Division of Research
School of Business Administration

January 1989

ON THE USE OF STRUCTURAL EQUATION MODELS IN EXPERIMENTAL DESIGNS

Working Paper #594

Richard P. Bagozzi
and
Youjae Yi
The University of Michigan

On the Use of Structural Equation Models

in Experimental Designs

Abstract

New procedures are developed and illustrated for the analysis of experimental data with particular emphasis placed on MANOVA and MANCOVA designs. The paper begins with one-way designs, including overall tests of significance, step-down analyses, and the use of latent variables. Next a general test of homogeneity is described and a procedure considered that is applicable even under heterogeneity conditions. Two-way designs are then derived as special cases of the more general n-way case. Finally, advantages and disadvantages of the new methods are considered.

*Richard P. Bagozzi is the Dwight F. Benton Professor of Marketing and Behavioral Science in Management and Youjae Yi is Assistant Professor of Marketing, School of Business Administration, The University of Michigan, Ann Arbor, MI 48109-1234.

The authors would like to thank the Editor and three anonymous JMR reviewers for their helpful comments on a previous version of the article. The financial assistance of The University of Michigan's School of Business Administration is also gratefully acknowledged.

INTRODUCTION

Structural equation models have been used extensively in substantive research in the social sciences for well over a decade. In marketing, scarcely an issue passes by in the major journals without one or more articles employing structural equation models. Indeed, it is safe to say that structural equation models are a mainstay of multivariate statistical analysis in marketing (e.g., Fornell 1987).

Nevertheless, virtually all the studies to date using these methods have occurred in nonexperimental, survey contexts. Various authors over the years have discussed the use of structural equation models for experimental data (e.g., Alwin and Tessler 1974; Bagozzi 1977; Bray and Maxwell 1982), but few applications are to be found.[1] Why is this so?

One reason may be that the procedures are relatively new and time is needed to overcome the inertia associated with the traditional methods, ANOVA and MANOVA. We think the problem, however, is deeper seated than this. Previous expositions have tended to address primitive and less useful models, not to consider the formal specifications needed to perform analyses, and to neglect a full accounting of why one should consider the use of structural equation models in experimental research. For example, Alwin and Tessler (1974) and Bagozzi (1977) treat only one-way ANOVA analyses and the case where the independent variable is interval-like with enough categories to ensure robustness of the estimation procedures.[2] Bray and Maxwell (1982, 1985) suggest that path analysis can be used for MANOVA designs but limit discussion to introductory remarks and rudimentary causal diagrams. Finally, in neither the social science nor marketing literatures does one find a clear rationale for preferring structural equation models to traditional analyses of experimental data.

The purpose of this paper is to formally develop and illustrate appropriate structural equation models for various experimental designs.[3] We begin with a recent exposition of a one-way MANOVA proposed by Kühnel (1988) and then extend it to accommodate a number of useful MANOVA designs. Next, we introduce a model for one-way MANCOVA analyses. Following this, a structural equation procedure is shown for testing the assumption of homogeneity in variances and covariances of multiple dependent variables, and a method for relaxing the homogeneity assumption is considered. The two-way MANOVA design is then derived. Finally, the assumptions, advantages, and disadvantages of the new methods are discussed. Throughout the presentation of experimental designs, examples are provided using data derived from real experiments.

ONE-WAY MANOVA

Basic Design

In the usual MANOVA design, one desires to simultaneously test mean differences across two or more groups on two or more dependent variables. The main advantage that MANOVA provides over ANOVA is the ability to a) control the overall alpha level at a chosen value, b) test mean differences in the dependent variables simultaneously while controlling for their interdependencies, and c) consider the relationships among the dependent variables, rather than examining each of them in isolation (e.g., Bock 1975; Bray and Maxwell 1985).

The analysis of MANOVA designs can be accomplished with structural equation models but requires a reparameterization of the specifications common to most statistical packages. The objective is to create a system of equations analagous to interconnected dummy variable regressions. The reparameterization expresses hypotheses among manifest, as opposed to latent, variables and focuses upon the means of, instead of covariances among, measures so as to properly analyze experimental effects.

Building on Sörbom's (1982) specification of structured means in simultaneous equation systems, Kühnel (1988) recently showed how programs such as LISREL can be used to test one-way MANOVA designs. Figure 1 presents our adaptation of Kühnel's model applied to the case where three dependent variables are present for two groups: experimental and control groups. We have used the notation common to LISREL (Jöreskog and Sörbom 1986), but a similar specification is possible with EQS (Bentler 1985).

[Figure 1 about here]

Notice that the experimental manipulation and control groups are represented with a dummy variable which is expressed as an exogenous latent variable (ξ_1) with a single indicator and no corresponding residual. This effectively transforms the latent variable into a manifest variable. The dummy variable could be a 0,1 indicator representing two groups (e.g., experimental and control groups), or several dummy variables could be used to represent n groups in the more general case. Note further that a pseudovariable (i.e., "one") is shown as an indicator of a second latent variable (ξ_2). The pseudovariable is a constant added either to the sample moment matrix as another variable having 1 in the diagonal and the means of all other variables as off-diagonal elements[4] or to the raw data as a column of 1s, which is needed for computing the correct likelihood function and standard errors of estimates (Sörbom 1974). This specification permits one to analyze the means of observed dependent variables as a function of the categorical indepedent variables. To do this, the augmented moment matrix must be analyzed and not the more usual correlation or covariance matrices. Because the moments will be sums of covariances and products of means, the parameter ϕ, shown in Figure 1 as the association between the dummy variable and the constant term, will be in general nonzero.

To test the multivariate null hypothesis of equality in means of the dependent variables across groups versus the alternative hypothesis that one or more groups have a mean different from the others, we can use the likelihood ratio chi square tests provided by the maximum likelihood or generalized least squares procedures in LISREL or EQS. The full model as specified in Figure 1 will be exactly identified and thus will fit any data perfectly (i.e., $\chi^2(0) = 0.00$, $p \approx 1.00$). A test of the null hypothesis analogous to the omnibus tests commonly used in traditional MANOVA analyses (e.g., the Pillai's V or Wilks' Λ) consists of an examination of the paths from the dummy exogenous variable to the dependent variables. That is, γ_1*, γ_2*, and γ_3* are the differences in the means of dependent variables between the two groups. In particular, if the means of the dependent variables are equal across groups, then $\gamma_1* = \gamma_2* = \gamma_3* = 0$. Thus, by testing the model of Figure 1 with these constraints imposed, we obtain an overidentified model. The difference in chi-square values between the overidentified and full models provides a test of the null hypothesis. A significant chi-square difference suggests that the mean vectors of dependent variables are different across groups, whereas a nonsignificant value implies a failure to reject the null hypothesis of equal means.

An Illustration

To demonstrate the use of structural equation models in MANOVA designs, we applied the procedures to data derived from an experiment on decision making (Bagozzi, Yi, and Baumgartner 1988). In this experiment, the authors desired to create conditions where choice behaviors would be directly and indirectly influenced by one's preferences. It was predicted that, under low impedance conditions where little effort and planning are needed, choices would be directly affected by preferences with little or no impact of an explicit volition (i.e., decision). Under high impedance conditions, in contrast, explicit decisions are required to overcome obstacles, and therefore preferences were expected to influence choices primarily through their effect on these decisions. In the current context, one question might be: What are the effects of impedance on choice behavior? That is, are choice behaviors different between low and high impedance groups? The three behavior measures represented alternative self-report and indicators of actual choices of the decision makers. Two different measures of decisions were obtained as well. Table 1 presents the means, standard deviations, and within-group correlations for the data.

First, the SPSS-X program was used for the traditional MANOVA analysis. We have chosen to use the test for Wilks' Λ because it is the most frequently employed statistic of four commonly found in literature. It might be argued that one should use Pillai's V because of its robustness to violations of assumptions (Olson 1976). However, in this and subsequent analyses, the F-tests based on these statistics are equal, although they will in general be different when three or more groups are examined. The results of this analysis suggest a rejection of the null hypothesis that the means of choice behaviors are equal across the low and high impedance groups: Wilks' $\Lambda = .727$, $F(3, 148) = 18.56$, $p < .001$.

Next, the LISREL program was employed for the structural equation analysis of the same data. The full model allowing for the differences in means, as specified in Figure 1, is exactly identified and gives a perfect fit to the data: $\chi^2(0) = 0.00$, $p = 1.00$. The restricted model, constraining the mean difference parameters to zero (i.e., $\gamma_1* = \gamma_2* = \gamma_3* = 0$), gives the following results: $\chi^2(3) = 48.21$, $p < .001$. These findings suggest that the null hypothesis of equal means be rejected. Therefore, the impedance manipulation affected significantly the behavior measures in one or more instances. In fact, the estimates for the mean difference were all significant: γ_1* = 3.84 (5.7), γ_2* = 1.42 (6.7), and γ_3* = 3.54 (7.2), with the _t_-values in parentheses. Note that these are equal to the mean differences revealed in the bottom of Table 1.

[Table 1 about here]

Extension #1: Latent Variable MANOVA

Kühnel (1988) considered only models where each variable has one and only one measure, and in this sense his approach is equivalent to the traditional analysis of MANOVA designs. We wish to extend the structural equation approach to accommodate multiple measures of one or more criteria. This represents a straightforward use of LISREL and is motivated by three considerations. First, if individual measures of the variables show excessive random error, the tests may be too lacking in power to detect valid experimental effects. Second, certain criterion variables may, in fact, be theoretically indicated by two or more measures. For example, measured variables might reflect variation in an underlying theoretical construct which is inherently unobservable. Third, one's goal may be more concerned with explanation and understanding of latent variables or constructs as opposed to prediction and description of observed variables or measures, per se. That is, one may desire to test hypotheses implied by a theory containing latent variables as opposed to focusing directly on individual measures. Each of these issues leads to a need for taking into account multiple measures of constructs and possible causal orderings among constructs. We focus upon the former issue in this subsection and treat the latter in the next.

Figure 2 presents a diagram of a simple latent structural equation MANOVA model appropriate to the data shown in Table 1. Here we have interpreted the three behavioral measures as indicators of a single, latent dependent variable.[5] The hypothesis is that the experimental manipulation affects the mean of the underlying theoretical construct as measured by three indicators. Specifically, γ_1*, is now difference in the means of the behavioral construct (η). The full model in Figure 2 gave the following goodness-of-fit measures: $\chi^2(4) = 5.77$, $p \approx .22$. It can be noted that the full model is not exactly identified, but overidentified (cf. Figure 1). The restricted model with the constraint (i.e., $\gamma_1^* = 0$) yielded the following results: $\chi^2(5) = 48.89$, $p \approx .00$. Performing the proper test, we find that one must reject the hypothesis of equal means of η across the two groups ($\chi^2_d(1) = 43.12$, $p < .001$). Indeed the estimate of γ_1* is 3.21 with $t = 6.43$. Thus, the experimental manipulation produced its hypothesized effect.

We have so far examined two types of MANOVA analyses: MANOVA on manifest variables (Figure 1) and MANOVA on latent variables (Figure 2). One benefit of the latter approach would be taking into consideration measurement errors explicitly in analyses. Notice that the mean difference of η, as revealed by γ_1* in Figure 2 (latent MANOVA), is greater than the average of the sums of $\gamma_1^* - \gamma_3^*$ in Figure 1 (manifest MANOVA), showing the gain due to a correction for attenuation as a consequence of random error. We should stress that such analyses are not possible with traditional MANOVA procedures.

[Figure 2 about here]

Extension #2: Step-down Analyses

Kühnel (1988) considered only the chi-square test analogue to the omnibus MANOVA tests. Of course, finding that differences do occur in the means for different groups is only the first step in any analysis. A second and essential step is that one explains the differences. Given that an ominibus test has indicated the difference in means, one may wish to determine which of the dependent variables are responsible for the global significance. An inspection of the γ parameters linking the dummy variable to the dependent variables is useful for uncovering which criteria the manipulation(s) affected. This test of statistical significance for each parameter individually is equivalent to the univariate ANOVA on the dependent variable.

An even more powerful and insightful breakdown that is useful to perform when there is an a priori ordering of the dependent variables is the step-down analysis. In the classic step-down analysis (e.g., Roy 1958; Stevens 1973), each dependent variable is analyzed in succession in order to control for a causal ordering among them. Step-down analyses provide useful information, since they test whether variation in a certain dependent variable is due to a direct association with the manipulation or due to its dependence on other dependent variables.

The first stage of step-down analysis begins with a MANOVA test performed on all dependent variables. If the omnibus test points to a rejection of equal means, then the final variable in a hypothesized chain of dependent variables is tested with the variance due to all remaining dependent variables partialled out as covariates. A significant omnibus test here suggests that the final criterion differs across groups, even controlling for the effects of the other criteria as covariates. If this happens, the testing stops, as no further unconfounded testing is possible. In contrast, a nonsignificant omnibus test signals that the final criterion does not differ significantly across groups, after controlling for the other criteria. Therefore, the difference in the final criterion, if any, is wholly due to the causal relations between the final criterion and the other criteria. If this happens, one moves backward in the hypothesized chain to the preceding criterion and performs a similar test. The process stops again when a significant omnibus test results.

The structural equation approach to MANOVA can be used to perform step-down analyses. To illustrate, we applied the procedures to the data in Table 1 where 5 dependent variables are now considered: decision measures 1 & 2 and behavior measures 1-3. Table 2 shows the findings for the initial stage in the step-down analysis: the omnibus test with all variables included but no causal ordering as yet implied among them. The results lead us to reject the hypothesis of equal means of the dependent variables across the two groups ($\chi_d^2(5) = 51.04$, $p < .001$). Indeed, the groups differ on the means of all five manifest variables (see the bottom of Table 2).

Now that we know that the groups differ on all variables, we can ask whether or not these differences are directly affected by the manipulation or indirectly affected as a result of a causal ordering among the variables. A plausible hypothesis is that decisions ---> behavior. Thus, we would like to test whether or not the groups differ on the behavioral criteria with decisions covaried out. We could do this by treating all five measures as manifest variables, as done in Table 2. But the intention of the experimenters was to treat the two measures of decisions and the three measures of behavior as redundant indicators of the respective constructs. Therefore, MANOVA was conducted on latent variables, rather than on manifest variables.

Figure 3a represents an appropriate null hypothesis for doing this (i.e., $\gamma_1^* = \gamma_2^* = 0$), the first stage in step-down analysis. The findings for the omnibus test of this model are presented in the top half of Table 3 where it can be seen that the hypothesis of equal means for the constructs of decisions and behaviors is again rejected. Note that this test is done on the latent variables, not the manifest variables, because the experimenters were interested in the differences in the means for the constructs between the groups.

Next we desire to test the mean difference after considering the ordering implied by Figure 3b (i.e., $\gamma_2^* = 0$). Then γ_2^* could be interpreted as the effect of the experimental manipulation on behavior when decisions have been controlled for. The results in the bottom of Table 3 reveal that the means of behaviors differ across groups even with the effects of decisions covaried out. Notice that the mean difference in behavior between the two groups has declined from $\gamma_2^* = 3.20$ to $\gamma_2^* = 2.74$ when we go from stage one to stage two. This reflects the dependence of behavior on decisions, which is reflected by the path from decision to behavior (i.e., β in Figure 3b). That is, a portion of the difference in behavior between two groups is explained by the difference in the preceding variable (i.e., the decision), rather than by the experimental manipulation. Nevertheless, the groups still differ significantly in behavior even after taking into account this dependence. It should be noted that the step-down analysis performed herein, which includes a formal correction for random error by using latent variables, cannot be accomplished with the traditional MANOVA procedures.

[Tables 2 & 3 and Figure 3 about here]

MANCOVA

The step-down analysis is, in reality, a special case of the analysis of covariance.[6] We can build upon this property to show that structural equation models can be utilized to perform analyses of the general MANCOVA model. Figure 4 illustrates the case for a one-way analysis with 3 dependent variables and a single covariate, ξ_3. We have shown ξ_3 with only one measure for simplicity, but the accommodation of multiple measures is straightforward.

To illustrate a MANCOVA analysis within the context of a structural equation model, we applied the model in Figure 4 to the data in Table 1. Preferences (X) are taken as the covariate. The findings from SPSS-X analysis show that we must reject the hypothesis of equal means: Wilks' $\Lambda = .715$, $F(3,147) = 19.54$, $p < .001$. The chi-square difference test from structural equation analysis performed on the model in Figure 4 suggested the same conclusion: $\chi_d^2(3) = 50.67$, $p < .001$. The estimates for the gamma parameters reveal that the experimental manipulation affected all three behavioral dependent variables, even after controlling for variation in preferences ($\gamma_1{}^* = 3.77$, $\gamma_2{}^* = 1.40$, $\gamma_3{}^* = 3.48$, with the t-values of 5.77, 6.85, and 7.39, respectively).

[Figure 4 about here]

HOMOGENEITY

Two important assumptions of the traditional MANOVA and MANCOVA procedures are that the dependent variables have a multivariate normal distribution in each group and that the distributions are equal across groups. With respect to the former assumption, the procedures have been shown to be relatively robust to violations of multivariate normality (e.g., Mardia 1971). With respect to the latter assumption, Pillai's V has been found to be fairly robust to violations in equality of distributions but only when sample sizes are equal across groups (e.g., Olson 1976). A test of the equality of variances and covariance can be performed within the context of traditional MANOVA analyses with Box's M test (Norusis 1988).

We performed this test on the data of Table 1 and found that Box's M = 339.07 with $\chi^2(6) = 331.71$ and $p < .001$. Therefore, we must reject the hypothesis that the variance-covariance matrix for the dependent variables is equal across groups. Thus, the assumptions of the traditional MANOVA analysis are violated. What can we do? One could accept on faith that the procedures are robust and interpret Pillai's V accordingly. But strictly speaking, the traditional MANOVA analysis may be misleading. If we were limited to the traditional procedures, not much recourse is available.

Fortunately, structural equation models can be used not only to test the homogeneity assumption, but even if it were rejected, to perform a proper test of experimental effects (Kühnel 1988). The test is appropriate whether or not equal sample sizes occur across groups. To do this, we must reformulate the experimental design as a multiple group analysis of the appropriate moment matrices. Figure 5 illustrates a proper specification for a MANOVA design with the three dependent variables and two groups. Because each ξ variable is defined as 1 through use of a psuedovariable in the moment matrix, it turns out that the dependent variables (x_is) are equal to the sum of the mean for their respective group and error. The variance-covariance matrix for the error terms is, in fact, the variance-covariance matrix for the dependent variables, given this specification. The multiple group specification for the full model shown in Figure 5 is exactly identified. To test for homogeneity, we must specify: $\Theta_\delta^{(1)} = \Theta_\delta^{(2)}$. That is, $\Theta_{\delta 11}^{(1)} = \Theta_{\delta 11}^{(2)}$, $\Theta_{\delta 22}^{(1)} = \Theta_{\delta 22}^{(2)}$, $\Theta_{\delta 33}^{(1)} = \Theta_{\delta 33}^{(2)}$, $\Theta_{\delta 21}^{(1)} = \Theta_{\delta 21}^{(2)}$, $\Theta_{\delta 32}^{(1)} = \Theta_{\delta 32}^{(2)}$, and $\Theta_{\delta 31}^{(1)} = \Theta_{\delta 31}^{(2)}$. This results in 6 overidentifying restrictions, and the test of equal variances and covariances is performed by taking the difference in chi-squares for the full and restricted models.

The multiple group approach shown in Figure 5 can be used to test the MANOVA hypotheses on the means of the dependent variables and is, in fact, equivalent to the dummy variable approach shown in Figure 1 except for one important difference. The dummy variable approach, like the traditional MANOVA analyses, assumes homogeneity. The multiple group approach does not. Indeed, the latter is valid for, and can be used to test, instances where the variances and covariances are totally equal, totally unequal, or partially equal across groups. To test for equality in means by use of the multiple group approach, we fix $\lambda^{(1)} = \lambda^{(2)}$. More specifically, $\lambda_1^{(1)} = \lambda_1^{(2)}$, $\lambda_2^{(1)} = \lambda_2^{(2)}$, and $\lambda_3^{(1)} = \lambda_3^{(2)}$ in Figure 5. This yields three overidentifying restrictions, and again the test of equal means is performed by taking the difference in chi-squares for the full and restricted models.

To illustrate, we applied the model in Figure 5 to the data in Table 1. Table 4 presents the findings. The top half of the table shows the goodness-of-fit tests for the full, equal variance-covariance, equal means, and equal variance-covariance/equal means models, respectively. The bottom half of the table shows the appropriate chi-square difference tests. As revealed in the table, we must reject the assumption of homogeneity in variances and covariances of the three dependent variables ($\chi_d^2(6) = 339.23$, $p < .001$). This is the structural equation model analogue to Box's M test. The hypothesis of equal means across groups is rejected whether we assume homogeneity ($\chi_d^2(3) = 47.88$, $p < .001$) or heterogeneity ($\chi_d^2(3) = 46.26$, $p < .001$).

The above results seem to suggest that the violation of the homogeneity assumption has little effect on the test of mean difference. However, a Monte Carlo study by Kühnel (1988) suggests that when the homogeneity assumption is violated, the traditional MANOVA can provide misleading results. Kühnel investigated two factors (equal versus unequal means and equal versus unequal variances and covariances) in his study and compared the power of the tests. When the variances and covariances were equal, the traditional MANOVA analysis (via SPSS-X) performed well: At the .05 significance level, the test yielded no false decisions as to the mean difference test. The group comparison test (via LISREL) by assuming homogeneity gave the same results. However, when the variances and covariances were not equal, the traditional MANOVA test performed poorly. The rate of Type I error (i.e., rejection of a correct null hypothesis) was 12%, and the Type II error rate (i.e., not rejecting a false null hypothesis) was 74%. When the LISREL test was conducted by allowing for different variances and covariances, the error rates were greatly reduced: 6% and 4% for Type I and Type II errors, respectively. Although more research is needed for generalization, these results suggest that structural equation model analyses can provide more powerful tests of mean differences, especially when the homogeneity assumption is violated.

It might appear that the multiple group approach is to be always preferred to the dummy variable approach because of its ability both to test for homogeneity and to apply to heterogeneous as well as homogeneous contexts. However, because programs such as EQS cannot perform multiple group analyses, the dummy variable approach may sometimes be the only way to conduct analyses (LISREL does have multiple group capabilities, however). Moreover, for complex designs with many interactions, the dummy variable approach is more convenient.

[Figure 5 and Table 4 about here]

TWO-WAY MANOVA

Higher order experimental designs offer advantages in economy and power. In this section, we will introduce a general structural equation approach for the analysis of two-way MANOVA designs. The extension to the n-way case follows a parallel development.

As with the one-way design, two approaches can be taken: the dummy variable method or the multiple group method. For a two-way design, one can employ two dummy variables to capture main effects and a product term of the two dummy variables for the interaction effects. The dummy variable approach requires that nonlinear constraints be imposed on the variance-covariance matrix for the entire model. The variance of the product of two variables which will in general be a function of the sums of products of means, variances, and covariances of the two variables (Goodman 1960). Current versions of LISREL cannot accommodate nonlinearity constraints, but COSAN can (Fraser 1980). The theoretical model underlying COSAN is described by McDonald (1978, 1980). Kenny and Judd (1984) provide an illustration of a simple two-way interaction using COSAN and simulated data but not for a MANOVA analysis. Given the complexity of the COSAN specification and the greater availability of LISREL, we have chosen to focus herein upon the multiple group approach which avoids the necessity of representing nonlinear constraints.[7]

Figure 6 displays a multiple group representation of the two-way MANOVA design. Here we have assumed that two levels of each of two independent variables are manipulated and two dependent variabels are measured. This yields four groups in total, but by expressing one of the independent variables as a dummy separately for each of two groups, we have the model shown in Figure 6. The hypothesis of no interaction effect between the two variables can be tested by comparing the full model shown in Figure 6 to the restricted model with the following constraints: $\gamma_1^{(1)*} = \gamma_1^{(2)*}$; $\gamma_2^{(1)*} = \gamma_2^{(2)*}$. That is, the effects of one independent variable on the dependent variables do not vary with the level of another independent variable. The difference in chi-square tests between the two models will be distributed chi-square with corresponding degrees of freedom equal to the differences in degrees of freedom for the models. To test for main effects, we compare the model with no interaction (i.e., $\gamma_1^{(1)*} = \gamma_1^{(2)*}$; $\gamma_2^{(1)*} = \gamma_2^{(2)*}$) to the model with no main effects and no interaction (i.e., $\gamma_1^{(1)*} = \gamma_1^{(2)*} = \gamma_2^{(1)*} = \gamma_2^{(2)*} = 0$). Again the difference in chi-square values for the two models provides the test statistic.

[Figure 6 about here]

To illustrate the procedures, we applied the model in Figure 6 to data obtained from a second experiment in decision making (Bagozzi, forthcoming).

In this experiment, low and high involvement and low and high emotions were manipulated to see their effects on two decisions: the assessment of the probability of an event occurring to the subjects and their evaluation of the event. Without going into the theory behind the hypotheses, it was predicted that a high probability and a favorable evaluation would occur only when both involvement and emotions are high. Thus, interactions between involvement and emotions are predicted for both dependent variables.

Table 5 presents a summary of the data for the analysis via the model shown in Figure 6. Before we apply this model to the data, however, we would like to test the homogeneity assumption. To do this, we performed a multiple group test of homogeneity with two dependent variables, analogous to the model displayed in Figure 5. According to this test by structural equation models, we cannot reject the hypothesis of homogeneity $\chi^2_d(9) = 14.17$, $p \approx .12$). This result compares favorably with the Box's M test by SPSS-X (Box's M = 14.15, $\chi^2(9) = 13.82$, $p \approx .13$).

First, the two-way MANOVA analyses were conducted by the traditional approach with SPSS-X. The results show that the main effects of emotion and involvement are significant: Wilk's $\Lambda = .951$, $F(2,155) = 3.98$, $p \approx .02$ (Emotion); Wilks' $\Lambda = .947$, $F(2,155) = 4.35$, $p \approx .02$ (involvement). Furthermore, the interaction effect between emotion and involvement is significant: Wilks' $\Lambda = .960$, $F(2,155) = 3.27$, $p \approx .04$.

The two-way MANOVA analyses by use of structural equation models are shown in Tables 6 and 7. Looking first at Table 6, which shows the results for the model with high/low emotion as the dummy variable and high and low involvement as the two groups (see Figure 6), we find that a significant interaction occurs, as hypothesized ($\chi^2_d(2) = 6.51$, $p \approx .04$). Compare this finding to the corresponding results from the traditional analysis shown earlier ($F(2,155)$ = 3.27, $p \simeq .04$). Notice further that the highest mean occurs for the high emotion/high involvement condition. The effect of emotion is 2.05 in the high involvement condition (see bottom of Table 6). Recall that this is the mean difference above the baseline which in this case fixes γ_1 in the low emotion condition to 0 in both groups. Similarly, Table 7 presents the findings for the model with high/low involvement as the dummy variable and high and low emotion as the two groups. As before, we discover a significant involvement x emotion interaction ($\chi_d^2(2) = 6.51$, $p \simeq .04$).

[Tables 5, 6 and 7 about here]

We have introduced and illustrated a structural equation approach to two-way MANOVA designs. A similar development applies to the two-way MANCOVA, paralleling the presentation made for the one-way case. Further, n-way MANOVA and MANCOVA analyses can be performed with structural equation models. The models represent straightforward extensions of those considered herein.

DISCUSSION

This study has considered structural equation models as an alternative to traditional analyses of MANOVA and MANCOVA. We began with a specification of the one-way MANOVA recently proposed by Kühnel (1988) and illustrated it on set of experimental data from a marketing study. The model was extended to accommodate latent variables and to perform step-down analyses. Next the general MANCOVA design was treated. Following this, a test for homogeneity was investigated, and a test of mean difference even under heterogeneity was derived. Finally, the case of the two-way MANOVA was developed.

The advantages of the structural equation approach over traditional analyses are the following. First, the new procedures are more general and do not make the restrictive assumption of homogeneity in variances and covariances of the dependent variables across groups. The procedures can handle the cases where homogeneity assumptions are violated. Thus, the procedures overcome a fundamental limitation in current methods of analysis. Second, the new procedures provide a natural way to correct for measurement error in the measures of variables and thus reduce the chances of making Type II errors. Standard analyses assume that measurement error is negligible. To the extent to which such assumptions are violated, the new procedures will be useful. Third, structural equation models allow for a more complete modeling of theoretical relations, whereas traditional analyses are limited to associations among measures. Fourth, covariates in step-down and MANCOVA analyses can be treated as latent variables with the new procedures, thereby permitting a correction for attenuation and increasing the chances that valid experimental effects will be detected. The traditional procedures cannot take into account measurement error in covariates. Finally, structural equation models constitute flexible, convenient procedures. They not only perform tests of experimental effects and homogeneity but are special cases of very general programs and easily implemented.

In the way of caveats, we note the following. Structural equation models assume that the dependent variables are multivariate normal. A similar assumption is made by the traditional procedures. However, new developments in asymptotic distribution free estimation, now available with LISREL and EQS, make this assumption unnecessary. Nevertheless, to take advantage of these developments, one needs a large sample. This is not always feasible in experimental designs. Finally, as the number of factors, the number of levels within factors, and the number of measures increase, the number of free parameters to estimate increases and the chances for improper solutions or nonconvergence will increase as well.

An examination of the above arguments suggests the conditions under which the use of structural equation models is appropriate and useful in experimental designs. First, when several theoretical constructs underlie dependent variables, one may focus on these latent variables by use of structural equation models. Second, when basic measurements tend to be quite unreliable individually, the structural equation method may be useful. Third, if one knows a priori theoretical relations among the dependent variables, one may explicitly incorporate these relations into analysis. Fourth, structural equation analysis will provide more powerful tests when consequences of violating the homogeneity assumption are serious, as when sample sizes are quite unequal across groups. Finally, it is appropriate when the sample size is large enough for convergence and proper solutions in estimation.

Marketers have used the experimental method to advantage over the years in consumer behavior, advertising, sales management, and other areas of research. Up until now, however, analyses have relied exclusively on classic ANOVA or MANOVA methods. We have shown that structural equation procedures can be used to test the same hypotheses that can be addressed by the traditional methods yet do so while taking into account measurement error and not making the restrictive assumption of homogeneity. The new procedures also offer advantages with respect to the scope of hypotheses that can be investigated and under certain conditions can even be performed without assuming multivariate normality. It is our hope that future experimental research in marketing will utilize structural equation models and thereby increase the power of analyses.

Footnotes

1. Some researchers in marketing have used structural equation models to analyze data derived from experiments (e.g., MacKenzie, Lutz, and Belch 1986; Ryan 1982). However, rather than employing the procedures to conduct analyses of experimental effects, these authors performed causal analyses on aggregate samples formed by collapsing across all cells in their designs. The validity of these approaches rests on the assumptions that the measurement properties and causal paths are invariant across cells. Because experimental manipulations are designed to influence one or more variables and employ different stimuli to do this, it is unlikely that the required invariances will hold when collapsing across cells. Further, information on the means of variables was ignored by focusing on covariances only (see also Bearden and Shimp 1982). In any event, these studies have not used structural equation models in the senses developed in this paper (i.e., to perform analyses of experimental effects on the means using analogues to and generalizations of ANOVA and MANOVA).

2. To be fair, we should note that the programs available at the time of publication of these articles did not permit multiple group analyses; therefore, analyses of unordered category designs could not be performed because of technological limitations.

3. Although our derivations and illustrations focus upon MANOVA and MANCOVA designs, it should be pointed out that ANOVA and ANCOVA are special cases and can be analyzed in a manner parallel to the development following in the present paper.

4. This is called "the augmented moment matrix," and it is the sample moment matrix when the constant of 'one' has been added as the last variable for every sample unit. The augmented moment matrix is needed whenever the

model involves intercept terms or means of variables (Jöreskog and Sörbom 1986, V.16).

5. Strictly speaking, Figure 2 is a hybrid model sharing features of the classic ANOVA and MANOVA representations. It is similar to ANOVA if we regard η as a single latent dependent variable. Yet it is similar to MANOVA if we focus on $y_1 - y_3$ as three manifest dependent variables. We might have labelled this (M)ANOVA to differentiate it from ANOVA and MANOVA, but have refrained from doing so in the paper so as not to introduce an additional point of confusion. We would like to point out that, if two or more endogenous latent variables occur, each with multiple measures, then the approach can unambiguously be labelled a structural equation model of MANOVA. We treat such a model in the following subsection.

6. The step-down analysis is a type of ANCOVA in that covariates are used for analysis of variance. But it is a special case in that some of the dependent variables are used as covariates in explaining other dependent variables, whereas the usual (M)ANCOVA uses some of the independent variables for explaining the dependent variable(s).

7. Wothke and Browne (1988) have recently provided a linear formulation of nonlinear constraints by reparameterizing the direct product model for the multitrait-multimethod (MTMM) matrix as a second order factor analysis. However, their procedures are suitable for correlation matrices, and it is unclear at this point how such procedures can be modified to analyze the mean differences, a focus of our study. We thank an anonymous reviewer for bringing our attention to Wothke and Browne (1988)'s work.

REFERENCES

Alwin, Duane F. and Richard C. Tessler (1974), "Causal Models, Unobserved Variables, and Experimental Data," American Journal of Sociology, 80 (July), 58-86.

Bagozzi, Richard P. (1977), "Structural Equation Models in Experimental Research," Journal of Marketing Research, 14 (May), 209-26.

_________ (forthcoming), "An Investigation of the Role of Affective and Moral Evaluations in the Purposeful Behavior Model of Attitude," British Journal of Social Psychology.

_________, Youjae Yi, and Johann Baumgartner (1988), "The Level of Effort Required for Behavior as a Moderator of the Attitude-Behavior Relation," unpublished working paper, The University of Michigan.

Bearden, William O. and Terence A. Shimp (1982), "The Use of Extrinsic Cues to Facilitate Product Adoption," Journal of Marketing Research, 19 (May), 229-39.

Bentler, Peter M. (1985), Theory and Implementation of EQS, A Structural Equations Program, Los Angeles, CA: BMDP Statistical Software.

Bock, R. Darrell (1975), Multivariate Statistical Methods in Behavioral Research, New York: McGraw-Hill.

Bray, James H. and Scott E. Maxwell (1982), "Analyzing and Interpreting Significant MANOVAs," Review of Educational Research, 52 (Fall), 340-67.

_________ (1985), Multivariate Analysis of Variance, Beverly Hills, CA: Sage.

Fornell, Claes (1987), "A Second Generation of Multivariate Analysis: Classification of Methods and Implications for Marketing Research," in Review of Marketing 1987, Michael J. Houston, ed. Chicago, IL: American Marketing Association, 407-50.

Fraser, Colin (1980), COSAN User's Guide. Centre for Behavioral Studies, University of New England Armidale, New South Wales, Austrailia.

Goodman, Leo A. (1960), "On the Exact Variance of Products," Journal of the American Statistical Association, 55 (September), 708-13.

Jöreskog, Karl G. and Dag Sörbom (1986), LISREL VI: Analysis of Linear Structural Relationships by Maximum Likelihood, Instrumental Variables, and Least Squares Methods, 4th ed., Mooresville, IN: Scientific Software.

Kenny, David A. and Charles M. Judd (1984), "Estimating the Nonlinear and Interactive Effects of Latent Variables," Psychological Bulletin, 96 (1), 201-10.

Kühnel, Steffen M. (1988), "Testing MANOVA Designs with LISREL," Sociological Methods & Research, 16 (May), 504-23.

MacKenzie, Scott B., Richard J. Lutz, and George E. Belch (1986), "The Role of Attitude Toward the Ad as a Mediator of Advertising Effectiveness: A Test of Competing Explanations," Journal of Marketing Research, 23 (May), 130-43.

Mardia, K. V. (1971), "The Effects of Nonnormality on Some Multivariate Tests and Robustness to Nonnormality in the Linear Model," Biometrika, 58 (April), 105-21.

McDonald, Roderick P. (1978), "A Simple Comprehensive Model for the Analysis of Covariance Structures," British Journal of Mathematical and Statistical Psychology, 31 (May), 59-72.

_________ (1980), "A Simple Comprehensive Model for the Analysis of Covariance Structures: Some Remarks on Applications," British Journal of Mathematical and Statistical Psychology, 33 (November), 161-83.

Norusis, Marija J. (1988), SPSS/PCT Advanced Statistics V2.0, Chicago, IL: SPSS.

Olson, Chester L. (1976). "On Choosing a Test Statistic in Multivariate Analysis of Variance," Psychological Bulletin, 83(4), 579-86.

Roy, J. (1958), "Step-down Procedure in Multivariate Analysis," Annals of Mathematical Statistics, 29 (December), 1177-87.

Ryan, Michael J. (1982), "Behavioral Intention Formation: The Interdependency of Attitudinal and Social Influence Variables," Journal of Consumer Research, 9 (December), 263-78.

Sörbom, Dag (1974), "A General Method for Studying Differences in Factor Means and Factor Structure Between Groups," British Journal of Mathematical and Statistical Psychology, 27 (November), 229-39.

__________ (1978), "An Alternative to the Methodology for Analysis of Covariance," Psychometrika, 43 (3), 381-96.

__________ (1982), "Structural Equation Models with Structured Means," in Systems Under Indirect Observations, Karl J. Jöreskog and Herman Wold, eds. Amsterdam: North-Holland, 183-95.

Stevens, James P. (1973), "Step-down Analyses and Simultaneous Confidence Intervals in MANOVA," Multivariate Behavioral Research, 8 (July), 391-402.

Wothke, Werner and Michael W. Browne (1988), "The Direct Product Model for the MTMM Matrix Parameterized as a Second Order Factor Analysis Model," unpublished working paper.

Table 1

DATA FOR ONE-WAY MANOVA AND MANCOVA EXAMPLES

Measure	High Impedance Group (n = 73)						Low Impedance Group (n = 79)					
Behavior 1	1.000						1.000					
Behavior 2	.774	1.000					.641	1.000				
Behavior 3	.736	.945	1.000				.580	.921	1.000			
Decision 1	.256	.425	.430	1.000			.255	.173	.171	1.000		
Decision 2	.263	.426	.430	.907	1.000		.263	.205	.181	.882	1.000	
Preference	.082	.190	.209	.325	.263	1.000	.310	.343	.344	.316	.271	1.000
Mean	.206	.274	.548	4.027	3.932	14.16	4.050	1.696	4.089	4.899	4.760	14.48
Std. Dev.	.726	.838	1.633	1.462	1.456	4.09	5.686	1.620	3.877	1.446	1.398	4.51

Table 2

FINDINGS FOR STEP-DOWN ANALYSIS OF MANOVA APPLIED TO DATA IN TABLE 1 TAKING TWO DECISION AND THREE BEHAVIORAL MEASURES AS DEPENDENT VARIABLES: THE INITIAL STAGE

Traditional MANOVA Analysis

Test	Value	*F*	df	Error df	*p*
Pillai's V	.287	11.74	5	146	.000
Wilks' Λ	.713	11.74	5	146	.000

Analysis by Structural Equation Methods

Full Model	Model with $\gamma_1^* = \gamma_2^* = \gamma_3^* = \gamma_4^* = \gamma_5^* = 0$
$\chi^2(0) = 0.00$	$\chi^2(5) = 51.04$
$p \simeq 1.00$	$p \simeq 0.000$
	Hence:
$\gamma_1^* = 0.87\ (3.69)^a$	
$\gamma_2^* = 0.83\ (3.58)$	$\chi_d^2(5) = 51.04$
$\gamma_3^* = 3.84\ (5.73)$	$p \simeq 0.000$
$\gamma_4^* = 1.42\ (6.72)$	
$\gamma_5^* = 3.54\ (7.23)$	

[a] *t*-values in parentheses.

Table 3

FINDINGS FOR STEP-DOWN ANALYSIS OF MANOVA APPLIED TO DATA IN TABLE 1 WITH DECISION AND BEHAVIORAL CONSTRUCTS AS DEPENDENT VARIABLES: THE FIRST AND SECOND STEP-DOWN STAGES

First Stage	
Full Model	Model with $\gamma_1^* = \gamma_2^* = 0$
$\chi^2(10) = 7.88$	$\chi^2(12) = 53.85$
$p \simeq 0.64$	$p \simeq 0.000$
$\gamma_1^* = 0.86\ (3.76)^a$	Hence:
$\gamma_2^* = 3.20\ (6.42)$	$\chi_d^2(2) = 45.97$
	$p \simeq 0.000$
Second Stage	
Full Model	Model with $\gamma_2^* = 0$
$\chi^2(10) = 7.88$	$\chi^2(11) = 39.57$
$p \simeq 0.64$	$p \simeq 0.000$
$\gamma_1^* = 0.86\ (3.76)^a$	Hence:
$\gamma_2^* = 2.74\ (5.56)$	$\chi_d^2(1) = 31.69$
	$p \simeq 0.000$

[a] *t*-values in parentheses.

Table 4

FINDINGS FOR MANOVA APPLIED TO DATA FOR TABLE 1 TAKING THREE BEHAVIORAL MEASURES AS THE DEPENDENT VARIABLE: ALTERNATIVE TEST BASED ON MULTIPLE GROUPS AND A TEST OF HOMOGENEITY

Model	Goodness-of-fit
A: Full, all parameters free	$\chi^2(0) = 0$, $p \simeq 1.00$
B: Equal Variance-Covariance ($\underset{\sim}{\Theta}_\delta^1 = \underset{\sim}{\Theta}_\delta^2$)	$\chi^2(6) = 339.23$, $p \simeq 0.000$
C: Equal Means ($\underset{\sim}{\lambda}^1 = \underset{\sim}{\lambda}^2$)	$\chi^2(3) = 46.26$, $p \simeq 0.000$
D: Equal Means and Variance-Covariance ($\underset{\sim}{\Theta}_\delta^1 = \underset{\sim}{\Theta}_\delta^2$, $\underset{\sim}{\lambda}^1 = \underset{\sim}{\lambda}^2$)	$\chi^2(9) = 387.11$, $p \simeq 0.00$
Test of	
Homogeneity of Variances and Covariances (B-A):	$\chi_d^2(6) = 339.23$, $p \simeq 0.000$
Equal Means under assumption of Homogeneity (D-B):	$\chi_d^2(3) = 47.88$, $p \simeq 0.000$
Equal Means under assumption of Heterogeneity (C-A):	$\chi_d^2(3) = 46.26$, $p \simeq 0.000$

TABLE 5

DATA FOR TWO-WAY MANOVA EXAMPLES

Measure	Low Involvement (n = 80)			High Involvement (n = 80)		
Probability	1.000			1.000		
Evaluation	.066	1.000		-.193	1.000	
Dummy[a]	.072	-.271	1.000	.336	.036	1.000
Means	7.188	3.788	.500	6.625	3.413	.500
Std. Dev.	2.977	.791	.503	3.066	1.039	.503

Measure	Low Emotion (n = 80)			High Emotion (n = 80)		
Probability	1.000			1.000		
Evaluation	-.029	1.000		-.061	1.000	
Dummy[b]	-.220	-.331	1.000	.045	-.068	1.000
Means	6.288	3.688	.500	7.525	3.513	.500
Std. Dev.	3.147	.949	.503	2.783	.928	.503

[a]A dummy variable for emotion (0 for low emotion, 1 for high emotion).

[b]A dummy variable for involvement (0 for low involvement, 1 for high involvement).

Note: The top portion gives the input data for analysis presented in Table 6, whereas the bottom is the input data for the results in Table 7.

Table 6

FINDINGS FOR TWO-WAY MANOVA APPLIED TO DATA OF TABLE 5: STRUCTURAL EQUATION ANALYSIS WITH MULTIPLE GROUP APPROACH (LOW AND HIGH INVOLVEMENT GROUPS)

Model	Goodness-of-fit
A: Full, all parameters free	$\chi^2(0) = 0.00$, $p \simeq 1.00$
B: No interaction effects ($\gamma_1^1 = \gamma_1^2$; $\gamma_2^1 = \gamma_2^2$)	$\chi^2(2) = 6.51$, $p \simeq .04$
C: No main effects of emotion and no interaction effects ($\gamma_1^1 = \gamma_1^2 = \gamma_2^1 = \gamma_2^2 = 0$)	$\chi^2(4) = 17.13$, $p \simeq .002$
Test of	
Interaction (B-A)	$\chi_d^2(2) = 6.51$, $p \simeq .04$
Main effects of emotion (C-B)	$\chi_d^2(2) = 10.52$, $p \simeq .01$

Key Parameter Estimates

Parameter	Low Involvement	High Involvement
γ_1*	.43 (.64)[a]	2.05 (3.17)
γ_2*	-.43 (2.50)	.07 (.32)

[a] t-value in parentheses.

Table 7

FINDINGS FOR TWO-WAY MANOVA APPLIED TO DATA OF TABLE 5: STRUCTURAL EQUATION ANALYSIS WITH MULTIPLE GROUP APPROACH (LOW AND HIGH EMOTION GROUPS)

Model	Goodness-of-fit
A: Full, all parameters free	$\chi^2(0) = 0.00$, $p \approx 1.00$
B: No interaction effects ($\gamma_1^1 = \gamma_1^2$; $\gamma_2^1 = \gamma_2^2$)	$\chi^2(2) = 6.51$, $p \approx .04$
C: No main effects of involvement and no interaction effects ($\gamma_1^1 = \gamma_1^2 = \gamma_2^1 = \gamma_2^2 = 0$)	$\chi^2(4) = 14.49$, $p \approx .01$

Test of	
Interaction (B-A)	$\chi_d^2(2) = 6.51$, $p \approx .04$
Main effects of involvement (C-B)	$\chi_d^2(2) = 7.98$, $p \approx .03$

Key Parameter Estimates

Parameter	Low Emotion	High Emotion
γ_1*	-1.38 (2.00)[a]	.25 (.40)
γ_2*	-.62 (3.11)	-.13 (.61)

[a] t-value in parentheses.

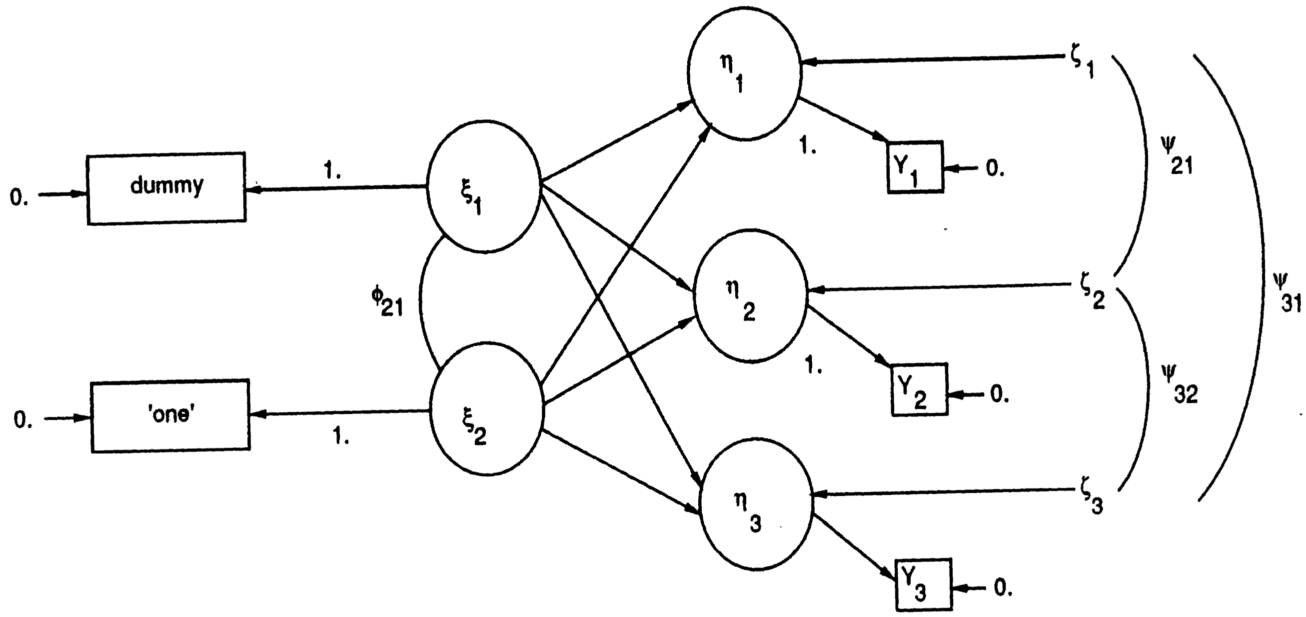


Figure 1

STRUCTURAL EQUATION MODEL SPECIFICATION OF ONE-WAY MANOVA
FOR THREE DEPENDENT VARIABLES: DUMMY VARIABLE APPROACH

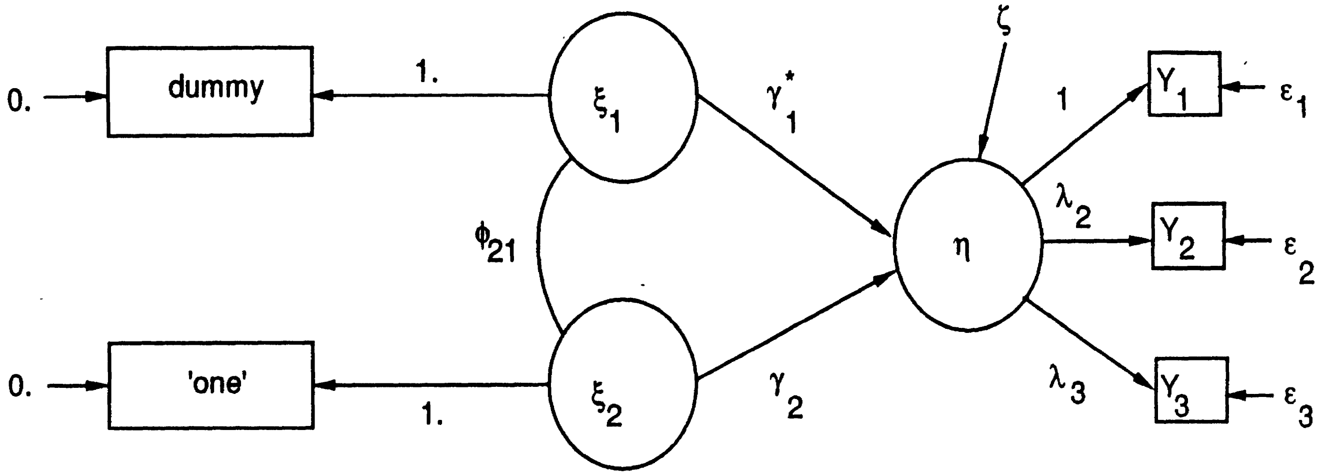
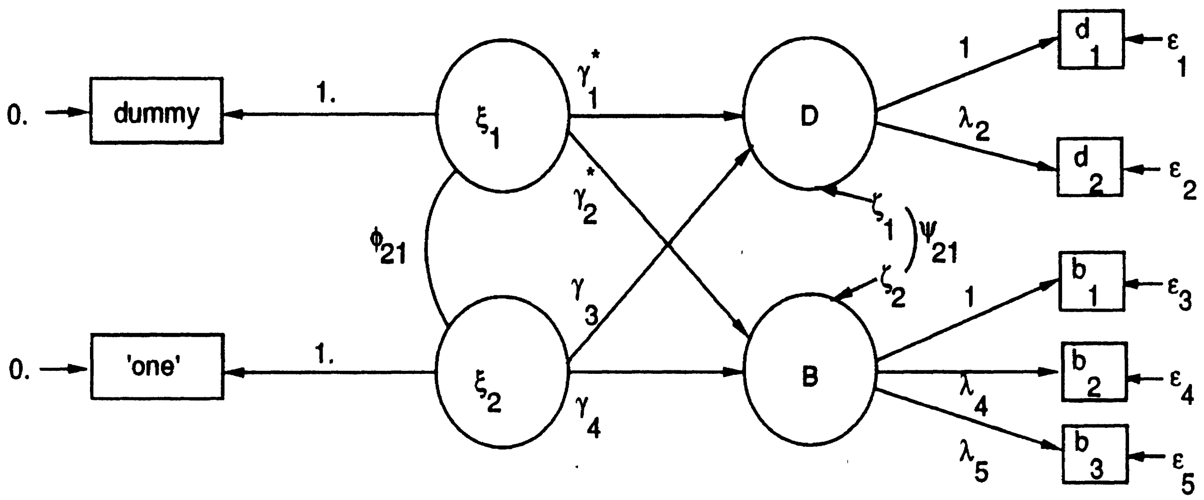


Figure 2

STRUCTURAL EQUATION MODEL SPECIFICATION OF ONE-WAY MANOVA FOR THREE MEASURES OF A SINGLE LATENT DEPENDENT VARIABLE: DUMMY VARIABLE APPROACH

A. Redundancy among sub-sets of dependent variables taken into account, but with unordered covariance between dependent variables



B. Covariance between dependent variables interpreted as a causal path

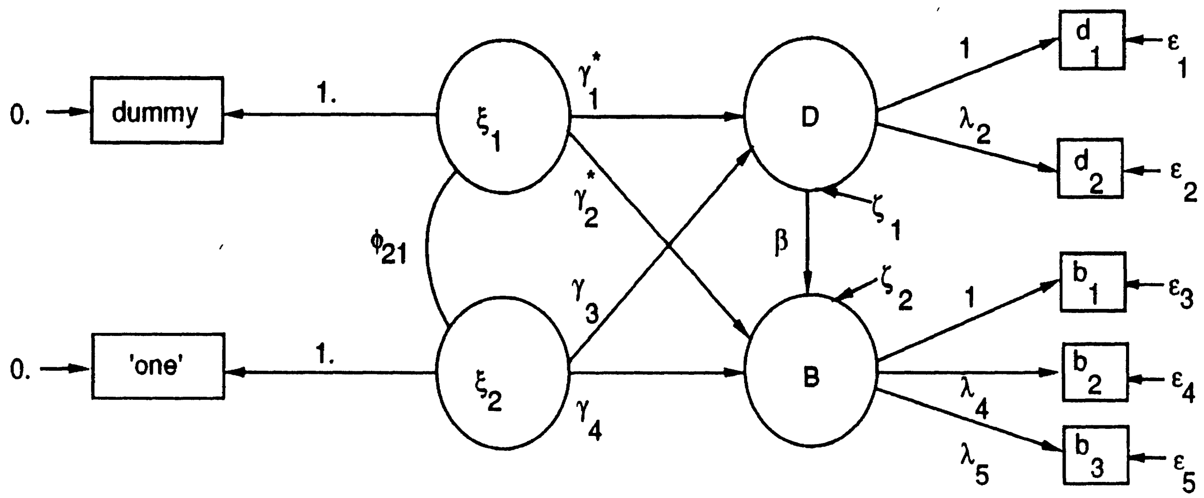


Figure 3

STRUCTURAL EQUATION MODELS FOR ONE-WAY MANOVA STEP-DOWN ANALYSIS WITH TWO LATENT DEPENDENT VARIABLES, EACH MEASURED WITH MULTIPLE INDICATORS: DUMMY VARIABLE APPROACH

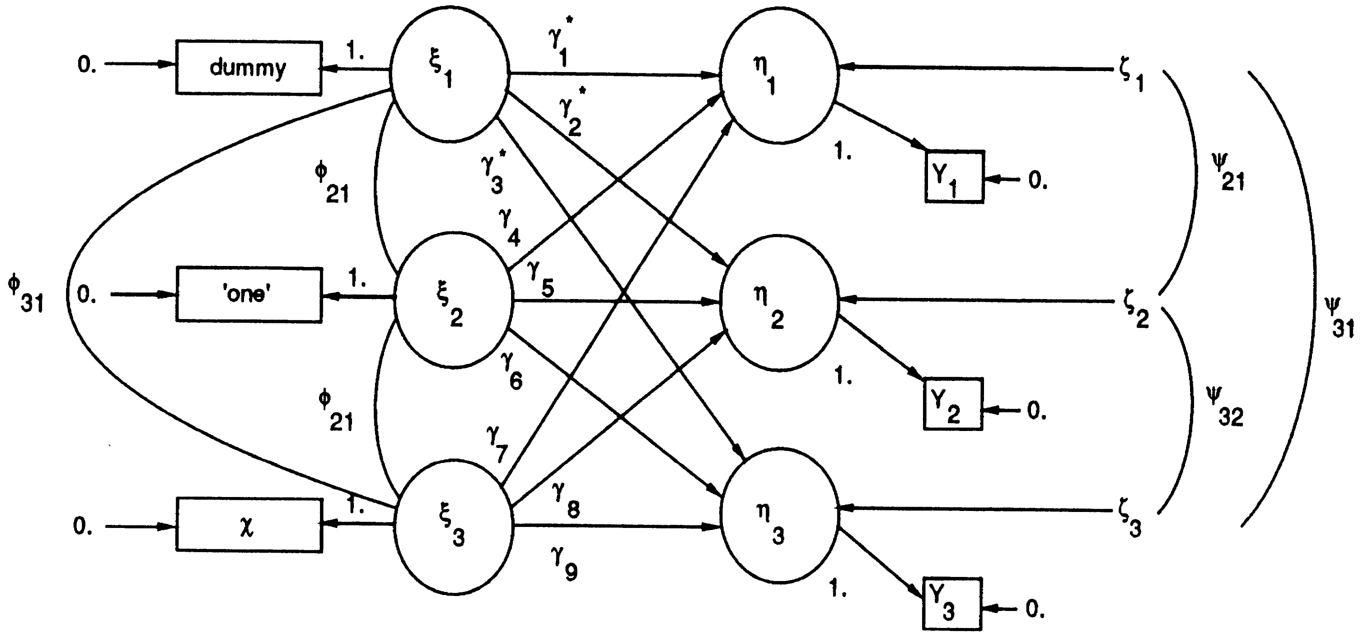
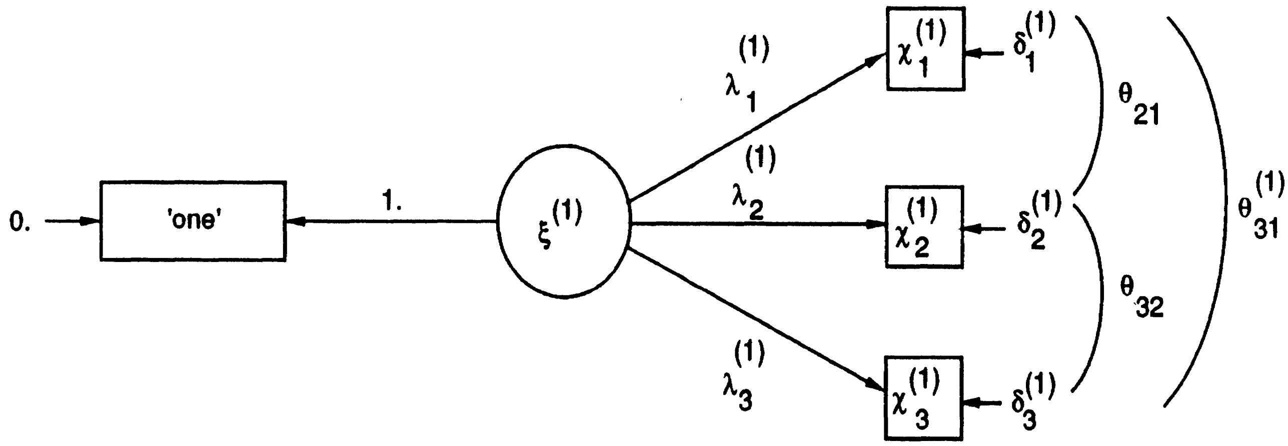


Figure 4

STRUCTURAL EQUATION MODEL SPECIFICATION OF ONE-WAY MANCOVA FOR THREE DEPENDENT VARIABLES AND ONE COVARIATE: DUMMY VARIABLE APPROACH

Group 1



Group 2

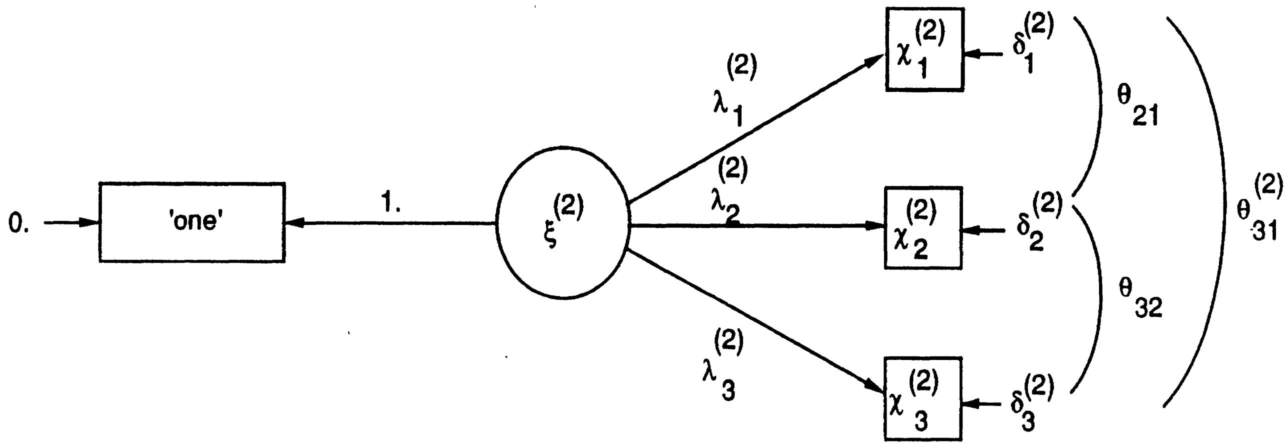
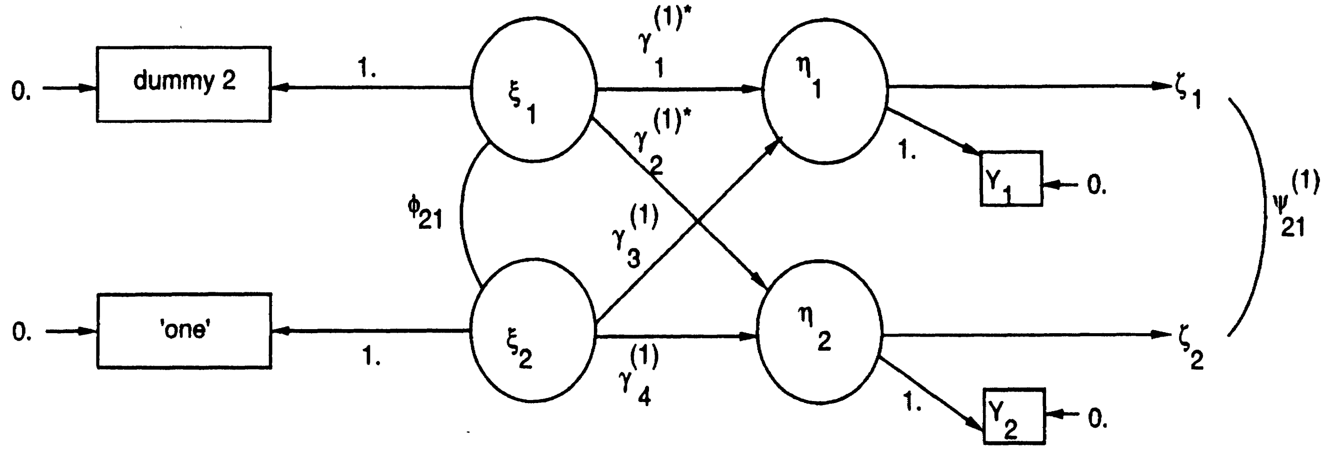


Figure 5

STRUCTURAL EQUATION MODEL SPECIFICATION OF ONE-WAY MANOVA
FOR THREE DEPENDENT VARIABLES: MULTIPLE GROUP APPROACH

Group 1



Group 2

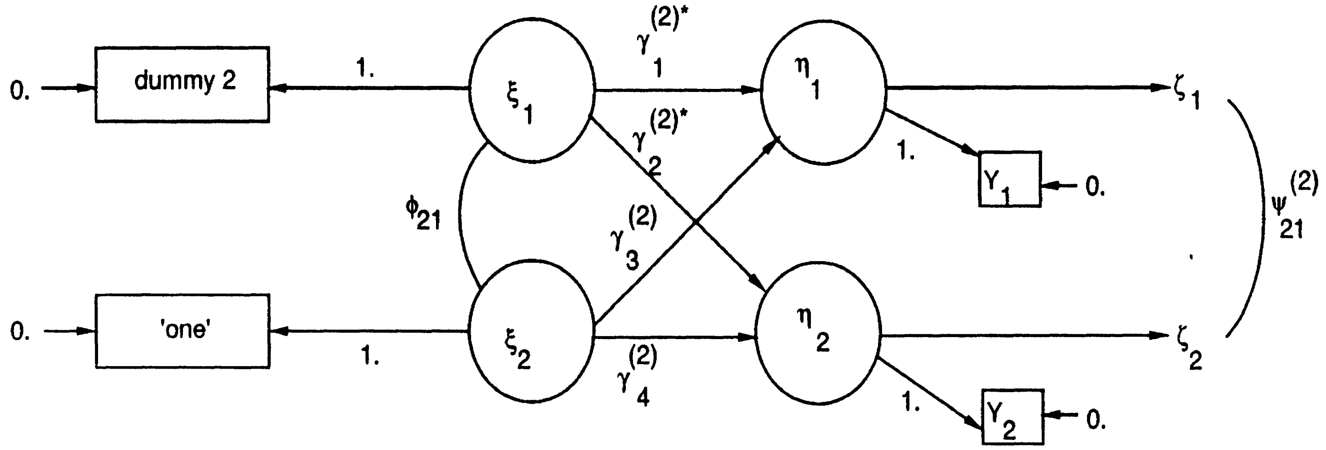


Figure 6

STRUCTURAL EQUATION MODEL SPECIFICATION OF TWO-WAY MANOVA FOR TWO DEPENDENT VARIABLES: MULTIPLE GROUP APPROACH