Scientific Report

THE COOLING OF THE PROTONOSPHERE

Pierre Bauer

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ABSTRACT

This work is a theoretical study of the nighttime behavior of the protonosphere. The continuity, momentum and heat equations are derived from the Boltzmann equation for the special conditions of the protonospheric plasma. The confinement of the protonospheric plasma by the magnetic field reduces the study to a one-dimensional problem along the magnetic field.

This set of basic time-dependent equations is solved along three geomagnetic mid-latitude field lines from an altitude of 1000 km in one hemisphere to the same altitude in the conjugate hemisphere. The "geomagnetic latitude of the field lines" refers to the geomagnetic latitude of the base of these lines at 1000 km. The boundary conditions at 1000 km are obtained from Explorer XXII measurements. The solution yields the temperature and density profiles along the three field lines as a function of time. Energy and particle fluxes are also deduced from the numerical solution. Due to the low collision frequencies of charged particles with neutral ones, the charged particle distribution is always close to the instantaneous diffusive equilibrium.

The results show the existence of a strong coupling between the temperature and the density of the plasma. The total energy being conducted down is sufficient to explain the maintenance of the electron temperature of the \( F_2 \) layer above the neutral temperature. The \( 30^\circ \) and \( 40^\circ \) geomagnetic latitude field lines are characterized by a large initial downward flux of ionization followed by a smaller upward flux over most of the night. As a consequence, the total plasma content of a flux tube at the end of the night is roughly equal to its value at the time of sunset. The distribution of the plasma along the field lines at the end of the night, however, differs considerably from the initial distribution. The existence of an upward flux of particles for the \( 30^\circ \) and \( 40^\circ \) field lines during the latter part of the night is explained qualitatively in terms of the charge exchange process between \( O^+ \) and \( H \) in the upper \( F \) region and in terms of the horizontal neutral wind directed toward the subsolar point.

The time response of the protonospheric plasma to changes in densities and temperatures at the boundaries depends sensitively upon the geomagnetic latitude because both the length and the average cross sectional area of the tubes increase with latitude; this in turn results in an increasing particle and energy content in the tubes.

The \( 50^\circ \) tube is characterized by a downward flux of particles, with a large maximum at the beginning of the night followed by a slow decrease to zero during the rest of the night. For the same \( 50^\circ \) tube, the importance of the gravitational energy of the protons is clearly evident by the appearance of a temperature maximum between the equatorial region and the base of the tube at the time of largest downward particle flux.
I. INTRODUCTION

Measurements of the electron temperature in the ionosphere during the last few years with Langmuir probes (e.g., Spencer et al., 1962; Nagy et al., 1963) and by the incoherent radar backscatter technique (e.g., Evans, 1965) have clearly shown the absence of thermal equilibrium between the electrons and the neutral particles in the mid-latitude upper F region during the night.

Early suggestions for the nighttime electron heating mechanism invoked energy sources such as soft (Brace et al., 1965) and/or energetic (Willmore, 1964) electron fluxes. Recently Geisler and Bowhill (1965) studied the thermal structure of the protonosphere. Utilizing the fact that the ionization and, consequently, the energy content of the ionization, cannot move across the field lines, they proposed that the energy stored in the protonosphere during the day is conducted down during the night, thus maintaining the nighttime electron temperatures in the F region above the temperature of the neutral particles. Their calculations based upon the simple model of an isothermal heat reservoir at 1000 km in thermal contact with a heat sink at 300 km led to the conclusion that, at geomagnetic mid-latitudes under sunspot minimum conditions the heat conducted down from the protonosphere during the early part of the night is of the order of $10^8$ eV cm$^{-2}$ sec$^{-1}$. Gliddon (1966) has solved a simplified heat equation for the protonospheric plasma, keeping the particle density constant in time and using an arbitrary step variation of the temperature at the boundaries. His calculations also indicated a downward flux in excess of $10^8$ eV cm$^{-2}$ sec$^{-1}$ for most of the night.
In order to facilitate calculations it is useful to represent the protonosphere by tubes of ionization directed along given field lines. The cross sectional area of the tube is chosen such that the flux through it is constant. In that case all bulk motions and the heat transport are confined to the tube which, therefore, acts as coupling agent between the two conjugate ionospheres. During the day some of the photoelectrons, created by the photoionization of the neutral particles in the ionosphere, are redistributed along the field tube, selectively heating the electron gas. Due to close contact between ions and electrons the protonospheric plasma as a whole is brought to a high temperature above that of the neutrals. The two processes which are of importance for the nighttime protonosphere are the cooling and the bulk motion of the plasma. These two phenomena interact with each other in a complex manner.

The purpose of this thesis is to study the protonosphere by solving numerically the coupled time dependent energy, continuity, and momentum equations describing the nighttime protonospheric plasma along a magnetic field tube going from an altitude of 1000 km in one hemisphere to an altitude of 1000 km in the opposite hemisphere. Known boundary values have been used for the temperature and the density at 1000 km.

This study will be limited to mid-latitude problems since the simpler case of low latitude tubes is well understood (Brace et al., 1967), while on the other hand, the problems become complex for high latitude tubes because the ionosphere is in a disturbed state due to the coupling with the interplanetary medium. Equinoctial conditions have been chosen in order to separate the purely protonospheric effects from those which are the result of asymmetric
ionospheres. The existing data correspond to the minimum of the solar cycle, and therefore this particular condition has been chosen. The calculations carried out in this thesis give the density and temperature profiles along different geomagnetic field lines as functions of time during the night, as well as the heat and particle fluxes through the base of the tubes at an altitude of 1000 km. The implications of these results for the question of ionosphere-protonosphere coupling are discussed in detail.
II. DERIVATION OF BASIC EQUATIONS

2.1 CONTINUITY EQUATION

In this chapter, the basic set of equations, which describe the behavior of the nighttime protonosphere, is derived from the Boltzmann equation given below (Burgers, 1960; Holt and Haskell, 1965):

\[
\frac{\partial f_j}{\partial t} + \nabla_r \int \vec{u}_j \cdot \vec{W} + \nabla_{\vec{w}_j} f_j \cdot \vec{F}_j = \left( \frac{\partial f_j}{\partial t} \right)_{\text{coll}}
\]  

(2.1)

where

\( f_j(\vec{r}, \vec{w}_j, t) d^3r d^3w_j \) = number of particles of type \( j \) in the spatial element \( d^3r \) and velocity element \( d^3w_j \)

\( t \) = time

\( \vec{r} \) = space

\( \vec{w}_j = \vec{u}_j + \vec{v}_j \) = velocity of \( j \) type particle

\( \vec{u}_j \) = random velocity

\( \vec{v}_j \) = mean velocity

\( \nabla_r \) = gradient with respect to space

\( \nabla_{\vec{w}_j} \) = gradient with respect to velocity

\( \vec{F}_j \) = external force applied to the \( j \) type particles

\( m_j \) = mass of the \( j \) type particle

\( (\partial f_j/\partial t)_{\text{coll}} \) = change in \( f_j(\vec{r}, \vec{w}, t) \) due to collisions.

The integral of the Boltzmann equation over the velocity space gives Eq. (2.2), if the creation and the removal of \( j \) type particles are negligible.

\[
\frac{\partial n_j}{\partial t} + \nabla_r \left( n_j \frac{u_j}{\partial \vec{w}_j} \right) = 0
\]

(2.2)

where

\( n_j \) = density of \( j \) type particle.
The above equation is a simple statement of the fact that the change in the density in a given volume element is equal to the net flux of particles flowing across the boundaries of this element. The logical choice for a volume element in the protonosphere is a portion of the field tube of cross section S and length ds along the field line. The volume Sds contains njSds particles; since there is no particle flux across the field lines, as will be shown in Section 2.2., the divergence term can be written (see Fig. 1), in the limit of ds → 0, as

$$\lim_{ds \to 0} \left( \nu_{j2} n_{j2} S_2 - \nu_{j1} n_{j1} S_1 \right) = \frac{d(n_j \nu_j)}{d \phi} ds$$

(2.3)

where

$$| \vec{v}_j | = \nu_j = \nu_i$$

Fig. 1. Volume element of a magnetic flux tube.
The geometry of the tube does not depend explicitly on time; therefore, the change with respect to time can be written as

$$\frac{\partial (n_0 \rho \beta \varepsilon)}{\partial t} = \int \frac{\partial n_0}{\partial t} \rho \beta \varepsilon \, ds$$  \hspace{1em} (2.4)

The continuity equation for the protonospheric plasma can thus be written in one dimension as

$$S \frac{\partial n_0}{\partial t} + \frac{\partial}{\partial s} \left( n_0 \nu \frac{\partial \rho \beta \varepsilon}{\partial s} \right) = 0$$

or

$$\frac{\partial n_0}{\partial t} = -n_0 \nu \frac{\partial (\rho \beta \varepsilon)}{\partial s} - \frac{\partial (n_0 \nu \rho \beta \varepsilon)}{\partial s}$$  \hspace{1em} (2.5)

2.2 MOMENTUM (TRANSPORT) EQUATION

The first moment of the Boltzmann equation (2.1) leads to the momentum equation, which is also referred to sometimes as the transport equation. The integral over the velocity space of the product of $n_j \gamma_j$ with the Boltzmann equation (2.1) gives:

$$n_0 \rho \beta \varepsilon \left( \frac{\partial \nu_j}{\partial t} \right) + n_0 \rho \beta \varepsilon \left( \nu_j \cdot \nabla \rho \beta \varepsilon \right) + \nabla \cdot \left( n_0 \rho \beta \varepsilon \left< \nu_j \nabla \rho \beta \varepsilon \right> \right)$$

$$- n_0 \left< \vec{F}_j \right> = \hat{P}_{j, \text{coll}}$$  \hspace{1em} (2.6)

where

$$n_j \rho \beta \varepsilon \left< \nu_j \nabla \rho \beta \varepsilon \right> \widehat{=} \psi_j = \text{pressure tensor}$$

$$\hat{P}_{j, \text{coll}} = \text{change in momentum of the } j\text{th species per unit}
\text{time per unit volume due to collisions with other species.}$$

The symbol $\left< \omega \right>$ denotes the average of a quantity $Q$ over velocity space. The use of the relation (2.7) below and the above definition of the pressure...
tensor permit the momentum equation (2.6) to be written as Eq. (2.8):

\[ \mu_d \frac{d}{dt} \left( \frac{d}{dt} \mathbf{v}_r \right) + \mu_d \mathbf{v}_r \cdot \nabla \mathbf{v}_r = \mu_d \frac{d}{dt} \mathbf{v}_d \quad (2.7) \]

\[ \eta_d \mu_d \frac{d}{dt} \mathbf{v}_d + \nabla_r \left( \chi \right) - \eta_d \frac{d}{dt} \mathbf{F}_d = \mathbf{P}_d \text{ coll} \quad (2.8) \]

At this point six assumptions are made to simplify Eq. (2.8). This equation will be used to study a specific problem; therefore, the validity of these assumptions in terms of the nighttime protonosphere is discussed below:

1. The first assumption is that the momentum equation is to be used to study large scale motions for which "quasi-equilibrium" prevails, i.e.:

\[ \frac{d}{dt} \mathbf{v}_d = 0 \quad (2.9) \]

Clearly this is not the case shortly after significant sharp changes in boundary conditions and other major disturbances. In fact, examination of the initial and of the boundary conditions to be used in later calculations shows that during the initial minutes the velocity is expected to reach a value in the order of 1000 m/sec.; this velocity requires a value of \( d\mathbf{v}/dt \) which is not negligible with respect to the other terms (e.g., gravity) but such non-negligible value cannot persist for any significant amount of time because otherwise the velocity would increase to physically unreasonable values.

2. It is assumed that the pressure is isotropic, e.g.,

\[ \nabla_r \cdot \left( \chi \right) = \nabla_r \mathbf{P}_d \quad (2.10) \]
where $p_j$ is the partial scalar pressure of the particles of type $j$. In the presence of an inhomogeneous magnetic field this assumption is justified only because the mean free path of the charged particles (see Table I) is small compared to the distances in which macroscopic quantities (including the magnetic field) undergo significant changes. It should also be noted that as the mean free path increases, the relevant quantities such as the gradient of the magnetic field or gravity decrease.

**TABLE I**

**MEAN FREE PATH ALONG THE MAGNETIC FIELD**

<table>
<thead>
<tr>
<th>$n_{cm^{-3}}$</th>
<th>$T^K$</th>
<th>$2 \times 10^3$</th>
<th>$3 \times 10^3$</th>
</tr>
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<td>$2 \times 10^4$</td>
<td>4 km</td>
<td>15 km</td>
<td>31 km</td>
</tr>
<tr>
<td>$1 \times 10^4$</td>
<td>8 km</td>
<td>31 km</td>
<td>62 km</td>
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*Computed by multiplying the thermal velocity along the magnetic field line by the self-collision time (Spitzer, 1965).

3. The collision term is assumed to be of the form

$$\vec{P}_{coll}^j = \sum_{\beta} \eta_j \eta_\beta \nu_j^\beta (\vec{v}_\beta - \vec{v}_j)$$  \hspace{1cm} (2.11)

where

$$\nu_j^\beta = \text{momentum transfer collision frequency between the } j \text{ and } \beta \text{ type particles.}$$

4. The equation for the external forces is:

$$\vec{F}_j = q_j (\vec{E} + \vec{v}_j \times \vec{B}) + m_j \vec{g}$$  \hspace{1cm} (2.12)
where

\[ q_j = \text{electric charge of the } j \text{ type particle} \]

\[ \vec{E} = \text{electric field} \]

\[ \vec{B} = \text{magnetic field} \]

\[ \vec{g} = \text{gravitational acceleration}. \]

5. A single ionic constituent is assumed for the nighttime protonosphere. This assumption is discussed in detail in Chapter III.

The presence of a single ionic constituent permits the collision term to be written as

\[ \vec{P}_{\|} = n_e m_e \langle \vec{v}_e - \vec{v}_i \rangle + n_e \frac{m_i}{m_e} \langle \vec{v}_i - \vec{v}_e \rangle \]  \hspace{1cm} (2.13)

where the subscripts \( e \), \( i \), and \( n \) refer to electrons, ions, and neutrals, respectively. The assumptions of ambipolar diffusion (Kendall and Pickering, 1967) and macroscopic neutrality leads to

\[ \mathcal{U}_{e||} = \mathcal{U}_{i||} \]  \hspace{1cm} (2.14)

\[ n_e = n_i \]  \hspace{1cm} (2.15)

and

\[ P_{e||} = n_e m_e \langle \vec{v}_e - \vec{v}_i \rangle \]

where the symbol \( || \) designates the component parallel to the field line.

6. Finally, it is assumed that the neutral gas is stationary; therefore

\[ \vec{P}_{\perp} = n_e m_e \langle \vec{v}_e \rangle + n_e \frac{m_i}{m_e} \langle \vec{v}_i \rangle \]  \hspace{1cm} (2.16)
By using the above assumptions Eq. (2.8) can be written as

\[ 0 = n_i q_i \left[ \vec{E}_i + \vec{v}_i^j \times \vec{B} \right] + n_j m_j \vec{v}_j + n_j m_j \vec{v}_j \left( -\vec{v}_j^\xi \right) + n_j m_j \vec{v}_j \left( \nabla \vec{v}_j - \vec{v}_j \right) - \nabla P_j \] (2.17)

where \( \gamma \) refers to the electrons when \( J \) refers to the ions and vice versa.

In order to reduce the problem ultimately to a one dimensional one, the following coordinates are chosen (Fejer, 1965):

- \( s = \) coordinate along the field line, directed antiparallel to \( \vec{B} \),
- \( \eta = \) coordinate perpendicular to both \( s \) and \( \vec{F}_j^\eta \) (defined below),
- \( \xi = \) the third coordinate, such that \( \xi, \eta, \) and \( s \) form a right-handed system.

The force \( \vec{F}_j^\eta \) is defined by the expression,

\[ \vec{F}_j^\eta = m_j \nu_j^\eta \vec{v}_j^\eta - q_j \vec{e} \times \vec{B} \]

\[ = q_j \vec{E}_j + m_j \vec{v}_j - \frac{1}{n_j} \nabla P_j + m_j \nu_j^\eta \left( \vec{v}_j^\eta - \vec{v}_j \right) \] (2.18)

The components of \( \vec{F}_j^\eta \) are

\[ F_j^\eta = m_j \nu_j^\eta \vec{v}_j^\eta \] (2.19)

\[ F_j^\eta \eta = 0 = m_j \nu_j^\eta \vec{v}_j^\eta + q_j \vec{v}_j^\eta \vec{B} \] (2.20)

\[ F_j^\eta \xi = m_j \nu_j^\eta \vec{v}_j^\eta - q_j \vec{v}_j^\eta \vec{B} \] (2.21)

By using the definition of the gyrofrequency

\[ \omega_j^\eta = \frac{q_j \vec{B}}{m_j} \] (2.22)
Eq. (2.20) can be rewritten as:

$$\nu_{j\eta} = - \frac{\omega_j}{\nu_j} \nu_{j\xi}$$  \hspace{1cm} (2.23)

This relation for $\nu_{j\eta}$ is then substituted into Eq. (2.21) to give

$$\mathcal{F}_{j\xi} = \left[ \frac{\omega_j^2 m_j}{\nu_j} + m_j \frac{\nu_j}{\nu_j} \right] \nu_{j\xi}$$  \hspace{1cm} (2.24)

The three components of the velocity can now be written as

$$\nu_{j\xi} = \mathcal{F}_{j\xi} \frac{1}{m_j \nu_j}$$  \hspace{1cm} (2.25)

$$\nu_{j\eta} = - \mathcal{F}_{j\xi} \frac{\omega_j}{m_j} \frac{1}{\omega_j^2 + \nu_j^2}$$  \hspace{1cm} (2.26)

$$\nu_{j\xi} = \mathcal{F}_{j\xi} \frac{\nu_j}{m_j} \frac{1}{\omega_j^2 + \nu_j^2}$$  \hspace{1cm} (2.27)

At all heights above about 100 km

$$\mathcal{F}_{j\xi} \ll |\omega_j|$$

which leads to the following simplified relations for the velocity components:

$$\nu_{j\xi} \approx \mathcal{F}_{j\xi} \frac{1}{m_j \nu_j}$$  \hspace{1cm} (2.28)

$$\nu_{j\eta} \approx - \mathcal{F}_{j\xi} \frac{\omega_j}{m_j}$$  \hspace{1cm} (2.29)

$$\nu_{j\xi} \approx 0$$  \hspace{1cm} (2.30)

From Eqs. (2.28) and (2.29) it follows that

$$\frac{\nu_{j\eta}}{\nu_{j\xi}} = - \frac{\mathcal{F}_{j\xi}}{\mathcal{F}_{j\xi}} \frac{\nu_j}{\omega_j}$$

In the protonosphere $\nu_{j\eta}$ is four to five orders of magnitude smaller than $\omega_j$. Hence, unless $\mathcal{F}_{j\xi}$ is about four orders of magnitude larger than $\mathcal{F}_{j\xi}$, it follows that $|\nu_{j\eta}| \ll |\nu_{j\xi}|$ or
\[ \mathbf{v}_{\mathbf{j}}^n \approx 0 \]  

(2.31) 

Since both vectors \( \mathbf{v}_p \) and \( \mathbf{\hat{g}} \) have non-negligible components along the magnetic field line (except possibly at the equatorial plane) and since the ion-electron collision term can only reduce the ion-electron relative velocity, it follows from the definition of \( \mathbf{P}_j \) (Eq. (2.18)) that \( P_{ij}^j \) cannot be four orders of magnitude larger than \( P_{jj}^i \), unless there are extremely intense electric fields perpendicular to the magnetic field in the protonosphere. However, there is no evidence for such fields. Therefore, the approximation expressed in Eq. (2.31) is justified.

The mean velocity can now be expressed in the following form

\[ \mathbf{v}_j = \mathbf{v}_j^s \mathbf{\hat{s}} = F_j^i \frac{1}{m_j} v_j^n \mathbf{\hat{s}} \]  

(2.32)

where \( \mathbf{\hat{s}} \) is the unit vector along the coordinate \( s \) (antiparallel to the direction of \( \mathbf{\hat{B}} \)). The above result means that there can be no net flux across the field line, and therefore the problem reduces to a one-dimensional one.

The specific equations for the electrons and ions are

\[ \begin{align*}
\mathbf{v}_{es} & \quad m_e \quad v_{en} = F_{es} = q_e E_s + m_e q_e \frac{\partial P_e}{\partial s} \\
\mathbf{v}_{is} & \quad m_i \quad v_{in} = F_{is} = q_i E_s + m_i q_i \frac{\partial P_i}{\partial s}
\end{align*} \]  

(2.33)  

(2.34)

where \( q_e = -q_i \) and \( g_s \) is the component of \( \mathbf{\hat{g}} \) along the field line.

Adding Eqs. (2.33) and (2.34) and using Eqs. (2.14) and (2.15) one obtains

\[ \mathbf{v}(m_i v_{in} + m_e v_{en}) = (m_i + m_e) q_s - \frac{1}{n} \frac{\partial (P_i + P_e)}{\partial s} \]  

(2.35)
where
\[ u = n_e = n_i \]
\[ n = n_e = n_i \]
The inequalities
\[ m_e \leq m_i \]
\[ m_e \frac{u}{c_n} \leq m_i \frac{u}{c_n} \]
lead to
\[ u m_i \frac{d}{d_s} = m_i g \sin I - \frac{1}{n} \frac{\partial (p_i + p_i)}{\partial s} \tag{2.36} \]
Since
\[ \frac{p}{c_i} = n_i k \frac{T_i}{c_i} \tag{2.37} \]
the final expression for the equation of motion is
\[ u m_i \frac{d}{d_s} = -m_i g \sin I - \frac{1}{n} \frac{\partial n_k (T_e + T_i)}{\partial s} \tag{2.38} \]
where
\[ I = \text{dip angle} \]
\[ T_i = \text{ion temperature} \]
\[ T_e = \text{electron temperature} \]

2.3 THE ENERGY EQUATION

The general expression for the heat equation is obtained by multiplying
the Boltzmann equation by \(1/2 m_j w_j^2\) and by integrating it over \(w_j\). This yields:
\[ \frac{\partial}{\partial t} \left( \frac{1}{2} n_j m_j \langle w_j^2 \rangle \right) + \nabla_f \left( \frac{1}{2} n_j m_j \langle w_j^2 \rangle \right) - n_j \overrightarrow{F} \cdot \langle \overrightarrow{w} \rangle = \mathcal{M}_{i \text{coll}} \tag{2.39} \]
where $M_{j \text{ coll}}$ is the rate of change in energy of the $j$th species per unit volume due to collisions with other species.

The only assumption which was made to obtain Eq. (2.39) is the following:

$$\langle \vec{F}_j \cdot \vec{W}_i \rangle = \vec{F}_j \cdot \langle \vec{W}_i \rangle$$

(2.40)

This equation is certainly true for forces due to electric or gravitational fields, since these forces are independent of $\vec{W}_i$. This relation also holds for a force due to a magnetic field because

$$q_i \left( \vec{W}_i \times \vec{B} \right) \cdot \vec{W}_i = 0$$

(2.41)

$$q_i \left( \langle \vec{W}_i \rangle \times \vec{B} \right) \cdot \langle \vec{W}_i \rangle = 0$$

The above equation is a statement of the well-known fact that the magnetic field does no work.

At this point it will be helpful to evaluate $\langle \vec{W}_j^2 \rangle$:

$$\langle \vec{W}_j^2 \rangle = \sum_{\alpha} \langle U_{j\alpha}^2 \rangle + 2 \sum_{\alpha} \langle U_{j\alpha} \rangle \nu_{j\alpha} + \sum_{\alpha} \nu_{j\alpha}^2$$

where the subscript $\alpha$ denotes the components along the coordinates $s$, $\eta$, and $\xi$. It was shown in Section 2.2 that the mean velocity $\bar{u}_j$ is approximately along the field and also by definition $\langle \bar{u}_j \rangle = 0$; therefore

$$\langle \vec{W}_j^2 \rangle = \sum_{\alpha} \langle U_{j\alpha}^2 \rangle + \nu_j^2$$

(2.42)

where

$$\nu_j = \nu_{j\alpha}$$

In order to evaluate the second term in Eq. (2.39), the heat flux $Q_{js}$ along the field line needs to be defined and evaluated:
\[ Q_{j}^d = \int_{\vec{w}_d} f_j(\vec{w}_d) \, w_j \, \frac{1}{2} \, m_j \, \vec{w}_j^2 \, d\vec{w}_j \]

\[ Q_{j}^d = \int_{\vec{w}_d} f_j(\vec{w}_d) \, u_j^2 + v_j^2 \, \frac{1}{2} \, m_j \, (u_j^2 + 2 \, u_j \, v_j + v_j^2) \, d\vec{w}_j \]

\[ = \int_{\vec{w}_d} f_j(\vec{w}_d) \, \frac{1}{2} \, m_j \, u_j^2 \, \frac{1}{2} \, d\vec{w}_j + \int_{\vec{w}_d} f_j(\vec{w}_d) \, m_j \, u_j \, v_j \, d\vec{w}_j + \]

\[ + \int_{\vec{w}_d} \frac{3}{2} \, f_j(\vec{w}_d) \, m_j \, u_j \, \frac{1}{2} \, v_j^2 \, d\vec{w}_j + \int_{\vec{w}_d} \frac{1}{2} \, f_j(\vec{w}_d) \, m_j \, v_j \, u_j \, \frac{1}{2} \, d\vec{w}_j + \]

\[ + \int_{\vec{w}_d} \frac{1}{2} \, f_j(\vec{w}_d) \, m_j \, v_j^3 \, d\vec{w}_j \]

(2.43)

The first term of the above equation is:

\[ \int_{\vec{w}_d} \frac{1}{2} \, f_j(\vec{w}_d) \, m_j \, u_j \, \frac{1}{2} \, u_j^2 \, d\vec{w}_j = \frac{1}{2} \, n_j \, m_j \, \langle u_j^2 \rangle \]

(2.44)

The term in Eq. (2.44) corresponds to the heat flux in a plasma with no net particle flux. For a fully ionized gas this term was evaluated by Spitzer (1965) and found to be equal to

\[ \frac{1}{2} \, n_j \, m_j \, \langle u_j^2 \rangle = - B_j \, T_j^{5/2} \, \frac{dT_j}{dz} \]

(2.45)

where

\[ B_j \, T_j^{5/2} = \text{thermal conductivity of the gas of type } j \]

\[ T_j = \text{temperature of the particles of type } j. \]
The above relation will be used in all the calculations to follow since Banks (1966a) has shown that above about 200 km the relation for a fully ionized gas is applicable. The second term is

\[
\int \frac{d}{d_j} \left( \bar{\mathbf{w}}_j \right) m_j u_j^2 v_j \, d \bar{\mathbf{w}}_j = \eta_j m_j v_j^2 \langle u_j^2 \rangle \tag{2.46}
\]

The third term is

\[
\int \frac{3}{2} f_j \left( \bar{\mathbf{w}}_j \right) m_j u_j^2 v_j \, d \bar{\mathbf{w}}_j = \frac{3}{2} \eta_j m_j v_j^2 \langle u_j \rangle = 0 \tag{2.47}
\]

since by definition \( \langle u_j \rangle = 0 \). The fourth and fifth terms are

\[
\int \frac{1}{2} f_j \left( \bar{\mathbf{w}}_j \right) m_j v_j^2 u_j \, d \bar{\mathbf{w}}_j = \frac{1}{2} \eta_j m_j v_j^2 \langle u_j^2 \rangle \tag{2.48}
\]

and

\[
\int \frac{1}{2} f_j \left( \bar{\mathbf{w}}_j \right) m_j v_j^3 \, d \bar{\mathbf{w}}_j = \frac{1}{2} \eta_j m_j v_j^3 \tag{2.49}
\]

The heat equation is now written in one dimension, by considering an element of field tube with a cross section \( S \) and length \( ds \). The use of the one-dimensional heat equation is justified since conduction is much higher along than across the field line and all the other flux terms are collinear to the drift velocity which itself is approximately directed along the field line.

Since the second term in Eq. (2.39) is the divergence of \( \mathbf{q}_j \), the one-dimensional energy equation can be written as

\[
S \frac{d}{dt} \left( \frac{1}{2} \eta_j m_j \langle u_j^2 \rangle \right) + \frac{2}{\rho} \left( - S B_j T_j^{5/2} \frac{\partial T_j}{\partial t} \right) + \frac{2}{\rho \beta} \left( S \eta_j m_j v_j \langle u_j^2 \rangle \right) + \frac{2}{\rho \beta} \left( \frac{1}{2} S \eta_j m_j v_j \langle u_j^2 \rangle \right) + \frac{2}{\rho \beta} \left( \frac{1}{2} S \eta_j m_j v_j^3 \right) - \eta_j F_j v_j \, S = S |\mathbf{v}_j|_{\text{coll}}. \tag{2.50}
\]
where Eqs. (2.45) through (2.49) have been used. Equation (2.50) can now be simplified by noting the following relation:

\[
\frac{\partial}{\partial t} \left( \frac{1}{2} \eta_d m_d v_d^2 \right) + \frac{\partial}{\partial \xi} \left( \frac{1}{2} \eta_d m_d v_d^2 \right) = \]

\[
= \frac{1}{2} \eta_d v_d^2 \left[ \frac{\partial}{\partial \xi} \left( \frac{S \eta_d m_d v_d^2}{\partial \xi} \right) \right] + \eta_d m_d v_d \left[ \frac{\partial v_d}{\partial \xi} + \frac{\partial \psi_d}{\partial \xi} \right] (2.51)
\]

The first term in the bracket on the right-hand side corresponds to the continuity equation (2.5) and is therefore equal to zero. By using the momentum equation (2.8) the second term on the right-hand side can be written as

\[
\eta_d m_d v_d \left[ \frac{\partial v_d}{\partial \xi} + \frac{\partial \psi_d}{\partial \xi} \right] = \psi_d \left[ S \eta_d m_d v_d \frac{\partial}{\partial \xi} \left( \frac{F_d}{\psi_d} - S \frac{\partial}{\partial \xi} \left( \psi_d \right) \right) \right] (2.52)
\]

the substitution of Eqs. (2.51) and (2.52) into Eq. (2.50) gives

\[
\frac{1}{2} \frac{\partial}{\partial \xi} \left( \frac{1}{2} \eta_d m_d v_d^2 \left( u_d^2 \right) \right) + \frac{\partial}{\partial \xi} \left( - \eta_d m_d v_d \frac{\partial \psi_d}{\partial \xi} \right) + \frac{\partial}{\partial \xi} \left( \frac{1}{2} \eta_d m_d v_d \left( u_d^2 \right) \right) - \frac{\partial}{\partial \xi} \left( \frac{1}{2} \eta_d m_d v_d \left( u_d^2 \right) \right) = \frac{1}{2} \eta_d m_d v_d \left( u_d^2 \right) (2.53)
\]

The equation is now further simplified by making the following assumptions:

1. The random energy,

\[
\frac{1}{2} \eta_d m_d < u_d^2 > = \frac{3}{2} \eta_d k T_d (2.54)
\]

where

\[ k = \text{Boltzmann constant.} \]

2. There is sufficient isotropy in the velocity distribution so that

\[
<u_d^2> = \frac{1}{3} < u_d^2 > (2.55)
\]
3. The following substitution can be made as was done previously in Section 2.2:

\[
\frac{\partial (\frac{\psi_{d+h}}{d} \Delta n)}{\partial t} = \frac{\partial (P_d)}{\partial \lambda} = \frac{\partial}{\partial \lambda} \left( n_d k T_{d \lambda} \right) \tag{2.56}
\]

These assumptions permit Eq. (2.53) to be written as

\[
S \frac{\partial}{\partial t} \left( \frac{3}{2} n_d k T_{d \lambda} \right) + \frac{\partial}{\partial \lambda} \left( -S B_d T_{d \lambda}^2 \frac{\partial T_{d \lambda}}{\partial \lambda} \right) + \frac{\partial}{\partial \lambda} \left( S \psi_d n_d k T_{d \lambda} \right) + \frac{\partial}{\partial \lambda} \left( \frac{3}{2} S \psi_d n_d k T_{d \lambda} \right) = S M_{\text{d \lambda \ coll}} - S \psi_d P_{\text{d \lambda \ coll}} \tag{2.57}
\]

The use of the continuity equation (2.5) and some further rearrangement yields

\[
\frac{3}{2} S n_d k \frac{\partial T_{d \lambda}}{\partial t} + \frac{3}{2} S n_d \psi_d k \frac{\partial T_{d \lambda}}{\partial \lambda} - \frac{\partial}{\partial \lambda} \left( S B_d T_{d \lambda}^2 \frac{\partial T_{d \lambda}}{\partial \lambda} \right) + n_d k T_{d \lambda} \frac{\partial}{\partial \lambda} \left( S \psi_d \right) = S M_{\text{d \ coll}} - S \psi_d P_{\text{d \ coll}} \tag{2.58}
\]

The term \( M_{\text{d \ coll}} \) needs to be evaluated next. \( M_{\text{d \ coll}} \) consists of the following three terms for the protonospheric plasma:

1. Collision of charged particles of one sign with charged particles of the other sign.

2. Collision of charged particles with neutral particles.

3. Collision of charged particles with "energetic" particles (e.g., photoelectrons).

This means that:

\[
M_{\text{e \ coll}} = M_{\text{e \ e}} + M_{\text{e \ m}} + M_{\text{e \ p}} \tag{2.59}
\]
\[ M_{ic}^{\text{col}} = M_{ic} + M_{in} + M_{ip} \]  

(2.60)

where \( M_{ab} \) is the change of energy of constituent \( a \) in a unit volume due to collisions with constituent \( b \). The subscripts \( e, i, n, \) and \( p \) denote electrons, ions, neutrals, and photoelectrons, respectively. At night

\[ M_{ep} \approx M_{ip} \approx 0 \]  

(2.61)

The amount of energy lost by the thermal electrons to neutrals is negligible; therefore

\[ M_{en} \approx 0 \]  

(2.62)

Also, because of symmetry it follows that

\[ M_{ei} = -M_{ie} \]  

(2.63)

By using the relation of overall neutrality, (Eq. (2.15)), and by taking into account the fact that the diffusion is ambipolar (Eq. (2.16)); the two energy equations for the electrons and the ions can now be written explicitly as

\[ \frac{3}{2} \eta k \frac{\partial T_e}{\partial t} + \frac{3}{2} \eta n K \frac{\partial T_e}{\partial \lambda} - \frac{1}{S} \frac{\partial}{\partial \lambda} \left( S B_e T_e^{5/2} \frac{\partial T_e}{\partial \lambda} \right) \]

\[ + \eta k \frac{T_e}{S} \frac{\partial (S \nu)}{\partial \lambda} = M_{ei} - \nu P_{e, \text{el}} \]  

(2.64)

\[ \frac{3}{2} \eta k \frac{\partial T_i}{\partial t} + \frac{3}{2} \eta n K \frac{\partial T_i}{\partial \lambda} - \frac{1}{S} \frac{\partial}{\partial \lambda} \left( S B_i T_i^{5/2} \frac{\partial T_i}{\partial \lambda} \right) \]

\[ + \eta k \frac{T_i}{S} \frac{\partial (S \nu)}{\partial \lambda} = M_{ic} + M_{im} - \nu P_{i, \text{el}} \]  

(2.65)
In the protonosphere, \( T_e \) and \( T_i \) are nearly equal, and the only term which is sensitive to \( T_e - T_i \) is \( M_{ei}(= -M_{ie}) \). Only the sum of the two energy equations is used in the calculations which are carried out in the following chapters and therefore, the assumption of equal electron and ion temperatures is a good approximation. In other words if the electron gas and ion gas are considered separately, it is very important to consider the difference between \( T_e \) and \( T_i \), since the energy exchange between the electrons and ions depends on this temperature difference. If, on the other hand, the plasma is considered as a whole, the energy exchange between ions and electrons is an internal process, and a common plasma temperature, \( T = T_e = T_i \), can be assumed to describe the loss and transport of the plasma energy to a good approximation provided \( T_e - T_i \) is small compared with \( T_e \). With this approximation, the sum of Eqs. (2.64) and (2.65) is

\[
3 n k \frac{dT}{dt} + 3 n \nu k \frac{dT}{d \alpha} - \frac{3}{S} \frac{dT}{d \alpha} (SBT^{5/2} \frac{dT}{d \alpha}) + \frac{2 n k}{S} \frac{dT}{d \alpha} (S \nu) = M_{in} - \nu \left( P_{e, \text{coll}} + P_{i, \text{coll}} \right) \tag{2.66}
\]

where

\[
T = T_e = T_i \\
B = B_e + B_i
\]

The term \((P_{e, \text{coll}} + P_{i, \text{coll}})\) in Eq. (2.66) is the rate of momentum change due to electron-neutral and ion-neutral collisions. Since the electron mass is much smaller than the ion mass, it is reasonable to assume [see Eq. (2.16)]
If $P_{es\ coll}$ is neglected, the energy equation becomes

$$
3 \, \eta \, k \, \frac{\partial T}{\partial t} + 3 \, \eta \, u \, k \, \frac{\partial T}{\partial y} - \frac{I}{S} \, \frac{\partial}{\partial \delta} \left( S \, B \, T^{5/4} \, \frac{\partial T}{\partial \delta} \right) + \\
+ \frac{2 \, \eta \, k \, T}{S} \, \frac{\partial}{\partial \delta} \left( S \, u \right) - \frac{M_{\gamma}}{\eta} - u \, P_{\gamma}
$$

(2.67)

The right-hand side of Eq. (2.67) can be expressed as the sum of the rate of energy change due to collisions between stationary gases, $L_{in}^0$, and the rate of energy change due to the relative motion of the two gases, $L_{\gamma n}^1$. The values adopted for $L_{in}^0$ in this study are discussed in detail in Section 4.4. The energy change due to the relative motion of the gases is given by Tanenbaum (1965) as:

$$
\frac{L_{\gamma n}^1}{L_{in}^0} = \gamma_{\gamma n} \, \eta \, u \, u^2
$$

(2.68)

where

$$
\mu = \frac{m_1 m_n}{(m_1 + m_n)} = \text{reduced mass.}
$$

$$
m_n = \text{neutral particle mass.}
$$

Substitution of Eq. (2.68) into Eq. (2.67) yields the final form of the energy equation

$$
3 \, \eta \, k \, \frac{\partial T}{\partial t} + 3 \, \eta \, u \, k \, \frac{\partial T}{\partial y} - \frac{I}{S} \, \frac{\partial}{\partial \delta} \left( S \, B \, T^{5/4} \, \frac{\partial T}{\partial \delta} \right) + \frac{2 \, \eta \, k \, T}{S} \, \frac{\partial}{\partial \delta} \left( S \, u \right) = L_{\gamma n}^0 + u \, \mu \, \gamma_{\gamma n}
$$

(2.69)

The basic set of equations, which were derived in this chapter to describe the nighttime protonospheric plasma, is summarized in Table II.
TABLE II

BASIC SET OF EQUATIONS

\[ v_m \nu_{in} = -m_g \sin I - \frac{1}{n} \frac{\partial}{\partial s} (2nkT) \]

\[ \frac{\partial}{\partial t} (Sn) + \frac{\partial}{\partial s} (Snv) = 0 \]

\[ 3nk \frac{\partial T}{\partial t} + 3nvk \frac{\partial T}{\partial s} - \frac{1}{S} \frac{\partial}{\partial s} (SBT^{5/2} \frac{\partial T}{\partial s}) + \frac{2nkT}{S} \frac{\partial}{\partial s} (Sv) - v^2 \nu_{in} = I_{\nu_{in}}^0 \]
III. CHOICE OF THE VARIOUS PARAMETERS AND BASIC ASSUMPTIONS

3.1 GENERAL ASSUMPTIONS

The set of equations derived in Chapter II is quite general; it describes the behavior of a single constituent ionosphere (e.g., the protonosphere). These equations can be applied to daytime as well as nighttime protonospheric problems if the local heat input due to the photoelectron flux is taken into account (Fontheim et al., 1968). It should be mentioned, in passing, that although the photoelectrons may constitute an important heat source, they are insignificant as a source of particles in the protonosphere. The purpose of this thesis is to study the cooling of the nighttime protonosphere and its interaction with the upper F-region of the ionosphere. In this work the investigation will be further limited to mid-latitude field tubes, as the coupling between the high latitude tubes and the interplanetary medium is not well understood and as the simpler equatorial problem has already been satisfactorily treated by Brace et al. (1967), and Mayr et al. (1967). In order to eliminate the problem of the coupling between conjugate hemispheres only equinoctial conditions are studied. The protonospheric field tubes are assumed to extend from an altitude of 1000 km in one hemisphere to the same altitude in the other hemisphere. Most of the experimental data available at the present are for sunspot minimum conditions; therefore, the calculations are carried out for this period of the sunspot cycle.
3.2 BOUNDARY CONDITIONS

The results of the Explorer XXII Langmuir probe experiment (Brace et al., 1967) provide data on electron temperature and density at an altitude of 1000 km as a function of local time and geomagnetic latitude. These results shown in Figs. 2, 3, and 4 were used as the boundary conditions for the solutions presented in Chapter V.

3.3 INITIAL CONDITIONS

No relevant experimental data exist on the electron temperature and density profiles along protonospheric field lines; this makes necessary the assumptions which will be outlined next.

The very low neutral particle density in the protonosphere causes the drag exerted on the charged particles to be small which in turn results in a fast response of the charged particle distribution to changing conditions. The density distribution of the charged particles, therefore, is expected to be close to the diffusive equilibrium one (see Section 5.7). This fast response also implies that after the lapse of a very short time the calculated behavior of the protonosphere becomes insensitive to the assumed initial conditions (see Section 5.8). These arguments make the choice of a diffusive equilibrium distribution for the initial conditions \( t = 0 \) justifiable.

The steps used in obtaining the diffusive equilibrium distribution will now be outlined. Diffusive equilibrium means that the diffusion flux is zero, i.e.,

\[ \mathbf{v} = 0 \quad (3.1) \]
Fig. 2. Averaged latitudinal and local time behavior of $T_e$ at 1000 km, near vernal equinox from Explorer XXII (Brace et al., 1967).
Fig. 3. Averaged latitudinal and local time behavior of $n_e$ at 1000 km, near vernal equinox from Explorer XXII (Brace et al., 1967).
Fig. 4. Averaged diurnal variation of $T_e$ and $n_e$ at magnetic latitudes of 0°, 35°, and 55° at 1000 km, near equinox from Explorer XXII (Brace et al., 1967).
The substitution of this condition into Eq. (2.38) yields,

\[ \nabla \cdot \mathbf{m}_c \cdot \nu = 0 = - \mathbf{m}_c \cdot q \cdot \mathbf{v} \cdot \nabla \mathbf{I} - \frac{2}{h} \frac{\partial}{\partial \phi} \left( n k T \right) \]  

(3.2)

where \( T = T_e = T_1 \), as has been discussed above.

Dividing Eq. (3.2) by \( T \) and rearranging leads to:

\[ \frac{\mathbf{m}_c \cdot q \cdot \mathbf{v} \cdot \nabla \mathbf{I}}{2 k T} = - \frac{1}{n T} \frac{\partial}{\partial \phi} \left( n T \right) \]  

(3.3)

Integration along the field line and further rearrangement gives:

\[ n = n_0 \left[ \frac{T_0}{T} \right] e^{T_0} \left[ - \int_0^{\phi} \frac{\mathbf{m}_c \cdot q \cdot \mathbf{v} \cdot \nabla \mathbf{I}}{2 k T} \, d\phi \right] \]  

(3.4)

where

\[ n_0 = \text{charged particle density at } s = 0 \]

\[ T_0 = \text{charged particle temperature at } s = 0. \]

The values of \( g, I, \) and \( T \) are functions of position. Therefore, Eq. (3.4) cannot be written in a simple analytic form. The simple computer program which was written to evaluate the diffusive equilibrium density distribution is given in Appendix A. It is interesting to note that the temperature appears in two different places with opposite effects in the expression for the diffusive equilibrium profile, Eq. (3.4). Usually high densities at high altitudes are associated with high temperatures. However, some numerical calculations of Eq. (3.4) carried out for the field tubes considered in this study show that, for constant boundary values of \( n_0 \) and \( T_0 \), the highest densities in the upper part of the tube correspond to the lowest temperatures.

In other words, for mid-latitude tubes the effect on the density of a temperature change is governed mainly by the factor \( n_0 T_0 / T \) in Eq. (3.4).
Geisler and Bowhill (1965) showed that high temperature gradients cannot be maintained in the protonosphere. The temperature profile chosen as the initial condition for the calculations is the one given by Geisler and Bowhill (1965) for sunspot minimum conditions and shown in Fig. 5. This question of initial temperature profiles and temperature gradients in the protonosphere is examined in more detail later (see Section 5.8). The assumed initial temperature profile is used with Eq. (3.4) to calculate the initial density profile, and these two profiles constitute the initial conditions needed for solving the two coupled equations which describe the time dependent behavior of the protonosphere.

3.4 GEOMETRY OF THE FIELD TUBES

A simple dipole magnetic field is assumed for the protonospheric field tubes. The components of a dipole field are:

\[
\begin{align*}
B_r &= M \frac{z \sin \theta}{r^3} \\
B_\theta &= M \frac{\cos \theta}{r^3} \\
B_\phi &= 0
\end{align*}
\]

(3.5) \hspace{1cm} (3.6) \hspace{1cm} (3.7)

where

- \( r \) = radial distance
- \( \theta \) = magnetic latitude
- \( M \) = magnetic dipole moment.

The magnitude of the magnetic field, \( B \), is:
Fig. 5. Typical temperature distributions along the field tube during the day (Geisler and Bowhill, 1965).
\[ B = \frac{M}{r^3} \left[ 1 + 3 \sin^2 \theta \right]^{\frac{1}{2}} \]  

Equation (3.8)

The equation of a line of force is:

\[ r = r_{eq} \cos^2 \theta \]  

Equation (3.9)

where \( r_{eq} \) = geocentric radial distance of the field line in the equatorial plane (i.e., \( \theta = 0 \)). Combining Eqs. (3.8) and (3.9), one obtains:

\[ B = \frac{M}{r^3} \left[ 4 - 3 \frac{r}{r_{eq}} \right]^{\frac{1}{2}} \]  

Equation (3.10)

The variation of the cross sectional area of a field tube, \( S \), is deduced from the relation:

\[ BS = \text{constant} \]  

Equation (3.11)

Substituting Eqs. (3.10) into Eqs. (3.11) gives

\[ S = S_0 \left( \frac{r}{r_0} \right)^3 \left[ 4 - 3 \frac{r}{r_{eq}} / \frac{r_0}{r_{eq}} \right]^{\frac{1}{2}} \]  

Equation (3.12)

where

\[ S_0 = \text{cross section of field tube at } r = r_0. \]

In this report the behavior of three field tubes will be studied. These will be referred to as the 30°, 40°, and 50° tubes, and they correspond to field tubes which cross the 30°, 40°, and 50° geomagnetic latitudes at an altitude of 1000 km respectively. Let \( r_0 \) correspond to 1000 km and let \( S(r_0) \) be unity; Eq. (3.12) can then be written as:

\[ S = \left( \frac{r}{r_0} \right)^3 \left[ 4 - 3 \frac{r_0}{r_{eq}} / \frac{r}{r_{eq}} \right]^{\frac{1}{2}} \]  

Equation (3.13)
The equation for the cross sectional area, \( S \), in terms of \( \theta \) is

\[
S = \left( \frac{\cos^2 \theta}{\cos^2 \theta_0} \right)^3 \left[ \frac{4 - 3 \cos^2 \theta_0}{4 - 3 \cos^2 \theta} \right] L
\]

\( (3.14) \)

where

\[
\theta_0 = \text{magnetic latitude corresponding to } r = r_0.
\]

The expression for the arc length along a line of force is

\[
\delta(r, \theta) = \frac{r_0}{2 \sqrt{3} \cos \theta_0} \left[ x_0 + \sin h x \cos h x - (x + \sin h x \cos h x) \right]
\]

\( (3.15) \)

where

\[
\sinh x = \sqrt{3} \sin \theta.
\]

Note that \( s(r_0, \theta_0) = 0 \); this means that the distance along the field line is measured from the base of the protonosphere, i.e., \( r_0 \) and \( \theta_0 \). The relation between the cross sectional area \( S \) and the arc length, \( s \), is obtained by using Eqs. (3.14) and (3.15) (see Fig. 6).

The variation of the dip angle, \( I \), as a function of the arc length is easily obtained from the relation:

\[
\tan I = \frac{B_r}{B_\theta} = \frac{2 \sin \theta}{\cos \theta} = 2 \tan \theta
\]

\( (3.16) \)

This equation, combined with Eq. (3.15) provides the necessary relation between the dip angle and the arc length.

3.5 INTERACTION BETWEEN THE CHARGED AND THE NEUTRAL PARTICLES

The two aspects of particle interactions considered in this section are:

(i) momentum transfer, and (ii) energy transfer. In the previous discussion
Fig. 6. Cross sectional area along the 40° tube.
the first of these has been characterized by $v_{in}$ and the second was taken into account by the energy loss term, $I_{in}^0$.

In the altitude region above 1000 km, H, He, and H$^+$ are the significant atomic and ionic constituents. An excellent review of the various interaction processes was given in two recent papers by Banks (1966b, c), the discussion in this section relies heavily on his work. In order to evaluate the relative importance of the H$^+$-He and the H$^+$-H reactions, these two processes are examined first.

The relation given here for the H$^+$-He reaction rates assumes that the polarization force alone represents the true interaction over the entire range of ion and neutral temperatures. This assumption is only strictly valid at low temperatures ($<300^\circ$K), but the lack of data at high temperatures makes it convenient to accept over the whole range the collision equations developed for low temperatures. The velocity averaged collision cross section for this process is then given by (cf. Banks, 1966c)

$$\sigma_{in} = 13 \cdot 3 \times 10^{-14} \left( \frac{\alpha_0}{\mu_A} \right)^{1/2} \left[ \frac{T_i}{A_i} + \frac{T_n}{A_n} \right]^{-1/2} \quad (3.17)$$

where

- $\alpha_0$ = the atomic polarizability in units of $10^{-24}$ cm$^3$
- $A_i$ = ion mass in atomic mass units
- $A_n$ = neutral particle mass in atomic mass units
- $\mu_A$ = reduced mass in atomic mass units.

Using Eq. (3.17) it can be shown (cf. Banks, 1966c) that the rate at which energy is transferred from the ions to the neutrals, if the two gases have Maxwellian
velocity distributions characterized by the temperatures $T_1$ and $T_n$ respectively, is:

$$L^{\circ}_{H^+He} = -6.8 \times 10^{-13} \eta(H^+) \eta(He) \frac{(\mu_A \alpha_{He} \rho_{He})^{\frac{1}{2}}}{(A_e + A_n)} (T_i - T_n) e \sqrt{cm^3/\lambda} c^4 (3.18)$$

where

$$n(H^+) = \text{hydrogen ion density in units of cm}^{-3}$$
$$n(He) = \text{neutral helium density in units of cm}^{-3}$$
$$\alpha_{He} = \text{polarizability of helium} (=0.21).$$

In the consideration of the $H^+H$ interaction it is important to note that above 100$^\circ$K, the charge exchange

$$H^+ + H \rightarrow H + H^+ (3.19)$$

becomes the dominant process. Under these conditions the velocity averaged cross section for momentum transfer becomes (Banks, 1966c):

$$\bar{\sigma}_{H^+H} = \left[ 14.4 - 1.17 \rho_{o3}' \Gamma \right] \times 10^{-16} \text{ cm}^2 (3.20)$$

where

$$\Gamma = T_e + T_1.$$ 

The expression for energy transfer is (Banks, 1966c):

$$L^{\circ}_{H^+H} = -1.4 \times 10^{-14} \eta(H^+) \eta(H) \Gamma^{1/2} (T_i - T_n) e \sqrt{cm^3/\lambda} c^4 (3.21)$$

where

$$n(H) = \text{neutral hydrogen atom density in units of cm}^{-3}.$$
The collision frequency $\nu_{in}$, defined by Eq. (2.11), is equal to the product of $m_i/(m_i+m_n)$ and the collision frequency given by Banks (1966c). Thus,

$$\nu_{in} = \frac{m_i}{m_i + m_n} \frac{4}{3} n(M) \left( \frac{8k}{\pi} \right)^{\frac{1}{2}} \left( \frac{T_i}{m_i} + \frac{T_n}{m_n} \right)^{\frac{1}{2}} \nu c \sigma_{in}$$  \hspace{1cm} (3.22)

where

$$n(M) = \text{density of the relevant neutral particle.}$$

The collision frequency for the $H^+\text{-He}$ interaction is obtained, therefore, by combining Eqs. (3.17) and (3.22); the result is

$$\nu_{H^+-\text{He}} \approx 1.1 \times 10^{-9} n(\text{He}) \text{ sec}^{-1}$$  \hspace{1cm} (3.23)

The corresponding collision frequency for the $H^+\text{-H}$ charge exchange interaction for $\Gamma = 2000^\circ \text{K}$ is

$$\nu_{H^+-\text{H}} \approx 5 \times 10^{-9} n(\text{H}) \text{ sec}^{-1}$$  \hspace{1cm} (3.24)

From a comparison of these two collision frequencies it is clear that almost five times as much helium as hydrogen is needed to result in comparable frequencies. Since, moreover, helium decreases more rapidly with altitude than hydrogen, the contribution of helium to the drag term in the nighttime protonosphere can to a good approximation be neglected.

Using a value of $\Gamma = 2000^\circ \text{K}$, a conservative estimate, the energy transfer rate due to $H^+\text{-H}$ reactions is:

$$\int_{\text{H}^+\text{-H}}^{0} \approx -7 \times 10^{-13} n(H^+) n(H) \left( T_{H^+} - T_H \right) eV \text{ cm}^{-3} \text{ sec}^{-1}$$  \hspace{1cm} (3.25)
This is to be compared to the loss rate due to $H^+$-He collision

$$
\lambda_{H^+He} \approx 5.5 \times 10^{-14} \eta(H^+) \eta(He) \left( T_{H^+} - T_{He} \right) \epsilon V \left( m_c \lambda c \right)^{-1}
$$

(3.26)

In this case again more than ten times as much helium as hydrogen is needed to make the two loss rates comparable. Therefore, the effect of helium can also be neglected as far as energy transfer is concerned.

The collision frequency and the energy transfer rate values used in the main set of calculations assume that the $H^+$-$H$ reactions are the only significant ones. This assumption is further justified by a number of parametric studies discussed in Chapter V, which investigate the influence of various parameters (e.g., collision frequency) on the final results.

3.6 Ion-Ion Interactions

Hanson and Ortenburger (1961) pointed out many years ago, that in a mixture of $O^+$ and $H^+$ ions, a considerable drag on the diffusion of $H^+$ is caused by the presence of the $O^+$ ions. Therefore the effect of a small amount of $O^+$ on the diffusion of $H^+$ in the lower part of the protonosphere will now be investigated by considering a situation where the density of $O^+$ is equal to 1% of the $H^+$ density at an altitude of 1000 km. Experimentally obtained values of the nighttime ion composition (Johnson, 1967) are shown in Fig. 7 which indicates that the relative abundance of $O^+$ is considerably less than the fraction chosen here for this evaluation. The equation for the $H^+$-$O^+$ collision frequency after Banks (1966c) is

$$
\lambda_{H^+O^+} = 1.7 \times 10^{-2} \eta(O^+) \frac{4 \pi \bar{v}_O}{M_{H^+}^{1/2} \bar{T}_{O^+}^{3/2}} \lambda c \lambda^{-1}
$$

(3.27)
Fig. 7. Typical nighttime ion densities from 100 km to 1000 km (Johnson et al., 1967).
where
\[ \Lambda = \frac{3}{\varepsilon^3} \frac{e^3}{T_i} \lambda_D \] \[ -e = \text{charge of the electron} \]

and \( \lambda_D \) is the Debye length
\[ \frac{1}{\lambda_D} = 4 \pi e^2 \left( \frac{n(O^+) + n(H^+)}{kT_i} \right) \]

Using the following (typical) parameters:
\[ \log \Lambda = 15 \]
\[ n(O^+) = 10^2 \text{ cm}^{-3} \]
\[ T_i = 1600 \text{ }^\circ \text{K} \]

the collision frequency is:
\[ \nu_{H^+O^+} \approx 2 \times 10^{-3} \lambda e c^{-1} \tag{3.28} \]

Substituting a value of \( n(H) = 4 \times 10^4 \text{ cm}^{-3} \) into Eq. (3.24) gives:
\[ \nu_{H^+H^+} \approx 2 \times 10^{-4} \lambda e c^{-1} \tag{3.29} \]

These results clearly indicate that the effect of \( O^+ \) on the drag is not negligible at 1000 km, if the values used in this calculation are representative. As stated earlier the experimental results indicate that the abundance of \( O^+ \) is even less than was assumed here. The density of \( O^+ \) also falls off much more rapidly than neutral hydrogen as a function of altitude. On account of these factors, the effect of \( O^+ \) was neglected in the main body of calculations to be discussed in Chapter V. Some further numerical analysis has been made to evaluate the effect of this assumption. The results of this analysis are presented in Section 5.11.
A similar effect could be expected from the He$^+$ ions, but due to the fact that they are much lighter than the O$^+$ ions, they cannot be considered to be stationary, as was implicitly done for the O$^+$ ions. Thus, even if the value of $\nu_{\text{He}^+-\text{He}^+}$ is of the same order as $\nu_{\text{H}^+-\text{O}^+}$, the drag force corresponding to He$^+$ is much less important, since

$$
\nu_{\text{He}^+-\text{He}^+} \left( \frac{\mathcal{V}_{\text{He}^+} - \mathcal{V}_{\text{He}^+}}{\mathcal{V}_{\text{He}^+}} \right) \ll \nu_{\text{H}^+-\text{O}^+} \left( \frac{\mathcal{V}_{\text{H}^+} - \mathcal{V}_{\text{O}^+}}{\mathcal{V}_{\text{H}^+}} \right)
$$

This justifies the assumption that the H$^+$-He$^+$ interaction is negligible for the conditions considered in this thesis.

### 3.7 Neutral Hydrogen Density

In order to calculate the collision frequency and the energy transfer rates, given respectively by Eqs. (3.24) and (3.25), one needs the neutral hydrogen density as a function of arc length. If an isothermal neutral atmosphere in diffusive equilibrium is assumed in the protonosphere, the value of the density at the base (i.e., 1000 km) needs to be known in order for the altitude variation to be calculated. The available experimental information on hydrogen density is very meager and the values presented by the various authors (e.g., Donahue, 1966) vary significantly. The CIRA model atmosphere (1965) corresponding to a value of the 10.7 cm solar radiation of $F = 75 \times 10^{-22}$ W/m$^2$ Hz indicates that the neutral hydrogen density stays almost constant during the night around a value of $3 \times 10^4$ cm$^{-3}$. This model atmosphere corresponds to a neutral particle temperature of about 750$^\circ$K. The value used for the neutral hydrogen density, in the main set of calculations
to be presented in Chapter V, is the one given by the CIRA model \( (3.17 \times 10^4 \text{ cm}^{-3}) \). However, since some authors suggest (Brace et al., 1967) that this value may be too small, calculations are also made in which \( n(\text{H}) \) at 1000 km is considered as a parameter and varied between \( 3.17 \times 10^4 \text{ cm}^{-3} \) and \( 3 \times 10^5 \text{ cm}^{-3} \). The results of these calculations are presented in Section 5.9.
IV. NUMERICAL ANALYSIS

4.1 REARRANGEMENT OF THE BASIC EQUATIONS

The protonospheric plasma is described by the three equations obtained in Chapter II (see Table II) which simultaneously determine the temperature T, the density n, and the drift velocity v. These equations can be reduced to two coupled equations by eliminating the drift velocity v. By using the continuity equation

\[ \frac{\partial (\rho v)}{\partial t} = - \nabla \cdot \frac{\partial n}{\partial t} - \nabla \cdot \frac{\rho v}{\partial t} \]  

(4.1)

the energy equation (2.69) can be written as:

\[ 3 \, n \, k \, \frac{\partial T}{\partial t} = \frac{\partial}{\partial s} \left( B \, T^{5/2} \, \frac{\partial T}{\partial s} \right) + B \, T^{5/2} \, \frac{\partial T}{\partial s} \, \frac{\partial \rho}{\partial s} \] 

(4.2)

Further rearrangement of Eq. (4.2) leads to

\[ \frac{3}{2} \, n \, k \, \frac{\partial T}{\partial t} = \frac{\partial}{\partial s} \left( B \, T^{5/2} \, \frac{\partial T}{\partial s} \right) + \frac{B}{2} \, T^{5/2} \, \frac{\partial T}{\partial s} \, \frac{\partial \rho}{\partial s} - \frac{B}{2} \, n \, k \, \frac{\partial T}{\partial s} \] 

(4.3)

The following relation for the drift velocity is obtained from Eq. (2.38):

\[ \nu = - \frac{g \, \sin \, I}{g_{\mu}} - \frac{1}{n \, m_c \, \mu_{\mu}} \, \frac{\partial}{\partial s} \left( 2 \, n \, k \, T \right) \]  

(4.4)
This equation is then substituted into Eq. (4.1) to give

\[ \frac{\partial m}{\partial t} = \frac{\partial}{\partial s} \left[ \frac{g m \sin I}{\nu_m} + \frac{2}{m_i \nu_i} \frac{\partial (\eta T)}{\partial s} \right] \]

\[ + \frac{\partial}{\partial s} \left[ \frac{g m \sin I}{\nu_m} + \frac{2}{m_i \nu_i} \frac{\partial (\eta T)}{\partial s} \right] \]  \hspace{1cm} (4.5)

Substitution of Eq. (4.4) into Eq. (4.3) gives

\[ \frac{3}{2} n k \frac{\partial T}{\partial t} = \frac{\partial}{\partial s} \left[ \frac{B}{2} \frac{T^{\frac{3}{2}}}{\nu_m} \frac{\partial T}{\partial s} \right] + \frac{B}{2} \frac{T^{\frac{3}{2}}}{\nu_m} \frac{\partial T}{\partial s} \frac{\partial \log S}{\partial s} + \]

\[ + kT \frac{\partial m}{\partial t} + \frac{3}{2} k \frac{\partial T}{\partial s} \left[ \frac{g m \sin I}{\nu_m} + \frac{2}{m_i \nu_i} \frac{\partial (\eta T)}{\partial s} \right] \]

\[ + \frac{1}{2} \left[ \frac{m_i m}{\nu_m} g \sin I + \frac{2}{m_i \nu_i} \frac{\partial (\eta T)}{\partial s} \right] \frac{k^2}{m_i \nu_i} (\frac{\partial (T \eta)}{\partial s})^2 + \]

\[ + kT \frac{\partial m}{\partial s} \left[ - \frac{g m \sin I}{\nu_m} - \frac{2}{m_i \nu_i} \frac{\partial (\eta T)}{\partial s} \right] \frac{1}{2} \frac{\partial}{\partial s} \]  \hspace{1cm} (4.6)

These two coupled nonlinear Eqs. (4.5) and (4.6) are solved numerically in order to study the nighttime behavior of the protonosphere.

4.2 NUMERICAL TECHNIQUE

The coupled equations to be solved have the density \( n \) and the temperature \( T \) as unknowns and the time \( t \) and length \( s \) as independent variables.

The given initial values are \( n(0,s) \) and \( T(0,s) \), and the given boundary values are

\[ n(t,0), T(t,0), \]

\[ n(t,s_{\text{max}}), T(t,s_{\text{max}}) \]

which are known for all values of \( t \).
The two equations can be written in a general form as:
\[
f_1 \left( n, T, n^\cdot, T^\cdot, n''^\cdot, T^\cdot, \beta, \alpha, t \right) = 0
\]
\[
f_2 \left( n, T, n^\cdot, T^\cdot, n''^\cdot, T^\cdot, n^\cdot, T^\cdot, \beta, \alpha, t \right) = 0
\]
where \( \frac{\partial h}{\partial x} \) represents \( \partial h/\partial x \). The space and time coordinates are divided into equal intervals \( \Delta s \) and \( \Delta t \) as indicated in the mesh shown in Fig. 8. The running indices \( i \) and \( j \) are used to indicate the "position" of a given coordinate with respect to the origin. For the sake of brevity \( i \) stands for \( i\Delta s \) and \( j \) for \( j\Delta t \). The values of \( n \) and \( T \) are computed for all values of \( i \) at the time \( j+1 \) from the knowledge of \( n \) and \( T \) for all values of \( i \) at the time \( j \).

In the numerical calculations the following two simplifications are made:

a. The space and time derivatives of a quantity \( Z \) are replaced by implicit differences, i.e.,

\[
\frac{\partial}{\partial t} (Z_{i,j+1}) \text{ is written in terms of } Z_{i,j} \text{ and } Z_{i,j+1}
\]

\[
\frac{\partial}{\partial s} (Z_{i,j+1}) \text{ is written in terms of } Z_{i,j+1} \text{ and } Z_{i+1,j+1}
\]

\[
\frac{\partial^2}{\partial s^2} (Z_{i,j+1}) \text{ is written in terms of } Z_{i,j+1}, Z_{i+1,j+1}, Z_{i+1,j+1}
\]

b. All nonlinear terms of the form \( G(Z_{i,j+1}, Z_{i+1,j}, Z_{i-1,j}) \) are replaced by a Taylor series expansion to the first order:

\[
G \left( Z_{i,j+1}, Z_{i+1,j}, Z_{i-1,j} \right) \approx G \left( Z_{i,j}, Z_{i+1,j}, Z_{i-1,j} \right) +
\]

\[
+ \left[ Z_{i,j+1} - Z_{i,j} \right] G^\cdot \left( Z_{i,j}, Z_{i+1,j}, Z_{i-1,j} \right)
\]

\[
+ \left[ Z_{i+1,j+1} - Z_{i+1,j} \right] G^\cdot \left( Z_{i,j+1}, Z_{i+1,j+1}, Z_{i-1,j+1} \right)
\]

\[
+ \left[ Z_{i-1,j+1} - Z_{i-1,j+1} \right] G^\cdot \left( Z_{i,j+1}, Z_{i+1,j}, Z_{i-1,j} \right)
\]

\[
\text{(4.7)}
\]
Only the first derivative with respect to time appears in the computations; thus only two values of \( j \) are involved, and furthermore all quantities are known at the time \( j \). These facts permit the suppression of the subscript \( j \); the known parameters \( n_{i\,j} \) and \( T_{i\,j} \) are denoted simply by \( n_i \) and \( T_i \), and the unknowns \( n_{i\,j+1} \) and \( T_{i\,j+1} \) will be denoted by \( X_i \) and \( X_{i+1} \), respectively.

The use of the implicit differences and the linearization for \( i = 1, 2, \ldots, \beta - 1 \) results in a set of \( 2(\beta - 1) \) linear equations with \( 2(\beta - 1) \) unknowns.

Here \( \beta \) is the number of intervals into which the length of the field line from an altitude of 1000 km in one hemisphere to the same altitude in the conjugate hemisphere is divided. However, it should be noted that each equation contains at most six unknowns \((X_{i-1}, X_i, X_{i+1}, Y_{i-1}, Y_i, Y_{i+1})\). This system of equations, written in matrix form, is:

\[
\begin{bmatrix}
  b_{ii} & c_{i1} & d_{i1} & e_{i1} & f_{i1} & g_{i1} \\
  b_{i2} & c_{i2} & d_{i2} & e_{i2} & f_{i2} & g_{i2} \\
  b_{21} & c_{21} & d_{21} & e_{21} & f_{21} & g_{21} \\
  b_{22} & c_{22} & d_{22} & e_{22} & f_{22} & g_{22}
\end{bmatrix}
\begin{bmatrix}
  X_i \\
  Y_i \\
  X_{i+1} \\
  Y_{i+1}
\end{bmatrix}
= \begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2 \\
  a_3
\end{bmatrix}
\]

(4.8)
Fig. 8. The finite difference mesh employed in the numerical calculations.
The first two equations contain \( X_0, Y_0, X_1, Y_1, X_2, Y_2 \). \( X_0 \) and \( Y_0 \) are known and, therefore, \( X_1 \) and \( Y_1 \) can be expressed as linear functions of \( X_2 \) and \( Y_2 \). These expressions are then used to replace \( X_1 \) and \( Y_1 \) in the second set of two equations, the only other place where they appear. In turn \( X_2 \) and \( Y_2 \) are expressed as linear functions of \( X_3 \) and \( Y_3 \). This process continues until an expression is obtained for \( X_{\beta-1} \) and \( Y_{\beta-1} \) as linear functions of \( X_{\beta} \) and \( Y_{\beta} \). This relation combined with the boundary conditions, which give \( X_{\beta} \) and \( Y_{\beta} \), is used to evaluate \( X_{\beta-1} \) and \( Y_{\beta-1} \), which in turn can be used to calculate \( X_{\beta-2} \) and \( Y_{\beta-2} \). These calculations are continued through decreasing values of \( i \) back to \( X_1 \) and \( Y_1 \).

This implicit numerical method described here was selected in order to minimize problems of instability. The basic difference between the implicit and explicit methods is indicated in Fig. 9.

The general criterion for stability of the implicit methods is

\[
\frac{\Delta t}{(\Delta s)^2} < \lambda
\]

where \( \lambda \) is a numerical constant. This criterion requires very small time steps which can lead to significant round-off errors.

Implicit methods, such as the Laasonen (1949) and the Crank-Nicholson (1947), are unconditionally stable in their applications to linear differential equations. However, in the application of implicit methods to nonlinear differential equations it is very difficult to establish a criterion of stability except for very simple problems (Richtmyer and Morton, 1967). In the case of coupled nonlinear partial differential equations the stability of the solution is studied experimentally (see Section 4.5).
Fig. 9. Diagram showing the basic difference between the implicit and the explicit differences. The circles refer to the point for which the partial differential equations are evaluated. The crosses refer to the location of the quantities being used in the differential equations.
4.3 IMPLICIT DIFFERENCE QUOTIENTS

In this section the implicit differences which are used to replace the derivatives are discussed.

Only first derivatives with respect to time appear in the equations; in the numerical calculations the first derivative of the temperature is written as

\[
\frac{1}{\Delta t} \left[ T(\mathcal{S}, t + \Delta t) - T(\mathcal{S}, t) \right]
\]

With the notation, described in Section 4.2, the above expression becomes

\[
\frac{1}{\Delta t} \left[ X_i - T_i \right]
\]

Similarly the first derivative with respect to the spatial coordinate is written as

\[
\frac{1}{\Delta \mathcal{S}} \left[ X_i - X_{i-1} \right]
\]

Finally the expression for the second derivative is:

\[
\frac{1}{(\Delta \mathcal{S})^2} \left[ X_{i+1} - 2X_i + X_{i-1} \right]
\]

Analogous expressions are used for the derivatives of the density.

The approximation of the derivatives of a quantity Z at the grid point \((i,j+l)\) by using the values of Z at the grid points \((i,j), (i,j+1), (i-1, j+1), (i+1,j+1)\), is referred to as the Laasonen scheme. The two well-known methods for the numerical solution of differential equations are the Crank-Nicholson scheme and the Laasonen scheme. Although the Crank-Nicholson scheme in general is more accurate, the Laasonen scheme is more suitable for the problem to be solved here, since its application requires a simpler linearization process.
4.4 TRUNCATION ERRORS

The two coupled equations contain first derivatives with respect to time and space and second derivatives with respect to space. It is easy to show that the difference relations (Section 4.3) used to approximate the second derivatives with respect to space lead to truncation errors of the order $O((\Delta s)^2)$, and the differences used to approximate the first derivatives with respect to time and space lead to errors of the order of $O(\Delta t)$ and $O(\Delta s)$, respectively.

The linearization procedure given by Eq. (4.7) leads to further truncation errors. The terms neglected in Eq. (4.7), are of the order of $[Z_{i,j+1} - Z_{i,j}]^2$, which is proportional to $(\Delta t)^2$. Hence, the error due to the linearization is of the order $O((\Delta t)^2)$. As an example, the truncation errors made in the approximation of

$$n(i,j+1) \frac{\partial^2(T(i,j+1))}{\partial s^2}$$

are given below:

$$n(i,j+1) \frac{\partial^2(T(i,j+1))}{\partial s^2} = \frac{n(i,j+1)}{\Delta s^2} \left[ T_{i+1,j+1} - 2T_{i,j+1} + T_{i-1,j+1} - \frac{\Delta s^4}{4!} \frac{\partial^4(T(i,j+1))}{\partial s^4} + \ldots \ldots \right] \quad (4.9)$$

With the notation described in Section 4.2, the above expression becomes

$$n(i,j+1) \frac{\partial^2(T(i,j+1))}{\partial s^2} = \frac{Y_i}{\Delta s^2} \left( X_{i+1} - 2X_i + X_{i-1} \right) + O[\Delta s^2] \quad (4.10)$$

The linearization leads to
\[ n_{(i, j+1)} \frac{\partial^2 (T_{(i, j+1)})}{\partial \Delta s^2} = \frac{1}{\Delta s^2} \left\{ \begin{array}{l} -n_i \; T_{i+1} + Y_i \; T_i + X_{i+1} n_i + (X_{i+1} - T_{i+1})(Y_{i-1} - n_i) \\ + 2n_i T_i - 2Y_i T_i - 2X_i n_i - 2(Y_i - T_i)(Y_{i-1} - n_i) \\ -n_i \; T_{i-1} + Y_i \; T_{i-1} + X_{i-1} n_i + (X_{i-1} - T_{i-1})(Y_{i-1} - n_i) \end{array} \right\} + \mathcal{O}[\Delta s^2] \]

After rearrangement the above expression becomes

\[ n_{(i, j+1)} \frac{\partial^2 (T_{(i, j+1)})}{\partial \Delta s^2} = \frac{1}{\Delta s^2} \left[ (Y_i - n_i)(T_{i+1} - 2T_i + T_{i-1}) + n_i (X_{i+1} - 2X_i + X_{i-1}) \right] \]

\[ + \frac{1}{\Delta s^2} \left[ (Y_i - n_i)(X_{i+1} - 2X_i + X_{i-1} - (T_{i+1} - 2T_i + T_{i-1})) \right] \]

\[ + \mathcal{O}[\Delta s^2] \] (4.11)

The use of the relations

\[ \frac{\partial^2 (T_{(i, j+1)})}{\partial \Delta s^2} = \frac{X_{i+1} - 2X_i + X_{i-1}}{\Delta s^2} + \mathcal{O}[\Delta s^2] \]

\[ \frac{\partial^2 (T_{(i, j)})}{\partial \Delta s^2} = \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta s^2} + \mathcal{O}[\Delta s^2] \]

\[ Y_i - n_i = \Delta t \frac{\partial (n_{(i, j+1)})}{\partial t} + \mathcal{O}[\Delta t] \]

\[ \frac{\partial^2 (T_{(i, j+1)})}{\partial \Delta s^2} - \frac{\partial^2 (T_{(i, j)})}{\partial \Delta s^2} = \Delta t \frac{\partial^3 (T_{(i, j+1)})}{\partial \Delta s^2 \partial t} + \mathcal{O}[\Delta t] \]

leads to

\[ n_{(i, j+1)} \frac{\partial^2 (T_{(i, j+1)})}{\partial \Delta s^2} = \frac{1}{\Delta s^2} \left\{ \begin{array}{l} (Y_i - n_i)(T_{i+1} - 2T_i + T_{i-1}) \\ + n_i (X_{i+1} - 2X_i + X_{i-1}) \end{array} \right\} \]

\[ + \mathcal{O}[\Delta t^2] + \mathcal{O}[\Delta s^2] \]
and more simply
\[ n(i, j+1) \frac{\partial^2 (T(i, j+1))}{\partial x^2} = \frac{1}{4 \Delta x^2} \left\{ \left( Y_i - n_i \right) \left( T_{i+1} - 2 T_i + T_{i-1} \right) + n_i \left( X_{i+1} - 2 X_i + X_{i-1} \right) \right\} + O\left[ \Delta t^2 + \Delta x^2 \right] \] (4.12)

Therefore the approximation of
\[ n(i, j+1) \frac{\partial^2 (T(i, j+1))}{\partial x^2} \]
by the first term of the right-hand side leads to a truncation error \( O[\Delta t^2 + \Delta x^2] \).

4.5 CHOICE OF STEP SIZES

The equations describe physical quantities along a field tube which is symmetrical about the equatorial plane. The choice of equinoctial conditions in the calculations means that the results should also be symmetric about the equatorial plane. If the field tube from the 1000 km altitude in one hemisphere to the same altitude at the conjugate hemisphere is divided into \( \beta \) intervals, the results corresponding to positions \( \alpha \) and \( \beta - \alpha \) have to be identical.

This requirement of symmetry is used as a criterion in the selection of step sizes along the tubes. In order to avoid any systematic effects due to the program itself, the numerical calculations are sequenced to run alternately from north to south and south to north along the tube. A value of 250 km was the first arbitrary choice for the 40° tube step size. By using time steps of 60 sec, it is found that after 30 min the maximum difference between two symmetrical points is not increasing and is about 1% for the dens-
ity and less than that for the temperature. A change in the step size to
125 km decreases the maximum difference to less than 0.1%. Both of these
step sizes appear to lead to stable solutions. Therefore, step sizes of 100
km, 125 km and 133 km were chosen for the 30°, 40°, and 50° tubes respec-
tively. The larger step sizes were chosen for the longer tubes in order to
keep the computing times reasonable and to minimize the round-off errors.
The value of the collision frequency $v_{in}$ has a controlling effect on the ef-
fectiveness of a given step size, because the difference equations (Section
4.2) contain typical terms such as

$$\frac{1}{U_{in}} \left( \frac{X_i - X_{i-1}}{\Delta \lambda} \right)$$

As was mentioned in the previous paragraph, the initial choice for the
time step was 60 sec. As a check whether this is a reasonable choice, cal-
culations were made with time steps of 120 sec. The maximum difference caused
by this change is less than 0.1% for the density and 0.2% for the tempera-
ture after the lapse of 18 min; therefore, a time interval of 60 sec appears
to be a reasonable choice.

From elementary physical considerations it follows that if the tempera-
ture profile is maintained constant, the density profile must tend toward
diffusive equilibrium, and stay constant, once it is reached. This can pro-
vide a possible check of the reliability of the program as it is possible to
determine independently the diffusive equilibrium profile corresponding to a
given temperature profile (Section 3.3). A given temperature profile and the
corresponding calculated density profile were taken as initial conditions,
and the selected boundary conditions were kept constant for the $40^\circ$ tube. The calculations were carried out for 30 min, at which time the difference between the density profile calculated in the above fashion shown in Fig. 4, and the diffusive equilibrium one is about 0.3%, which is a definite indication of the reliability of the numerical technique.

A second test was carried out for the $40^\circ$ tube in which the initial density profile was chosen to be different from the diffusive equilibrium one. It was the purpose of this test to check how fast the density profile approaches diffusive equilibrium if the temperature profile is kept constant. The results of these calculations (see Fig. 10) show that the density profile approaches diffusive equilibrium in a short time.

All of the tests discussed in this section indicate the reliability of the numerical technique employed.
Fig. 10. Test of convergence of the density distribution toward diffusive equilibrium along 40° tube.
V. RESULTS

5.1 INTRODUCTION

The results of the calculations of the nighttime behavior of the 30°, 40° and 50° protonospheric tubes are presented in this chapter. The method of analysis was described in Chapter IV, and the parameters used were given in Chapter III. This nighttime study covers about 10 hr after sunset. The calculations directly give the temperature and density as a function of altitude and time; particle and energy fluxes can be easily obtained from the results.

5.2 TEMPERATURE PROFILES

The time dependent behavior of the temperature in the three tubes is shown in Figs. 11, 12, and 13. The general behavior of the three tubes is similar; differences in the speed of response are basically due to the different heat content of the various tubes.

In general, the temperature tends to decrease rapidly along the entire tube in the same way as the boundary values do at 1000 km. However, at the same time an important temperature gradient builds up in the lower part of the tube, which means that a significant portion of the heat is conducted away by the electron gas. The boundary temperatures decrease uniformly during the first 4 hr for the 30° and 40° tubes, and during the first 6 hr for the 50° tube. In the case of the 30° and 40° tubes, when the temperature values at the boundary become approximately constant, the lower portion of the temperature profile also stabilizes while the temperature in the equatorial region
Fig. 11. Calculated temperature distributions along the 30° tube.
Fig. 12. Calculated temperature distributions along the $40^\circ$ tube.
Fig. 13. Calculated temperature distributions along the 50° tube.
keeps decreasing, thus accounting for the heat which is lost from the tube through conduction and collisions with the neutrals. Due to the greater heat content of the 50° tube the temperature at the boundary decreases more slowly than for the other tubes, and the final temperatures are higher. In the equatorial region of the 50° tube a temperature minimum develops about an hour after sunset, which disappears slowly a few hours later. The cause of this behavior is discussed in Section 5.4.

5.3 DENSITY PROFILES

The behavior of the boundary values coupled with the temperature variation result in a three phase variation of the density in the 30° and 40° tubes (Figs. 14 and 15). During the first phase the decrease in density at the boundary combined with the general cooling results in an overall decrease of density in the tubes as well as an increase in the absolute value of the density gradient. During the second phase, the boundary value of the density, after reaching a minimum, starts to increase; this tends to increase the density all over the tube, but this is counteracted by the cooling which still tends to increase the magnitude of the density gradient. The cooling finally becomes insignificant while the boundary density keeps increasing with a resulting density increase over the whole tube. It will be shown in Section 5.4 that a downward flux of particles is associated with the first phase, a transition period with the second phase, and the last phase, which is the largest one, corresponds to an upward flux of particles.

The variation of the density at the boundary for the 50° tube is simpler (Fig. 16); the initial decrease is followed by a steady situation. The cool-
Fig. 14. Calculated density distributions along the 30° tube.
Fig. 16. Calculated density distributions along the 50° tube.
ing and the initial decrease in the boundary values combine to give an overall decrease in the density and an increase in the absolute value of the density gradient. After a lapse of a few hours the density profiles stabilize, since both the temperatures and the boundary values tend to do the same. A significant downward flux during the night is associated with this behavior of the 50° tube (see Section 5.4). It should be noted, however, that due to the great particle content of this tube, the change in the density profiles corresponding to a given particle flux is considerably less than the changes in the other tubes. The behavior of the density and temperature profiles of the 50° tubes are complementary: due to the large particle content (i.e., heat content) of the tube the temperature cannot fall rapidly and this in turn leads to a slow change in the equilibrium density profile.

5.4 PARTICLE FLUX

The temperatures and densities are quantities which are directly obtained from the numerical solutions. The particle, convection and conduction fluxes are derived quantities involving derivatives and are therefore less accurate and are also extremely sensitive to experimental boundary values. Some of the fluctuations in the calculated fluxes (e.g., 30° tube particle flux) could be eliminated by rearranging the boundary values within the quoted error flags, and therefore, no physical significance should be attached to these fluctuations.

The particle fluxes at 1000 km give a clear indication of the changes in the charged particle content of the field tubes. These fluxes corresponding to the three field tubes are shown in Figs. 17, 18, and 19. The value of these
Fig. 17. Calculated nighttime variation of the particle flux at 1000 km for the 30° tube.
Fig. 18. Calculated nighttime variation of the particle flux at 1000 km for the 40° tube.
Fig. 19. Calculated nighttime variation of the particle flux at 1000 km for the 50° tube.
fluxes is obtained by computing the change in the total electron content of the tube per unit time. This approach is permissible since only equinox conditions are considered.

The calculations show a downward particle flux for the 30° and 40° tubes during the first three to four hours of the night with a sharp maximum of the order of $2.5 \times 10^9 \text{ cm}^{-2} \text{ sec}^{-1}$ for the 40° tube and a smoother maximum of $8 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ for the 30° tube. The difference in the magnitude of the flux is the result of the difference in the content of the tubes. The calculations also show the presence of an upward flux in both tubes during the latter part of the night; flat maxima of the order of $7.5 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ and $3.5 \times 10^8 \text{ cm}^{-2} \text{ sec}^{-1}$ are found for the 40° and 30° tubes respectively.

The integrated downward flux during the early part of the night is almost equal to the integrated upward flux for the 30° tube and within 20% for the 40° tube. In other words, the breathing of these tubes does not result in any significant change of their total particle content, and therefore, under the given conditions, the protonosphere cannot supply directly a sufficient number of particles to maintain the mid-latitude nighttime F region as suggested by some authors (e.g., Yonezawa, 1965; Hanson and Patterson, 1964).

A large downward flux, slowly decreasing to zero during the night, is found for the 50° tube. There is a sharp maximum of the order of $9 \times 10^9 \text{ cm}^{-2} \text{ sec}^{-1}$ associated with this flux. It was mentioned in Section 5.2 that a significant temperature trough develops about the equatorial region shortly after sunset for the 50° tube. A similar but much smaller trough is also present in the 40° tube. It was just shown in this section that a large downward motion is
present at the same time, particularly from the 50° tube. The temperature maximum between the 1000 km altitude and the equatorial plane can be explained as the result of the transfer of gravitational energy into kinetic energy by the particles moving down from the equatorial region. Using a simple, qualitative argument the change in the potential energy of a particle moving down from the equatorial plane is \( m \Delta g \). If the following values are used:

\[
\begin{align*}
    m &= 1.7 \times 10^{-24} \text{ g;} \\
    g_{eq} &= 125 \text{ cm sec}^{-2}; \\
    g_{1000} &= 733 \text{ cm sec}^{-2}; \\
    r_{eq} &= 18000 \text{ km;} \\
    r_{1000} &= 7400 \text{ km},
\end{align*}
\]

the change in the potential energy of a hydrogen ion is of the order of \( 4 \times 10^{-13} \) ergs. Some of this energy is in turn transferred via collisions to the electrons, the ambient ions, and the neutrals. This process of the conversion of gravitational energy into kinetic energy is the mechanism which causes the temperature maximum in the lower part of the field tube.

The following approximations of the various terms of the energy equation \( \text{(2.69)} \) provide an insight into the manner in which this temperature maximum develops. Initially the plasma is isothermal in the equatorial region, hence

\[
\frac{\partial}{\partial t} \left( B \frac{T^{5/2}}{2} \frac{\partial T}{\partial \lambda} \right) = 0
\]

A typical value for the rate of change of the temperature is

\[
\frac{\partial T}{\partial t} = 10^{-1} \, \text{°K sec}^{-1}
\]

Typical values for the other terms are

\[
\left( \frac{L}{\eta k} \right)_{eq} = 10^{-3} \, \text{°K sec}^{-1}
\]
\[ \nu^2 \frac{N \nu \sin \theta}{K} \approx 10^{-3} \, ^\circ K \, s^{-1} \]

These terms are negligible compared with \( \frac{\partial T}{\partial t} \). The only remaining significant term is

\[ \frac{2}{S} \frac{\partial T}{\partial \beta} \left( S \nu \right) \]

Therefore, the approximate energy equation reduces to

\[ 3 \frac{\partial T}{\partial t} = - \frac{2}{S} T \frac{\partial}{\partial \beta} \left( S \nu \right) \]

Now consider the following two cases: at the equatorial plane:

\[ \frac{\partial S}{\partial \beta} = 0 \]

and

\[ T \frac{\partial \nu}{\partial \beta} \approx 10^{-1} \, ^\circ K \, s^{-1} \]

therefore

\[ \frac{\partial T}{\partial t} < 0 \]

Away from the equatorial region

\[ \frac{\partial \nu}{\partial \beta} \approx 0 \]

and

\[ \nu \frac{T}{S} \frac{\partial S}{\partial \beta} \approx - 10^{-1} \, ^\circ K \, s^{-1} \]

therefore

\[ \frac{\partial T}{\partial t} > 0 \]
In other words the temperature has a tendency to decrease near the equatorial region and to increase in the lower regions, which is just what the calculations indicate.

5.5 CONVECTION FLUX

The particle fluxes discussed in the previous section transport energy into and out of the protonosphere. This convection flux is obtained by simply multiplying the particle flux by the value of \((3/2)kT\) at 1000 km; they are shown in Figs. 20, 21, and 22. The general behavior of this flux is clearly the same as that of the particle flux. The maxima of the heat convected down are \(2 \times 10^8\) eV cm\(^{-2}\) sec\(^{-1}\), \(8.6 \times 10^8\) eV cm\(^{-2}\) sec\(^{-1}\) and \(2.7 \times 10^9\) eV cm\(^{-2}\) sec\(^{-1}\) for the 30\(^\circ\), 40\(^\circ\) and 50\(^\circ\) tubes respectively; the maxima of the heat convected up are \(1.1 \times 10^8\) eV cm\(^{-2}\) sec\(^{-1}\) and \(4.5 \times 10^7\) eV cm\(^{-2}\) sec\(^{-1}\) for the 30\(^\circ\) and 40\(^\circ\) tubes respectively. Since the temperature decreases monotonically during the night there is a downward flux of relatively high temperature particles during the early part of the night and an upward flux of low temperature particles later on. The net flux of energy convected down in this manner during the night is \(7.2 \times 10^{11}\) eV cm\(^{-2}\) and \(1.8 \times 10^{12}\) eV cm\(^{-2}\) for the 30\(^\circ\) and 40\(^\circ\) tubes respectively. The numbers quoted for the fluxes are for one kind of particles only; to get the energy carried by the entire plasma these numbers must be multiplied by a factor of two.

5.6 CONDUCTION FLUX

The energy conducted out of the protonosphere by the electron gas is given by

\[
B T^{5/2} \frac{\partial T}{\partial z}
\]
Fig. 20. Calculated nighttime variation of the energy fluxes at 1000 km for the 30° tube.
Fig. 21. Calculated nighttime variation of the energy fluxes at 1000 km for the 40° tube.
Fig. 22. Calculated nighttime variation of the energy fluxes at 1000 km for the 50° tube.
evaluated at an altitude of 1000 km. The fluxes from all three tubes are in excess of $10^8$ eV cm$^{-2}$ sec$^{-1}$ for several hours after sunset, which is in agreement with the values obtained previously by a number of authors in a less direct manner (Geisler and Bowhill, 1965; Glidson, 1966; Nagy and Walker, 1967). The maximum fluxes are $2.25 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$, $6.2 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$ and $3.2 \times 10^9$ eV cm$^{-2}$ sec$^{-1}$ for the 30°, 40°, and 50° tubes, respectively. It should be noted that the initial temperature profile assumed for the 30° tube apparently has too high a gradient near 1000 km resulting in an initial decrease in the conduction flux.

5.7 DIFFERENCE BETWEEN THE DYNAMIC AND DIFFUSIVE EQUILIBRIUM DENSITY PROFILES

In order to get an upper limit of the differences between the calculated dynamic and the corresponding diffusive equilibrium density profiles, very high values are selected for the collision frequency in the calculation to be discussed in this section. This study is carried out for the 40° tube. The dynamic profiles are calculated using collision frequencies corresponding to a hydrogen density ten times the CIRA value. These results are plotted in Fig. 23 along with the corresponding diffusive equilibrium profiles calculated using Eq. (3.4). Forty minutes after sunset the flux has the maximum value of $1.9 \times 10^9$ cm$^{-2}$ sec$^{-1}$. At the same time the difference between the two density profiles is also a maximum and has a value of about 10%. One hundred twenty minutes after sunset, the flux is $7 \times 10^8$ cm$^{-2}$ sec$^{-1}$ and the difference in the profiles is down to 4%. Finally at 240 minutes, when the flux is about $1 \times 10^8$ cm$^{-2}$ sec$^{-1}$ the two profiles are practically identical, the difference being less than 1%.
Fig. 23. Comparison of the calculated density distributions with the corresponding instantaneous diffusive equilibrium distributions for the 40° tube.
This study shows that the dynamic profiles are close to the diffusive equilibrium profiles and that the difference between the profiles is proportional to the magnitude of the flux.

5.8 EFFECT OF INITIAL TEMPERATURE GRADIENTS

In order to see what effect the choice of the initial temperature profile has on the final results, calculations were carried out using an initial temperature profile with high temperature gradients. A profile with a variation of 2000°K along the 40° tube was chosen for these trial calculations. The results, shown in Fig. 24, indicate that the equatorial temperature drops by 700°K in the first 10 min; within 1 hr the temperature relaxes to the same order of magnitude as the values calculated by using the "reasonable" initial temperature profile.

This thesis shows that the exact form of the assumed initial conditions does not determine the general nature of the results. Another important result of this calculation is that it is apparently impossible to maintain very high temperatures and/or temperature gradients in the nighttime protonosphere at mid-latitudes. This restriction does not apply to high L values, since, when the electron densities in the tubes are very low, the conduction is significantly lower; therefore no significant amount of energy is transported down into the ionosphere. Thus the above conclusions need not be contradictory to experimental results which indicate high temperatures at high L values (Serbu and Maier, 1966; Sagalyn and Smiddy, 1968).
Fig. 24. Calculated temperature profiles for the case of a high initial temperature gradient for the 40° tube.
5.9 NEUTRAL PARTICLE DENSITY EFFECTS

The actual value of the neutral hydrogen density is not well known as was discussed earlier. Therefore it was considered to be important to study the control that this density has on the results. Calculations were carried out for the 40° tube and for density values which were five and ten times the adopted CIRA value. The density exerts a strong control on the temperature since the energy lost to the hydrogen atoms is directly proportional to the neutral hydrogen density.

The results of these calculations, shown in Fig. 25, are as expected. The heat conducted down decreases with increasing hydrogen density. The temperature profile calculated for a hydrogen density of $5n(H)_{CIRA}$ is reasonable; the maximum heat conducted down is about one half of the value calculated for $n(H)_{CIRA}$. On the other hand the results obtained for $10n(H)_{CIRA}$ appear to be nonphysical. The temperature tends to fall faster than the boundary value. This results in a negative temperature gradient near 1000 km, which means that energy must be conducted upward into this region. This requires the presence of a heat source around 1000 km and there is no reason to expect such a source to exist. Thus it appears from these results that both $n(H)_{CIRA}$ and $5n(H)_{CIRA}$ are acceptable and give similar results, and that the value of $10n(H)_{CIRA}$ is too high. There are other indications (Jacchia, 1967; Donahue, 1966), that the low value of the hydrogen density is the true one.

5.10 EFFECT OF THE ION-NEUTRAL COLLISION FREQUENCY

The magnitude of the ion-neutral collision frequency controls the speed with which the density profiles respond to changing conditions. A test cal-
Fig. 25. Effect of the magnitude of the neutral densities upon the temperature distribution along the 40° tube.
calculation has been performed for the 30° tube in which the values used for $\nu_{in}$ were ten times the ones used in the general study. After 120 min, the time of the maximum particle flux, the effect of the increased collision frequency on the density profile is less than 3% and the effect on the temperature profile is less than 1%. The comparison between the particle fluxes calculated by using the different collision frequencies is shown in Fig. 26.

This study further emphasizes the point made in Section 5.7, that the collision frequencies are small enough to result in density distributions which are close to the diffusive equilibrium ones.

5.11 DRAG EFFECTS OF $O^+$ IONS

It has just been shown that a change in the ion-neutral collision frequency in a uniform manner over the whole tube has a negligible effect. The presence of $O^+$ ions results in an increased collision frequency in the lower protonosphere. In order to see the effect of such an increase a set of calculations was carried out for the 40° tube, in which $\nu_{in}$ was changed in a manner to simulate the presence of $O^+$ ions. The collision frequency was chosen to be ten times the normal value at an altitude of 1000 km and then progressively reduced as a function of altitude to reach the usual value at 2000 km. This resulted in changes in the temperature and density of less than 2%. The maximum difference between these results and those obtained for the regular ion-neutral collision frequency occurs at the time of maximum downward flux. Figure 27 shows moreover, that the fluxes compare very well except for only a small time delay introduced by the increased drag in the lower part of the tube.
Fig. 26. Effect of the collision frequency upon the particle flux at 1000 km.
Fig. 27. Effect of simulated O\(^+\) drag upon the particle flux.
VI. IONOSPHERE-PROTONOSPHERE COUPLING

6.1 INTRODUCTION

The results of the calculations presented in the previous chapter indicate the presence of a downward flux of ionization in all three tubes during the beginning of the night and an upward flux in the 30° and 40° tubes during the latter portion of the night. This is a somewhat unexpected result and in this section two possible mechanisms which may be responsible for such behavior are discussed in a qualitative manner.

The particle and heat fluxes into and out of the protonosphere are controlled by the conditions at the boundary; therefore, the discussion can be based on either an accounting for the fluxes or an explanation of the changing boundary conditions.

6.2 EFFECT OF THE CHARGE EXCHANGE PROCESS

The charge exchange reactions between atomic hydrogen and oxygen

\[ \text{H}^+ + \text{O}(^3\text{P}) \rightarrow \text{H} + \text{O}^+(^4\text{S}) + \Delta E \]  

(6.1)

control the hydrogen ion distribution in the 500-700 km region of the ionosphere (Hanson and Ortenburger, 1961). The numerical value of \( \Delta E \) may be positive or negative depending on the \( J \) value of the atomic oxygen \( ^3\text{P} \) state. The chemical equilibrium relation for the atomic hydrogen ions follows from Eq. (6.1):

\[ n(\text{H}^+) = \frac{9}{8} \frac{n(\text{H})}{n(O)} \]  

(6.2)
The factor $9/8$ is the result of the different statistical weights of the states resulting from the reactions; the exponential factor $\exp(-\Delta E/kT)$ was considered to be unity.

By sheer coincidence the altitude distribution of atomic hydrogen ions in an isothermal ionosphere is the same for both the chemical and diffusive equilibrium conditions if the temperature of all species is the same, as shown in Appendix C. This means that in order to obtain an understanding of the general behavior of the hydrogen ions at 1000 km one does not have to know exactly where the transition region is. Only a knowledge of the time variation of the neutral atomic hydrogen and oxygen and oxygen-ion densities in the 500-700 km region is needed. The neutral atomic hydrogen density is practically constant, increasing only very slightly during the night (CIRA, 1965). The oxygen ion density also stays fairly constant throughout the day and night (Reddy et al., 1967), and the atomic oxygen density decreases during most of the night as a result of the cooling of the atmosphere. Therefore, it follows from Eq. (6.2) that $n(H^+)$ will increase during the night provided any possible decrease in $n(O^+)$ is slower than the decrease in $n(O)$, which is definitely the case. In other words, since $n(O)$ decreases, the oxygen ions tend to give their charges to the hydrogen atoms; but due to the fact that the hydrogen ions are lighter than the oxygen ions, they tend to redistribute themselves upward, giving rise to an upward flux of protons.

In order to investigate whether the boundary values are reasonable, two calculations have been carried out. In one it was assumed that hydrogen ions are in chemical equilibrium at 700 km, i.e., their density satisfies Eq. (6.2),
at that altitude while above 700 km they distribute themselves in diffusive equilibrium. In the other calculation the altitude of chemical equilibrium was taken to be 500 km. The neutral densities at 500 and 700 km were taken from the CIRA (1965) model atmosphere, and the oxygen ion density from Reddy et al. (1967). The hydrogen ion densities at 500 and 700 km can then be obtained from Eq. (6.2). The average ion mass in the region up to 1000 km is given by the expression

\[
\langle m^+ \rangle = \frac{m_o(0^+) \eta_o(0^+) + m(H^+) \eta_o(H^+)}{\eta_o(0^+) + \eta_o(H^+)} \exp \left\{ -\int_{\tau_0}^{\frac{z}{kT}} \frac{g x}{kT} \left[ m(H^+) - m(0^+) \right] \right\}
\]

where \( z_0 \) is either 500 or 700 km, \( z \) is the altitude at which the average ion mass is calculated and \( \eta_o(H^+) \) is given by Eq. (6.2). The ion temperature \( T \) is taken to be isothermal up to 1000 km, and all ion species are assumed to be in thermal equilibrium. \( \eta(H^+) \) can then be calculated from the expression

\[
\eta(H^+) = \eta(H^+) \exp \left\{ -\int_{\tau_0}^{\frac{3}{kT}} \left[ m(H^+) - \langle m^+ \rangle \right] \frac{g x}{kT} \right\}
\]

where \( \langle m^+ \rangle \) is given by Eq. (6.3). The values of the hydrogen ion density at 1000 km obtained from Eq. (6.4) have then been compared with the electron density values given by Brace et al. (1967), which were used as boundary values for the protonospheric calculations reported in this thesis. Figure 28 shows that the calculated hydrogen ion densities at 1000 km compare qualitatively very well with the experimental results. They show a minimum during
Fig. 28. Comparison of the densities obtained from the charge exchange theory with those obtained from Explorer XXII.

1. Diurnal variation of the density at 1000 km from Explorer XXII.
2. Diurnal variation of the density at 1000 km from the charge exchange theory with diffusive equilibrium starting at 500 km.
3. Diurnal variation of the density at 1000 km from the charge exchange theory with diffusive equilibrium starting at 700 km.
the day and a maximum during the night and are in fairly good agreement during
the transition periods. However, the calculated densities have larger rela-
tive variations than the experimental ones. This can be explained by the
following two arguments:

a. The theoretical results are based on equilibrium calculations. The
resulting day-to-night density variations therefore represent an
upper bound of the actual variation since the plasma has a finite
response time and never quite relaxes to the diffusive equilibrium
distribution.

b. Jacchia (1967) has shown that the CIRA tables tend to give much
larger oxygen density variations between day and night than the ob-
served variations. Since according to Eqs. (6.2) and (6.4) \( n(\text{H}^+) \)
is a function of \( n(\text{H})/n(\text{O}) \), it follows that too large a variation
in \( n(\text{O}) \) will result in too large a variation in \( n(\text{H}^+) \).

6.3 EFFECT OF ATMOSPHERIC WINDS

Global and time variations of the neutral particle pressure gradients
in the thermosphere result in significant horizontal winds. Kohl and King
(1967) have calculated these atmospheric winds as a function of latitude and
local time using the neutral particle model of Jacchia (1965). Such a general
theory can, of course, be only an approximation to the real conditions, but
it can still give an indication of the general behavior. The very few experi-
mental results available to date (Carru et al., 1968) provide no conclusive
comparison with the theory. The motion of the neutral particles in the upper
F region will cause the ions to move along the field line with a speed of the order of $v_n \cos I$ (Kohl and King, 1967). These authors present wind values for only one altitude, 300 km; however they also describe the manner in which the wind values vary with altitude, so that their results can be extended to 1000 km. An approximate expression for the upward flux of charged particles due to meridional neutral winds is:

$$\overline{F_w} \approx w_{ns} n \cos I$$  \hspace{1cm} (6.5)

where

- $F_w =$ charged particle flux along the field line due to winds
- $w_{ns} =$ meridional neutral wind velocity.

The results of calculations based on Eq. (6.5) for the 40° tube are shown in Fig. 29.

6.4 COMPARISON OF THE MECHANISMS CAUSING THE UPWARD FLUX

The two processes, outlined in the previous sections, were shown to be capable of causing an upward motion of charged particles from the F region into the protonosphere. The hydrogen-oxygen charge exchange reaction appears to be the more important process but certainly both mechanisms contribute to the total upward flux.

The results presented in Chapter V showed the existence of a latitudinal as well as a temporal variation in the fluxes. Experimental observations by Reddy et al. (1967) found large changes in the low latitude electron densities from day to night at 640 km. If these changes are interpreted as changing
Fig. 29. Comparison between the particle flux deduced from the protonospheric study with the particle flux deduced from the neutral wind theory.
atomic oxygen ion densities at these altitudes, then according to Eq. (6.2) these changes have the effect of reducing the relative variation in the hydrogen ion density, because the temporal variation of the neutral atomic oxygen is not significantly latitude dependent. Near the equator the wind is directed from east to west and consequently there is no component of the wind along the field lines.

The calculations also indicated the lack of an upward flux at high latitudes (50°); this behavior is consistent with both of the proposed mechanisms. At high latitudes a decrease in the electron densities at 500-700 km is observed (Reddy et al., 1967). If this implies that the $O^+$ ion density also decreases at these altitudes, the decreasing neutral atomic oxygen density does not cause an increase in the $H^+$ ion density as can be seen from Eq. (6.2). Therefore, the upward flux is not present at these high latitudes. The upward flux due to atmospheric winds is proportional to $\cos I$; therefore, this mechanism also disappears at high latitudes. One has to be careful in extending the discussion of the results obtained in this thesis because at latitudes above about 50° the predominant ion at 1000 km becomes atomic oxygen (e.g., Mayr et al.) and therefore some of the basic assumptions are no longer valid.

6.5 ENERGY BALANCE

The results presented in the previous chapter indicate that a major portion of the heat removed from the protonosphere is via electron thermal conduction. The energy of the ion gas can be removed by one of the following three processes:
a. ion conduction

b. ion-neutral collisions

c. ion-electron collisions

The conductivity of the ion gas is about 10 times less than that of the electron gas; therefore, this cannot be a significant mechanism. An order of magnitude estimate of the significance of the ion-neutral collisions can be obtained by integrating the loss over a given field tube. Assuming a value of 2000°K for $T_i-T_n$ for the 40° tube, the integrated energy loss by the ions to the neutrals is about $1.4 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$, as compared to $\frac{3}{3} \times 6 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$, for the heat conducted out of this tube by the electrons. These numbers indicate that the energy transfer through ion-electron collisions must be significant; order of magnitude calculations for this process indicate that a value of only 30°K for $T_i-T_e$ results in an integrated energy transfer rate comparable to the electron conduction loss. The schematic diagram shown in Fig. 30 is a representation of the energy balance in the protonosphere. This qualitative conclusion of higher ion than electron temperatures in the nighttime protonosphere should be taken with caution since it is very strongly dependent on the assumed neutral hydrogen density.

6.6 COMPARISON WITH EXPERIMENTAL RESULTS AND OTHER STUDIES

Nagy and Walker (1967), using a cylindrical Langmuir probe carried on the NASA 11.03 rocket, launched from Wallops Island, Virginia ($\approx$50° geomagnetic latitude) at 00.33 Eastern Standard time on June 30, 1965, measured
Fig. 30. Block diagram indicating the energy balance between the electrons, the ions and the neutrals in the protonosphere.
electron temperature and density profiles between 200 and 1600 km. They found that the temperature and density at 1000 km were 1300°K and $8 \times 10^3$ cm$^{-3}$, respectively; the heat conducted through the 1000 km level was evaluated from the temperature gradient and found approximately equal to $1.5 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$; the corresponding values in the present study for the 50° tube are 1670°K, $1.2 \times 10^4$ cm$^{-3}$ and $8 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$, respectively. The lower experimental values obtained by the rocket experiment are likely to be due to seasonal variations. The presence of a seasonal variation was noticed by Evans (1967a, 1967b), using results from the Millstone Thomson scatter facility. This seasonal variation is partly due to the dissymmetry of the Millstone field tube (similar to the Wallops Island field tube) with respect to the geographic equator and is characterized by low temperatures and low heat fluxes during the summer. Evans' measurements of the nighttime downward heat flux at 500 km averaged over 6 hr exhibit a decrease of one order of magnitude between the winter and the summer; his equinoctial measurements for sunspot minimum yield a 6 hr average of $2 \times 10^9$ eV cm$^{-2}$ sec$^{-1}$. Over a similar period of time the results of this study predict fluxes for the 50° tube in the range $3.2 \times 10^9$ eV cm$^{-2}$ sec$^{-1}$ to $8 \times 10^8$ eV cm$^2$ sec$^{-1}$.

Petit (private communication) using the bistatic Thomson scatter facility at Nançay (40° geomagnetic latitude) measured a downward heat flux of $5.5 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$ at 325 km on November 23, 1966 about 3 hr after sunset; the corresponding theoretical heat flux calculated at 1000 km is $3 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$, which is fairly close to the measured value considering the differences in local time season, solar cycle, and altitude.
Serbu and Maier (1966) have measured temperatures and densities as functions of geocentric distance with a retarding potential analyzer on IMP2. Their measurements covered the range 2R_E to 15R_E, and therefore a comparison can be made with the calculated densities and temperatures in the equatorial plane for the 40° and 50° tube. The measured densities for October 8, 1964 are $2 \times 10^3$ cm$^{-3}$ for 2R_E (≈40° tube) and $10^3$ cm$^{-3}$ for 2.7R_E (≈50° tube); the calculated values for the 40° tube range between $6 \times 10^3$ and $10^4$ cm$^{-3}$ in the equatorial plane; the calculated values for the 50° tube range between $3.5 \times 10^3$ and $7 \times 10^3$ cm$^{-3}$ in the equatorial plane. The measured temperatures for October 11, 1964 are 2600° for the 40° tube and 3000° for the 50° tube. The corresponding calculated values range between 2900° and 1500° for the 40° tube and 2900° and 2300° for the 50° tube. It should be emphasized that the absolute values of the measured densities may be in error by a factor of 2. The comparison is also made difficult by the lack of information about the local time and the exact geographic location. Nevertheless, there is a general agreement.

Sagalyn and Smiddy (1964) measured ion densities over the altitude region 240 to 1860 km during the flight of a Blue Scout rocket launched on April 19, 1961 at 0107 Eastern Standard time from Cape Kennedy (≈40° geomagnetic latitude). The measured ion densities at 1000, 1500, and 1800 km during the ascent were $2.5 \times 10^4$ cm$^{-3}$, $2 \times 10^4$ cm$^{-3}$ and $1.8 \times 10^4$ cm$^{-3}$, respectively. These values have been plotted in Fig. 15, and they compare very well with the values calculated 8 hr after sunset. It should be realized that the densities measured by Sagalyn and Smiddy correspond to a different period of the solar cycle.
Taylor et al. (1965) have obtained positive ion densities from an ion spectrometer carried aboard the OGO A satellite during the fall of 1964. Their measurements can be compared with the calculated equatorial densities. The following ion densities are taken from their results and correspond to the 50° tube:

\[
\begin{align*}
n(H^+) &= 4 \times 10^3 \text{ cm}^{-3} \quad \text{(Nighttime)} \quad \text{October 31, 1964} \\
n(H^+) &= 3.5 \times 10^3 \text{ cm}^{-3} \quad \text{October 15, 1964} \\
n(H^+) &= 1.8 \times 10^3 \text{ cm}^{-3} \quad \text{October 23, 1964} \\
n(H^+) &= 3.5 \times 10^3 \text{ cm}^{-3} \quad \text{(Daytime)} \quad \text{September 23, 1964}
\end{align*}
\]

The corresponding calculated values range from \(3.5 \times 10^3\) to \(7 \times 10^3\) cm\(^{-3}\). A comparison can also be made for the 40° tube:

\[
\begin{align*}
n(H^+) &= 2.8 \times 10^3 \text{ cm}^{-3} \quad \text{(Nighttime)} \quad \text{October 31, 1964} \\
n(H^+) &= 3.7 \times 10^3 \text{ cm}^{-3} \quad \text{October 15, 1964} \\
n(H^+) &= 1.8 \times 10^3 \text{ cm}^{-3} \quad \text{October 23, 1964}
\end{align*}
\]

The calculated values for the 40° tube range between \(6 \times 10^3\) cm\(^{-3}\), and \(10^4\) cm\(^{-3}\). The measured densities are in general lower than the calculated ones.

Carlson (1967) (private communication), using the Thomson scatter facility at Arecibo (30° geomagnetic latitude), measured electron densities and temperatures up to 1500 km. His measurements for March 30-31, 1967 from 23:09 to 00:30 Atlantic Standard time yielded an isothermal temperature of 1000°K and densities of \(3.5 \times 10^4\) and \(1.5 \times 10^4\) at 1000 km and 1500 km, respectively. The calculated temperatures are in complete agreement. The calculated densities, however (for the same local time), tend to be higher and with a smaller gradient. The difference may be due to the fact that the measurements were
not made during solar cycle minimum. It is significant that Carlson measured
a constant temperature at night while the theory predicts a rapid relaxation
of the temperature to a constant value at night for the low geomagnetic
field tubes.

Gordon (1967) using the Thomson scatter facility at Jicamarca (equator)
measured the electron densities up to 10,000 km. On February 1, 1965 at
15:24 Eastern Standard time his measurements at 10,000 km (≈50° tube) indi-
cated a density of $7 \times 10^3$ cm$^{-3}$ which is close to the value calculated at
sunset. In a study similar to this thesis Gliddon (1966) used a single heat
equation with a constant particle density and predicted conduction fluxes of
$10^8$ eV cm$^{-2}$ sec$^{-1}$ for a period of 5 hr at a geomagnetic latitude of 38° and
fluxes in excess of $10^9$ eV cm$^{-2}$ sec$^{-1}$ for nearly 10 hr at 50° geomagnetic
latitude. In view of the simplifications made by Gliddon these numbers can
be considered to be in rough agreement with the results presented here.

It should be emphasized that the comparison with experimental results
is made difficult by the small number of data between an altitude of 1000 km
and 2 earth radii; even when it is possible to find some measurements in the
range indicated it is often difficult to take into account the differences
in time, season and geometry between the experiments and the theoretical
study.
VII. CONCLUSION

In this thesis it has been shown that there exists a strong coupling between the charged particle temperature and density distribution in the protonosphere. The density distribution at all times is close to the diffusive equilibrium one. The movements of ionization which accompany the changing density distributions due to changing temperatures in turn have a significant effect upon the temperature, particularly through the conversion of gravitational energy into thermal energy.

The protonosphere represents an important source of heat for the upper F region during the night. Heat is conducted down at a rate exceeding $10^8$ eV cm$^{-2}$ sec$^{-1}$, $2 \times 10^8$ eV cm$^{-2}$ sec$^{-1}$ and $10^9$ eV cm$^{-2}$ sec$^{-1}$ for the 30°, 40°, and 50° tubes respectively, during more than 4 hr after sunset. The ionization and energy content of a protonospheric tube is a rapidly increasing function of the geomagnetic latitude and makes the geomagnetic latitude an important parameter in the study of ionosphere-protonosphere interactions.

The calculations showed the presence of large downward fluxes of ionization, with maxima around $8 \times 10^8$ cm$^{-2}$ sec$^{-1}$ and $2.5 \times 10^9$ cm$^{-2}$ sec$^{-1}$ for the 30° and 40° tubes respectively, at the beginning of the night, followed by smaller upward fluxes extending over the largest part of the night; this latter phenomenon is likely to be the result of the charge exchange process taking place in the upper part of the F region between H and O$^+$, which gives rise to an upward flux of protons. The horizontal neutral wind directed toward the subsolar point, resulting from the cooling of the earth atmosphere,
can also contribute to the upward flux by its component along the field lines. The breathing mentioned above can be viewed as the movement of a hot plasma depositing part of its energy in the upper F region, which then moves back up into the protonosphere as a cool plasma; in other words this is a heat exchange mechanism. The change in the total particle content of these two tubes during the night is small, which means that the nighttime F region is not maintained by particles diffusing down from the protonosphere. The calculations also indicate the presence of a significant downward particle flux, but no upward flux, for the 50° tube. This behavior was shown to be consistent with the two mechanisms which were proposed to explain the upward fluxes at 30° and 40°.

Due to the charge exchange reaction $\text{H}^+ + \text{H} \rightarrow \text{H} + \text{H}^+$, the cross section for the ion energy loss to neutral hydrogen is relatively large; therefore a precise knowledge of the hydrogen density is necessary to evaluate the energy balance of the nighttime protonosphere. The values suggested by the various authors for the neutral hydrogen density vary over one order of magnitude. The calculations carried out in this thesis indicated that the lower range of these proposed density values gave acceptable results, whereas the upper range led to negative temperature gradients at 1000 km which are not believed to be physical. The uncertainty in the neutral hydrogen density also leads to some uncertainty in the collision frequency. A number of test calculations were carried out to show that the results have only a very weak dependence on the value of the collision frequency used. It was also established that large electron temperatures and/or temperature gradients
cannot be maintained in the mid-latitude field tubes, because the excess energy is evacuated relatively fast through conduction.

It had been shown (Geisler and Bowhill, 1965) that in order to explain the absence of thermal equilibrium between charged and neutral particles in the nighttime upper F region it was necessary to have not only an important reservoir of energy but also a regulated distribution of this energy, and this thesis shows that the protonosphere and its behavior satisfy these two conditions.
REFERENCES


APPENDIX A

The computer program given here, is the one used for the calculation of the diffusive equilibrium densities.

The MAD Language was used in writing the computer program.
REFERENCES ON

DIMENSION (G,SIN,TE,F1,F2,F3)(100)
INTEGER TMAX,1
VECTOR VALUES IN=*8E10*#
READ FORMAT $1110,5E10*#,TMAX,K,M,TD,NO,DELS
READ FORMAT IN=G(0)....G(TMAX)
READ FORMAT IN, SIN(O)....SIN(TMAX)
READ FORMAT IN, TE(O)....TE(TMAX)
F1(0)=G(0)*SIN(O)*M/(2*K)/TE(O)
F2(0)=0
THROUGH ALPHA, FOR I=1, 1, E.TMAX+1
F1(I)= G(I)*SIN(I)*M/(2*K)/TE(I)
F2(I) =F2(I-1)+DELS/2*(F1(I-1)+F1(I))
F3(I)= NO*TD/TE(I)*EXP.(-F2(I))
ALPHA
PRINT RESULTS F3(I)
TRANSFER TO READ
END OF PROGRAM
APPENDIX B

The computer program used to solve numerically the protonospheric heat and continuity equations is given here together with the flow diagram which describes the essential steps of the computation.
MAC (17 MAY 1967 VERSION) PROGRAM LISTING ... ... ...

REFERENCES ON
DIMENSION(TYUN(2CC))
DIMENSION(GIN,A1,A2,A3,C1,C2,C3(2CC))
DIMENSION KMI(2CC)
ERASABLE CUM(2II)(1X,YI)(2CC)
DIMENSION (QGL,QD1,G,S1,NL,M,DKMMI)(2CC)
DIMENSION(X,Y)(2CC)
DIMENSION(AR1(2CC))
INTEGER JJ,ALT,TIME
VECTOR VALUES IN=\$E10*K
VECTOR VALUES IN1=\$REIC*K*
VECTOR VALUES IN3=\$REIC*5*
SETFEND
READ
READ FORMAT \$?IC, 4E10*\$ALT, TIME, DT, SC, K, B
READ FORMAT \$E10*\$LC,CH,TA
READ FORMAT IN, AR(C),AR(I)
READ FORMAT IN, GC(O), GE(A)
READ FORMAT IN, SIN(C), SIN(ALT)
READ FORMAT IN, NL(C), NL(ALT)
READ FORMAT IN, T, T(ALT)
READ FORMAT IN1, XX(I1), XX(ALT)
READ FORMAT IN, NO(I1), NO(ALT)
READ FORMAT IN1, YY(I1), YY(ALT)
READ FORMAT IN, M(C), M(ALT)
NU=\$NS*\$H/\$NUO
INTEGER JJ7
DIMENSION ARCI(0)
THROUGH SETU$v$, FOR JJ7=1,1,JJ7,GF,69
SETUP
ARC(1J7)=100*JJ7
PLTXM,(3C,)
PAXIS, (2,2,2,2) ARCD, 3,11, C, C, 500, 111)
PAXIS, (1,2,2,2) ARCD, 3,11, C, C, 500, 111)
PAXIS, (2,2,2,2) TEMPS, 4,2, C, SC, 80C, 100, 222,
PAXIS, (1,2,2,2,2) DENSIT, 6,25, SC, 50C, 1000, 222,
THROUGH GAMA, FOR J = 1,1,1,E.TIME+1
A1(1) = 0
A3(1) = 0
A2(1) = XX(J)
C1(1) = C
C3(1) = C
C2(1) = YY(J)
PRINT FORMAT \$10H,10,3HJ=14,,\$10,5 H ALTITUDE,T24,1HX,T4
1 2,1HY,4*J
THROUGH ETA, FOR I = 1,1,1,E.(ALT)
DGI(I)= 1./SO*(G(I)-SIN(I+1)/NU(I)+GL(I)+SIN(I)/NU(I))
QDL(I)= 1./SO*(AR(I+1)-AR(I+1))
GNI(I) = G(I)*G(I)/NU(I)
QM(I) = K/SO*(M(I)+M(I)+1./M(I)/NU(I))
KMI(I) = K/M(I)/NU(I)
F1 = DLOG(I)*GNI(I)+DGI(I)
F2 = GNI(I)
F3 = DLOG(I)*KMI(I)+QM(I)
F4 = KMI(I)
...
**ALPHA**

\[
\begin{align*}
&X(0) = T(0) \\
&Y(0) = N(0) \\
\end{align*}
\]

**CAN**

\[
\begin{align*}
&F = \frac{W*10^5}{D} \\
&C_{CD} = 2.74 + 10^{0.01T(T-1)} + 0.17 \\
&GAMA \\
&\text{PRINT RESULTS FLO,CONV,CCN} \\
&\text{PUNCH FORMAT IN,T(0),...T(ALT)} \\
&\text{PUNCH FORMAT IN,N,ALT) \\
\end{align*}
\]

**END**

\[
\begin{align*}
&\text{PITEND,} \\
&\text{END OF PROGRAM}
\end{align*}
\]
APPENDIX C

It was shown in Chapter VI that the hydrogen-ion density, in the case of chemical equilibrium is given by:

$$ n(H^+) = \frac{\gamma}{8} \frac{n(H)}{n(O)} \frac{n(O^+)}{n(O)} \quad (6.2) $$

Assuming an isothermal ionosphere and thermal equilibrium between the species the diffusive equilibrium distribution of the various species is:

$$ n(H) = n_o(H) \exp\left[ -\frac{(z - \bar{z}_o)}{8 \text{H}_o^+} \right] \quad (C.1) $$

$$ n(O) = n_o(O) \exp\left[ -\frac{2(z - \bar{z}_o)}{\text{H}_o^+} \right] \quad (C.2) $$

$$ n(O^+) = n_o(O^+) \exp\left[ -\frac{(z - \bar{z}_o)}{\text{H}_o^+} \right] \quad (C.3) $$

where

$$ n_o(M) = \text{density of the M constituent at } z = z_o $$

$$ \text{H}_o^+ = \frac{2kT}{m_o^+ g} = \text{atomic oxygen-ion scale height} $$

$$ m_o^+ = \text{atomic-oxygen ion mass} $$

Combining Eqs. (6.2), (C.1), (C.2), and (C.3) leads to

$$ n(H^+) = n_o(H^+) \exp\left[ -\frac{Z - \bar{z}_o}{8 \text{H}_o^+} \right] \quad (C.4) $$

The diffusive equilibrium distribution of hydrogen ions in an isothermal ionosphere where atomic-oxygen ions are predominant, is:
\[
\eta \left( H^+ \right) = \eta_0 \left( H^+ \right) \exp \left[ - \frac{g \left( 2m_{H^+} - m_{O^+} \right) (Z - Z_0)}{kT} \right]
\]

\[
\eta \left( H^+ \right) = \eta_0 \left( H^+ \right) \exp \left[ - \frac{\gamma}{\delta} \frac{(Z - Z_0)}{H_{O^+}} \right] \quad \text{(C.5)}
\]

Comparison of Eqs. (C.4) and (C.5) indicates that under these simplifying and special conditions the distribution of atomic-hydrogen ions is the same for both chemical and diffusive equilibrium.