Wall and Gas Stream Temperature Profiles
Obtained by Approximate Methods.

Winer predicted a peak value of some $1000^\circ R$ above the adiabatic flame

temperature.

Two-Dimensional Model

The "flame velocities" predicted by the one-dimensional, "plug flow" model were smaller than the experimentally observed values by a factor of about 200. Thus the effects which are neglected in the one-dimensional model must lead to higher flame velocity limits. It is easy to see how a consideration of the two-dimensional nature of the gas flow would lead to higher predicted stability limits. The gas in the boundary layer close to the hot tube wall would reach the ignition temperature (about $1700^\circ F$) far upstream from the ignition point of the bulk gas stream in the one-dimensional model. Once the reaction occurred in the boundary layer close to the tube wall it would spread into the main gas stream by radial gas flow, radial conduction, and turbulent mixing of the hot gases in the boundary layer with the bulk gases. Once the reaction spread into the bulk gas region, the mixing would be very rapid causing the flame front to spread across the whole tube. Thus, the important factors in the two-dimensional model are: (1) the heating of the low velocity boundary layer gases by conduction from the hot tube wall to the reaction point, (2) heating of the higher velocity gases by mixing, conduction, and radial flow, and (3) the spreading of the flame throughout the turbulent region by turbulent mixing. Unfortunately, the mathematical expression of these concepts is not nearly so simple as the verbal one. There are problems in deciding how thick the boundary layer should be at various temperatures, what the pressure profile across the gas stream should be, what the eddy diffusivities for mass and heat transfer should be, etc. The

plan of action for the solution of the equations for the two-dimensional model was the same as that previously mentioned for the one-dimensional model, involving recursive calculation of wall and, now, two-dimensional gas stream profiles until there was little or no change in either profile. The equations for the heat balance on the wall elements were the same as in the one-dimensional case except for the heat flux to the gas stream. Thus, the whole problem was to find a technique for solving a mass-energy balance system of equations on the turbulent gas stream which takes all of the above mentioned effects into account.

Rather than going into the complete derivation of the equations used, I will merely set forth here the assumptions made in deriving these equations, the method of attack for the digital computer solution of the equations, and the limitations of the method of solution as I see them. The complete derivation of the equations may be found in Appendix A, Section 3. The following assumptions were made in attempting to solve the two-dimensional mathematical model:

(1) $\quad U_{max} = \bar{U}/(.0286\ N_{Re} + 0.516)$ $\hfill$ (I)

(2) $\quad V(J) = U_{max} \left[\dfrac{R_W - R(J)}{R_W} \right]^{1/7}$ $\hfill$ (II)

(3) The pressure was assumed to be constant in the radial direction.

(4) It was assumed that back mixing was negligible.

(5) The transport properties were assumed independent of the chemical reaction.

The proposed method for solution of the difference equations would be
to first assume a wall temperature profile. Material and energy
balances would then be made for each of the vertical increments
successively, starting with the top (inlet) element. Since the
radial flow profile of each element fits the same equation, the
velocity profiles between two successive vertical increments will
differ only by a constant multiplying factor, which will in turn
reflect the total amount of heat absorbed by the second increment.
While we are actually interested in calculating the radial temperature
profile in the element on the basis of radial conduction and radial
and vertical flow, the above multiplier must be evaluated correctly
in order that the radial flow quantities meet the condition that
there be zero radial flow at the central increment. This was done
by assuming a value of the multiplicative constant, going through
the radial "loop" in the computer, and then re-evaluating the constant
if the central radial flow term was not within the acceptable limits
of zero. Having finally met this condition the program was then to go
to the next increment. Actually, the material and energy balances were
made so as to include half of the previous vertical increments in com-
bination with half of the unknown increments since this could improve
the computer stability. Having thus computed the heat-mass balance
problem for the two-dimensional, gas flow system, the wall problem is
solved by the same techniques as in the one-dimensional problem and
the iteration is repeated over and over until there is no change in
T_w or T_{gas}. It was estimated, conservatively, that if the above program

could be made functional it would require approximately two hours

execution time on the IBM 709. Therefore, it was decided to attempt

to check the problem for a stable flow rate, given a wall temperature

profile. Owing to the complexity of the computer program, the problem

was never solved. However, it is thought that this basic problem of the

mathematical description of chemical reaction in turbulent flow in an

insulated ceramic tube would be worthwhile for investigation as a thesis

topic.

While the two-dimensional model described above does not

submit readily to mathematical treatment, the effects of boundary

layer heating can be at least bounded by making a crude approximation.

If it is assumed that the chemical reaction occurs instantaneously and

completely when the wall temperature reaches a certain level, the chemical

reaction becomes merely a step function in the gas stream temperature

profile. The problem is solved in a manner similar to the one-dimensional

model except that the heat input due to reaction comes all at once. The

gas stream is still treated in terms of "plug flow" as far as heat transfer

is concerned. A computer program was written in the MAD language for the

IBM 7090 computer in an attempt to solve the energy balance for the

system, making the above approximation. The equations used are the

same as those derived in Section 1 of Appendix A for the one-dimensional

model except for the reaction term which was omitted and replaced by a

$3500^{\circ}F$ increase in temperature at the reaction point. The computer

program calculated a gas temperature from a given wall temperature

profile and then used this gas temperature profile to calculate a new

set of wall temperatures in thermal equilibrium with it. Another gas

stream temperature profile was then calculated on the basis of these new wall temperatures. The iteration process described above was to be repeated until convergence was obtained.

This technique should calculate stability limits which form an upper bound to those which would have been calculated using the rigorous, two-dimensional model since it assumes that the reaction takes place completely as soon as the outer edge of the boundary layer reaches the reaction temperature. Similarly, the original "plug flow" should lead to a lower bound since it assumes that the reaction does not occur until all of the gas stream has reached the reaction point. The computer program written to solve the heat balance equations is presented in Section 4 of Appendix A. It was found that in order to maintain stability of the finite difference procedure it was necessary to make the time increment so small that convergence became very slow. Extrapolation of the runs made seemed to indicate that several thousand dollars worth of computer time would be required for each flow rate and fuel-air ratio using this technique. At this point use of the program was suspended.

The initial assumed wall temperature profile and the gas temperature profile calculated from it are shown as dashed lines in Figure 14. The assumed wall temperature profile is taken from experimental data. It should be noted that, given this wall temperature profile, the calculated gas stream temperatures will be effected only by the fuel-air ratio in the propane-air mixture.

VIII. CONCLUSIONS AND RECOMMENDATIONS

The conclusions which may logically be drawn from the experimental and theoretical work performed in this study have already been set forth at different places in this dissertation. They are listed here in the interests of consolidating them for the reference of the reader. On the basis of these conclusions certain recommendations are made for the extension of work in this area.

Conclusions:

1) Turbulent, propane-air flames can be stabilized in a self-perpetuating fashion in smooth, insulated ceramic tubes. Preheating is required.

2) A high-purity, non-porous Al_2O_3 tube backed by several inches of phosphate-bonded Zr_2O_3 cement will meet the materials requirements imposed by operation of the system.

3) The stable flow rates observed are intermediate between those observed for turbulent gas streams and those observed for stabilization by means of bluff body flame holders.

4) The flow stability limit increases with increasing fuel-air ratio up to the stoichiometric fuel-air ratio.

5) The point of highest temperature in the wall temperature profiles moved farther from the inlet section of the tube as the flow rate increased.

6) The "plug flow" mathematical model gave a very poor representation of the system.

7) The data seem to lend general confirmation to some form of a two-dimensional model.

Recommendations:

1) Data similar to that taken in this study should be taken for tubes of the same length to diameter ratio but with varying diameters. This should give some indication of the effect of the surface and gas boundary layer on the stability limits.

2) More work should be done on the mathematical description of the system both with respect to digital computer solution of simplified or limiting cases and the possible use of analog computation to solve the equations.

3) Data could be taken on the effect of varying the length to diameter ratio.

4) The effect of using different tube materials could be investigated.

5) Another possible source for more investigation would be to change the reacting materials used.

APPENDICES

APPENDIX A

DERIVATION OF FINITE DIFFERENCE EQUATIONS AND COMPUTER PROGRAMS

Section 1

DERIVATION OF FINITE DIFFERENCE EQUATIONS FOR THE ONE-DIMENSIONAL MODEL

Consider the physical situation as shown in Figure 13. Let the Jth increment be from x to x + DX. We may then write for the energy balance on the gas stream element:

Input-

$$\pi R^2 G C_p \left[\frac{T(J-1,t) + T(J,t)}{2} \right] + 2\pi R \cdot DX \cdot h_g \left[X(J,t) - T(J,t) \right] +$$

$$2 KRX \cdot A(J,t) \cdot B(J,t) \cdot \Delta H \cdot \rho^2 \cdot \pi R^2 DX \cdot e^{-E/R_0 \cdot T(J,t)} \tag{19}$$

Output-

$$\pi R^2 G C_p \left[\frac{T(J+1,t) + T(J,t)}{2} \right] \tag{20}$$

Accumulation-

$$2\pi R^2 \cdot DX \cdot \rho \cdot C_p \left[\frac{T(J, t+1) - T(J,t)}{DT} \right] \tag{21}$$

By applying the identity, Input - Output ≡ Accumulation, and simplifying, we have:

$$T(J, t+1) = T(J,t) + \frac{DT}{2 DX \cdot \rho \cdot C_p} \left\{ G C_p T(J-1,t) + 4 \frac{DX \cdot h_g}{R} \left[X(J,t) - T(J,t) \right] \right.$$

$$\left. - G C_p T(J+1,t) + 2 KRX \cdot A(J,t) \cdot B(J,t) \cdot \rho^2 \cdot DX \cdot \Delta H \cdot e^{-E/R_0 \cdot T(J,t)} \right\} \tag{22}$$

In a similar fashion, we may make the energy balance on the wall element:

Input-

$$-k_w \, \pi \left(2R \cdot DR + DR^2\right) \cdot \left. \frac{\partial X(J,t)}{\partial x}\right|_{x-\frac{DX}{2}} + 2\pi \, R \cdot DX \cdot \sigma \sum_I \mathcal{F}_{I \to J} \left[X^4(I,t)\right] \tag{23}$$

Output-

$$-k_w \, \pi \left(2R \cdot DR + DR^2\right) \cdot \left. \frac{\partial X(J,t)}{\partial x}\right|_{x+\frac{DX}{2}} + 2\pi \, R \cdot DX \cdot \sigma \sum_I \mathcal{F}_{J \to I}\left[X^4(J,t)\right] +$$

$$h_g \cdot 2\pi \cdot R \cdot DX \cdot \left[X(J,t) - T(J,t)\right] \tag{24}$$

Accumulation-

$$\pi \left(2R \cdot DR + DR^2\right) \cdot DX \cdot \rho \cdot C_P \left[\frac{X(J,t+1) - X(J,t)}{DT}\right] \tag{25}$$

Thus,

$$X(J,t+1) = X(J,t) + \frac{DT}{\left(2R \cdot DR + DR^2\right)} \cdot \frac{k_w}{\rho \, C_P} \left\{ \frac{\left(2R \cdot DR + DR^2\right)}{(DX)^2} \left[X(J+1,t) + X(J-1,t) - 2X(J,t)\right] \right.$$

$$\left. + \frac{2R \cdot \sigma}{k_w} \sum_I \mathcal{F}_{I \to J}\left[X^4(I,t) - X^4(J,t)\right] + \frac{2R \, h_g}{k_w} \left[X(J,t) - T(J,t)\right] \right\}$$

$$\tag{26}$$

Where $\mathcal{F}_{I \to J} = \mathcal{F}_{J \to I}$ is defined in **Mathematical Models**, Equation (15).

We are interested in a program for determining the steady state gas temperature profile given a wall temperature profile. We can thus ignore the equations for the energy balance on the wall. Since we are at steady state, the accumulation term is zero and we have,

$$T(J+1) = T(J-1) + \frac{4 \cdot DX \cdot h_g}{G \cdot C_P \cdot R} \left[X(J) - T(J)\right] + 2KRX \cdot A(J) \cdot B(J) \frac{\Delta H \rho^2 \cdot DX}{G \cdot C_P} e^{-E/R_0 T(J)} \tag{27}$$

as the basic equation. In order to express the computer program unambiguously, we must define the following:

ACT = The energy of activation in the Arrhenius rate equation, BTU/lb.mol.

AZ = The initial oxygen concentration, lb. O_2/lb. gas mixture.

BZ = The initial propane concentration, lb. C_3H_8/lb. gas mixture

CP = The heat capacity of the bulk gas stream, BTU/lb.^oR.

DELH = The heat of reaction, BTU/lb. C_3H_8

DELX = The incremental unit of tube length, ft.

KRX = The reaction rate coefficient, cu.ft./lb.hr.

RHO = The average gas stream density, lb./cu.ft.

RO = The gas constant, BTU/lb.mol.^oR.

U = The overall heat transfer coefficient, BTU/hr.sq.ft.^oR.

The program evaluates CP and RHO as a function of the conversion from data given in Hougen, Watson, and Ragatz[17]. The heat transfer coefficient, U, is a function of temperature, mass flow rate, density, and diameter to various powers as indicated in Perry[32]. Since it was found necessary to decrease the size of DELX in the region where the reaction took off, the program contains a loop which decreases the size of the increment by a factor of 10 whenever the difference between successive temperatures is too large. Provision is also made for increasing the increment size when the reaction ceases.

Section 2

ONE-DIMENSIONAL COMPUTER PROGRAM

The flow diagram for the program is shown on the following pages. It is followed by the program itself written in the MAD (Michigan Algorithm Decoder) computer language for the IBM 709.

An example of the program output is shown in Figure 14. The given wall temperature profile is also indicated on the plot.

START → READ X(1)...X(130) AZ, BZ, DELX, B, SIG, DEIR, R, KRX → READ DELH, ACT, RO, DT, KCEE → PRINT AZ, BZ, DELX, G, HEADING → INITIALIZE A, B, T, N → (1)

(1) → NEW, J.E. 15, 1, J.G. 116 → SIGM = φ(A(J)) → RHO = φ(A(J), B(J), T(J)) → CP = φ[A(J),B(J),T(J),RHO] → (2)

(2) → U = φ[G,CP,T(J),R] → NSC = 8.79+.0112T(J) → B(J+1)=φ[B(J),A(J),T(J)] → A(J+1)=φ[A(J),B(J+1)] → (3)

(3) → T(J+1)=T(J-1)+ $\frac{4(DELX)(U)[X(J)-T(J)]}{G \cdot CP \cdot R}$ +2KRX·A(J)·B(5)·DELH·(RHO)^2·DELX·EXP(-ACT/RO·T(J)) → (4)

(4) → PRINT J, T(J), X(J), A(J), B(J) → IS T(J+1)>1700 — NO → NEW → BOO → T2=T(J) T1=T(J-1) A2=A(J) A1=A(J-1) B2=B(J) B1=B(J-1) FOOL=J+0. ZORRO=0. → ZOO → GOO, K=1, 1, K.G.400 → (5)

IS T(J+1)>1700 — YES

(5) → IS ZORRO=1.0 — NO → T(0)=(9T2+T1)/10 A(0)=(9A2+A1)/10 B(0)=(9B2+B1)/10 → POOH → T(1)=T2 A(1)=A2 B(1)=B2 → STGM=φ(A(K)) RHO =φ[...] → (6)

IS ZORRO=1.0 — YES → POOH

(6) → IS T(K)>2500 — NO → CP = φ[T(K),...] → U = φ[...] B(K+1)= NSC=φ A(K+1)= T(K+1)=φ[...DX...] DX = DELX/(10)^N → IS B(K+1)<0 — NO → (7)

IS T(K)>2500 — YES → CP=.33

IS B(K+1)<0 — YES → B(K+1)=0

(7) → IS A(K+1)<0 — NO → FOO(K)=FOOL+ $\frac{K}{10^N}$ → PRINT FOO(K) T(K+1),A(K+1) B(K+1) → IS |T(K+1)-T(K)| >100 — YES → FOOL = FOO(K-1) N = N+1 T1=T(K-1);T2=T(K) A1=A(K-1);A2=A(K) B1=B(K-1);B2=B(K) → ZOO

IS A(K+1)<0 — YES → A(K+1)=0

IS |T(K+1)-T(K)| >100 — NO → (8)

(8) → IS T(K+1)<T(K) — YES → A(K+1) = 0. B(K+1) =.0027 → IS |T(K+1)-T(K)|<1 AND T(K+1)<3620 — YES → START

IS T(K+1)<T(K) — NO → MOO

IS |T(K+1)-T(K)|<1 AND T(K+1)<3620 — NO → GOO

MOO → IS |T(K+1)-T(K)| <.01 — YES → FOOL = FOO(K) A2=A(K) T(0)=T(K-10) B2=B(K) A(0)=A(K-10) T2=T(K) B(0)=B(K-10) → ZORRO=1.0 → ZOO

IS |T(K+1)-T(K)| <.01 — NO → GOO → START

THOMAS D. BATH	Q111N	005	030	000	EXPLICIT
THOMAS D. BATH	Q111N	005	030	000	EXPLICIT

```
$ COMPILE MAD, EXECUTE, DUMP, PUNCH OBJECT, PRINT OBJECT
              DIMENSION T(2000),X(2000),A(2000),B(2000),FOO(2000)
START         READ FORMAT IND, X(1)...X(130)
              READ FORMAT INC, AZ, BZ, DELX, G, SIG, DELR, R, KRX, DELH, AC
             1T, RO, DT, KCER
              PRINT FORMAT INVAL, AZ, BZ, DELX, G
              PRINT FORMAT ZOOP
              A(15)=AZ
              B(15)=BZ
              T(14)=530.
              T(15)=530.
              ZORRO=0.0
              N=1
              THROUGH NEW, FOR J=15, 1, J.G.116
              SIGM= 8.146+.243*A(J)/AZ
              RHO    =1.281/T(J)*(208.98+54.55*A(J)/AZ-39.06*B(J)/BZ)/SIGM
              CP     =1.281/T(J)/RHO    *((53.11+10.43*A(J)/AZ-11.5*B(J)/BZ)+T
             1(J)/1E3*(11.033+3.005*A(J)/AZ+3.215*B(J)/BZ)-T(J).P.2/1E6*(1.
             2028+.53*A(J)/AZ+.913*B(J)/BZ))/SIGM
              U=.00542*G.P.0.80*CP.P.0.30*(T(J)/460.).P.0.23/((2.*R).P.0.2)
              NSC=8.79+0.0112*T(J)
              T(J+1)=T(J-1)+4.*DELX*U*(X(J)-T(J))/(G*CP*R)+2.*KRX*A(J)*B(J)
             1*DELH*RHO.P.2*DELX*EXP.(-ACT/(RO*T(J)))/(G*CP)
              B(J+1)=B(J)-KRX*RHO.P.2*A(J)*B(J)*EXP.(-ACT/(RO*T(J)))*DELX/G
              A(J+1)=A(J)        +5.09*(B(J+1)-B(J))
              PRINT FORMAT OUTA,J,T(J),X(J),A(J),B(J)
              WHENEVER T(J+1).G.1700.
              T2=T(J)
              A2=A(J)
              B2=B(J)
              T1=T(J-1)
              A1=A(J-1)
              B1=B(J-1)
              FOOL=J+0.
              TRANSFER TO ZOO
              END OF CONDITIONAL
NEW           CONTINUE
              TRANSFER TO 800
ZOO           THROUGH GOO, FOR K=1,1, K.G.400
              WHENEVER ZORRO.E.1.0,TRANSFER TO POOH
              T(0)=(9.*T2+T1)/10.
              A(0)=(9.*A2+A1)/10.
              B(0)=(9.*B2+B1)/10.
POOH          T(1)=T2
              A(1)=A2
              B(1)=B2
              SIGM= 8.146+.243*A(K)/AZ
              RHO    =1.281/T(K)*(208.98+54.55*A(K)/AZ-39.06*B(K)/BZ)/SIGM
              WHENEVER T(K).G.2500.
              CP=0.330
              OTHERWISE
              CP    =1.281/T(K)/RHO   *((53.11+10.43*A(K)/AZ-11.5*B(K)/BZ)+T
             1(K)/1E3*(11.033+3.005*A(K)/AZ+3.215*B(K)/BZ)-T(K).P.2/1E6*(1.
             2028+.53*A(K)/AZ+.913*B(K)/BZ))/SIGM
              END OF CONDITIONAL
              U=.00542*G.P.0.80*CP.P.0.30*(T(K)/460.).P.0.23/((2.*R).P.0.2)
              NSC=8.79+0.0112*T(K)
              DX=DELX/10.0.P.(N)
```

```
        T(K+1)=T(K-1)+4.*DX*U*(3610.-T(K))/(G*CP*R)+2.*KRX*A(K)*B(K)*
       1DELH*RHO.P.2*DX*EXP.(-ACT/(RO*T(K)))/(G*CP)
        B(K+1)=B(K)-KRX*RHO.P.2*A(K)*B(K)*EXP.(-ACT/(RO*T(K)))*DX/G
        A(K+1)=A(K)+5.09*(B(K+1)-B(K))
        WHENEVER B(K+1).L.0.0, B(K+1)=0.0
        WHENEVER A(K+1).L.0.0,A(K+1)=0.0
        FOO(K)=FOOL+K/10.0.P.(N)+0.
        PRINT FORMAT COO, FOO(K), T(K+1), A(K+1), B(K+1)
        WHENEVER.ABS.(T(K+1)-T(K)).G.100.
        FOOL=FOO(K-1)
        N=N+1
        T1=T(K-1)
        A1=A(K-1)
        B1=B(K-1)
        B2=B(K)
        T2=T(K)
        A2=A(K)
        TRANSFER TO ZOO
        END OF CONDITIONAL
        WHENEVER T(K+1).L.T(K)
        A(K+1)=0.0
        B(K+1)=.0027
        TRANSFER TO WOO
        END OF CONDITIONAL
        TRANSFER TO MOO
WOO     WHENEVER.ABS.(T(K+1)-T(K)).L.1.0.AND.T(K+1).L.3620.
        TRANSFER TO START
        END OF CONDITIONAL
        TRANSFER TO GOO
MOO     WHENEVER .ABS.(T(K+1)-T(K)).L.0.1
        FOOL=FOO(K)
        A2=A(K)
        B2=B(K)
        T2=T(K)
        T1=T(K-10)
        A1=A(K-10)
        B1=B(K-10)
        T(0)=T1
        A(0)=A1
        B(0)=B1
        N=N-1
        ZORRO=1.0
        TRANSFER TO ZOO
        END OF CONDITIONAL
GOO     CONTINUE
        VECTOR VALUES COO=$F11.6,F10.5,8H 3610.00,2F8.4*$
        VECTOR VALUES IND=$(10F7.0)*$
        VECTOR VALUES INC= $2F8.4,F8.6,F8.1,E8.3,2F8.5,E8.1,E8.0/F8.0
       1,F8.3,F8.6,F8.3*$
        VECTOR VALUES INVAL= $6H1AZ = F8.4,6H BZ = F8.4,8H DELX = F8.
       16,5H G = F8.1*$
        VECTOR VALUES ZOOP=$46H     J        T(J)         X(J)        A(J)
       1      B(J)*$
        VECTOR VALUES OUTA=$I4,F17.5,F8.2,2F8.4*$
        INTEGER I,J,K,N
BOO     TRANSFER TO START
        END OF PROGRAM
```

Section 3

DERIVATION OF FINITE DIFFERENCE EQUATIONS
FOR THE TWO-DIMENSIONAL MODEL

Consider the following geometrical situation (a longitudinal cross section of the ceramic tube):
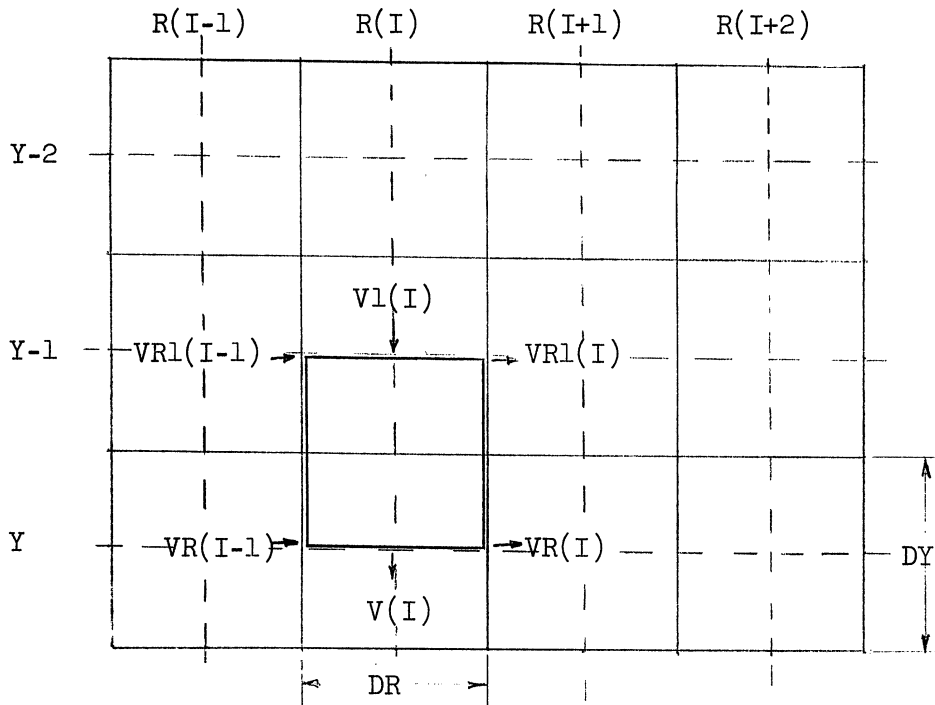

Figure 16. Geometrical Basis for Two-Dimensional
Mathematical Model.

Since we are dealing with a steady-state phenomenon, the basic balance equation is

INPUT = OUTPUT

In order to avoid doubly subscripting all variables we will denote

$VAR(I,Y-1) \equiv VAR1(I)$. The equations for the two-dimensional gas stream balances are:

Material Balance-

(Input)

$$2\pi\left[\left(\frac{R(I)+R(I-1)}{2}\right)^2 - \left(\frac{R(I)+R(I+1)}{2}\right)^2\right] V1(I) \cdot \rho 1(I) = \text{Axial Flow In} \qquad (28)$$

$$2\pi \cdot DY\left[\frac{R(I)+R(I-1)}{2}\right] \cdot \left[\frac{\rho(I-1)VR(I-1) + \rho 1(I-1)VR1(I-1)}{2}\right] = \text{Radial Flow In} \qquad (29)$$

(Output)

$$2\pi\left[\left(\frac{R(I)+R(I-1)}{2}\right)^2 - \left(\frac{R(I)+R(I+1)}{2}\right)^2\right] V(I) \cdot \rho(I) = \text{Axial Flow Out} \qquad (30)$$

$$2\pi \cdot DY\left[\frac{R(I)+R(I+1)}{2}\right] \cdot \left[\frac{\rho(I)VR(I) + \rho 1(I)VR1(I)}{2}\right] = \text{Radial Flow Out} \qquad (31)$$

If we define

$$\left[\left(\frac{R(I)+R(I-1)}{2}\right)^2 - \left(\frac{R(I)+R(I+1)}{2}\right)^2\right] \equiv AER(I) \qquad (32)$$

We have,

$$AER(I) \cdot V1(I) \rho 1(I) + \left[\frac{R(I)+R(I-1)}{2}\right] \cdot \left[\frac{\rho(I-1)VR(I-1) + \rho 1(I-1)VR1(I-1)}{2}\right] DY =$$

$$\left(\frac{R(I)+R(I+1)}{2}\right)\rho 1(I)VR1(I) + \left[AER(I) V(I) + \left(\frac{R(I)+R(I+1)}{2}\right) VR(I)\right] \rho(I) \qquad (33)$$

Since we are assuming,

$$V(I) = C \cdot V1(I) \qquad (34)$$

Then only VR(I) and $\rho(I)$ are unknown in the above equation.

Energy Balance-

(Input) $2\pi \cdot AER(I) \cdot V1(I) \cdot \rho 1(I) \cdot CP1(I) \cdot T1(I)$ (35)

= Axial Heat Flow In

$$2\pi \left(\frac{R(I)+R(I-1)}{2}\right)\left[\frac{\rho(I-1)VR(I-1)CP(I-1)T(I-1)+\rho 1(I-1)VR1(I-1)CP1(I-1)T1(I-1)}{2}\right]$$ (36)

= Radial Heat Flow In

$$2\pi \ AER(I) \cdot KRX \cdot \left(\frac{A(I)+A1(I)}{2}\right) \cdot \left(\frac{B(I)+B1(I)}{2}\right) \rho^2(I) \cdot DV \cdot \Delta H \cdot e^{-E/R_0\left(\frac{T(I)+T1(I)}{2}\right)}$$ (37)

= Heat Input from Reaction

$$2\pi \left(\frac{R(I)+R(I-1)}{2}\right)\left\{\frac{\left[K1(I)+K1(I-1)\right]\cdot\left[T1(I-1)-T1(I)\right]+\left[K(I)+K(I-1)\right]\cdot\left[T(I-1)-T(I)\right]}{4 \cdot DR}\right\}$$

= Radial Heat Input from Conduction (38)

(Output)

$$2\pi \ AER(I) \cdot V(I) \cdot \rho(I) \cdot CP(I) \cdot T(I)$$ = Axial Heat Flow Out (39)

$$2\pi \left(\frac{R(I)+R(I+1)}{2}\right)\left[\frac{\rho(I)VR(I)CP(I)T(I)+\rho 1(I)CP1(I)VR1(I)T1(I)}{2}\right]$$ = Radial Heat Flow Out (40)

$$2\pi \left(\frac{R(I)+R(I+1)}{2}\right)\left[\frac{\left[K1(I)+K1(I+1)\right]\cdot\left[T1(I)-T1(I+1)\right]}{4 DR}\right.$$ = Radial Heat Output from Conduction (41)

$$\left.\frac{\left[K(I)+K(I+1)\right]\cdot\left[T(I)-T(I+1)\right]}{4 DR}\right]$$

We now have the basic equations which must be solved in conformance with the boundary conditions indicated earlier in this work.

In order to attempt the solution of the above pair of equations on the digital computer, the following approximations were made:

A. First Approximation on T(I) -

   1) Assume Equations (36) and (40) are of negligible
      importance compared to the rest of the terms.

   2) In Equation (37) assume: $A(I) = A1(I)$, $B(I) = B1(I)$,
      $\rho(I) = \rho1(I)$, and in the exponential $T(I) = T1(I)$.

   3) In Equation (38) assume $K(I) = K1(I)$.

   4) In Equation (39) assume $V(I) = C \cdot V1(I)$, $\rho(I) = \rho1(I) \dfrac{T1(I)}{T(I)}$,
      and $CP(I) = CP1(I)$.

   5) In Equation (41) assume $K(I) = K1(I)$, $K(I+1) = K1(I+1)$
      and $T(I+1) = T1(I+1)$.

The above set of assumptions reduce the master (Input = Output) energy

balance equation to one explicit for $T(I)$. It can thus be solved for

$T(I)$, $A(I)$, $B(I)$, $\rho(I)$, and $K(I)$. These quantities replace the previous

approximations of $T1(I)$, etc. whenever possible and the explicit equation

is again used to calculate $T(I)$, $A(I)$, ... This cycle is repeated until

the $T(I)_j - T(I)_{j+1} \leq$ some specified error. The proposed program then

refers to (33) with the substitution of (34) in it and calculates the

$VR(I)$. If $VR(IMAX) \neq 0$, then the constant C must be re-evaluated by

$$C = C_{init} + \frac{VR(IMAX)}{V(IMAX) \cdot IMAX} \qquad (42)$$

and the above process of evaluating $T(I)$ is repeated. Once C is properly

evaluated, the whole previous process can be repeated with the radial flow

of fluid taken into account. In general, when a set of equations is sol-

ved by repeated iterations as in the above process, there is always a

question as to whether convergence with the correct answer will be reached.

It would seem that a more direct scheme than the above would be very use-

ful here.

Section 4

COMPUTER PROGRAM ASSUMING INSTANTANEOUS WALL
REACTION AT A GIVEN TEMPERATURE

The following program is similar to the previous one-dimensional

program except that the chemical reaction is included as a step function

in temperature and the iteration for steady-state wall temperatures is

performed.

$ COMPILE MAD, EXECUTE, DUMP, PRINT OBJECT, PUNCH OBJECT

MAD (16 APR 1962 VERSION) PROGRAM LISTING ... ...


```
        DIMENSION T(150),X(150),V(150),PHE(150),F(150),EPS(150),QRAD(
       1150),Y(150),Z(150)
        READ FORMAT INA,X(1)...X(140),V(0)...V(125)
        VECTOR VALUES INA=$(14F5.0)*$
        PRT=0
START   READ FORMAT INB,AZ,BZ,DX   ,G,SIG,DR   ,R,DT,KCER,TIN,ERROR,FLI
       1P
        VECTOR VALUES INB=$11F6.0,I6*$
        PRT=PRT+1
        PRINT FORMAT VERDI,AZ,BZ,DX   ,G,DT,TIN
        VECTOR VALUES VERDI=$6H   AZ =F8.6,6H   BZ =F8.6,22HSTOICHIOMET
       1RIC MIXTURE/10H    DX   =F8.6,6H   G =F8.2,6H   DT =F8.6,7H   T
       2IN =F8.2*$
        COUNT=0
        T(14)=TIN
        T(15)=TIN
LISZT   COUNT=COUNT+1
        THROUGH RAVEL,FORJ=1,1,J.G.140
RAVEL   Z(J)=X(J)
        THROUGH BACH,FORJ=15,1,J.G.125
        WHENEVER X(J).G.2000.,TRANSFER TO CZERNY
        RHO=34.3/T(J)
        CP=1.281*(52.04+17.253*T(J)/1E3-2.471*T(J)*T(J)/1E6)/34.3
        U=.00542*G.P.0.80*CP.P.0.30*(T(J)/460.).P.0.23/((2.*R).P.0.2)
BACH    T(J+1)=T(J-1)+4.*DX  *U*(X(J)-T(J))/(G*CP*R)
        TRANSFER TO DUKAS
CZERNY  T(J+1)=T(J)+3500.
        T(J+2)=T(J+1)
        SKIP=J
        THROUGH GRIEG,FORJ=J+2,1,J.G.125
        RHO=1.281*208.98/T(J)
```

-70-

```
        TEMPUS=0
        CP=.330
        U=.00542*G.P.0.80*CP.P.0.30*(T(J)/460.).P.0.23/((2.*R).P.0.2)
GRIEG   T(J+1)=T(J-1)+4.*DX  *U*(X(J)-T(J))/(G*CP*R)
DUKAS   THROUGH HANDEL,FORJ=16,1,J.G.126
        NSC=8.79+0.0112*X(J)
        EPS(J)=0.7+0.3*(X(J)-760.)/X(J)
        QRAD(0)=0.
        THROUGH HAYDN,FORK=1,1,K.G.140
        PHE(K)=V(.ABS.(J-K))
        EPS(K)=0.7+0.3*(X(K)-760.)/X(K)
        WHENEVER PHE(K).E.0.0
        F(K)=0.0
        TRANSFER TO HAYDN
        END OF CONDITIONAL
        F(K)=1./(1./PHE(K)+(1.-EPS(K))/EPS(K)+(1.-EPS(J))/EPS(J))
HAYDN   QRAD(K)=QRAD(K-1)+SIG*2.*R/KCER*F(K)*(X(K).P.4-X(J).P.4)/1E8
        Y(J)=X(J)+DT/(2.*DR*R+DR*DR)/NSC*((2.*R*DR+DR*DR)/(DX*DX)*(X(
       1J+1)-2.*X(J)+X(J-1))-2.*R/KCER*U*(X(J)-T(J))+QRAD(140))
        WHENEVER PRT.E.1.AND.(Y(J).L.400..OR.Y(J).G.4000..OR.(((TEMPU
       1S+9)/10)*10-TEMPUS).E.0)
        PRINT RESULTS Y(J),T(J),X(J), QRAD(140)

        END OF CONDITIONAL
HANDEL  CONTINUE
        GOOD =0.0
        THROUGH FRANCK,FOR J=20,20,J.G.1ø0
        WHENEVER.ABS.(Y(J)-X(J)).LE.ERROR
        GOOD=GOOD+1.0
FRANCK  END OF CONDITIONAL
        WHENEVER GOOD.E.5.0
        PRINT FORMAT SUK, TEMPUS
        VECTOR VALUES SUK=$36H ERROR CONVERGED ON TIME STEP NUMBERI6/
       130H    J       Y(J)           T(J)       *$
        THROUGH DVORAK, FOR J=16,1,J.G.126
        PRINT FORMAT MAHLER,J,Y(J),T(J)
        VECTOR VALUES MAHLER=$I6,2F12.4*$
DVORAK  CONTINUE
        OTHERWISE
        THROUGH WAGNER,FORJ=16,1,J.G.126
WAGNER  X(J)=Y(J)
        TEMPUS=TEMPUS+1
        TRANSFER TO DUKAS
        END OF CONDITIONAL
        WHENEVER SKIP.E.FLIP
        PRINT FORMAT MOZART,COUNT
        VECTOR VALUES MOZART=$17H ITERATION STEP =I6//45H THIS IS THE
       1 SOLUTION FOR THE ABOVE FLOW RATE*$
        TRANSFER TO START
        OTHERWISE
        FLIP=SKIP
        THROUGH CHOPIN,FORJ=16,1,J.G.126
CHOPIN  X(J)=(Y(J)+Z(J))/2.
        TRANSFER TO LISZT
        END OF CONDITIONAL
        INTEGER COUNT,J,SKIP,TEMPUS,K,FLIP,PRT
        END OF PROGRAM
```

TABLE II

EXPERIMENTAL DATA

Run No.	1	2	3	4	5	6	7	8	9	10
T1(°F)	700	200	550	380		1100	200	95		850
T2(°F)	1700	1390	1720	1390	unstable	1770	1800	1650	unstable	2070
T3(°F)	2020	2070	1950	2030		2000	2300	2270		2140
T4(°F)	2000	2120	2000	2100		1960	2380	2460		2180
T5(°F)	1840	1970	1800	1980		1700	2200	2310		2020
T6(°F)	1540	1720	1530	1740		1440	1800	1900		1700
Fuel-Air Ratio, φ	.053	.052	.0515	.058	.049	.058	.058	.0615	.059	.056
Mass Flow Rate, w	0.79	0.92	0.70	0.77	1.00	0.735	1.06	1.30	1.48	.78

Run No.	11	12	13[a]	14[b]	15	16	17	18	19	20
T1(°F)	400		2160	700	700		550	250		
T2(°F)	2070	unstable	2070	2300	1650	unstable	1700	1770	unstable	unstable
T3(°F)	2400		2000	2440	2100		2000	2300		
T4(°F)	2420		2020	2460	2050		1970	2330		
T5(°F)	2300		1890	2330	1920		1840	2130		
T6(°F)	2020		1700	2070	1440		1380	1770		
Fuel-Air Ratio, φ	.058	.055	.058	.0565	.058	.053	.056	.0661	.0632	.055
Mass Flow Rate, w	1.11	1.30	.73	1.21	.77	1.075	.79	1.0	1.44	1.08

[a] Flame stabilized in pipe fitting at top of tube
[b] Crack in tube

Run No.	21	22	23	24	25	26	27	28	29	30
T1(°F)				660	440		500			600
T2(°F)	unstable	unstable	unstable	1550	1580	unstable	1550	unstable	unstable	1620
T3(°F)				2070	2200		2040			2080
T4(°F)				2140	2200		2070			2090
T5(°F)				1980	1980		1900			1770
T6(°F)				1720	1690		1580			1430
Fuel-Air Ratio, φ	.046	.050	.054	.058	.056	.060	.058	.055	.054	.059
Mass Flow Rate, w	.62	.83	1.00	.77	1.0	1.25	.77	1.00	1.00	0.77

Run No.	31	32	33[c]	34	35	36	37	38	39	40
T1(°F)			2080	1040		1730	120	70	235	
T2(°F)			2030	1890		1770	1650	1600	1270	
T3(°F)			1970	2160		1570	1910	1910	1940	
T4(°F)	unstable	unstable	1910	2150	unstable	1530	2040	2110	2080	unstable
T5(°F)			1830	1960		1300	1800	1970	1910	
T6(°F)			1470	1580		1000	1360	1550	1570	
Fuel-Air Ratio, φ	.052	.055	.058	.059	.054	.050	.052	.055	.054	.0485
Mass Flow Rate, w	1.00	1.00	.77	.84	.93	.50	.75	.90	.825	.50

[c]Temperature profile discarded due to crack in tube

Run No.	41	42	43	44	45[d]
T1(°F)		300	1150		1900
T2(°F)		1950	1860		1850
T3(°F)		2430	2120		1800
T4(°F)	Flow rate too low Apparatus	2470	2170	unstable	2000
T5(°F)		2360	2080		1700
T6(°F)		2040	1820		1500
Fuel-Air Ratio, φ	.050	.0608	.0640	.049	.055
Mass Flow Rate, w	.44	1.26	1.30	.60	.62

[d]Temperature profile bad.

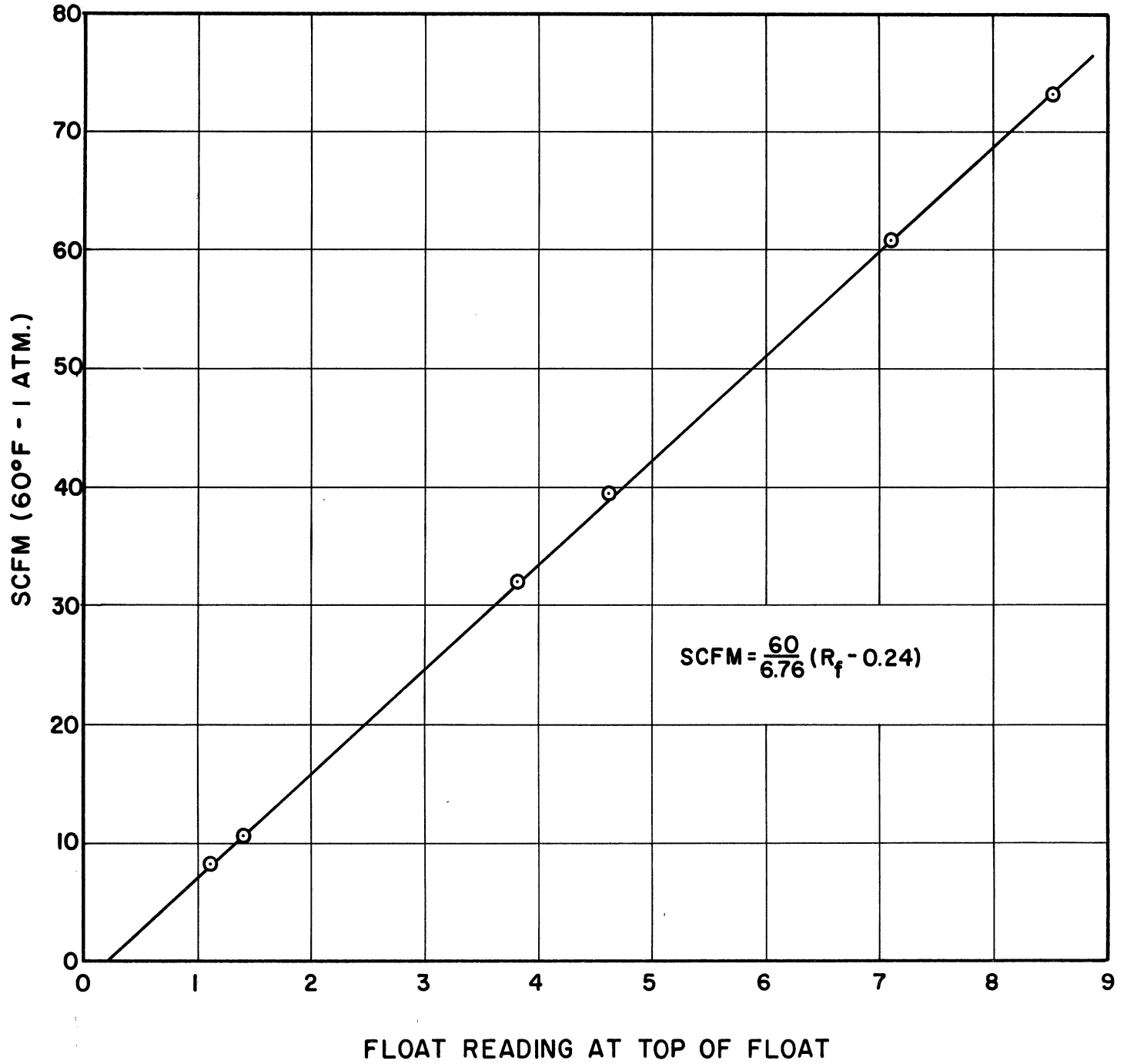APPENDIX C

ROTAMETER CALIBRATION CURVES

Figure 17. Calibration Curve for High Capacity Air Rotameter. (Tube B6-35-10/70-CG; Float-BNSVT-64 Pressure Regulated at 50 psig.)
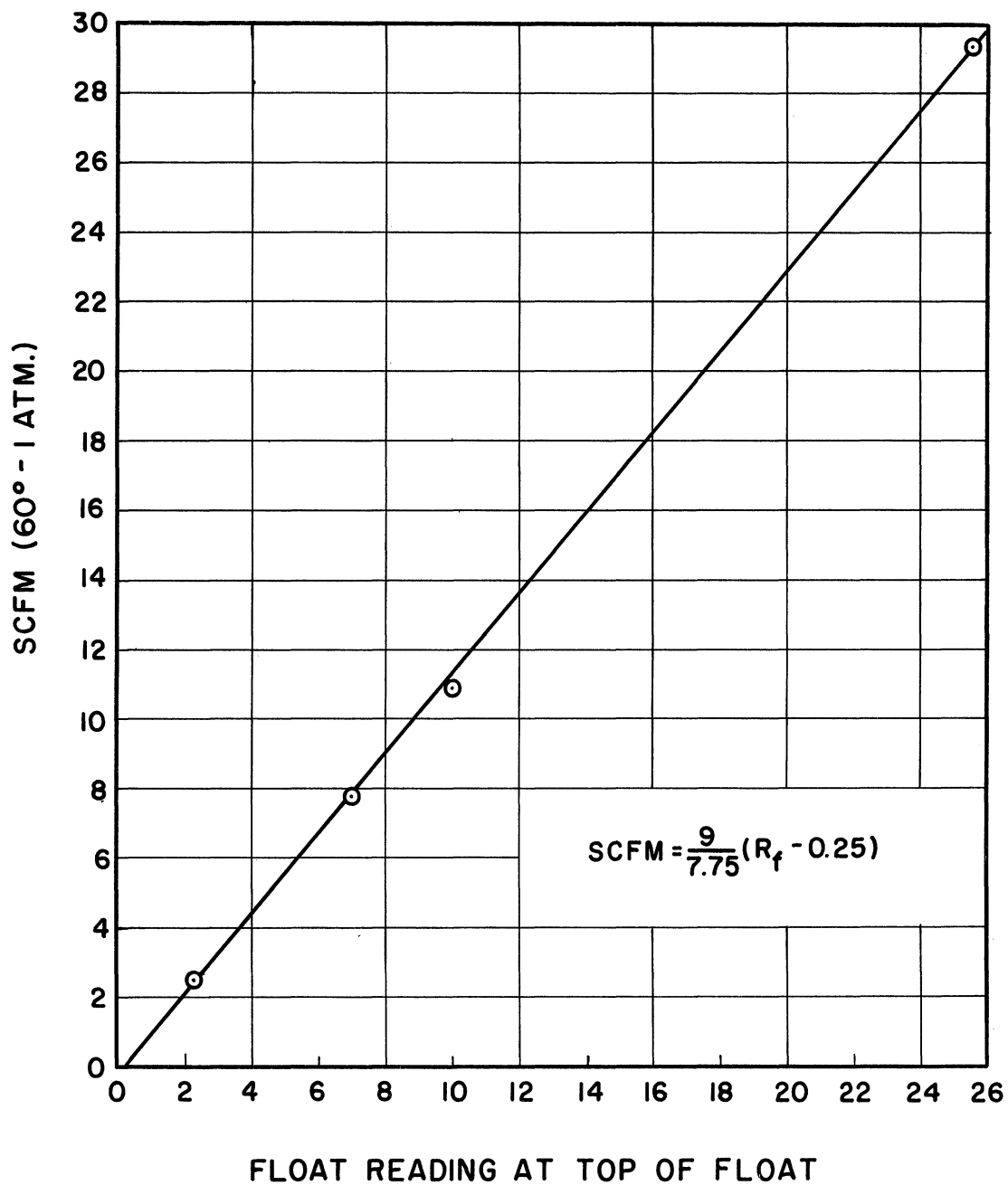
Figure 18. Calibration Curve for Low Capacity Air
            Rotameter. (Tube B5-21-10/70-CG; Float
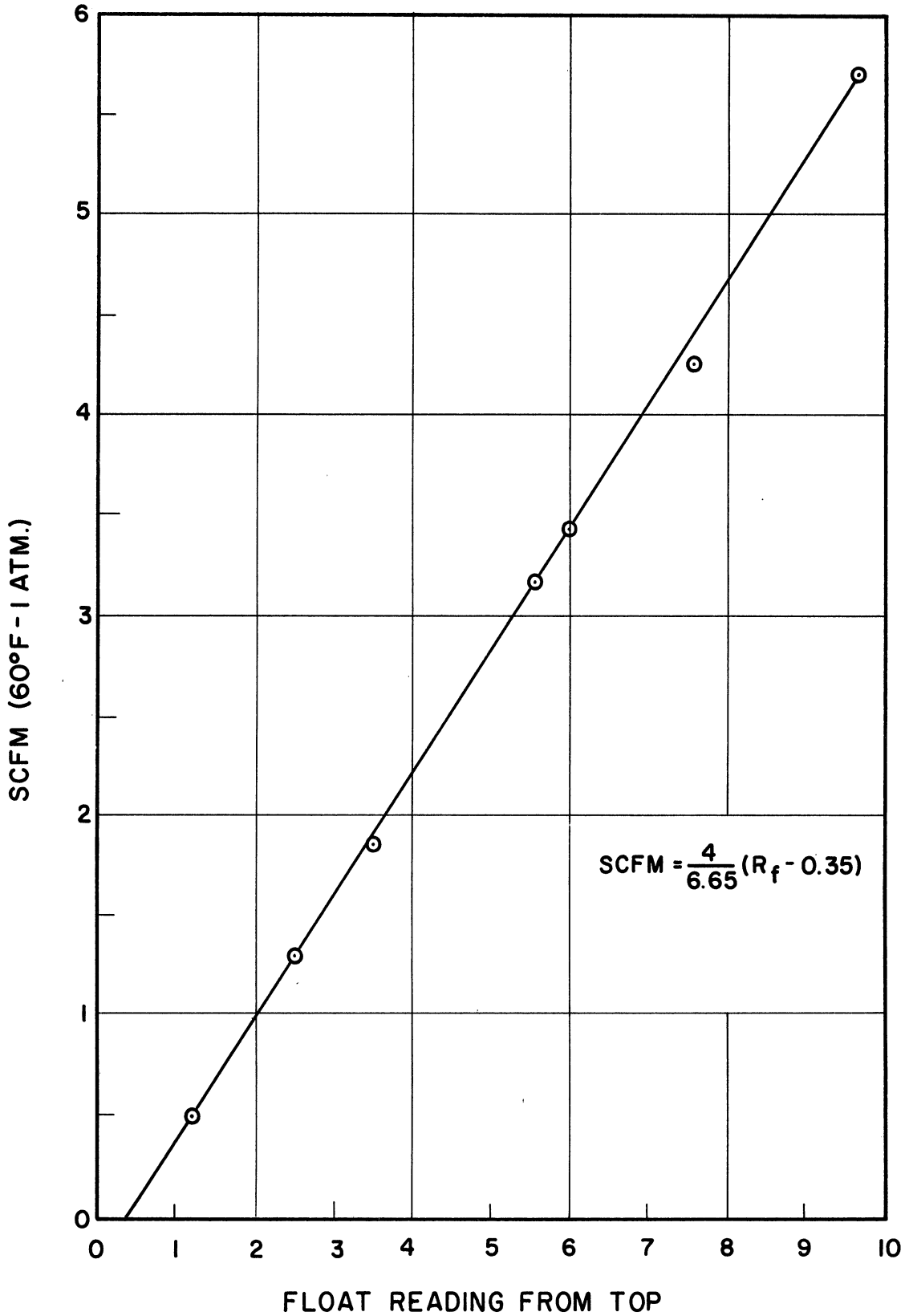   BSVT-53 Pressure Regulated at 50 psig.)

Figure 19. Calibration Curve for High Capacity Propane
Rotameter. (Tube B4-27-10/77; Float BSVT-45
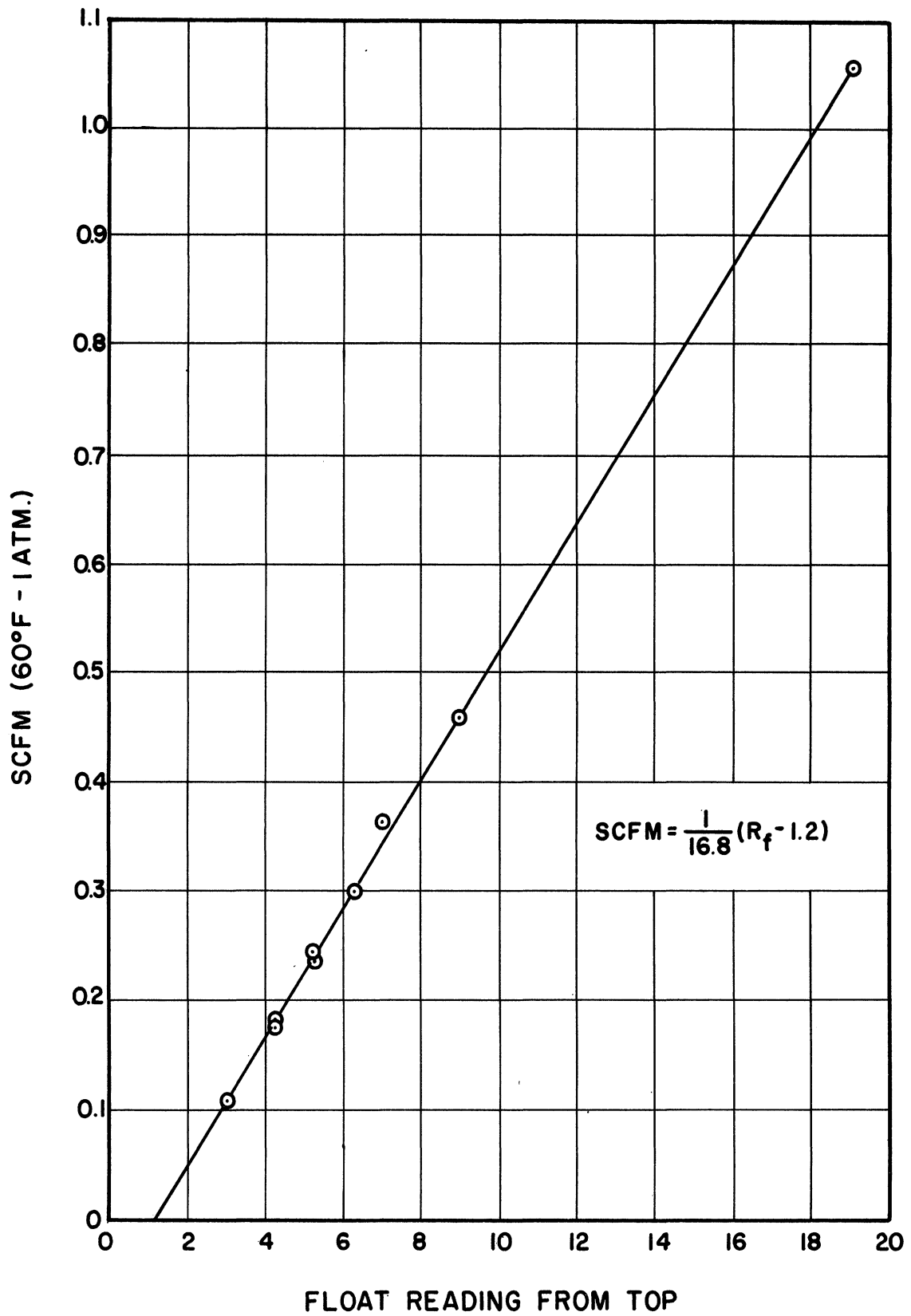Pressure Regulated at 40 psig.)

Figure 20. Calibration Curve for Low Capacity Propane
Rotameter. (Stainless Steel Float Pressure
Regulated at 40 psig.)

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