

THE UNIVERSITY OF MICHIGAN  
DEPARTMENT OF NAVAL ARCHITECTURE AND MARINE ENGINEERING

FUNDAMENTALS OF SHIP DESIGN ECONOMICS

Lecture Notes

by

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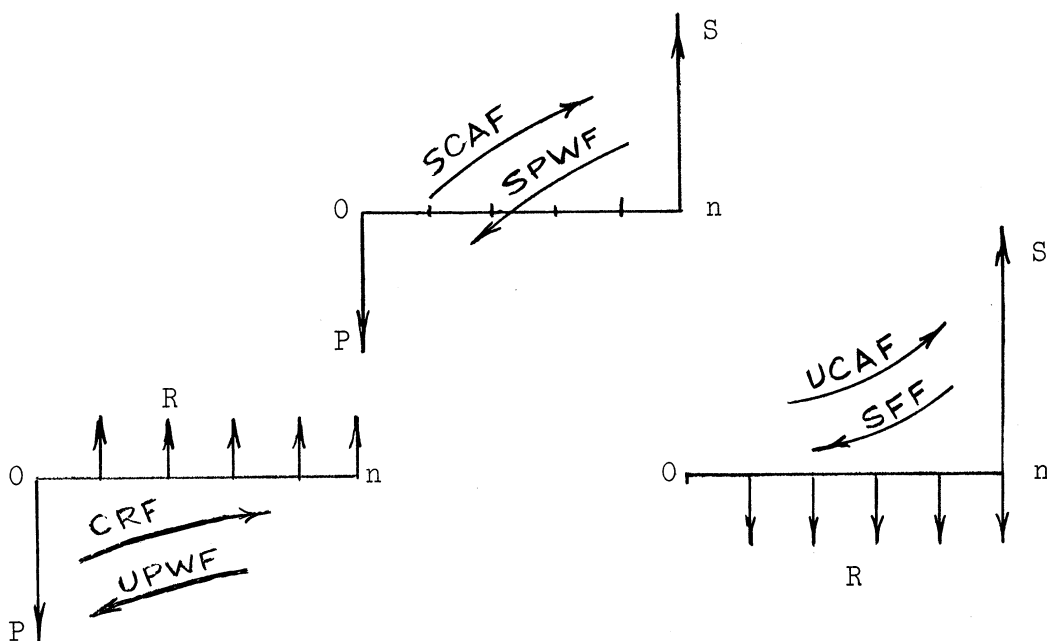


Ann Arbor, Michigan  
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## NOMENCLATURE

AAC	Average Annual Cost (including capital)
BCR	Benefit-Cost Ratio
CC	Capitalized Cost (present worth of perpetual service)
CRF	capital recovery factor
CRF'	capital recovery factor after tax
EiRR	Equated Interest Rate of Return
i	nominal interest rate, compounded annually
i'	interest rate after tax
L	resale or scrap value
n	number of years
P	principal, investment, or present worth of future amounts
PW	present worth of both invested and future costs
R	annual return (revenue less operating costs), annual repayments on a loan (returning capital plus interest)
R'	annual return after tax
RFR	Required Freight Rate
S	a future sum of money
SCAF	single payment compound amount factor
SFF	sinking fund factor
SPWF	single payment present worth factor
t	tax rate in proportion to annual profits before tax
UCAF	uniform series compound amount factor
UPWF	uniform series present worth factor
Y	annual operating costs (wages, repairs, insurance, overhead, supplies, fuel, and so on)



## PREFACE

My object in preparing these notes is to introduce students of naval architecture and marine engineering to engineering economy as a tool in ship design. The emphasis is on the principles of engineering economy. Methods of making practical application of these principles to commercial ship design are dealt with as well, but no effort is made to provide factual cost data. You will find such information available in industry, and years of experience may bring you the facility of making reliable projections into the future.

There is no intent here to teach business administration. Overlapping areas exist, however, and I shall try to indicate which decisions are clearly management's prerogative, which may belong either to management or engineering, and which belong to engineering alone.

Problems in communication frequently arise in matters of the sort dealt with here. Businessmen, engineers, and economists all may use different words for the same thought--or the same word for different thoughts. I make no claim that my particular definitions are better than anyone else's. But I do aim to be clear and consistent, and hope you will concentrate on the thoughts, not the semantics, herein.

## CONTENTS

	Page
NOMENCLATURE.....	ii
PREFACE.....	iii
INTRODUCTION.....	1
THE TIME-VALUE OF MONEY.....	3
PRESENT WORTH.....	4
INTEREST RELATIONSHIPS.....	5
PRESENT WORTH OF COMPLEX CASH FLOWS.....	14
AVERAGE ANNUAL COST.....	18
FINDING AN UNSPECIFIED INTEREST RATE.....	22
CORPORATE PROFITS TAX.....	27
CHOOSING AN INTEREST RATE.....	32
ECONOMIC CRITERIA.....	33
UN SOUND CRITERIA.....	42
CAPTIVE FLEETS.....	50
APPENDIX: INTEREST TABLES.....	51

## INTRODUCTION

Economics is defined in many ways. For our purposes, it is the task of allocating a finite supply of investment funds in the face of infinite possibilities. Most large companies and corporations decide between alternative investment opportunities principally on a basis of predicted profitability.

Engineering also is defined in many ways. Most definitions refer to using scientific knowledge for the benefit of society. And in a free economy, society expresses its collective needs through its individual purchases--which places businessmen between engineer and consumer. In short, the engineer must alternately deal, and communicate, with both scientists and business managers.

Engineering economy, then, is an approach to design aimed at meeting society's needs with a maximum effectiveness in the use of resources: man-power, materials, and investment funds.

In general, if he wants to be convincing to the business manager, the engineer must demonstrate that his proposed design promises to be more profitable than any alternative design. Furthermore, the engineer must realize that the wise business manager will consider several competing investment opportunities. An oil company, for example, may find a pipeline more profitable than a tanker. Or it may find the most profitable alternative is to rely on independent fleets, investing the capital so saved in a new refinery. These concepts are in keeping with both good economics and good engineering.

Economy is also something of a universal solvent, allowing the engineer to weigh the relative merits of design alternatives involving different units. For example: a choice between two engines, one heavier but more compact than the other, involves both weight and volumetric units. Converting both to resulting costs, present and future, allows rational quantitative judgment. There are, of course, certain considerations such as pride of ownership that defy reduction to cash value. These intangible factors are normally management's responsibility, although a complete engineering analysis should at least include mention of them.

Many engineers equate sophisticated technology with good design. They slight the economic aspects of their work and the result is over-design. The best wheelbarrows, commercially, are hardly goldplated. The same is true of ships.

In summary of the above, the well-engineered project has a basis of careful economic thought. Such thought is applied not only to the preliminary, major decisions but to every detailed decision along the way. And, finally, when ready to present to management the proposal is couched in terms of profitability. A good naval architect, then, must know how to make economic studies and must unendingly develop his ability to estimate future costs of building and operating ships.

## THE TIME-VALUE OF MONEY

Misers to the contrary, money does you no good until you spend it on the necessities or pleasures of life. Most of us would rather be given a dollar today than the promise of one in the future, simply because we are humanly impatient to have our necessities and pleasures. This time-value of money means that you should be rewarded if you are willing to lend or invest your spare cash since you are thereby postponing the pleasures it can bring you. The reward, or rent money, we call interest, which is generally expressed as an annual charge in percent of the funds invested.

Interest can be thought of in three ways:

1. Contracted interest is the type you are most familiar with. Savings deposits, bank loans, mortgages, and bonds all carry mutually agreed-upon interest rates.

2. Implied interest is appropriately considered when funds are tied up without any resulting visible reward. For example, if you hide a sum of money in your mattress, your decision is in effect costing you the interest you would gain from a savings bank. This is lost-opportunity interest.

Returned interest is a measure of the gain (if any) from risk capital invested by an entrepreneur. This is sometimes called internally generated interest or interest rate of return. It is the one true measure of profitability, expressing the returns from a venture as equivalent to returns from a bank at some rate of interest. Since most nations impose a tax on corporate profits, we must differentiate between interest rates of return before and after tax (abbreviated  $i$  and  $i'$ ).



## PRESENT WORTH

If you are satisfied with an interest rate of 5 percent, then \$1.00 today is exactly equivalent in desirability to \$1.05 a year hence. Conversely, \$1.00 a year from now is worth only \$0.95 now. That is, the present worth of \$1.00 one year hence is only \$0.95, assuming an interest rate of 5 percent. Thus, interest relationships can be used to transpose dollars to equivalent values in other periods of time. Future cash flows are often analyzed by discounting all future annual amounts to their individual present worths and finding the sum of those values. This important concept will be illustrated shortly.

Present worth is also called present value.

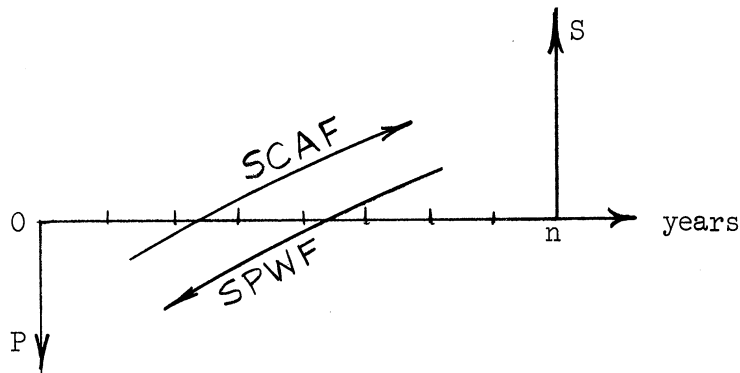
## INTEREST RELATIONSHIPS

We shall presume that you are aware of the logic behind compound interest. In engineering economy studies we almost always assume annual compounding and the interest rates we talk about are annual rates. As far as decision making in ship design is concerned, the assumption of annual compounding yields accurate conclusions. This is despite the fact that money may change hands almost every day.

There are basically six compound interest relationships that we must understand. The first two apply to single-payment arrangements. First, some abbreviations:

- P: present sum, principal, invested cost, or present worth of future money
- S: future sum of money
- R: uniform annual amounts of money
- i: interest rate compounded annually
- n: number of years in the future.

Plotting time on a horizontal scale and money on a vertical scale:



$$\left. \begin{array}{l} S = [\text{SCAF}]P \\ P = [\text{SPWF}]S \end{array} \right\} \text{SCAF} = \frac{1}{\text{SPWF}}$$

where

SCAF is single compound amount factor  
 SPWF is single present worth factor.

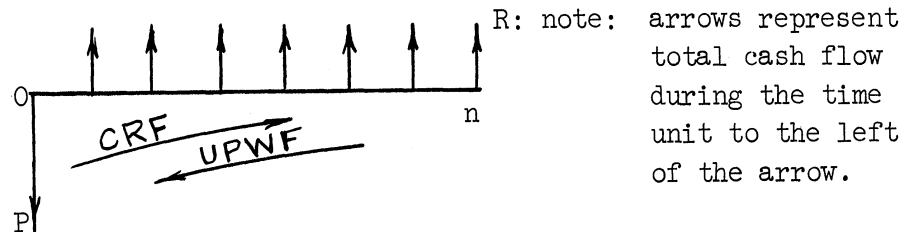
The algebraic values (proof of which you can find in standard texts) are as follows:

$$\text{SCAF} = (1 + i)^n$$

$$\text{SPWF} = \frac{1}{(1 + i)^n}$$

Numerical values of SPWF for various combinations of  $i$  and  $n$  are given in the Appendix. Values of SCAF can be found by taking the reciprocal of SPWF.

Where uniform annual amounts ( $R$ ) are balanced against a present value or investment, we have these two relationships:



$$\left. \begin{aligned} R &= [\text{CRF}]P \\ P &= [\text{UPWF}]R \end{aligned} \right\} \text{UPWF} = \frac{1}{\text{CRF}}$$

where

CRF is capital recovery factor  
UPWF is uniform present worth factor.

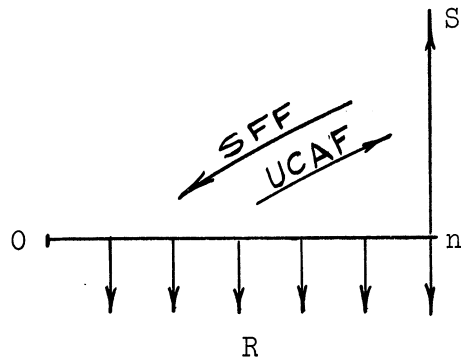
The algebraic value of CRF is:

$$\frac{i(1 + i)^n}{(1+i)^n - 1}$$

Numerical values of CRF are found in the Appendix. Reciprocal values give UPWF.

A common application of CRF is in installment buying. If you wish to find the periodic amounts ( $R$ ) that will repay a debt ( $P$ ) in  $n$  periods, at an interest rate ( $i$ ), simply multiply  $P$  times CRF. Conversely, if you have an opportunity to buy a facility that promises to return  $R$  dollars per year for  $n$  years, you can find how much you could afford to pay for it ( $P$ ) by multiplying  $R$  times UPWF, the latter based on the years of life of the facility and the interest rate you desire.

Where uniform annual amounts (R) are balanced against a single future amount (S), we have these two relationships:



$$\left. \begin{aligned} R &= [\text{SFF}] S \\ S &= [\text{UCAF}] R \end{aligned} \right\} \text{UCAF} = \frac{1}{\text{SFF}}$$

where

SFF is sinking fund factor  
UCAF is uniform compound amount factor.

The algebraic value of SFF is

$$\frac{i}{(1+i)^n - 1}$$

Numerical values of SFF are found in the Appendix.  
Reciprocal values give UCAF.

If you want to accumulate by some future date a certain amount of cash (S) by regularly banking R dollars, find R by multiplying S by SFF. Conversely, if you know you can afford to bank R dollars per year, you can find the compounded future amount (S) owed you by the bank by multiplying R by UCAF.

Some miscellaneous notes are appropriate at this point:

a. Although we use annual compounding in most engineering studies, the algebraic relationships and numerical values are valid for other compounding periods. For example, if the interest rate is one percent per month and we compound monthly for two years, the numerical value of any interest factor is the same as for annual compounding at one percent per year for 24 years. Do not be fooled into thinking, however, that one percent monthly interest compounded monthly is equal in desirability to 12 percent annual interest compounded annually.

b. All vertical arrows in the cash flow diagrams could be reversed without changing the interest relationships. We represent money paid out by a downward pointing arrow, money coming in as an upward pointing arrow. But remember that money out to a lender is money in to a borrower.

c. Professor Wilbert Steffy's shorthand method is used here for specifying interest rates and years applied to interest formulas. For example, the capital recovery factor for 10 percent interest and 5 years would be represented by:  $[CRF]_5^{0.10}$ . In numerical solutions, as a further short cut, the numerical value may be immediately appended:  $[CRF]_5^{0.10}$  0.2638.

d. As a matter of incidental concern, the sinking fund factor plus the interest rate is numerically equal to the capital recovery factor. Also, the uniform present worth factor is numerically equal to the sum of all the single present worth factors up to and including the total number of years involved. You can discover other relationships for yourself.

e. Accountants and bankers must make their calculations to the last penny. In engineering economy, however, no such accuracy is required. Engineers try to predict future costs and there is no point in predicting profitability levels to five significant figures when the inputs are seldom accurate in the second figure.

f. An engineer uses engineering economy largely to help in decision making. Decisions are between alternatives; no alternatives, no decisions. History offers no alternatives. Therefore an engineer does not analyze past performances except to help predict the future.

g. As a matter of convenience, in most of our studies, we start the measurement of time at the moment of delivery of the ship even though that may be several years in the future. This simplification is perfectly valid unless one alternative promises a greatly different time of delivery than the others. In precise studies, payments made before delivery may be compounded to an increased amount at the time of delivery.

h. In choosing between alternatives, relative values are more important than absolute values. Costs that are much the same between alternatives can usually be ignored. It's the differences that count.

Summary

Good engineering involves economic as well as scientific analysis. Design optimization requires an understanding of the factors influencing the potential success of the ship as an investment. Profitability is best measured as equivalent interest rate of return. Interest relationships are therefore appropriately applied to ship design economic analyses, which invariably require that we relate future amount to initial investment. The six basic interest factors (SCAF, SPWF, CRF, UPWF, SFF, and UCAF) can be used in various combinations to analyze cash flow pattern, no matter how complex.

Problems

In the following problems, assume an interest rate of 10 percent unless otherwise noted. Repeat solutions using other arbitrary interest rates if further practice is desired.

1 Calculate numerical values of the following interest factors. Assume a life of 5 years. Check your answers against the interest tables.

- a. SCAF
- b. SPWF
- c. CRF
- d. UPWF
- e. SFF
- f. UCAF

Solution to (1a)

The object is to find the numerical value of the single compound amount factor, SCAF, for 10 percent interest (i) and 5 years (n). Annual compounding is implied.

$$[\text{SCAF}]_n^i = (1 + i)^n$$

$$[\text{SCAF}]_5^{.10} = (1 + 0.1)^5 = (1.1)^5 = \underline{1.61} \text{ (ans.)}$$

(Value from interest table:  $\frac{1}{0.6209} = 1.61$ )

Note: Raising 1.1 to the fifth power can be done in various ways:

a. Multiply 1.1 by itself (n-1) times.

b. Use LL scale on sliderule.

c. Use logarithms:

$$\log(1.1)^5 = 5 \log 1.1 = 5 \times \underbrace{0.0414}_{\text{from log table}} = 0.2070$$

$$(1.1)^5 = \underbrace{1.61}_{\text{from log table}}$$

Solution to (1b)

$$[SPWF]_n^i = \frac{1}{(1+i)^n}$$

$$[SPWF]_5^{.10} = \frac{1}{(1+0.1)^5} = \frac{1}{\underbrace{1.61}} = \underline{0.62} \text{ (ans.)}$$

denominator same as  
SCAF found above

Value from interest table: 0.6209.

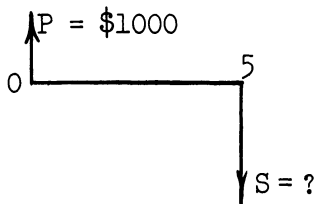
Note: The purpose of the first problem is to convince you of the credibility of your interest tables. Also, there will be occasional combinations of  $i$  and  $n$  not covered by the tables. In normal cases, however, you should use the tables rather than calculating every factor. Using the tables is both faster and safer.

2 In the problem above, check the relationship:

$$CRF = SFF + i$$

3 If you borrow \$1000 today and repay the loan in a lump sum 5 years from now, how much will you return to the lender?

Solution



$$S = \underbrace{[SCAF]_5^{.10}}_{1.61} \$1000 = \underline{\$1610} \text{ (ans.)}$$

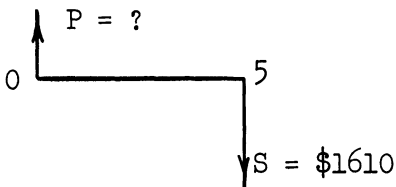
This is the shortcut way  
of saying:

$$[SCAF]_5^{.10} \text{ (which equals 1.61)}$$

4 Find the present worth of the future amount found in Problem No. 3.

5 Find the present worth of the future amount found in Problem No. 3 using an interest rate of 3 percent.

Solution

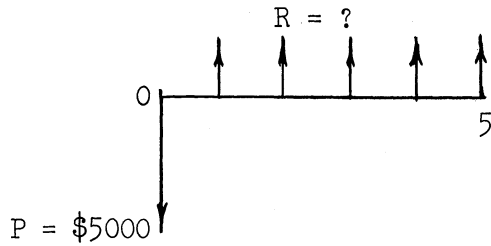


$$P = [SPWF]_5^{.03} 0.8626 \$1610 = \underline{\$1390} \text{ (ans.)}$$

Comparing this answer to the one above, we see that lowering the interest rate increases the present worth of future amounts. Conversely, increasing the rate decreases present worth.

- 6 What is the total present worth of \$100 five years hence and \$200 three years hence? (answer: \$212)
- 7 If you borrow \$5000 and repay the debt in 5 equal annual payments, how large will these be?

Solution

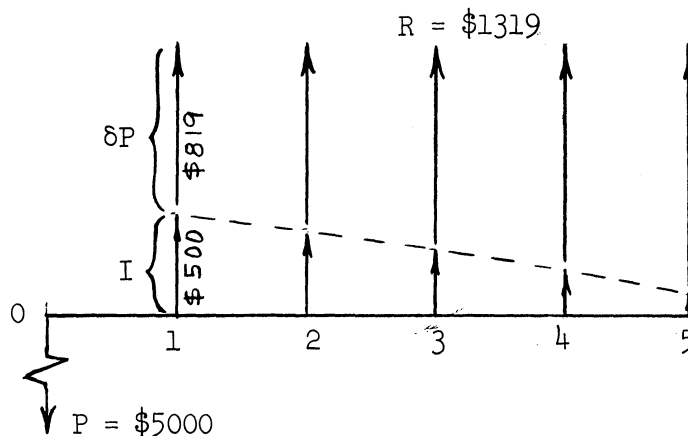


$$R = [CRF]_5^{.10} 0.2638 \$5000 = \underline{\$1319} \text{ (ans.)}$$

Let us see how a bookkeeper would keep account of the annual payments of \$1319 found above:

End of Year	Residual Debt Before Payment at End of Year	Interest Charge Due at 10%	Total Annual Payment	Reduction in Debt
1	\$5000	\$500	\$1319	\$819
2	4181	418	1319	901
3	3280	328	1319	991
4	2289	229	1319	1090
5	1199	120	1319	1199
	0			\$5000

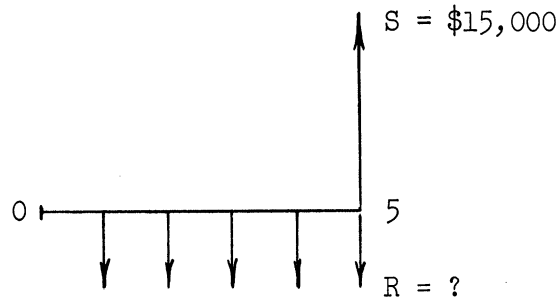
Based on the above, we can show on a cash flow diagram how the distribution between interest and debt reduction varies from year to year:





- 8 Find the present worth of the annual payments found in Problem No. 7.
- 9 Find the present worth of the annual payments found in Problem No. 7 using an interest rate of 20 percent. (answer: \$3940)
- 10 You think you will need \$15,000 in a lump sum 5 years hence. How much must you invest each year at 10 percent interest in order to meet this objective?

Solution



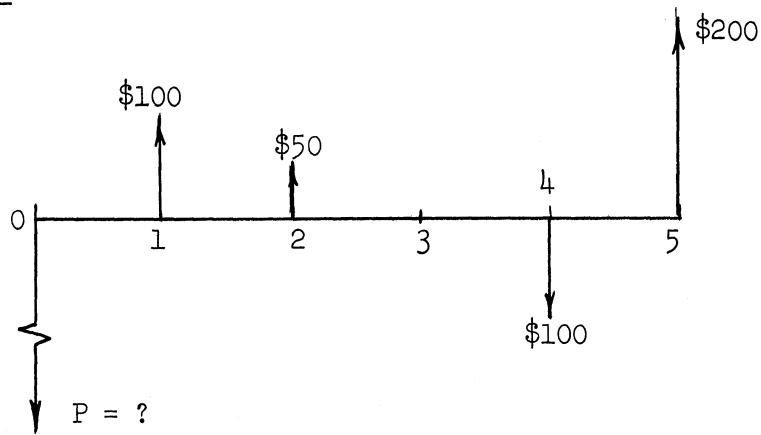
$$R = [SFF]_5^{.10} 0.1638 \$15,000 = \underline{\$2460} \text{ (ans.)}$$

- 11 Find the present worth of \$15,000 five years hence.
- 12 Find the present worth of the annual amounts found in Problem No. 10. Compare this answer with that found in Problem No. 11.
- 13 What is the present worth of \$1000 per year for each of the next 5 years? Find this in two ways: using UPWF, and tabulating values based on SPWF.
- 14 Find the present worth of the following cash flow.

During year 1	\$100
during year 2	\$ 50
during year 3	0
during year 4	\$100 loss
during year 5	\$200

(cont'd)

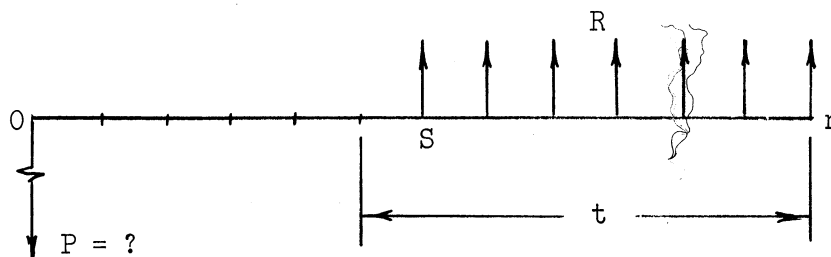
Solution



Year	Amount	$[\text{SPWF}]_n^{.10}$	Product
1	\$100	0.909	\$ 91
2	50	0.826	40
3	0	0.751	0
4	-100	0.683	-68
5	200	0.621	124
Total Present Worth			<u>\$188 (ans.)</u>

## PRESENT WORTH OF COMPLEX CASH FLOWS

The final problem in the previous section shows the universal solution to finding the present worth of complex cash flows: simply tabulate the individual annual amounts, multiply each by the appropriate SPWF, and add up the products. Frequently, however, alternative solutions are more convenient. Consider the problem of finding the present worth of  $R$  dollars per year for each of  $t$  years, starting in year  $S$  :



This can be done in several ways. One is to find the present worth at year  $(S-1)$ , then transfer that present worth back to year zero:

$$P \text{ at year } (S-1) = [\text{UPWF}]_t^i R$$

$$P = [\text{SPWF}]_{S-1}^i \{P \text{ at year } (S-1)\}$$

combining:

$$P = [\text{SPWF}]_{S-1}^i [\text{UPWF}]_t^i R$$

Another method is to find the uniform present worth for  $n$  years, then subtract the uniform present worth for  $(S-1)$  years:

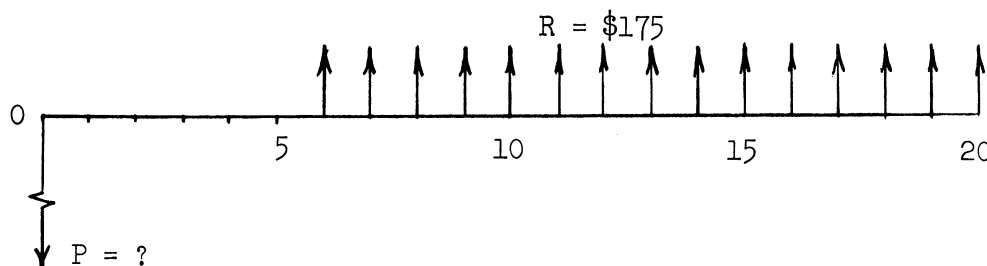
$$P = [\text{UPWF}]_n^i R - [\text{UPWF}]_{S-1}^i R$$

or

$$P = \{[\text{UPWF}]_n^i - [\text{UPWF}]_{S-1}^i\} R$$

Example 15 illustrates the foregoing approach.

- 15 Find the present worth of \$175 per year for 15 years starting in the 6th year. Assume an interest rate of 10 percent.



Using the first method:

$$P \text{ at year 5} = [\text{UPWF}]_{15}^{.10} \frac{1}{0.1315} \$175 = \$1331$$

$$P = [\text{SPWF}]_5^{.10} 0.6209 \$1331 = \underline{\$828} \text{ (ans.)}$$

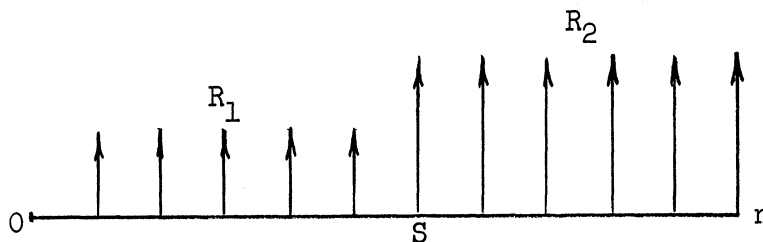
Using the second method:

$$P = \left\{ [\text{UPWF}]_{20}^{.10} \frac{1}{0.1175} - [\text{UPWF}]_5^{.10} \frac{1}{0.2638} \right\} \$175$$

$$P = (8.51 - 3.78) \$175 = 4.73 \$175 = \underline{\$828} \text{ (ans.)}$$

Devise and solve by a third method.

A slightly more complex problem involves a step in annual amounts:

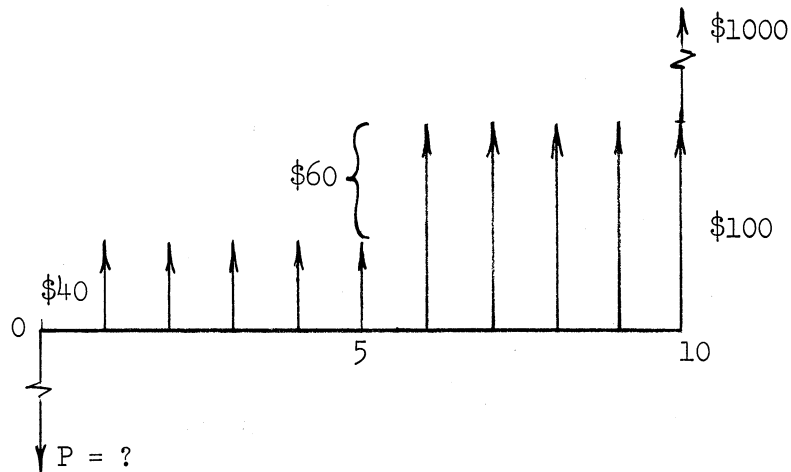


The present worth could, as before, be found in any of several ways. One solution would be to find the present worth of  $R_2$  dollars per year for  $n$  years, then subtract the present worth of  $(R_2 - R_1)$  dollars per year for  $(S-1)$  years:

$$P = [\text{UPWF}]_n^i R_2 - [\text{UPWF}]_{S-1}^i (R_2 - R_1)$$

- 16 Find the present worth of \$40 per year for the first 5 years, \$100 per year for years 6 through 10, and \$1000 at the end of the 10th year. Interest is 10 percent.

(continued)



let

- $P_1$  = present worth of \$100/yr for 10 years  
 $P_2$  = present worth of \$ 60/yr for 5 years  
 $P_3$  = present worth of \$1000 at end of 10th year.

$$P = P_1 - P_2 + P_3$$

$$P = [\text{UPWF}]_{10}^{.10} \frac{1}{0.1627} \$100 - [\text{UPWF}]_5^{.10} \frac{1}{0.2638} \$60 + [\text{SPWF}]_{10}^{.10} 0.3855 \$1000$$

$$P = \$614 - \$227 + \$385 = \underline{\$772} \text{ (ans.)}$$

Repeat this problem using 20 percent interest. Check your answer using tabular form.

17 Find the worth of a single payment during the 3rd year equivalent to the following cash flow:

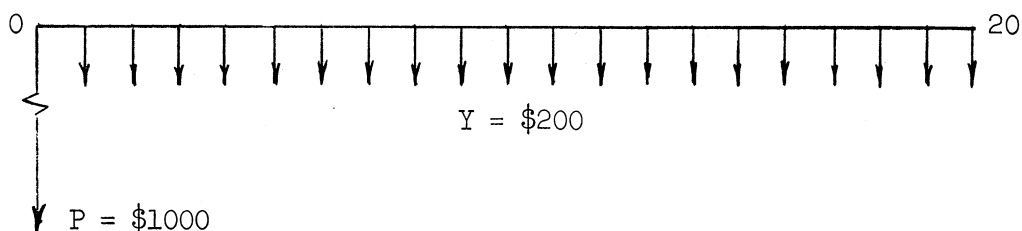
Year	Thousands of Dollars
1	2000 outflow
2	2000 outflow
3	2000 outflow
4 through 13, each	1000 return
14 through 28, each	800 return

(The answer is \$1,860,000)

- 18 Find the present worth of the following cash flow. Do this by iteration (tabular form) and by at least one other method.

<u>Year</u>	<u>Amount</u>
1 through 4	\$100
5 through 10	\$300
11	\$100 loss
12 through 15	\$200

- 19 What is the present worth of a facility that will cost \$1000 to start with and cost another \$200 per year for 20 years? There is no income. Interest rate is 6 percent.



There are two things to note here. One is that we are really talking about costs, not worths. The same principles nevertheless apply. Secondly, the initial investment is already designated "P" ; we shall therefore call present worth of both present and future amounts "PW":

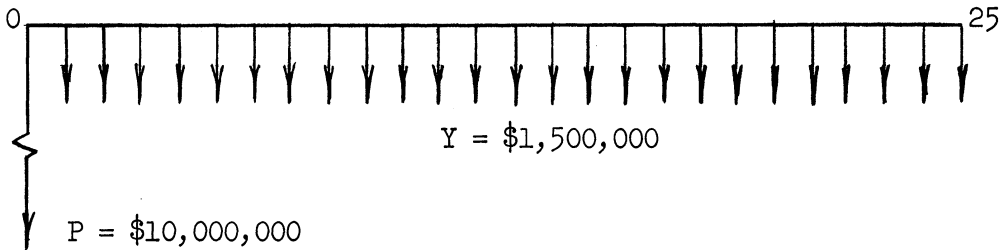
$$\begin{aligned}
 PW &= P + [UPWF]_n^i Y \\
 PW &= \$1000 + [UPWF]_{20}^{.06} \frac{1}{0.0872} \$200 \\
 PW &= \$1000 + 2295 = \underline{\$3295} \text{ (ans)}
 \end{aligned}$$

Repeat the above using an interest rate of 3 percent; also add an income from resale of \$1000 at the end of the 20th year.

AVERAGE ANNUAL COST

The previous section has used present worth to measure the cost or value of various cash flows. An alternative is to find the average annual cost, which converts all costs to an equivalent uniform annual amount, taking due recognition of the time-value of money. The following examples illustrate the principle of average annual cost.

- 20 Find the average annual cost for a ship that requires an initial investment of \$10,000,000 and annual operating costs of \$1,500,000. The ship is expected to last for 25 years. An interest rate of 18 percent is specified. ✓



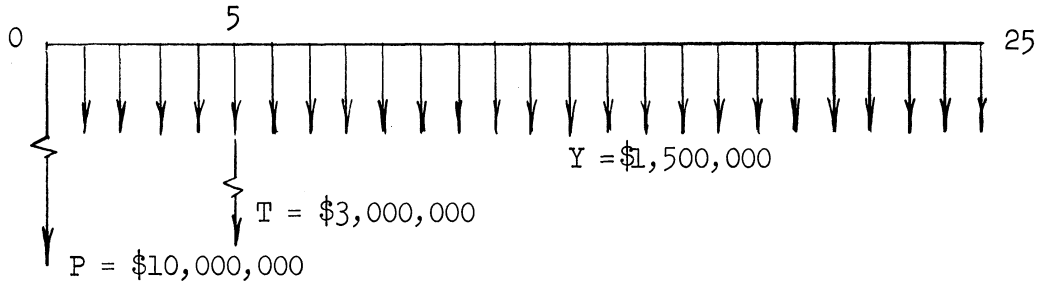
The average annual cost of operation is, of course, \$1,500,000. To this must be added the average annual cost of capital. This is found by converting the invested cost (P) to an annual cost by multiplying by the CRF corresponding to the specified years and interest rate:

AAC for operations = .....	\$1,500,000
AAC for capital = $[\text{CRF}]_{25}^{.18}$	$0.1829 \times \$10\text{M} = \$1,829,000$
Total AAC	<u>\$3,329,000</u> (ans.)

In algebraic form:

$$\text{AAC} = Y + [\text{CRF}]_n^i P$$

- 21 Recalculate Problem 20 with the added assumption of a special expense (T) of \$3,000,000 during the fifth year:

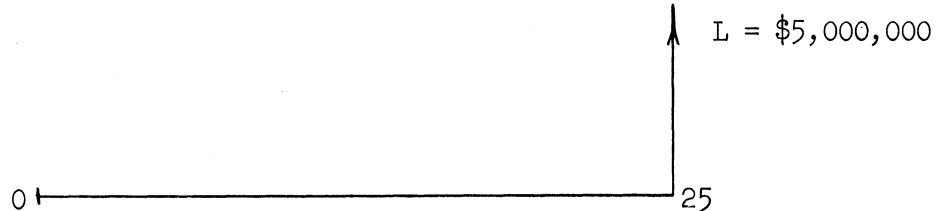


The average annual cost of a single expense such as T is found by first calculating its present worth and then multiplying by CRF. The complete average annual cost would then be:

$$AAC = Y + [CRF]_{25}^{.18} P + [SPWF]_5^{.18} [CRF]_{25}^{.18} T$$

(To be completed by the student)

- 22 Find the average annual cost (better phrased: equivalent annual return) of a predicted \$5,000,000 resale 25 years hence. Interest rate is 18 percent.



There are two ways to do this. One is to treat L just as T was treated in the previous problem:

$$AAC = -[SPWF]_{25}^{.18} [CRF]_{25}^{.18} L$$

An easier method is to multiply L by SFF:

$$AAC = -[SFF]_{25}^{.18} L$$

Note that incomes are treated as negative costs.

(You should complete this problem by both methods.)



Complex cash flow can be converted to average annual costs in two steps: 1. Find the total present worth as illustrated in Problem 14; 2. Multiply the total present worth by the capital recovery factor:

$$AAC = [CRF] \left\{ \sum_0^n S[SPWF] + P \right\}$$

23 Find the average annual cost of the following, if interest rate is 15 percent: (See sketch at bottom of page.)

Solution

First, use the tabular form to find the present worth of the future costs (and don't forget the initial cost of \$500):

Year	Amount	$[SPWF]_n^{.15}$	Product
1	\$100	0.8696	\$ 87
2	200	0.7561	151
3	-200	0.6575	-132
4	300	0.5718	172
Total PW of future amounts			278
P			500
Total PW			\$778

$$AAC = [CRF]_4^{.15} 0.3503 \$778 = \underline{\$272} \text{ (ans.)}$$

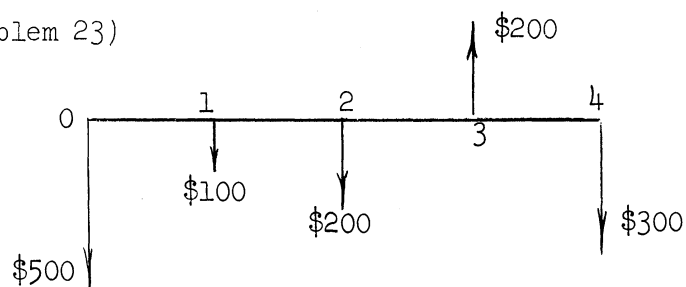
(Repeat, using an interest rate of 10 percent.)

24 Find the average annual cost of a ship that has an invested cost of \$6,000,000, a predicted resale value of \$3,000,000 after 5 years, and operating costs as listed in the table below. The interest rate is 20 percent.

Year:	1	2	3	4	5
Operating Costs:	\$1M	\$1.2M	\$3M	\$3M	\$3.2M

(M: million)

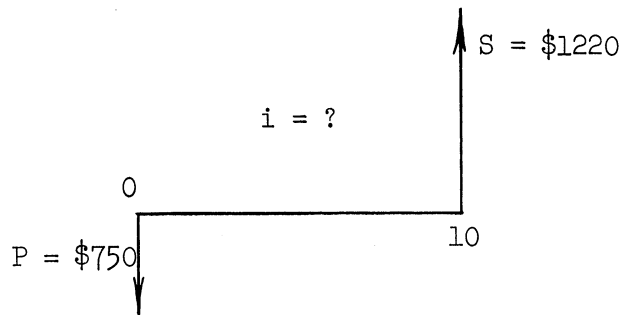
(Sketch for Problem 23)



- 25 Find the average annual cost of a passenger ship that has an invested cost of \$28,000,000, a life expectancy of 30 years and a disposal value of \$4,000,000. The operating costs are \$3,000,000 per year for the first 10 years and \$3,500,000 per year in the final 10 years. There are also rehabilitation expenses of \$5,000,000 in the 5th, 10th, 15th, 20th and 25th years. Interest rate is 22 percent.

## FINDING AN UNSPECIFIED INTEREST RATE

The principle of this section is best presented in an example such as this: Suppose a banker offers to give you a sum of \$1220 in 10 years if you will deposit \$750 in his bank today. What interest rate does he have in mind?



We know  $P$ ,  $S$ , and  $n$ . Dividing  $P$  by  $S$  gives us the single present worth factor the banker used:

$$P = [\text{SPWF}] S$$

$$\frac{P}{S} = \frac{\$750}{\$1220} = [\text{SPWF}]_5^i$$

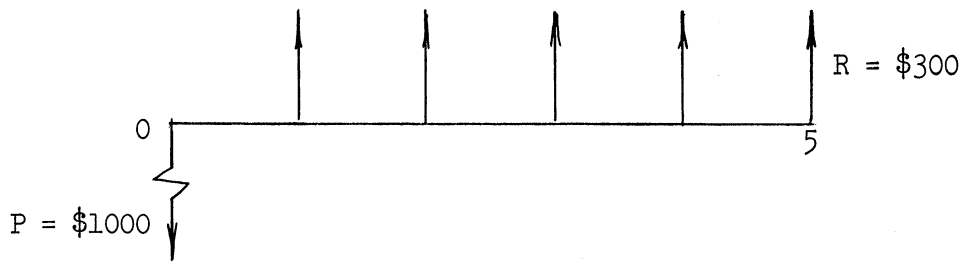
$$[\text{SPWF}]_5^i = 0.614$$

If we examine our SPWF table we will find that the calculated value of 0.614 is appropriate to 10 years and 5 percent interest. We can then infer that 5 percent interest was what the banker had in mind.

26 A banker offers to give you a sum of \$121.60 five years from now if you will give him \$100 today. What interest rate is implied?

27 A retirement plan involves your paying annual premiums of \$445 for the next 50 years. At the end of that period you will be given \$50,000. What interest rate is implied?

28 A banker says he'll give you \$300 for each of 5 years if you'll lend him \$1000 at the beginning of the first year. What interest rate is implied?

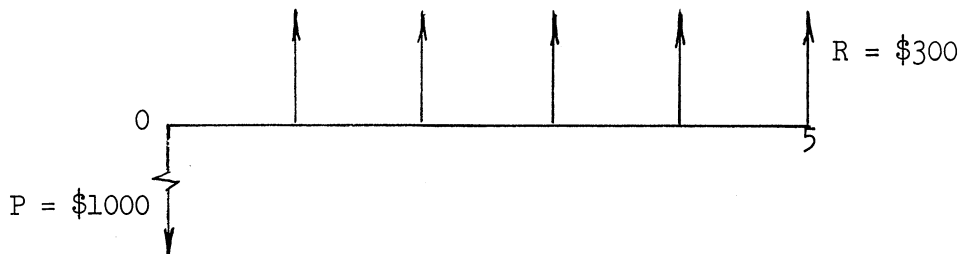


$$[\text{CRF}]_5^i = \frac{\$300}{\$1000} = 0.30$$

$$i = \underline{15.3\%} \text{ (ans.)}$$

Note: Figure 1 may be used in place of interest tables when CRF is known.

- 29 Suppose you can predict that a \$1000 investment in a business venture will return \$300 to you each year for 5 years. How profitable is this in terms of equivalent rate of interest?



As you may see, this cash distribution is exactly the same as that shown in the previous problem. The principle is the same and the method of solving is the same. The only difference is that the returns are not guaranteed.

$$[\text{CRF}]_5^i = \frac{R}{P} = \frac{\$300}{\$1000} = 0.30$$

$$i = \underline{15.3\%} \text{ (ans.)}$$

In short, the predicted returns from this investment are equivalent to repayment of the principal at 15.3 percent interest. When used in this way, the ratio of R to P is often called rate of return, and the derived interest may be called interest rate of return.

Note: Returns are not the same as revenue. They equal revenue minus operating costs. More on this later.

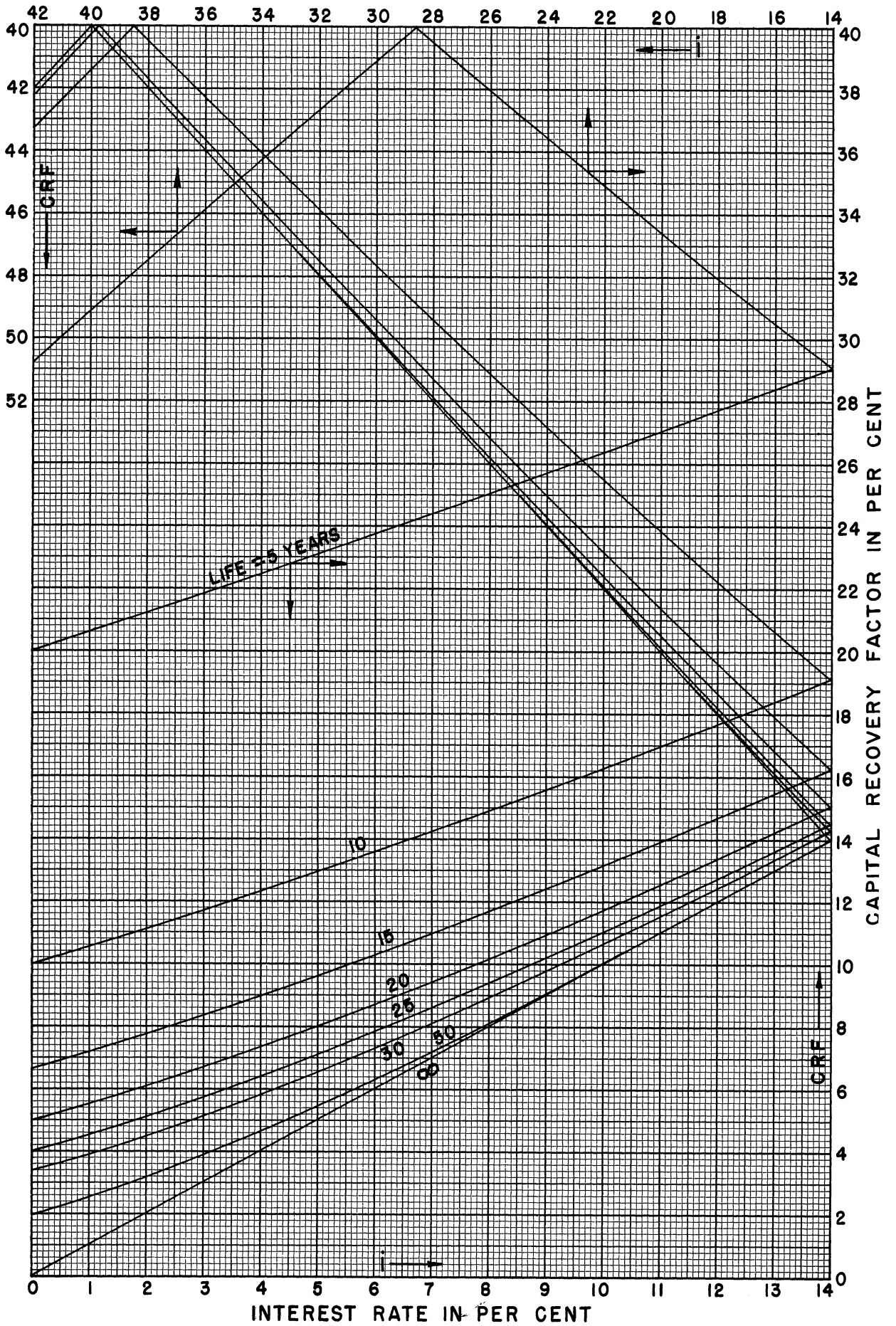
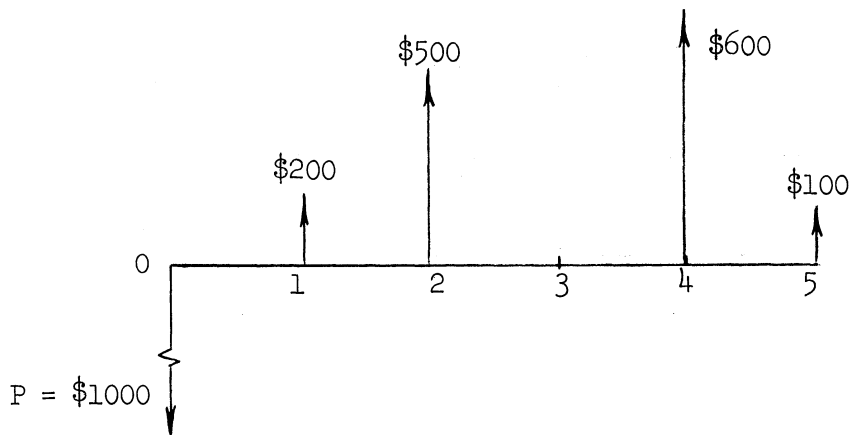


Figure 1. CRF vs Interest Rate.

- 30 Would you rather buy \$500,000 worth of bonds paying 5 percent interest or buy a tugboat with the \$500,000? The tug will last 25 years and bring you \$35,500 returns each year. Assume there are no taxes involved in either case and the tug's salvage value is nil.
- 31 What is the equivalent rate of interest for an investment of \$1000 that promises returns of \$200 the first year, \$500 the second, zero the third, \$600 the fourth, and \$100 the fifth?



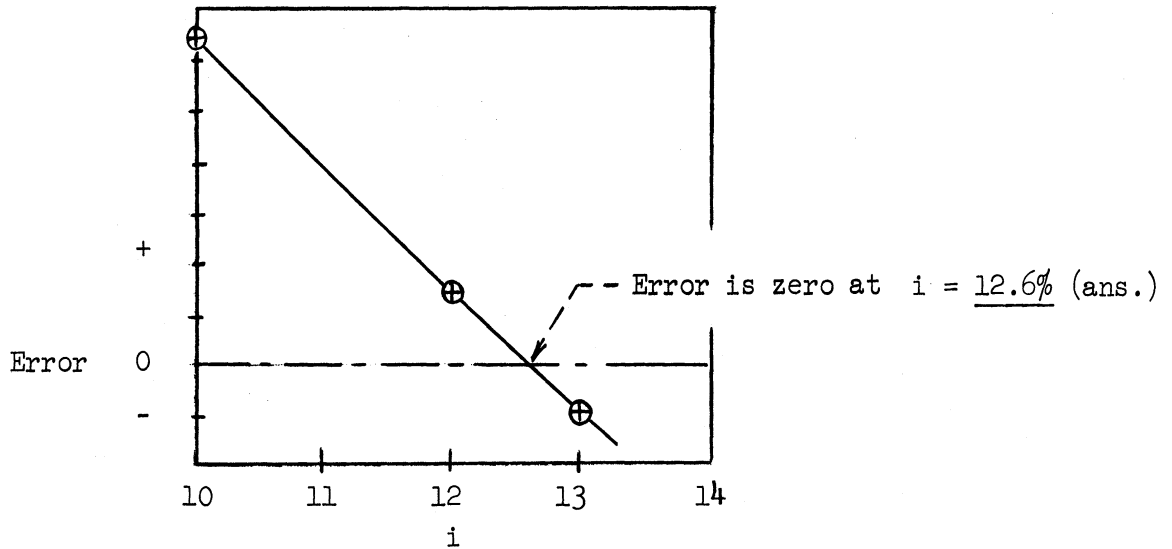
Solution

The key idea for a problem of this type is as follows: There is one and only one interest rate that will make the present worth of the future returns exactly equal to the investment. If we find that interest rate, we have the answer. This is essentially what we did by a shortcut method (using CRF) in the previous problems. For complex patterns we must resort to a trial-and-error approach. That is, we guess at an interest rate, find present worth and compare with investment. We repeat this process with other rates until we get agreement between invested cost and present worth of future returns.

Year	Return	i = 10%		i = 12%		i = 13%	
		$[\text{SPWF}]_n^{.10}$	P	$[\text{SPWF}]_n^{.12}$	P	$[\text{SPWF}]_n^{.13}$	P
1	\$200	.9091	180	.8929	178	.8850	177
2	500	.8264	413	.7972	398	.7831	392
3	0	.7513	-	.7118	-	.6931	-
4	600	.6830	410	.6355	381	.6133	368
5	100	.6209	62	.5674	57	.5428	54
Total			1065		1014		991
Error			+65		+14		-9

(continued)

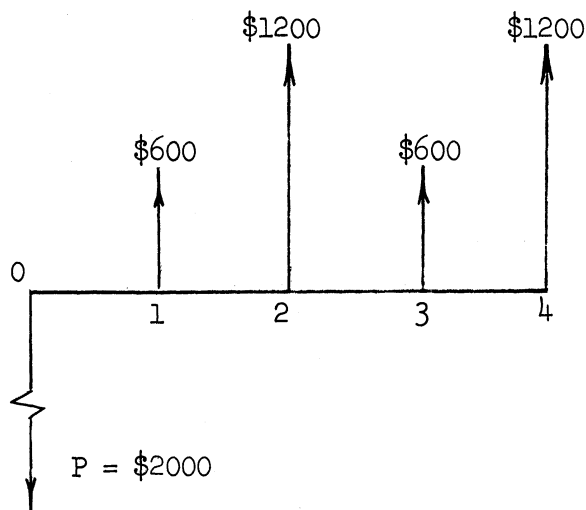
Plotting error in PW vs interest rate:



This method has a potential enigma. When one or more of the years produce a loss, more than one interest rate may bring the present worth of all returns into balance with the investment. Such a situation is unlikely in ship design studies, however.

The trial-and-error procedure described above is called here equated interest rate of return. Other writers call it discounted cash flow, profitability index, etc.

32 Find the equated interest rate of return for the investment and return shown below:



## CORPORATE PROFITS TAX

Most of the major maritime nations tax the profits of corporations. The tax laws not only vary between countries but are likely to change over the years within each country. We cannot of course deal with all possible tax situations here. We can however present the principles involved in the simplest sort of tax. You can then develop analytical solutions to suit your own needs. The monograph "Profitability Before and After Tax" deals with the complexities of U.S. tax laws and also serves as an example of how these simple principles can be applied to complex situations.

An understanding of taxes is important for reasons that will be explained shortly.

Figure 2 shows the distribution of annual revenue from a capital investment such as a ship. Taxes are actually computed on a corporate - wide basis. But, for our purposes, we cut the pie as though it applied to a single ship. Figure 2 is for the simple case of an all-equity investment, and straight-line depreciation without fast writeoff.

To find the return after tax ( $R'$ ); we must subtract from revenue the operating costs ( $Y$ ) and the tax itself. The tax is usually taken as a percentage of what the government calls profit: return ( $R$ ) minus depreciation allocation ( $P \div n$ ).

(When bank loans are involved, interest payments to the bank are treated as operating costs, that is they are subtracted before tax.)

The mathematics of Figure 2 can easily be manipulated to develop the following relationship between capital recovery factors before and after tax. The prime symbol denotes the after-tax condition:

$$CRF' = CRF (1-t) + \frac{t}{n} \quad (1)$$

or

$$CRF = \frac{CRF' - \frac{t}{n}}{(1-t)} \quad (2)$$



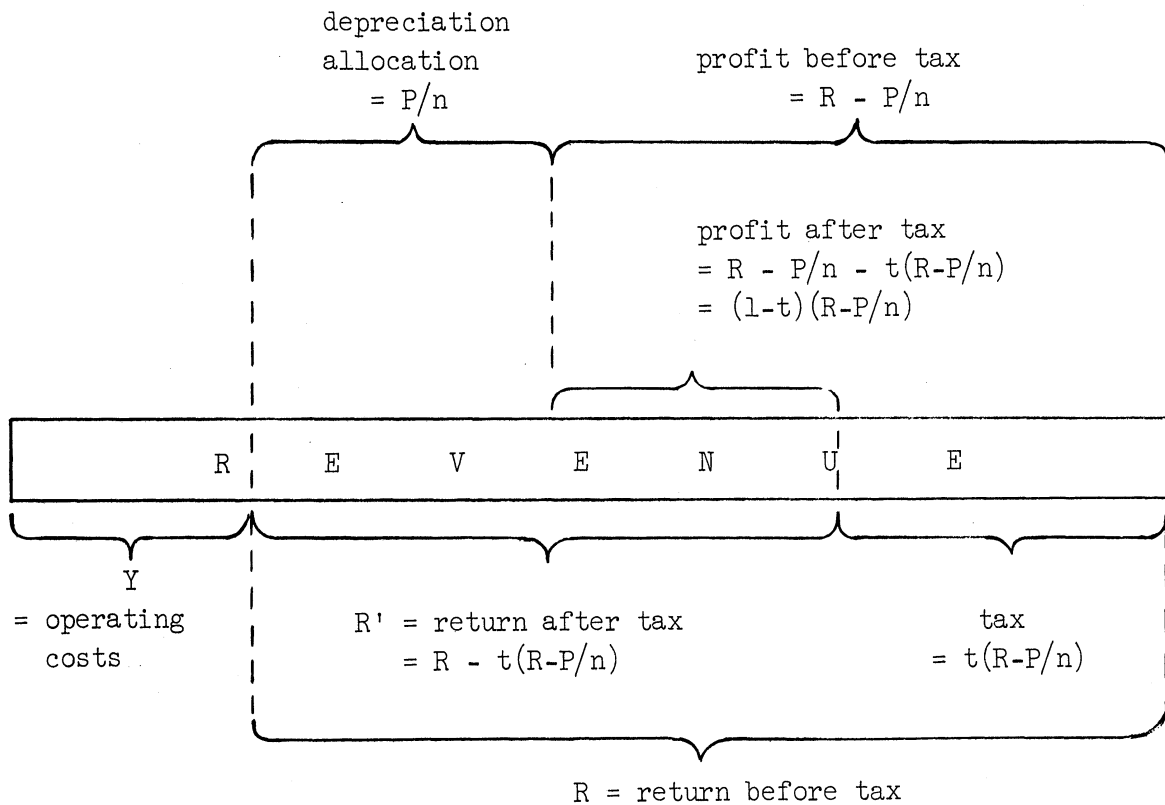


Figure 2. Division of Revenue.  
(all amounts are on an annual basis)

Note: The words profit and profitability are not synonymous. A \$1,000,000 investment that generates profits of \$2 per year is not as profitable as a \$10 investment that yields \$1 profit per year.

The following examples illustrate the use of Equations (1) and (2).

- 33 An owner specifies an after-tax interest rate of 15 percent and a life of 25 years for his proposed ship. The tax rate is 48 percent and straight-line depreciation is used. What interest rate should you use in relating present and future cash flows before tax?

Solution

$$\begin{array}{ll} i' = 15\% & t = 0.48 \\ n = 25 & i = ? \end{array}$$

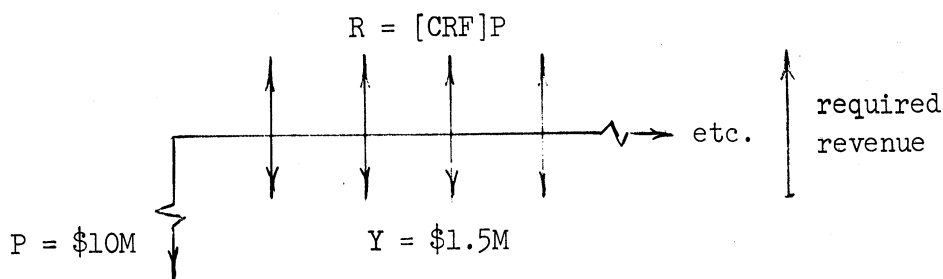
The solution requires three steps. First: find CRF' from interest tables. Second: convert to CRF by means of Equation (2). Third: find i from Figure 1 or from interest tables.

$$\begin{aligned} [CRF'] \cdot \frac{1.15^{25} - 1}{0.15} &= 0.1547 \\ CRF &= \frac{CRF' - \frac{t}{n}}{1 - t} \\ CRF &= \frac{0.1547 - \frac{0.48}{25}}{1 - 0.48} = 0.261 \\ i &= \underline{25.9\%} \text{ (ans.)} \end{aligned}$$

- 34 Continuing the above, how much annual revenue would be required if the first cost of the ship is \$10,000,000 and the annual operating costs are \$1,500,000?

Solution

$$\begin{array}{l} P = \$10,000,000 \\ Y = \$1,500,000 \end{array}$$



$$\begin{aligned} \text{Required revenue} &= Y + [CRF] P \\ &= \$1,500,000 + 0.261 \times \$10,000,000 \\ &= \$1,500,000 + \$2,610,000 \\ &= \underline{\$4,110,000} \text{ (ans.)} \end{aligned}$$

35 Starting with the \$4,110,000 annual revenue calculated above, set up a table and derive the after-tax interest rate of return. In short, reverse the procedure of Problems 33 and 34. Keep Figure 2 in view.

Solution

Item	\$ per Year	Notes
Revenue	4,110,000	
Operating costs	<u>1,500,000</u>	
Return before tax	2,610,000	
Depreciation allocation	<u>400,000</u>	1
Taxable profit	2,210,000	
Tax at 48%	<u>1,060,000</u>	
Profit after tax	1,150,000	
Return after tax (R')	1,550,000	2
CRF'	0.155	3
i'	<u>15%</u>	4
	(ans.)	

Notes

1. Depreciation allocation =  $P \div n = \$10,000,000 \div 25$
2. Return after tax = Return before tax minus tax  
also = Profit after tax plus depreciation
3.  $CRF' = R' \div P$
4. Interest after tax is found from Figure 1.

The validity of Equations (1) and (2) is shown by the two examples above.

- 36 A shipowner stipulates an after-tax level of profitability equivalent to 13 percent interest rate. If ship's life is 20 years and tax rate is 48 percent, find interest rate before taxes. (Answer: 22.4%)
- 37 Based on above, find top price owner should be willing to pay for a gadget that would last 10 years and save a net amount of \$1000 per year before tax. Gadget has zero scrap value. (Answer: \$3860)
- 38 Based on #1, convert the scrap value of the ship (\$400,000) to an average annual return before tax.

- 39 A ship costs \$15,000,000 and is expected to last 30 years. Its annual gross income is \$4,000,000 and its annual operating costs total \$750,000. There are no interest charges. Tax rate is 48 percent. Find capital recovery factors before and after tax, using tabular form.
- 40 Check CRF relationships above by means of Equations (1) or (2).

## CHOOSING AN INTEREST RATE

There are often circumstances when cost studies require that a minimum acceptable interest rate of return be specified. Specifying a rate is primarily a responsibility of management. Nevertheless, engineers should develop some insight along these lines.

In stipulating an interest rate, a business manager will weigh alternative investment opportunities and relative risks. Furthermore, if free-market economic conditions exist, he knows that such condition will set reasonable upper and lower limits on returned interest over the life of the ship.

Most business managers think about after-tax levels of profitability, so stipulated interest rates should be adjusted upwards (as in Problem 33) before applying to before-tax cash flows.

What constitutes a reasonable interest rate of return for a commercial ship is a matter of opinion that varies in different parts of the world. In the United States, owners of captive fleets generally aim for after-tax interest rates of about 12 percent. Independent operators, with their greater risks, aim for about 15 percent. These levels are based on idealized cost studies; actual returns are usually somewhat lower. In all too many cases they are considerably lower.

Some engineers advocate lower rates than those mentioned above. They argue that half or more of the investment comes from a bank loan with interest rates of only about 6 percent. Thus, if half the investment requires 6 percent interest and half requires 12 percent, the overall interest would be 9 percent. There is some merit in this argument, but how far you should go is a matter of business judgement. Since bank repayment takes priority, the return of profits on equity capital is less certain than if no bank loan were involved. Which means that expected returns on equity capital must thereby be higher than normal.

Service vessels owned by governments seldom earn income. The time-value of money invested in them is real, nevertheless. Low rates, about 4 percent, are used in the United States. Many of us suspect the rate should be considerably higher.

## ECONOMIC CRITERIA

Figure 3 outlines the major combinations of circumstances under which we may have to analyze a commercial ship design. The bottom of each column shows, by abbreviation, the one or more economic criteria that are suited to the circumstance.

Where revenues are predictable, the optimum ship can (to save time) be chosen on a before-tax basis; the best alternative before tax normally being the best after tax.

In this discussion, specific terms are capitalized; generic terms are not. For example: The Present Worth method uses present worth discount procedures.

All valid criteria adhere to the following principles:

- I. A commercial ship is an investment that earns its returns as a socially useful instrument of transport.
- II. The best measure of engineering success is profitability; and the only meaningful measure of profitability is the returned profit (after tax) expressed as interest on the investment.

In keeping with the above principles, our recommended criteria either predict the returned interest rate, or make use of a specified rate. Most of these criteria have already been discussed but all are included below for convenience.

EiRR: Equated Interest Rate of Return

In those rare cases where we can predict quantitative fluctuations in returns, we can use the trial-and-error procedure illustrated in Problem 31 to find that rate of interest that will make the present worth of the returns equal to the investment (EiRR). As a shortcut, if the cash flow patterns are similar for all alternatives, take average returns and use the following method.

CRF: Criterion Based on Capital Recovery Factor

If we can predict future returns, and if we can assume them to be reasonably uniform, we can find the capital recovery factor ( $R \div P$ ) and then find the corresponding interest rate of return from Figure 1. This is really a shortcut way of finding the equated interest

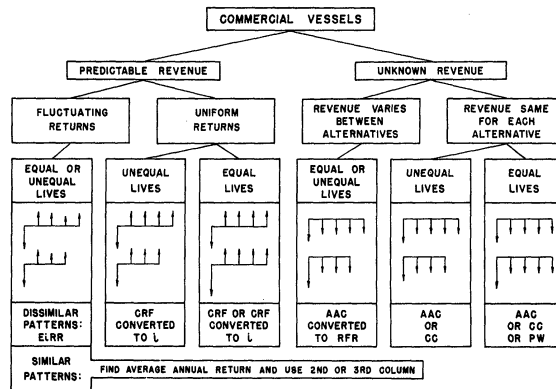
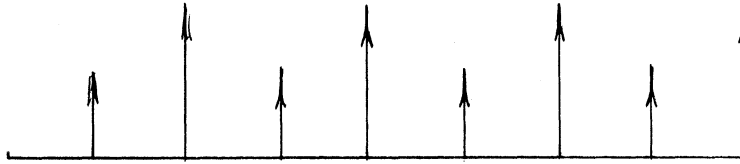


Figure 3. Suitable Criteria Under Various Economic Circumstances.

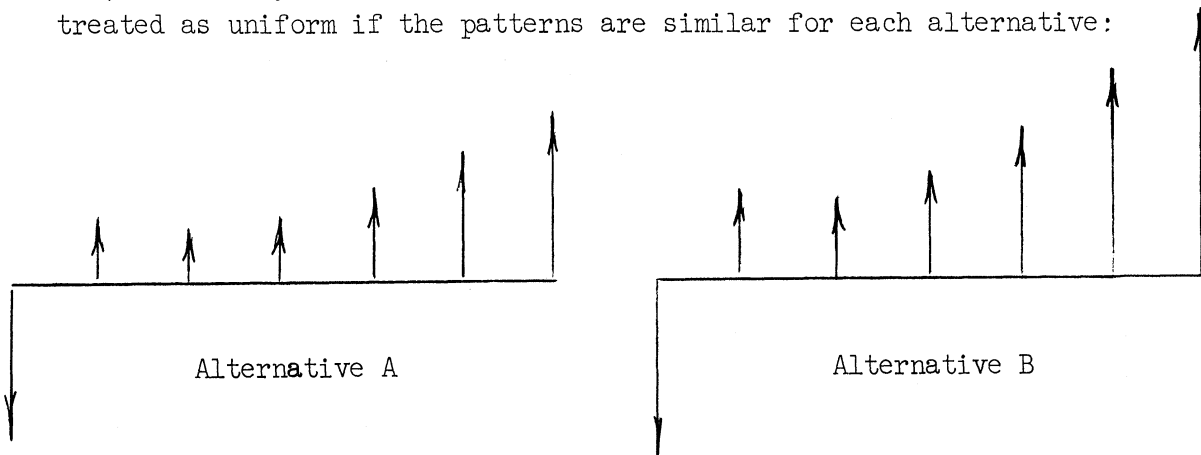
Abbreviations

- EiRR: Equated Interest Rate of Return
- CRF: Capital Recovery Factor
- i : Interest Rate
- AAC: Average Annual Cost
- RFR: Required Freight Rate
- CC : Capitalized Cost
- PW : Present Worth

rate of return. Research has shown that the CRF approach is often valid even when returns fluctuate. For example, uniform fluctuations such as this



are quite accurately treated as uniform returns of average value. And, as already mentioned, non-uniform returns may be averaged and treated as uniform if the patterns are similar for each alternative:



Where all alternatives have equal lives, CRF by itself is a suitable criterion; the highest CRF will automatically correspond to the highest interest rate. Where lives differ, however, CRF may be misleading and interest rate should be found.

The reciprocal of CRF is the payout time, or years to pay back the investment. It has the same limitations as CRF as a criterion and should not be used where lives differ. In unsettled political environment, however, payout time is intrinsically significant. All other things being equal, the owner would select the alternative that promised quickest return of his investment. A high CRF would of course be equally meaningful.

In discussing both EiRR and CRF, we have referred to known returns. These concepts can as well be applied to cases where returns are not known but annual cost savings are. Each dollar saved will increase the returns by a like amount. For example, we can use CRF to weigh the merits of a proposed heat exchanger that will add a known amount to the first cost of the ship but will produce predictable annual savings in fuel cost. Usually we would require that the incremental CRF be at least as great as the ship's overall CRF.



AAC: Average Annual Cost Method

The Average Annual Cost method is appropriate when we cannot predict income but know it will be the same between alternatives. The most profitable ship will be the one that can produce the service at the lowest average annual cost. It should also eventually set the freight rate.

This method requires a stipulated interest rate and may result in wrong decisions if the interest rate is unrealistic, or if the influence of corporate profits tax is overlooked.

The AAC technique has already been described and will not be repeated here.

RFR: Required Freight Rate

The AAC method is valid only when all alternatives have equal (but unknown) annual revenues. In bulk trades, where cargo is relatively unlimited, different possible ships may promise varying annual transport capabilities. Revenues then may vary between alternatives. In such cases, we can use the Required Freight Rate method to find the optimum design. Required Freight Rate is the income per unit of cargo that a shipowner must collect in order to earn returns equivalent to the repayment of his investment at a reasonable rate of interest. The optimum ship is the one that can provide the service at minimum RFR. This criterion is based on costs to the customer, not the owner; this sets it apart from the popular but unsound criterion of Operating Cost per Ton, which is discussed later.

The mathematics of RFR for any given trade route are:

$$\text{RFR} = \frac{\text{AAC}}{\text{C}}$$

where

C = Cargo carried each year

AAC = Average annual cost

Again, proper interest rates must be used if the results are to be valid.

PW: Present Worth Method

Proposed ships may be compared on a basis of their total present worths including investment. For example, in the case of uniform annual costs:

$$PW = P + [UPWF] Y$$

The Present Worth method may be used under the same economic circumstances as AAC : unknown but equal revenues for all alternatives. There is one further restriction however: PW must not be used where the alternatives have unequal lives, unless we find the least common multiple and grind out the total present worth of each combination.

CC: Capitalized Cost

The Capitalized Cost is the present worth of providing a perpetual service. The method is infrequently used and can do nothing that cannot be done more easily by the Average Annual Cost method.

Criteria for Non-Commercial Craft

We can treat naval craft or other governmental vessels of no measurable benefit exactly as we do commercial vessels of unknown but equal revenues. That is, use Average Annual Cost or, alternatively if lives are equal, Present Worth.

In the case of state-owned vessels of measurable benefit, we can use the Benefit Cost Ratio (BCR) :

$$BCR = \frac{U_0 - U_I}{F_I + Y_I - F_0 - Y_0}$$

where subscript 0 refers to original or existing facility and subscript I refers to improved or new facility.

U = Average annual cost to public benefiting  
from facility

F = Average annual cost of investment = [CRF]P

Y = Annual cost of operating facility

Problems

41 A proposed ship has a life expectancy of 20 years. The predicted uniform annual before-tax returns are \$2,000,000. The estimated first cost is \$12,500,000. Would it be economic to add a heat exchanger that would have an installed cost of \$130,000? The unit would save \$25,000 per year in fuel; it would not affect crew costs or cargo capacity. Annual insurance costs would be 1 percent of investment; repairs cost \$2,000 per year for the unit. Salvage value would be nil.

Solution

$$\text{For the ship: CRF} = \frac{R}{P} = \frac{\$2\text{M}}{\$12.5\text{M}} = 0.16$$

Incremental annual returns from the heat exchanger:

Fuel saving	+ \$25,000
Insurance at 1%	- 1,300
Repairs	- 2,000
Net incremental return	+ 21,700

For the heat exchanger:

$$\text{CRF} = \frac{\$21,700}{\$130,000} = \underline{0.167} \text{ (ans.) (vs 0.16 for ship)}$$

Conclusion: Addition of the heat exchanger would be barely worth while.

42 Repeat Problem 41 but assume the ship's first cost is dropped to \$10,000,000.

Comparing the results of the last two problems leads to an important point: High technical efficiency is most appropriate in ships of low profitability. The owner of the highly profitable ship should use his spare cash to build another ship, not add refinements. If you argue that perhaps he can't afford another ship but can afford a heat exchanger, his reply is that that's his worry not yours.

43 What economic criterion or criteria would be suitable for finding the superior alternative below. Explain

Alternative	A	B
Investment	\$8000	\$12,000
Life, years	12	20
Annual return after tax	\$2000	\$ 2,400
Resale value	\$ 200	\$ 6,200

44 A company, which generally earns profits of 18 percent (equivalent interest rate) before tax, has the following alternatives. Which would you recommend?

Alternative	A	B	C*
Annual gross revenue	\$4000	\$4,500	
Invested cost	\$10,000	\$9,000	
Life, years	8	10	
Annual operating costs	\$2000	\$2,700	
Resale value	0	0	

\* Alternative C is to turn down both A and B and seek other investments.

45 A ship needs a new compressed air tank. Tank A will cost \$4000 and last 10 years. Tank B will cost \$6000 and last 20 years. The ship itself is expected to last another 20 years, also. Desired rate of interest is 20 percent before tax. Which tank would you recommend?

Solution

Revenues are unknown but equal. Lives are unequal ∴ Use the AAC criterion.

$$\text{Tank A: AAC} = [\text{CRF}]_{10}^{.20} 0.2385 \$4000 = \$955$$

$$\text{Tank B: AAC} = [\text{CRF}]_{20}^{.20} 0.2054 \$6000 = \$1230$$

Choose Tank A (ans.)

46 Rework Problem 45 using an interest rate of 5 percent.

47 Find the indifference rate for Tanks A and B (Problem 45). That is, find the interest rate at which both tanks would be equally desirable.

48 A Baratarian shipowner expects his new ore carrier to move 200,000 tons of cargo per year on a given trade route. The ship will cost \$6,000,000 to build and will last 25 years. Its average annual operating costs will be \$1,000,000. The Baratarian corporate profits tax rate is 33 percent. What revenue per ton must he charge if he is to earn 12 percent interest on his investment, after taxes?

Solution

The problem is to find Required Freight Rate, which equals the average annual cost per ton of cargo. In finding average annual cost, the invested cost must be converted to an annual cost by a capital recovery factor gaged to produce the stipulated after-tax level of profitability. The first step is to find CRF:

$$[CRF']_{25}^{.12} = 0.1275$$

$$CRF = \frac{CRF' - \frac{t}{n}}{1 - t} = \frac{0.1275 - \frac{0.33}{25}}{1 - 0.33} = 0.1705$$

The second step is to find AAC:

AAC for operations.....	\$1,000,000
AAC for capital: [CRF]0.1705 \$6,000,000.....	<u>\$1,023,000</u>
Total AAC	\$2,023,000

Dividing AAC by annual cargo capacity we find Required Freight Rate:

$$RFR = \frac{\$2,023,000}{200,000} = \underline{\$10.11 \text{ per ton}} \text{ (ans.)}$$

49 If the above owner can reduce average building costs by 15 percent through buying several identical ships, what will the new RFR become?

50 Rework Problem 48 assuming an increase in the Baratarian tax rate to 50 percent.

51 Rework Problem 48 assuming a resale value for the vessel of \$1,000,000 at the end of 25 years.

52 Reverting to the original conditions of Problem 48, how profitable would the investment be if the attainable freight rate were only \$9.00 per ton?

53 Find the superior alternative if desired interest rate before taxes is 18 percent:

Alternative	A	B
Life, years	25	30
Invested cost	\$10,000,000	\$11,000,000
Annual operating costs	\$ 1,500,000	\$ 1,250,000
Salvage value	\$ 1,000,000	\$ 1,000,000

54 In Problem 53, would the Present Worth criterion be as suitable as the one you chose? Explain.

55 Which of these proposed ships would you recommend? All have an expected life of 25 years. (Costs shown are in \$1000, weights in 1000's of tons.)

Ship	A	B	C	D	E	F
Invested Cost	4000	4200	4500	4900	5400	6000
Annual oper. costs	600	610	620	630	640	650
Cargo per year	50	55.5	62	64.5	67	70
Annual revenue	1000	1110	1240	1290	1340	1400

56 Explain why you did or did not add depreciation and/or interest charges in the above analysis.

## UNSOUND CRITERIA

The marine industry harbors several persistent economic criteria that do not give proper weight to the time-value of money. For example, some studies compare alternatives by adding the cumulative total operating costs to the first cost. This is present worth with zero interest.

Another common misconception is to design for maximum annual profit. Remember, however, that a small investment yielding a modest profit is more profitable than a large investment yielding a slightly greater profit. In the design stage, profit and profitability are not the same. Once the ship is built, however, the investment is no longer a variable and maximum profit means maximum CRF hence maximum profitability.

One of the most common criteria is the Minimum Cost method or its cousin Minimum Cost per Ton. These methods can also be called Cost of Service. They are akin to AAC and RFR with one important exception: they use unrealistically low interest rates (or none at all) and overlook the corporate profits tax. This slighting of the time value of money means that high capital costs are (in appearance) easily offset by future savings. This leads to over-design; which prompted Basingstoke to call it the Sales Pitch criterion. Another flaw in the Minimum Cost concept is that it is totally blind to market conditions and available freight rates.

Here are three related problems that illustrate the danger of using the Cost of Service criterion. All are based on a proposal that a fuel-saving device be included in the specifications for a new cargo ship. The device would cost \$200,000 installed and save \$25,000 each year over the 20-year life of the ship. Insurance costs per year for the device would be 1 percent of its first cost, maintenance and repair: 2 percent.

57 Analyze the above proposal using the Minimum Cost approach.  
Assume an interest rate of 5 percent.

### Solution

In this method, depreciation and interest are usually shown as separate annual charges. The annual interest charge would not be 5 percent of the investment as you might at first think; this is

because the residual debt is continually reduced and the stipulated interest rate applies to a diminishing amount. Therefore, as a percentage of the initial debt, the annual interest charge is only about half the interest rate.

Since the capital recovery factor is made up of capital return (depreciation) plus interest payment, we can find the annual interest cost (as a percent of the investment) by subtracting the depreciation allocation from CRF :

Annual Allocation as Percent of Investment

Capital recovery factor	= [CRF] <sup>.05</sup>	= 0.0802
Depreciation	= $1/n^{20} = 1/20$	= 0.0500
<hr/>		
Annual interest allocation		= 0.0302 (say 3%)

Annual depreciation allocations, interest allocations, insurance, and maintenance and repair costs are now all given in terms of the investment. We can summarize as follows:

Costs per Year

Depreciation	0.05P
Interest	0.03P
Insurance	0.01P
Maintenance and repair	0.02P
<hr/>	
Total	0.11P
Total capital - related costs per year	
= 0.11 \$200,000	= \$22,000
Fuel cost saving per year	= \$25,000
<hr/>	
Net saving per year	= \$ <u>3,000</u> (ans.)

Based on costs alone the proposal appears beneficial. But let's analyze its after-tax level of profitability in the following problem.



58 What after-tax interest rate of return is implied in the problem above? Assume a tax rate of 48 percent.

Solution

$$\begin{array}{rcl} \text{Depreciation allocation} & = & 0.05P \\ \text{Annual interest allocation} & = & 0.03P \\ \hline \text{Annual return} & = & 0.08P \\ \therefore \text{CRF} & = & 0.08 \end{array}$$

$$\begin{aligned} \text{CRF}' &= \text{CRF}(1-t) + \frac{t}{n} \\ &= 0.08(1-0.48) + \frac{0.48}{20} \\ &= 0.0656 \end{aligned}$$

From Figure 1:

$$i' = \underline{2.75\%} \text{ (ans.)}$$

Would you ask a business man to risk his capital for this level of profitability?

59 Given all of the other conditions mentioned in Problems 57 and 58, what annual fuel cost savings would be needed to justify the invested cost of the heat exchanger if the shipowner stipulates an after-tax interest rate of 11 percent?

Solution

$$\begin{aligned} i' &= 11\% \\ \text{CRF}' &= 0.1256 \\ \text{CRF} &= \frac{\text{CRF}' - \frac{t}{n}}{1-t} \\ &= \frac{0.1256 - \frac{0.48}{20}}{1-0.48} = 0.195 \end{aligned}$$

$$\begin{aligned} R &= \text{Annual return from the device} \\ R &= [\text{CRF}] 0.195 \$200,000 = \$39,000 \end{aligned}$$

also:

$$\begin{aligned} R &= F - (N + M) \\ \therefore F &= R + N + M \end{aligned}$$

where

F = Annual saving in cost of fuel  
 N = Annual cost of insurance  
 M = Annual cost of maintenance and repair

$$N = 0.01P$$

$$M = 0.02P$$

$$N+M = 0.03P = 0.03 \$200,000 = \$6,000$$

$$F = \$39,000 + \$6,000 = \underline{\$45,000} \text{ (ans.)}$$

(This compares with the predicted annual fuel saving of \$25,000.)

The three foregoing examples illustrate how the Cost of Service criterion can lead to over-design in the case of feasibility studies. When applied to optimization studies, however, the criterion leads to under-design. The next three examples illustrate this point.

60 Choose the optimum SHP for a proposed tanker, given the costs and revenues tabulated below. The owner wants to earn an after-tax interest rate of 12 percent. The ships are expected to last for 20 years and the tax rate is 48 percent.

(Costs and weights are in 1000's)

Tanker	A	B	C	D
SHP	10,000	15,000	20,000	25,000
Speed in knots	12.83	14.59	15.90	16.93
Cargo: tons/year	315.0	351.3	376.6	395.2
Investment: \$	13,425	14,010	14,511	14,977
Annual oper. costs: \$	1174	1347	1509	1668

Solution

Revenues, which are unknown, would vary between alternatives because of differences in annual transport capability. Required Freight Rate (RFR) would be the appropriate criterion. The first step is to find CRF :

$$i' = 12\%$$

$$[CRF']_{20}^{.12} = 0.1339$$

$$CRF = \frac{CRF' - \frac{t}{n}}{1 - t} = \frac{0.1339 - \frac{0.48}{20}}{1 - 0.48} = 0.2115$$

(continued)

Applying this capital recovery factor to each invested cost gives us the annual capital cost for each design: Continuing the table, we have:

Tanker	A	B	C	D
Annual cap. costs: \$	2840	2962	3070	3167
Annual oper. costs:\$	1174	1347	1509	1668
AAC	4014	4309	4579	4835
Cargo: tons per year	315.0	351.3	376.6	395.2
RFR	\$12.75	\$12.26	<u>\$12.16</u> (min.)	\$12.23

Conclusion: Tanker C (20,000 SHP) is optimum by virtue of promising the lowest Required Freight Rate.

61 Rework Problem 60 using the Minimum Cost (Cost of Service) criterion. Assume 6 percent interest.

Solution

The annual capital costs are now:

$$[CRF]_{20}^{.06} = 0.0872P$$

In the usual manner of presentation:

Depreciation	0.0500P
Interest	<u>0.0372P</u>
Total annual capital cost	0.0872P

(Annual cost of insurance and maintenance and repair are already included in the operating costs in this instance.)

Using this CRF, we find the cost per ton of cargo by modifying the table as follows:

Tanker	A	B	C	D
Annual capital costs: \$	1171	1221	1266	1306
Annual oper. costs: \$	1174	1347	1509	1668
Fullydistributed costs:\$	2345	2568	2775	2974
Cargo: tons per year	315.0	351.3	376.6	395.2
Costs per ton cargo	\$7.45	<u>\$7.31</u> (min.)	\$7.37	\$7.53

On this basis, we would choose Tanker B (15,000 SHP). This would be a mistake, however, because under normal market conditions Tanker C would be more profitable.

62 Assume Owner X buys Tanker B and Owner Y buys Tanker C above. Assume the freight rate is an average of RFR found for Tankers B and C in Problem 60. Find the after-tax interest rate of return in each case.

63 Assume a freight rate of \$12.16 per long ton. Analyze proposed Tankers A, B, C, and D on the Maximum Annual Profit basis. (Profit = revenue - operating costs - depreciation allocation.)

Solution

Tanker		A	B	C	D
Annual revenue:	\$	3830	4270	4579	4803
Annual oper. costs:	\$	1174	1347	1509	1668
Depreciation (.05P):	\$	671	701	726	748
Annual profit:	\$	1985	2222	2344	<u>2387</u> (max.)

64 Under the conditions of Problem 63, which tanker would be the most profitable?

These examples have illustrated the shortcomings of the Minimum Cost and Maximum Profit criteria, both of which fail to weigh correctly present and future monies. There are other criteria that have similar weaknesses, and some do not even use monetary units. The concept of deadweight times speed divided by horsepower is an example. Another kind of disorder may arise when basically valid criteria are applied under the wrong circumstances. For example, the Present Worth or Average Annual Cost criteria may lead to a poor decision if used where incomes (or savings) are known. Consider Problems 65 and 66:

65 Choose the better alternative:

Alternative:	A	B
Invested cost (P)	\$100	\$150
Life, years	10	10
Annual return (R)	\$ 25	\$ 35

(continued)

Solution

Since revenues are known and lives are the same, CRF is a valid criterion:

Alternative:	A	B
CRF = R ÷ P:	$\frac{0.25}{(\text{max.})}$	0.233

Conclusion: Alternative A is more profitable.

66 Rework Problem 65 on the basis of present worths. Assume a before-tax interest rate of 10 percent.

Solution

For Alternative A, the present worth of the future returns equals:

$$[\text{UPWF}]_{10}^{.10} \frac{1}{0.1627} \$25 = \$153$$

For Alternative B:

$$[\text{UPWF}]_{10}^{.10} \frac{1}{0.1627} \$35 = \$215$$

Finding total present worths:

Alternative	A	B
Present worth of future savings	\$153	\$215
Invested cost	\$100	\$150
Net gain in present worth	\$ 53	\$ 65
		(max.)

Conclusion: The PW criterion would, in this case, lead to an incorrect decision. (AAC would have done so too.)

67 Suppose you are asked to find the optimum ship design in a situation where the owner has given you a predicted freight rate. You analyze several possible designs and find the best using CRF. Its value is 0.70. Would you be satisfied with CRF as a criterion? Discuss.

- 68 Suppose you are asked to decide whether to add to a new ship a device costing \$1000 and saving \$700 per year. The device will last as long as the ship. Would you be satisfied with CRF as a criterion? Discuss.
- 69 How would you compare two proposals both of which would reduce both the building cost and the operating cost of the ship?

## CAPTIVE FLEETS

Managers of subsidiary fleets, wholly owned by some industrial corporation, are apt to aim for minimum cost rather than maximum profitability in their ships. They reason that the fleet is not really producing any income and is simply a service facility to the corporation. In truth, however, each ship is producing a hidden income: annual cost of chartering the services of an independent operator minus the annual operating costs for the captive vessel.

Put another way, the corporation has a choice. It can invest in a ship and pay relatively small annual operating costs, or it can invest in another mine, oil refinery, or whatever, and pay large annual amounts for chartering. If this annual charter cost is smaller than the average annual cost of the captive ship, chartering would be more profitable. The CRF used in finding average annual cost should reflect alternative investment opportunities.

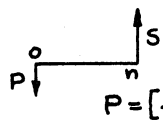
An analogous approach is to find the equivalent interest rate based on

$$\text{CRF} = \frac{A - Y}{P}$$

where

A = annual chartering costs, including internal administration  
Y = annual operating costs for captive ship  
P = invested cost in captive ship

70 A corporation needs transport services that an independent operator could provide on long-term contract for \$12,500,000 per year. Internal administrative costs would add \$50,000 per year. As an alternative, the corporation could enlarge its own fleet at a first cost of \$30,000,000 plus annual operating costs of \$5,000,000. The ships would last 20-years. The corporation has other investment opportunities that would yield interest rates of 12 percent after tax. Which choice would you recommend?



APPENDIX

Single Payment Present Worth Factors

$$SPWF = \frac{1}{(1+i)^n}$$

$$P = [SPWF]S$$

$$S = [SCAF]P$$

$$SCAF = \frac{1}{SPWF}$$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	.9901	.9804	.9709	.9615	.9524	.9434	.9346	.9259	.9174	.9091
2	.9803	.9612	.9426	.9246	.9070	.8900	.8734	.8573	.8417	.8264
3	.9706	.9423	.9151	.8890	.8638	.8396	.8163	.7938	.7722	.7513
4	.9610	.9238	.8885	.8548	.8227	.7921	.7629	.7350	.7084	.6830
5	.9515	.9057	.8626	.8219	.7835	.7473	.7130	.6806	.6499	.6209
10	.9053	.8203	.7441	.6756	.6139	.5584	.5083	.4632	.4224	.3855
15	.8613	.7430	.6419	.5553	.4810	.4173	.3624	.3152	.2743	.2394
20	.8195	.6730	.5537	.4564	.3769	.3118	.2584	.2145	.1784	.1486
25	.7798	.6095	.4776	.3751	.2953	.2330	.1842	.1460	.1160	.0923
30	.7419	.5521	.4120	.3083	.2314	.1741	.1314	.0994	.0754	.0573
50	.6080	.3715	.2281	.1407	.0872	.0543	.0339	.0213	.0134	.0085

n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	.9009	.8929	.8850	.8772	.8696	.8621	.8547	.8475	.8403	.8333
2	.8116	.7972	.7831	.7695	.7561	.7432	.7305	.7182	.7062	.6944
3	.7312	.7118	.6931	.6750	.6575	.6407	.6244	.6086	.5934	.5787
4	.6587	.6355	.6133	.5921	.5718	.5523	.5337	.5158	.4987	.4823
5	.5935	.5674	.5428	.5194	.4972	.4761	.4561	.4371	.4190	.4019
10	.3522	.3220	.2946	.2697	.2472	.2267	.2080	.1911	.1756	.1615
15	.2090	.1827	.1599	.1401	.1229	.1079	.0949	.0835	.0736	.0649
20	.1240	.1037	.0868	.0728	.0611	.0514	.0433	.0365	.0303	.0261
25	.0736	.0588	.0471	.0378	.0304	.0245	.0197	.0160	.0129	.0105
30	.0437	.0334	.0256	.0196	.0151	.0116	.0090	.0070	.0054	.0042
50	.0054	.0035	.0022	.0014	.0009	.0006	.0004	.0003	.0002	.0001

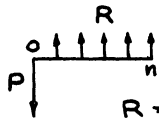
  

n	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	.8264	.8197	.8130	.8065	.8000	.7937	.7874	.7813	.7752	.7692
2	.6830	.6719	.6610	.6504	.6400	.6299	.6200	.6104	.6009	.5917
3	.5645	.5507	.5374	.5245	.5120	.4999	.4882	.4768	.4658	.4552
4	.4665	.4514	.4369	.4230	.4096	.3968	.3844	.3725	.3611	.3501
5	.3855	.3700	.3552	.3411	.3277	.3149	.3027	.2910	.2799	.2693
10	.1486	.1369	.1262	.1164	.1074	.0992	.0916	.0847	.0784	.0725
15	.0573	.0507	.0448	.0397	.0352	.0312	.0277	.0247	.0219	.0195
20	.0221	.0187	.0159	.0135	.0115	.0098	.0084	.0072	.0061	.0053
25	.0085	.0069	.0057	.0046	.0038	.0031	.0025	.0021	.0017	.0014
30	.0033	.0026	.0020	.0016	.0012	.0010	.0008	.0006	.0005	.0004
50	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

n	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	.7634	.7576	.7519	.7463	.7407	.7353	.7299	.7246	.7194	.7143
2	.5827	.5739	.5653	.5569	.5487	.5407	.5328	.5251	.5176	.5102
3	.4448	.4348	.4251	.4156	.4064	.3975	.3889	.3805	.3724	.3644
4	.3396	.3294	.3196	.3102	.3011	.2923	.2839	.2757	.2679	.2603
5	.2592	.2495	.2403	.2315	.2230	.2149	.2072	.1998	.1927	.1859
10	.0672	.0623	.0577	.0536	.0497	.0462	.0429	.0399	.0371	.0346
15	.0174	.0155	.0139	.0124	.0111	.0099	.0089	.0080	.0072	.0064
20	.0045	.0039	.0033	.0029	.0025	.0021	.0018	.0016	.0014	.0012
25	.0012	.0010	.0008	.0007	.0006	.0005	.0004	.0003	.0003	.0002
30	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001	.0001	.0000





APPENDIX

Capital Recovery Factors

$$CRF = \frac{i(1+i)^n}{(1+i)^n - 1}$$

$$R = [CRF]P$$

$$P = [UPWF]R$$

$$UPWF = \frac{1}{CRF}$$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0100	1.0200	1.0300	1.0400	1.0500	1.0600	1.0700	1.0800	1.0900	1.1000
2	.5076	.5155	.5226	.5305	.5376	.5455	.5529	.5606	.5685	.5760
3	.3401	.3466	.3534	.3604	.3671	.3741	.3811	.3880	.3951	.4021
4	.2564	.2625	.2691	.2755	.2820	.2886	.2952	.3019	.3086	.3155
5	.2062	.2121	.2183	.2246	.2309	.2374	.2439	.2505	.2571	.2638
10	.1056	.1113	.1172	.1233	.1295	.1359	.1424	.1490	.1558	.1627
15	.0721	.0778	.0838	.0899	.0963	.1030	.1098	.1168	.1241	.1315
20	.0554	.0612	.0672	.0736	.0802	.0872	.0944	.1018	.1095	.1175
25	.0454	.0512	.0574	.0640	.0710	.0782	.0858	.0937	.1018	.1102
30	.0387	.0447	.0510	.0578	.0651	.0726	.0806	.0888	.0973	.1061
50	.0255	.0318	.0389	.0465	.0548	.0634	.0725	.0817	.0912	.1009

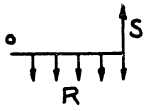
n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.1100	1.1200	1.1300	1.1400	1.1500	1.1600	1.1700	1.1800	1.1900	1.2000
2	.5839	.5917	.5994	.6074	.6150	.6231	.6308	.6388	.6467	.6545
3	.4092	.4164	.4236	.4308	.4380	.4453	.4526	.4599	.4673	.4747
4	.3223	.3292	.3362	.3432	.3503	.3574	.3646	.3717	.3790	.3863
5	.2706	.2774	.2843	.2913	.2983	.3054	.3126	.3198	.3270	.3344
10	.1698	.1770	.1843	.1917	.1993	.2069	.2146	.2225	.2305	.2385
15	.1391	.1468	.1547	.1628	.1710	.1794	.1878	.1964	.2051	.2139
20	.1256	.1339	.1424	.1510	.1598	.1687	.1777	.1868	.1960	.2054
25	.1187	.1275	.1364	.1455	.1547	.1640	.1734	.1829	.1925	.2021
30	.1150	.1241	.1334	.1428	.1523	.1619	.1715	.1813	.1910	.2008
50	.1106	.1204	.1303	.1402	.1501	.1601	.1701	.1801	.1900	.2000

n	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	1.2100	1.2200	1.2300	1.2400	1.2500	1.2600	1.2700	1.2800	1.2900	1.3000
2	.6625	.6705	.6785	.6865	.6944	.7025	.7105	.7187	.7266	.7348
3	.4822	.4897	.4972	.5047	.5123	.5199	.5275	.5352	.5429	.5507
4	.3936	.4010	.4085	.4159	.4234	.4310	.4386	.4462	.4539	.4616
5	.3417	.3492	.3567	.3642	.3719	.3795	.3872	.3949	.4027	.4106
10	.2467	.2549	.2632	.2716	.2801	.2886	.2972	.3059	.3147	.3235
15	.2228	.2317	.2408	.2499	.2591	.2684	.2777	.2871	.2965	.3060
20	.2147	.2242	.2337	.2433	.2529	.2626	.2723	.2820	.2918	.3016
25	.2118	.2215	.2313	.2411	.2510	.2608	.2707	.2806	.2905	.3004
30	.2107	.2206	.2305	.2404	.2503	.2603	.2702	.2802	.2901	.3001
50	.2100	.2200	.2300	.2400	.2500	.2600	.2700	.2800	.2900	.3000

n	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	1.3100	1.3200	1.3300	1.3400	1.3500	1.3600	1.3700	1.3800	1.3900	1.4000
2	.7429	.7510	.7591	.7673	.7755	.7838	.7920	.8002	.8085	.8167
3	.5584	.5662	.5740	.5818	.5896	.5975	.6053	.6134	.6214	.6293
4	.4694	.4772	.4850	.4929	.5008	.5087	.5167	.5246	.5327	.5408
5	.4185	.4264	.4344	.4424	.4505	.4585	.4667	.4749	.4831	.4913
10	.3323	.3413	.3502	.3593	.3683	.3774	.3866	.3958	.4050	.4143
15	.3155	.3250	.3347	.3443	.3539	.3636	.3733	.3831	.3928	.4026
20	.3114	.3213	.3311	.3410	.3509	.3608	.3707	.3806	.3905	.4005
25	.3104	.3203	.3303	.3402	.3502	.3602	.3701	.3801	.3901	.4001
30	.3101	.3201	.3301	.3401	.3500	.3600	.3700	.3800	.3900	.4000



APPENDIX Sinking Fund Factors

$$SFF = \frac{i}{(1+i)^n - 1}$$

$$R = [SFF]S$$

$$S = [UCAF]R$$

$$UCAF = \frac{1}{SFF}$$

n	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4975	.4950	.4926	.4902	.4878	.4854	.4831	.4808	.4785	.4762
3	.3300	.3268	.3235	.3203	.3172	.3141	.3111	.3080	.3051	.3021
4	.2463	.2426	.2390	.2355	.2320	.2286	.2252	.2219	.2187	.2155
5	.1960	.1922	.1884	.1846	.1810	.1774	.1739	.1705	.1671	.1638
10	.0956	.0913	.0872	.0833	.0795	.0759	.0724	.0690	.0658	.0627
15	.0621	.0578	.0538	.0499	.0463	.0430	.0398	.0368	.0341	.0315
20	.0454	.0412	.0372	.0336	.0302	.0272	.0244	.0219	.0195	.0175
25	.0354	.0312	.0274	.0240	.0210	.0182	.0158	.0137	.0118	.0102
30	.0287	.0246	.0210	.0178	.0151	.0126	.0106	.0088	.0073	.0061
50	.0155	.0118	.0089	.0066	.0048	.0034	.0025	.0017	.0012	.0009

n	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4739	.4717	.4695	.4673	.4651	.4630	.4608	.4587	.4566	.4545
3	.2992	.2963	.2935	.2907	.2880	.2853	.2826	.2799	.2773	.2747
4	.2123	.2092	.2062	.2032	.2003	.1974	.1945	.1917	.1890	.1863
5	.1606	.1574	.1543	.1513	.1483	.1454	.1426	.1398	.1371	.1344
10	.0598	.0570	.0543	.0517	.0493	.0469	.0447	.0425	.0405	.0385
15	.0291	.0268	.0247	.0228	.0210	.0194	.0178	.0164	.0151	.0139
20	.0156	.0139	.0124	.0110	.0098	.0087	.0077	.0068	.0060	.0054
25	.0087	.0075	.0064	.0055	.0047	.0040	.0034	.0029	.0025	.0021
30	.0050	.0041	.0034	.0028	.0023	.0019	.0015	.0013	.0010	.0008
50	.0006	.0004	.0003	.0002	.0001	.0001	.0001	.0000	.0000	.0000

n	21%	22%	23%	24%	25%	26%	27%	28%	29%	30%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4525	.4505	.4484	.4464	.4444	.4425	.4405	.4386	.4367	.4348
3	.2722	.2697	.2672	.2647	.2623	.2599	.2575	.2552	.2529	.2506
4	.1836	.1810	.1785	.1759	.1734	.1710	.1686	.1662	.1639	.1616
5	.1318	.1292	.1267	.1242	.1218	.1195	.1172	.1149	.1127	.1106
10	.0367	.0349	.0332	.0316	.0301	.0286	.0272	.0259	.0247	.0235
15	.0128	.0117	.0108	.0099	.0091	.0084	.0077	.0071	.0065	.0060
20	.0047	.0042	.0037	.0033	.0029	.0026	.0023	.0020	.0018	.0016
25	.0018	.0015	.0013	.0011	.0009	.0008	.0007	.0006	.0005	.0004
30	.0007	.0006	.0005	.0004	.0003	.0003	.0002	.0002	.0001	.0001
50	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

n	31%	32%	33%	34%	35%	36%	37%	38%	39%	40%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.4329	.4310	.4292	.4274	.4255	.4237	.4219	.4202	.4184	.4167
3	.2484	.2462	.2440	.2418	.2397	.2376	.2355	.2334	.2314	.2294
4	.1594	.1572	.1550	.1529	.1508	.1487	.1467	.1447	.1427	.1408
5	.1085	.1064	.1044	.1024	.1005	.0986	.0967	.0949	.0931	.0914
10	.0223	.0212	.0202	.0192	.0183	.0174	.0166	.0158	.0150	.0143
15	.0055	.0051	.0046	.0043	.0039	.0036	.0033	.0031	.0028	.0026
20	.0014	.0012	.0011	.0010	.0009	.0008	.0007	.0006	.0005	.0005
25	.0004	.0003	.0003	.0002	.0002	.0002	.0001	.0001	.0001	.0001
30	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000

EPILOGUE

A Closing Reflection on the Minimum Cost Criterion



"Gentlemen—we've achieved the ultimate in cost reduction . . . we've gone out of business!"

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