Research Paper

THE RESOLUTION OF BANKRUPTCY BY AUCTION: ALLOCATING THE RESIDUAL RIGHT OF DESIGN

WORKING PAPER #98013

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The Resolution of Bankruptcy by Auction: Allocating the Residual Right of Design

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ABSTRACT

In this paper we examine the value of the right to choose the *method* of sale of corporate assets. The question we ask is simple - who should have the right to decide *how* the assets of the firm should be sold? We show that this right is valuable and its value comes from recognizing conflicting incentives of claimants at the time of sale. We characterize the choices of the senior and junior claimants and show that they have distinct preferences on a set of common auction procedures. In addition, different claimants have different preferences regarding the allocation of resources towards attracting bidders for the auction. As a consequence, we show that the optimal allocation of the right cannot be independent of the realized values of information at the time of sale. Our analysis points out that when the process of disposition of a firm's assets in bankruptcy is disassociated from that of the distribution of cash flows accruing from the sale, the incentive problems commonly analyzed in the bankruptcy literature do not disappear but, instead, show up in the choice of the selling mechanism. In particular, the choice of a selling mechanism is formally identical to the choice of a risky project.

1 Introduction

An efficient bankruptcy resolution procedure has been a holy grail for both practitioners and academics in the past two decades. Debates on the issue have centered around two complementary focal points: (i) inefficiencies associated with reorganization procedures like Chapter 11 of the U.S. bankruptcy code and (ii) the use of market based or auction procedures to transfer control of the firm's assets.[1] The first line of inquiry focuses on the substantial legal and delay costs that many Chapter 11 cases entail in addition to deviations from absolute priority (APR) that are quite common in reorganization settings.[2] Consequently, several critics, including Baird (1986) and Jensen (1991), have argued that such costly reorganization procedures should be replaced by mandatory sale procedures utilizing competitive auctions.[3] This suggestion has been supplemented by proposals in Roe (1983), Jackson (1986), Bebchuk (1988), Aghion, Hart and Moore (1992) and Bradley and Rosenzweig (1995). Stripped to their bare essence all these proposals involve bidders making bids for either the entire firm or for particular outstanding claims. Most of these proposals involve cash bids but some, noting the presence of liquidity constraints, also advocate more sophisticated schemes which entail claimants voting on the most desirable proposal(s) in conjunction with bids made with both cash and paper.

The arguments made in support of auction or market based procedures are simple. A principal feature of such procedures is that they serve to separate the reorganization/ liquidation decision from the procedure of settlement of outstanding claims. In doing so, such mandated sales are supposed to eliminate costly bickering over seniority of claims and the value of the firm's assets and future cash flows that pervades a reorganization process. Perhaps Jensen (1991) phrases this central message most succinctly: "One way to solve the information and incentive problem would be to allow any party - outsiders as well as current claimant- to make an all-cash bid for the control rights to the company. At the close of the *auction*, the highest bidder would immediately assume control of the company and its operations." In addition, supporters point out that with such schemes in place, it is a simple matter to avoid violations of APR, since cash proceeds from the sale can be easily distributed in order of seniority. Finally, from the point of view of economic efficiency, they

[1]A recent survey of the issues involved in financial distress is provided in John (1993).

[2]Weiss (1990) documents significant deviations from APR in a sample of U.S. bankruptcy cases. Bradley and Rosenzweig (1992) critique the entire chapter 11 process.

[3]Perhaps the earliest manifestation of these ideas is found in Haugen and Senbet (1978) who argue for the possibility of market based procedures to eliminate deadweight costs associated with bankruptcy.

point out that such schemes, in the absence of liquidity constraints, ensure that the firm's assets end up in their highest valued use. This criterion of *ex post* efficiency is by no means guaranteed in a reorganization setting where costly bargaining is coupled with asymmetric information between various claimants and where holdout problems abound.[4]

In this paper, we reexamine some of the claims of the proponents of the auction method of resolving bankruptcy.[5] We find that prescribing a sale alone, without specifying the sale procedure in detail, does not eliminate incentive conflicts between senior and junior claimants. Instead, some of the incentive conflicts that arise in the reorganization setting show up, in this setting, in the choice of sale procedures themselves. We show that senior and junior claimants have conflicting preferences over common auction procedures and, consequently, the right to *design* a sale procedure itself has value *over and above* the rights to any cash flows arising out of the sale. Thus, merely specifying rights to cash flows - as debt and equity contracts do - is not a complete specification of ownership rights. The *residual right* to design the method of sale has to, in addition, be allocated to some party in order to get past incentive conflicts. Our inquiry, then, is concerned with the optimal allocation of this residual right.[6]

In order to focus on the *resolution* of bankruptcy, we assume that a firm with outstanding risky debt enters bankruptcy with an exogenous probability. The bankruptcy resolution process is simply modeled as a sale of the firm's assets to outside bidders.[7] To focus on the incentive issues, we abstract away from any asymmetric information problems between senior and junior claimants. Consequently, we use an independent private values structure of valuations.[8] Our assumptions ensure that the corporate assets are always transferred to the party who values them the most. That is, sales procedures satisfy the criterion of *ex post* efficiency.[9] However, as we establish, not all choices yield the same outcomes in terms of eliciting efficient investment incentives *ex ante*.

We first analyze claimants' preferences over sales mechanisms when the number of bidders, n, is exogenously specified. We establish that the first-price sealed-bid auction is the globally best

[4]A model of bargaining between claimants in bankruptcy is provided in Bergman and Callen (1991).

[5]In a different vein Easterbrook (1990) also criticizes auction based solutions to bankruptcy.

[6]The right to design that we focus on is closely related to the concept of the rights to ownership outlined in Grossman and Hart (1986). In their approach, ownership implies the right to choose optimal actions in contingencies left unspecified in an incomplete contract. In our case, the residual right to choose the sales mechanism is vested with some party in advance.

[7]As we point out later, allowing existing claimants to bid in the auction may violate *ex post* efficiency. However, we expect out qualitative results to hold even when existing claimants can bid.

[8]As Myerson (1981) establishes, our results carry over to the case of common values when the signals received by bidders are independent.

[9]Bhattacharyya and Singh (1995) analyze the case when selling procedures satisfy the criterion of *ex ante* efficiency.

mechanism from the point of view of the senior claimant. Equity, however, is shown to prefer an ascending-bid auction to the first-price sealed-bid auction. As a result,the *ex ante* values of debt and equity are affected by the allocation scheme chosen. Despite such disparate preferences, we establish that when the allocation of the right to choose a sale procedure is specified in advance, *ex ante* contracting ensures that *ex ante* investment incentives remain unaffected by the choice of allocation schemes. Consequently, the choice of selling procedures can be delegated to either claimant or an outside party without adverse effects on *ex ante* investment incentives.

Next, we examine the case when the designated seller is allowed to choose, in addition, the amount of resources devoted to attracting bidders to the sale. With the choice of n now effectively endogenized, we show that, while preferences over auction forms remains unchanged, *ex ante* firm values are sensitive to the actual allocation of the right to sell. As we demonstrate, senior claimants have a general tendency to hold a sale "too early", while junior claimants always prefer to sell "too late". This result is established without recourse to the popular instrument of invoking non-pecuniary benefits of control to equity. While it is widely appreciated that a senior claimant may rush into a sale since she does not care about the cash flows to the junior claimant, we show that the freedom to choose a sale mechanism allows the senior claimant to act optimally if her claim is relatively high. A low debt claim, on the other hand, is never sufficient to entice equity to act in the interest of the firm as a whole. Despite this feature, in order to promote efficient *ex ante* investment incentives, it may be optimal to allow equity control over the selling procedure for low levels debt overhang. Based on our analysis, we show that a bankruptcy resolution procedure that *always* allocates the selling rights to one claimant can be improved upon. Our conclusion is in line with the concern voiced by Schwartz (1997) over the rigidities associated with state-mandated bankruptcy resolution procedures.

Finally, we demonstrate that the intuition of a senior claimant's desire to hold a fire-sale is crucially dependent on the assumption that all bidders invited to the sale have the same *ex ante* distribution of values. In fact, when bidders with higher expected values are located earlier than bidders with lower expected values, we show that a senior claimant may optimally choose to wait for more bidders to materialize than a junior claimant. This is because the senior claimant benefits from promoting competition at the lower end of the support of valuations. Such competition is of no value to a junior claimant.

The rest of the paper is organized as follows. Section 2 lays out the basic model. Section 3 analyzes the case when the number of bidders is exogenously specified while section 4 examines the case when the number is optimally selected. The last section concludes. Longer proofs are relegated to the appendix.

2 Model

A firm is set up at date 0 with a required investment of I. Part of this investment is financed by debt with face value F to which the assets of the firm are indentured. At time 1 the state of world $\omega \in \Omega$ is realized which determines the value for the firm under current management, denoted by $L(\omega)$. There also exist some outside bidders who can potentially improve the value of the firm. The resolution of bankruptcy is assumed to be accomplished by selling the firm to one of these bidders.

We concentrate on the case when the firm is clearly insolvent, that is, when the value of the firm's future cash flows under current management is less than the face value of debt.[10] As mentioned earlier, the resolution of bankruptcy is constrained to be by a sale of the assets of the entire firm to outside managers.[11] We assume that there are $n \geq 2$ potential outsiders who may be interested in buying the firm in the bankrupt state. We first analyze the problem for an exogenously specified n and later analyze the implications of endogenizing the choice of n. In both cases, these outside managers can improve the future cash flows of the firm and the value of the firm's assets to these outside managers is given by

$$V_i = L(\omega) + \epsilon_i \qquad \text{for } i \in \{1, n\}.$$

The total value of the firm to the bidder, thus, has a common component and a private component. We assume that the common component, $L(\omega)$, is public information. The private component, $\epsilon_i \; \forall \; i$ is independently and identically distributed over an interval $[0, H(\omega)]$ with a density function $f(\cdot)$, which satisfies the increasing hazard rate property.

The set of states in which the firm is insolvent is denoted by Ω_I and the set of states in which

[10]When the value of the firm under current management is higher than F, the firm can issue new securities and retire the debt. For more details of this argument, see Haugen and Senbet (1978).

[11]The analysis is considerably more complicated when the claimholders are allowed to directly participate as potential buyers in the process. However, the main qualitative results of the paper will continue to hold in this setting. The reader is referred to analyses in Singh (1998), Burkart (1995) and Berkovitch, Israel and Zender (1994) to see the nature of the complications that arise when existing claimholders are allowed to be buyers in the process. Briefly, a claimholder has, in an ascending-bid auction, the incentives to bid above his valuation to induce outside bidders to go higher. The main implication of this behavior is that *ex post* efficient bankruptcy resolution may not be possible even with all-cash bids.

the firm is solvent is denoted by Ω_S, where $\Omega_S \cup \Omega_I = \Omega$ and $d\phi(\omega)$ is the probability density of ω. We assume that the realization of ω is exogenous and does not depend on any action taken by the parties in the interim. On the declaration of bankruptcy, the value of the firm need not fall to $L(\omega)$ when there is a chance that there exist alternate bidders who can add value to the firm's operations. Consequently, we denote the value of the firm at this point by $BV(\omega)$ where the abbreviation stands for "bankruptcy value". Note that $BV(\omega)$ potentially depends on the method employed to sell the firm. We denote the value of the firm in a solvent state by $CV(\omega)$. Both equity and debt holders are assumed risk neutral and the interest rate is normalized to zero.[12] Thus, conditional on a bankruptcy regime in place, the value of a firm at time 0 with an outstanding debt obligation of F due at time 1 is, then, given by,

$$V_0(F) = \int_{\Omega_S} CV(\omega)d\phi(\omega) + \int_{\Omega_I} BV(\omega)d\phi(\omega).$$

Given our assumptions, in bankruptcy, the firm is put up for sale to outsiders and the proceeds are distributed to claimants by strict adherence to APR.[13] In what follows, we demonstrate that the *form* of sale procedure adopted affects the values of existing claims and, consequently, the allocation of the right to choose is critical. Although we analyze the effects of allocation of such decision rights, we abstract away from the issue of the specific institutional features that may be required to implement such allocations. We would, however, like to point out that such rights are typically claimed by one or the other party based on notions of ownership rights and/or the rights conferred by the bankruptcy code. By way of example, note that the British bankruptcy code allows creditors the right to appoint "administrative receivers" who may sell assets in order to repay the claims of creditors. In the U.S., too, the management of a bankrupt firm is supposed to act in the interest of the creditors once the firm has entered into bankruptcy proceedings.

Finally, we need to specify the criterion used to judge the efficacy of various allocations of the residual right. Note that, in the presence of dissipative costs incurred during the resolution of bankruptcy, merely ensuring that the firm's assets end up in the hands of the party who values them the most is not sufficient to guarantee *ex ante* investment efficiency. Consequently, we call a bankruptcy procedure superior if it produces a higher value of $BV(\omega)$ than another procedure.

[12] The assumption of zero interest rate is relaxed later in the paper where the firm has to expend time or resources to obtain bidders.

[13] We should point out that the assumption of adherence to APR rules out bargaining between equity and debtholders in the event of bankruptcy.

Given our assumption of exogenous triggering of bankruptcy, maximizing the value of $BV(\omega) \forall \omega \in \Omega_I$ assures us that the *ex ante* value of the firm, $V_0(F)$, is maximized. Clearly, a commitment to a superior bankruptcy resolution procedure ensures that the firm's expected rate of return on any project it undertakes is higher. Hence, a superior bankruptcy procedure produces a higher level of *ex ante* investment efficiency and, therefore, is value maximizing.[14]

In what is to follow, we shall usually suppress the dependence on ω for the sake of notational convenience. Unless explicitly indicated otherwise, the variables will stand for particular realizations of these quantities.

3 The Case of Exogenous Number of Bidders

For the purposes of this section, we assume that the number of bidders attracted to the auction of the firm's assets is exogenous. We denote this number by n. Further, we assume that the auctioneer does not have the commitment power to post a reserve price that is not subgame perfect. This implies that the reserve price employed in the auction will be $L(\omega)$, since this value can be obtained by letting the current management run the firm in the absence of a greater bid. Our assumption on the inability of the auctioneer to set a higher reserve price serves two functions. First, the assumption ensures that any mechanism chosen is renegotiation proof.[15] Second, the assumption guarantees that the firm ends up in the hands of the party which values it the most. In other words, the auction procedure chosen has to be *ex post* efficient.[16] In this section, we demonstrate that even when sales mechanisms are restricted to be in a class that maximizes the bankruptcy value of the firm, senior and junior security holders have different preferences on the set of such procedures.

3.1 Value Maximizing Sales Procedures

As we have mentioned earlier, in order to ensure adequate *ex ante* incentives to invest, permissible bankruptcy procedures should seek to maximize BV. Consequently, we first provide a characteri-

[14]White (1980) and Bulow and Shoven (1978) were among the first to emphasize the *ex ante* costs of a bankruptcy procedure and point out the possibility of underinvestment caused by inefficient procedures. White (1994) persuasively argues for the case that such *ex ante* costs can be very substantial. Cornelli and Felli (1996) also discuss *ex ante* efficiency properties of bankruptcy procedures.

[15]The mechanism is renegotiation proof in the sense that the seller has no incentives to renegotiate. In accordance with standard practice we assume, however, that bids made in the auction are contractually enforceable.

[16]Most analyses in the area of optimal bankruptcy procedures impose *ex post* efficiency as a desirable feature of the optimal mechanism. An exception is Bhattacharyya and Singh (1995) who also consider in detail the case where the auctioneer has the commitment power to set reserve prices optimally. The power to impose reserve prices does not change the qualitative results in any way.

zation of procedures which maximize the value, BV, subject to the constraint of ensuring that the firm is sold to one of the potential bidders.[17] The following remark provides a full characterization.

Remark 1 *In the class of all mechanisms that ensure sale, any* ex post *efficient sale mechanism that imposes zero expected payments on a bidder of the lowest possible type maximizes expected revenues to the seller.*

The result is a simple extension of the well known *Revenue Equivalence Theorem* which states that the expected revenue of any incentive compatible mechanism is completely determined by its allocation rule and the expected utility of the lowest possible type.[18] This result lends formal credence to the argument in Jensen (1991) that an auction of the firm's assets would serve to establish a value for the firm as an entity. This is because all the standard, bid-based auction procedures which award the object to be sold to the highest bidder and ask for payment only by the winner are clearly members of the optimal class. Thus, there is no loss of expected revenue to the seller if we confine attention to the class of mechanisms envisioned by Jensen (1991).

We shall refer to all selling mechanisms which ensure *ex post* efficiency and involve no payments by losing bidders as *competitive mechanisms.* The commonly used bid-based procedures such as the ascending-bid auction,[19] and the first-price sealed bid auction,[20] when used without reserve prices or entry fees, are clearly members of this class. In what follows, we will confine our attention to this class since *Remark 1* assures us that this focus is without a loss in terms of efficiency of the selling mechanism.

3.2 The Preferences of Claimants

Given that all competitive mechanisms yield a maximum for BV, it follows that the choice of selling mechanism can be delegated to either claimant without any loss of *ex ante* investment efficiency. However, this does not imply that the right to choose the method of sale has no value even when the choice is constrained to be within the set of mechanisms that ensures value maximization. As a result, to ensure consistent *ex ante* valuation of claims, the bankruptcy code needs to either specify a particular mechanism or a specific rule for allocating the right of design of the selling mechanism to either claimant.

[17] Note that this is a natural restriction to impose in the context of a mandated sale procedure.
[18] The proof of this result is omitted for brevity. A proof is available from the authors on request.
[19] Or, the strategically equivalent second-price auction.
[20] Or the equivalent descending-bid or Dutch auction.

The reason that the right of design has value even though all mechanisms in the permissible set have the same expected value, lies in the variability associated with the different mechanisms in the set. The following result demonstrates that it is possible to identify a sale mechanism that has the least variance.

Proposition 1 *With risk neutral bidders, the first-price sealed-bid auction dominates, in the sense of second order stochastic dominance, all other competitive mechanisms.*

The intuition behind the proposition lies in the fact that it is possible to decompose the variability of revenues in an auction mechanism into two components.[21]

$$Var(x(v)) = E_{v_m}\left[Var(x(v)|v_m)\right] + Var_{v_m}\left[E(x(v)|v_m)\right] \quad (1)$$

where x(v) is the revenue generated by a given auction for a given vector of valuations and v_m is the maximum of the valuations. For example, x(v) in a first-price sealed-bid auction is just the bid of the winning bidder while, in the ascending-bid auction, it is the valuation of the highest losing bidder. The second component in equation 1 measures the variability of the expected revenues across different values of the winning bidder. The expected payments made by all bidders combined in any member of the competitive class is the same conditional on each value of v_m.[22] Consequently, the second component is constant across all competitive mechanisms. The first component, on the other hand, stems from the variability of the total payments conditional on the winning bidder's valuation. This clearly depends on the actual rules of the auction since, for example, this component is zero in any auction where the winning bidder pays his own bid and others do not pay. Competitive selling mechanisms which call for a winning bidder's payments to depend on anything other than his own bid will, thus, have a positive value for this component.[23]

Given this property, it is easy to show that senior claimants strictly prefer the first-price sealed-bid auction to all other competitive mechanisms. The key observation is that senior claimants

[21] Note that, in order to prove second order stochastic dominance, it is not sufficient to order the mechanisms by variance. The formal proof does not rely on the properties of the second moment.

[22] See proof of proposition 1.

[23] Vickrey (1961) first demonstrated that, with uniform distributions, the variance of revenues in a first-price auction is less than that produced in an ascending-bid auction. Maskin and Riley (1984) show that his result generalizes to other distributions and risk-averse utility functions in general. Our result is stronger and demonstrates that the first-price auction is the best that the seller can do in the universe of competitive mechanisms. A simple extension of our proof technique shows that the superiority of the first-price auction extends to the case with reserve prices.

effectively have a concave utility function over the cash flows produced by the auction. This fact is presented without proof in the following corollary.

Corollary 1 *Among all competitive mechanisms, debtholders have a strict preference for a first-price sealed-bid auction.*

Although all such selling mechanisms give the same expected revenue (lemma 1) to the firm, the corollary shows that the expected cash flow to debtholders is the highest under the first-price sealed-bid auction.[24] Consequently, the expected cash flow for equityholders is minimized under the first-price sealed-bid auction. We state this result without proof in the next corollary.

Corollary 2 *Among all competitive mechanisms, equityholders are strictly worse off under a first-price sealed-bid auction than any other sale procedure in the class. Specifically, equityholders are strictly better off with an ascending-bid auction.*

The intuition underlying the results above can be illustrated with a simple example. Assume $L = 0$, $H = 1$, and two bidders with independent valuations distributed uniformly over this interval. The face value of debt F is 0.5. In the first-price auction, the symmetric equilibrium bidding strategy can be shown to be that of bidding half one's valuation. The maximum possible bid is, therefore, 0.5. Thus, equityholders obtain nothing if the chosen mechanism is a first-price auction. In contrast, if the chosen mechanism is an ascending-bid auction, each bidder's dominant strategy is to bid up to his value. In this scenario, equityholders obtain positive cashflows if the valuation of both bidders is above 0.5, the *ex ante* probability of which is 0.25. This fact, combined with the fact that expected revenue in both formats is the same, implies that the debtholders prefer the first-price sealed-bid auction while the equityholders prefer the ascending-bid auction.

What is not often appreciated is that the issues in the choice of a sales procedure are formally identical to those that arise in the choice of an investment project by a claimant. From Jensen and Meckling (1976), we know that junior claimants have an incentive to take on high risk projects to appropriate value from senior claimants. Similarly, equityholders in our analysis have incentives to pick a sale procedure which generates greater variability in revenues since they care only about one end of the distribution of revenues. Since debtholders do not care about the same end of the

[24]Strictly speaking, our use of the term "first-price" is somewhat misleading. It is easy to see that all auctions where the winning bidder pays a function of his own bid and others do not pay have the same revenue profile. Thus, it is not important whether the winning bidder pays his own bid or, say, twice his own bid.

distribution, their preferred choice is the opposite. As Black and Scholes (1973) originally pointed out in their seminal paper, the payoffs to equity are of the form of a call option written on the cash flows accruing to the firm, with an exercise price equal to the face value of debt. The choice by the equityholders of the ascending-bid auction serves to increase the variance of the payoffs of the underlying asset and goes to increase the value of the call option they hold.

3.3 Implications

We have established, then, that senior and junior claimants have very different preferences over the set of selling mechanisms that may be employed in resolving bankruptcy by auction. This implies that the right to choose a sale procedure itself has value to a claimholder. Put differently, the *ex ante* valuation of any claim is dependent on the exact procedure employed to sell a firm in bankruptcy. In order to ensure consistent valuation of claims, therefore, such an explicit specification is needed to augment the recommendation that assets be auctioned off in bankruptcy.

Consistent *ex ante* valuation is not the only casualty when the exact sale procedure is left unspecified. The motivation behind proposing an open auction procedure to resolve bankruptcy is rooted in its prospects for reducing or eliminating various turf battles that emerge under present day bankruptcy procedures. We have shown that unless the sale procedure is specified in complete detail, such turf battles merely show up in a different form in that various claimants would jockey for their own preferred selling mechanism. Consequently, an incomplete specification of the selling procedure does not succeed in eliminating such conflicts.

Finally, this section has established that once a sale mechanism has been specified in advance, it does not matter who carries out the auction itself. In particular, there would be no disagreement over the implementation of any such procedure irrespective of whether senior of junior claimants, or even a court appointed trustee, ran the sale. In particular, then, the delegation of such a duty has no valuation consequences *ex ante* and the choice of any competitive mechanism would have the same impact on *ex ante* investment incentives. Unfortunately, this feature is entirely an artifact of the assumption of an exogenous number of bidders. In the next section, we address the issue of an endogenous selection of the number of bidders.

4 The Case of Endogenous Number of Bidders

The assumption of an exogenous number of bidders in an auction, though convenient for analytical tractability, is clearly unrealistic. Attracting bidders for assets with unique characteristics is not a simple process. In the case of corporate assets, for example, costly and time consuming procedures must be followed to ensure access to corporate data through due diligence procedures. In addition, investment bankers are usually retained to search for buyers who may have the highest interest in the assets up for sale. Consequently, the seller has to determine how much of such costs should be incurred in order to generate the optimal amount of interest before the auction is held. It seems plausible, then, that different claimants would have different incentives when it comes to undertaking a costly search for bidders and, consequently, choose to invite different numbers of bidders to the auction. Thus, our analysis in the last section needs to be modified to take into account such different incentives.

A commonly invoked rationale for not allowing debtholders to assume complete control of bankruptcy resolution procedures is that, as senior claimants, they would not have adequate incentives to engage in an exhaustive enough search for bidders. One popular version of this argument takes the form that, as soon as a bid is received that tops the face value of the claims of the senior claimant, she would not incur any more efforts to augment the revenue receipts for the firm as a whole. Thus, it is claimed, courts and/or equityholders should have a role to play in the resolution of bankruptcy to ensure that the assets of the firm are not sold off in a "fire sale". Consequently, the involvement of courts or equityholders is claimed to reduce *ex ante* costs of bankruptcy. In this section, we investigate the validity of this kind of a claim to see who should have control over selling procedures.

An interesting feature of this problem is that the expenses associated with selling the firm's assets typically do not come from the pockets of the party conducting the sale but, instead, from the firm's own resources. Thus, for example, it could be that search costs are borne by the firm, either through depletion of its cash reserves or from sales of some assets piece-meal. On the other hand, the use of resources to attract bidders need not take the form of cash expenditures, since merely waiting for potential bidders to study the firm itself involves costly delay. The costs of such delay are composed of both a decrease in the present value of realized sale revenues and any destruction of future value caused by the inefficient operation of the firm by present management up to the moment of sale. For analytical tractability, we will focus on the case where delay costs each period are proportional to the value of the firm. However, our qualitative results should extend to the other forms of costs mentioned above.

To obtain a benchmark for comparison, we first consider the case where the bankrupt firm's assets are sold by a party who is interested in maximizing BV. In addition to providing a benchmark consistent with the principle of a value maximizing sale, this case is equivalent to the choice that would be made by a trustee who is motivated to maximize the sale value of the firm.[25] We denote by $V(n)$ the expected value of the proceeds raised in a sale with n bidders whose valuations are distributed independently over the support $[L, H]$ as before. We denote by N, the maximum number of bidders who may possibly show interest in the sale. The cumulative probability distribution on the number of bidders who show interest by any given time t is denoted by $P(n|t)$ for $n = 1, \ldots, N$ and $t \in \mathcal{R}_+$ with an associated probability mass function $p(n|t)$. Thus,

$$P(n|t) = \sum_{k=2}^{n} p(k|t)$$

We assume that an increase in waiting time has two effects. First, waiting longer increases the chances of getting a larger number of interested bidders. We model this effect by assuming that $P(n|t_2)$ first order stochastically dominates $P(n|t_1)$ for all $t_2 > t_1$. The distribution function is also assumed to display the *Monotone Likelihood Ratio Property*(MLRP)[26] and the *Concavity of the Distribution Function Condition* (CDFC).[27] These assumptions, standard in the literature, assure that an increase in waiting always increases the probability of a higher number of bidders and that such an increase exhibits a notion of diminishing marginal returns. We denote the expected value of a sale that is conducted at period t by $U(t)$. Thus,

$$U(t) = \sum_{n=2}^{N} V(n)p(n|t)$$

Second, waiting reduces the present value of expected revenues from any sale. We represent such a loss in value by the discounted payoff $\delta^t U(t)$ for a sale held at time t, where $\delta < 1$ denotes the per unit cost of delay. Under these assumptions, we have the following characterization result.

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[25] We shall return to this interpretation later.

[26] This implies that $\frac{p_t(n_1|t)}{p(n_1|t)} \leq \frac{p_t(n_2|t)}{p(n_2|t)}$ $\forall n_1 < n_2$ and $\forall t$. In fact, this condition is sufficient for FOSD.

[27] Formally, the condition is that $\frac{\partial^2}{\partial t^2} P(n|t) \geq 0 \; \forall n, \; \forall t$.

Lemma 1 *A value maximizing sales mechanism entails a waiting period given by t^* which satisfies*

$$\frac{V_t(t^*)}{V(t^*)} = -\log(\delta)$$

The result above is straight forward to interpret. It says that a seller's expected revenues reaches its global optimum, in present value terms, when the percentage change in value obtained from waiting an extra instant falls to the level of the cost of waiting.

It should be clear, then, that an analogue of this result holds for other interpretations of costs of conducting an optimal sale. So long as the expected return from undertaking costly actions which are expected to boost expected sales proceeds is higher than the cost of such actions, it pays to engage in such actions. In our case, however, the costs of any such action are shared by claimants in a particular fashion. Thus, if either claimant is allocated the right to run the sales mechanism, his/her own cost-benefit calculation is unlikely to be the same as the one for the entire firm.

Finally, at the level of the entire firm, the optimal waiting time, t^*, is shown to be independent of the particular sale mechanism employed. This follows from remark 1, where we demonstrated revenue equivalence of all competitive mechanisms.

4.1 Debtholders as Sellers

Having established our benchmark, we now turn our attention to the question of the chosen t when the senior claimant is allocated the right to run the sale. We denote by t_D, the waiting time chosen by the debtholders.[28] For ease of presentation we restrict attention to two common mechanisms: (1) the first-price sealed-bid auction and (2) the ascending-bid auction.[29] We show that while, in general, debtholders would choose to wait for a time period less than t^*, the power to choose the auction form may help them to reach a decision that is optimal for the firm as a whole. Note that this implication is antithetical to the notion that debtholders always have incentives to engage in a "fire sale".

Proposition 2 *1. The senior claimants' choice of t is weakly lower than t^* for any selling mechanism employed. In particular, the optimal t_D is strictly lower than t^* for an ascending-bid auction and is weakly lower for a first-price sealed-bid auction.*

[28] t_D will in general depend on the mechanism chosen. For expositional ease, we drop the dependence of t_D on the mechanism. The particular mechanism should be clear from the context.

[29] Similar arguments can be used to show that our results hold in the general class of competitive mechanisms.

2. The incentive to wait longer increases with the face value of debt.

An increase in t leads to a rightward shift in the distribution for the number of bidders that may show an interest in the firm. This, in turn, leads to a rightward shift in the distribution of payoffs from any auction-based mechanism. The debtholders, however, are not concerned as much about the shift at the right of the distribution as about the shift at the left, since they are senior claimants. For the second-price auction, the support of the payoffs to the auction coincides with that of values of the bidders and the proposition above shows that the proportional improvement in the value of the debt is always less that of the whole firm. However, when debtholders have access to a selling mechanism that ensures that the support of the distribution of payoffs is skewed to the left, they get to appropriate more of the value increase that accrues from greater competition. Any mechanism in which the payment from the winning bidder(s) depend(s) on the bid of the highest valued bidder in a positive fashion has the property that the payoff distribution is truncated on the right. This is because each bidder will, perforce, shade his bid. Such shading is the highest when the winning bidder pays his own bid. Hence, for a high enough value of F, it is possible that the payoffs to debt are the same as that to the whole firm. The implications of this result are described below.

Proposition 3 *For a given maximum possible number of bidders, N, there exist an $\hat{F}(N)$ such that for all $F > \hat{F}$ debtholders choose $t_D = t^*$.*

Thus, for a high enough face value of debt, it is in the debtholders' own interest not to rush into a "fire sale". [30] Note that, however, for the debtholders' choice to be firm value maximizing, they need to be given control over the form of the auction in addition to the decision of when to hold the auction. This is all the more true for situations in which N is small and, hence, the extent of bid-shading is greater. The fact that senior claimants, if delegated the *entire* choice of the sales mechanism, may behave as if they were the the sole owners of the firm has important implications for the allocation of rights between senior and junior claimants in bankruptcy. The result shows that an asymmetric allocation of such rights may very well be preferable when the level of impairment of debt is (probabilistically) relatively high.

[30] The result does, however, depend on the finiteness of N. In most realistic situations of bankruptcy resolution, the value of N is likely to be relatively low. We thank the referee for pointing to the importance of this feature.

4.2 Equityholders as Sellers

We have already established that equityholders prefer, for any value of n, an ascending-bid auction over a first-price auction, since the former has greater variability associated with it. Given a choice of the auction form, therefore, they would adopt the ascending-bid form. In addition, waiting for an additional bidder involves the potential of benefits while the costs of waiting are shared between debtholders and equityholders. Under these circumstances, it is not surprising that equityholders will always choose to wait longer than the benchmark time of t^*.

Proposition 4 *1. Equityholders always choose to wait for a period of time, t_E, strictly higher than t^*.*

2. The incentive to wait longer increases with the face value of the debt.

The proposition above points out that although debt may very well not have sufficient incentives to generate an optimally competitive auction, allocating selling rights to equity does not necessarily solve the problem of efficiency. Even without artifacts like control benefits, equity has the incentive to delay a sale to a point that cannot be justified by efficiency considerations alone. The natural question to ask, then, is when is the inefficiency that occurs when equity is allocated the right to choose the selling mechanism likely to be lower than that involved with the debt retaining such a right? This is the question we turn to next.

4.3 Optimal Allocation of Selling Rights

We have already demonstrated above that for a high enough face value of debt, the senior claimant has the incentive to choose the efficient value of t. Consequently, for relatively high levels of the face value of debt, it is clear that allocating selling rights to debt is efficient. For low levels of F, debtholders choose too low a value of t_D, while equityholders always go overboard. However, when F is low enough, the extent to which equityholders choose to overshoot is relatively small. Hence, for low enough values of F it is better for equityholders to have the right to run the sale. This intuition is formalized below.

Proposition 5 *There exists an F^* such that the BV is higher if the equityholders are allocated the selling rights for all $F < F^*$ and the debtholders are allocated the right otherwise, where F^* is* *given by,*

$$\delta^{t_D(F^*)}U(t_D(F^*)) = \delta^{t_E(F^*)}U(t_E(F^*))$$

By optimally allocating the decision to one of the claimants, the firm is able to avoid extreme forms of opportunistic behavior, even though neither party, in general, has incentives to make a value maximizing choice of t.[31] For a given state of the world (ω), such an allocation process has to, perforce, be a function of the face value of debt in relation to the range of possible values obtainable from conducting a sale. Consequently, a blanket rule that allocates selling rights to either party irrespective of the face value of debt outstanding or the realized state of the world can be improved upon.[32]

Shleifer and Vishny (1992) also point out that realizable values for the assets of a firm in bankruptcy can vary depending upon why the firm entered bankruptcy.[33] They point out that when bankruptcy is triggered principally by systemic or industry-wide factors, liquidity considerations may prevent the highest valued buyer from acquiring the firm's assets. On the other hand, a bankruptcy brought about primarily by idiosyncratic performance characteristics need not suffer from this problem. While our approach does not model liquidity constraints *per se*, we share with them the notion that different states of the world may have associated with them very different parameters for the valuation distributions of outside bidders. Hence, in a generalization of our approach, the right to design a sale mechanism may be best delegated by an outside authority whose primary task is to determine the state of the world by eliciting information from claimants and other informed parties. This suggests the desirability of a court or a contractually appointed arbitrator to perform an information solicitation role in addition to any role of mere contract enforcement.[34] However, generalizing our approach to the case of asymmetric information is beyond the scope of this paper.

[31] Note that the values of t_D and t_E used above are the values chosen by the respective claimants under the auction form they individually prefer.

[32] Schwartz (1997) also echoes our concern that some bankruptcy resolution procedure may be appropriate in one set of states and another appropriate in other states.

[33] Stromberg (1996) provides indirect empirical support to their arguments.

[34] Schwartz (1997) argues that contracts between claimants may be able to do better in the area of bankruptcy resolution than state-mandated procedures. Thus, the role for the legal system may be simply one of contract enforcement. Our analysis suggests that, while such contracts may indeed allow for potentially greater flexibility in bankruptcy resolution procedures, there may still be the need for a party to function as an impartial solicitor of information. We are, however, sympathetic to his central concern that the state-mandated procedure may not allow enough flexibility in bankruptcy resolution.

Another way to interpret the result above is to observe that the modalities of an optimal bankruptcy resolution mechanism must be dependent on the circumstances under which a firm enters bankruptcy. If the bankruptcy regime is such that management can destroy much value before being forced to declare bankruptcy, then F is likely to be nearer H. In such a "loose" regime, it is efficient to give debtholders control over bankruptcy resolution procedures. On the other hand, if bankruptcy is triggered relatively early in the process, or is strategically declared to force the hands of other claimants,[35] it would be reasonable to give control of the resolution process to the junior claimant. Bankruptcy resolution schemes, therefore, should not be one-size-fits-all kinds of schemes which focus only on the solution methods while leaving out considerations of the bankruptcy triggering mechanisms. In a fashion, reorganization procedures are meant to allow for such flexibilities. Our point is that auction-based resolutions also need to allow for some degree of flexibility which renders them capable of distinguishing between cases which should be treated differently.

The importance of the flexibility that can be obtained simply by the optimal delegation of the *form* of the sale mechanism can be easily illustrated. To see this, let's assume that the auction form is constrained to be an ascending-bid auction while the actual mechanics of running the auction are delegated to a claimant. Our results above show that, under such circumstances, both t_D and t_E differ from t^*. Hence, with an endogenous choice of t, the specification of the auction mechanism to be employed does not, by itself, allow for the achievement of *ex ante* investment efficiency. Instead, optimal delegation of the form itself is an important ingredient to achieving this aim. The additional flexibility engendered by allowing for a choice of form is crucial to enhancing the bankruptcy value of the firm.

4.4 A Trustee as Seller

In the previous sections, we showed that the optimal allocation of selling rights to debt or equity depends on the level of the face value of debt and that neither party will, in general, have incentives to make a value maximizing choice of t. The interesting question to ask at this stage, then, is whether there exists an allocation scheme which will guarantee the efficient choice of t under all circumstances.

As we mentioned earlier, the simple answer is a "yes", since we could always have an outsider

[35]For example, Dow Corning declared bankruptcy in the face of massive customer litigation in order to fashion a survival plan.

conduct the sale. If the outsider were given incentives so that his compensation for the activit were directly linked to the present value of the sale price obtained, he would always choose to wa t^* periods. Thus, a sale conducted by an outsider could always attain efficiency.[36]

It is tempting to conclude that such an outsider may correspond to the notion of a trustee whic is found in varying forms in all existing bankruptcy codes. However, in our view, such an automati identification would be misleading for a number of reasons. Perhaps the most fundamental reaso that we would disagree with this interpretation is that it is possible in all models of asymmetri incentives to make agency conflicts disappear by conjuring up another player who has either benign interests or has incentives to act for the common good. Consequently, agency problems like debt-equity conflicts would never arise if every decision that was subject to conflict were handled by an outside person. We do not think that this is a reasonable solution. We bolster our objections on this account by detailing several additional factors.

First, no matter how the formal incentives of such a trustee are set up, there always remains the possibility of side-payments. Consequently, the trustee ends up acting as an agent of one of the principals and extracts rents based on his power. Second, trusteeship typically also entails running of the firm while the sale process is going on. We think it is likely that the trustee's running of operations may very well be value destroying by itself.[37] Third, although we have not considered this case formally, different parties may very well have asymmetric information about sale prospects. In such a case, it may be quite likely that a trustee's information is of lower quality than that of either interested claimant. In such a case, the trustee's decision may very well be worse than that of one of the claimant's. On account of all these reasons, we find the trustee interpretation of our benchmark case an unlikely fit under all circumstances. Nevertheless, in circumstances where the concern with our objections is demonstrably low, a sale run by a trustee would be sufficient to attain *ex ante* investment efficiency in concert with *ex post* allocational efficiency.

4.5 Multiple Claimants

Our analysis has been conducted in the context of two claimants: debt and equity. In most cases of corporate bankruptcy, the capital structure of the firm is significantly more complicated and involves multiple layers of creditors with varying degrees of seniority. Our analysis suggests that,

[36] In the U.S., trustees today are compensated in part on the basis of revenues they raise for the bankrupt estate.

[37] Weiss and Wruck (1996) document severe value destruction during trustee supervised operations of Eastern Airlines.

in such cases, when resolution of bankruptcy is performed by auctioning the firm and distributing proceeds by adherence to APR, the control of the sale procedure be left in the hands of a claimant junior to senior debt and senior to equity. Such a junior claimant can be shown to have incentives to choose a value of t higher than the choice of the senior debt and strictly lower than that of equity. However, the precise auction form chosen by such a claimant cannot be characterized in general. Perhaps the two-thirds approval process by each class of claimants other than equity that is a feature of today's Chapter 11 procedures, is a step that attempts to bestow power to junior creditors in a similar vein.

4.6 Asymmetric Bidders

Our results on the choice of t by each claimant have been established under the assumption of an identical support of valuations for each bidder. An alternative, and perhaps more realistic, description may involve later bidders being inferior to earlier arrivals in the sense of their having a lower support of values (or lower expected values). In such a scenario, these later bidders provide intense competition only on the lower part of the support. Debtholders, instead of the equityholders, welcome such competition since it does not necessarily involve leakage of value to junior claimants. The next proposition shows that, in this setting, it is the equityholders who have greater incentives to conduct an early sale. Under these circumstances, then, the intuition behind debtholders being more eager to sell than the equityholders breaks down completely.

Proposition 6 *If later bidders are sufficiently inferior to earlier bidders, equityholders may choose a value of t_E strictly lower than either t_D or t^*.*

It is the equityholders who are now more prone to holding a "fire sale" for the firm. Thus, the intuition that senior claimants have inadequate incentives to conduct an efficient sale is crucially dependent on the implicit assumption that potential bidders are *ex ante* similar in their valuations. Absent this assumption, the intuition may have little force, as we demonstrate above. In general, then, the optimal allocation of decision rights to claimants is a rather complicated issue which must take into account the parameters of the problem at hand. [38]

[38]Note, however, that in a setting with asymmetric support for buyers' valuations, competitive mechanisms may not qualify as optimal mechanisms. The general nature of optimal mechanisms under this setting is, as far as we know, an unsolved problem to date.

5 Conclusion

We have shown that market-based bankruptcy resolution schemes that claim the virtue of dis pensing with inter-claimant conflict in the bankruptcy process do so only by mandating away suc conflicts. This is because, any market-based sales mechanism has its own specific distributional at tributes and different claimants have distinct preferences over these. Just as present day bankruptc resolution procedures engender claimant conflicts over reorganization plans, the proposed auction based resolutions which seek to maintain adherence to APR, will engender claimant conflicts ove specific selling mechanisms. Consequently, a proposed scheme needs to either specify the sellin mechanism in detail or identify a specific claimant who will choose his own preferred mechanism.

Furthermore, we have demonstrated that a resolution procedure based on the delegation of a choice of mechanism cannot be independent of the expected degree of impairment of a creditor's claim. An efficient allocation process for the right to choose a selling procedure must depend on expectations regarding the likelihood of creditor impairment. Given inherent inter-claimant conflict in this regard, it is likely that an independent court can add value to the process over and above what could be contractually achieved by the affected parties *ex ante.*

At a more general level, we establish that the first-price sealed-bid auction has the least variability of revenues among competitive mechanisms. This demonstration has implications outside the area of bankruptcy design. For example, whenever sales methods are restricted to common bid-based procedures, a risk averse seller will have incentives to use a first-price sealed-bid auction.

Finally, we hope to have drawn attention to an area that gets considerable attention among practitioners who spend a lot of time and energy designing methods of sale of various corporate assets.[39] Much of the analysis in this area, however, has been undertaken without the benefit of a cogent body of work examining the various choices.[40] We view our work as a partial analysis of such choices and hope that other researchers will be motivated by this analysis to work in this area.

[39] For example, Baldwin and Bhattacharyya (1991) describes the tremendous effort that went into the design of the method of sale for Conrail.

[40] Notable exceptions are found in Cramton and Schwartz (1991) who study auction procedures for takeover bidding in the context of private and common values and Bulow and Klemperer (1996) who compare auctions with negotiations. Recent and emerging work in the areas of privatization and auctioning state-owned properties have also focused on related issues.

6 Appendix

As in most of the main body, we suppress here the dependence of variables on ω. That is, our proofs hold for every realization of ω. To avoid notational clutter, we also normalize $L(\omega)$ to zero where clarity is not compromised.

Proof of Proposition 1: To establish second order stochastic dominance of the first-price auction, we use the definition of Rothschild and Stiglitz (1979). Thus, x_f dominates x_o if and only if

$$x_o(v) =_{dist} x_f(v) + \epsilon(v) \qquad \text{with } E(\epsilon|x_f) = 0$$

where $x_f(v)$ is the cash flow produced under the first-price auction for a specific realization of the vector of values. Similarly, x_o is the cash flow of some other auction in our class. Instead of using the equivalence in distributions we use the stronger pointwise equivalence. Thus, define

$$\epsilon(v) = x_o(v) - x_f(v)$$

All we need to show is that the conditional expectation of $\epsilon = 0$.

$$\begin{aligned} E[\epsilon(v)|x_f(v)] &= E[x_o(v) - x_f(v)|x_f(v)] \\ &= E[x_o(v) - x_f(v_m)|v_m] \\ &= E[x_o(v)|v_m] - x_f(v_m) \\ &= 0 \end{aligned}$$

The second equality is obtained by recognizing that, in the first-price auction, the revenue obtained is just a function of the maximum of the value realizations. We obtain the last equality by showing that $E[x(v)|v_m]$ is constant across all competitive mechanisms. This follows the proof strategy in Myerson (1981). From the revelation principle we know that, for every competitive mechanism, there exists a direct mechanism which satisfies incentive compatibility. Thus, given a direct mechanism which allocates the object with the probability vector $\xi(v)$ and has an associated payment function $x(v)$ the incentive compatibility condition is given by:

$$\int_{v_{-i}} (v_i\xi_i(v) - x_i(v))\, f_{-i}dv_{-i} \geq \int_{v_{-i}} (v_i\xi_i(v_{-i}, s_i) - x_i(v_{-i}, s_i))\, f_{-i}dv_{-i} \qquad \forall s_i, v_i, i$$

The above can be simplified to,[41]

$$\int_{v_{-i}} (v_i\xi_i(v) - x_i(v))\, dF_{-i}(v_{-i}) = \int_0^{v_i} \int_{v_{-i}} \xi_i(v_{-i}, s_i) dF_{-i}(v_{-i}) ds_i$$

Separating the terms, the expected payment by bidder i with value v_i is given by,

$$\int_{v_{-i}} x_i(v_i, v_{-i}) dF_{-i}(v_{-i}) = \int_{v_{-i}} v_i\xi_i(v) dF_{-i}(v_{-i}) - \int_0^{v_i} \int_{v_{-i}} \xi_i(v_{-i}, s_i) dF_{-i}(v_{-i}) ds_i \qquad (2)$$

The expected revenue obtained by the seller for $\max(v_i) = v_m$, in any such direct mechanism is,

$$\begin{aligned}
&= n\ E[x_i(v)|v_m] \\
&= n\ \text{Prob}(v_i = v_m | \max(v) = v_m) E[x_i(v)|v_i = v_m] \quad \text{using } x_i = 0 \text{ for losers} \\
&= \int_{v_{-i}, \max(v_{-i}) < v_m} x_i(v_{-i}, v_m) \frac{dF_{-i}(v_{-i})}{F(v_m)^{(n-1)}} \qquad \text{using Prob}(v_i = v_m) = \frac{1}{n} \\
&= \int_{v_{-i}, \max(v_{-i}) < v_m} x_i(v_{-i}, v_m) \frac{dF_{-i}(v_{-i})}{F(v_m)^{(n-1)}} \\
&\qquad + \int_{v_{-i}, \max(v_{-i}) \geq v_m} x_i(v_{-i}, v_m) \frac{dF_{-i}(v_{-i})}{F(v_m)^{(n-1)}} \quad \text{using } x_i = 0 \text{ for losers} \\
&= \frac{1}{F(v_m)^{(n-1)}} \int_{v_{-i}} x_i(v_{-i}, v_m) dF_{-i}(v_{-i}) \\
&= \frac{1}{F(v_m)^{(n-1)}} \left(v_m \int_{v_{-i}} \xi_i(v_m, v_{-i}) dF_{-i}(v_{-i}) - \int_0^{v_m} \int_{v_{-i}} \xi_i(v_{-i}, s_i) dF_{-i}(v_{-i}) ds_i \right).
\end{aligned}$$

The last equality is obtained from equation (2). Thus, the expected revenue for a given v_m is constant over all direct mechanisms which implement a competitive mechanism, as the above expression depends only on the probability function $\xi(\cdot)$, which is the same across all competitive mechanisms. ∎

[41] See Myerson (1981) for details.

Proof of Lemma 1 An all equity firm maximizes: $\delta^t\, U(t)$.[42] The first derivative is given by,

$$\delta^t \left[U_t(t) + \log(\delta)U(t)\right] \tag{3}$$

Setting the first derivative to zero gives us the desired result:.

$$\frac{U_t(t^*)}{U(t^*)} = -\log(\delta)$$

A zero value of the first derivative is sufficient for a global maximum if $\forall t < t^*$ the first derivative is positive and f $\forall t > t^*$ it is negative. As δ^t in equation (3) is positive $\forall t$, it is sufficient to show $[U_t(t) + \log(\delta)U(t)]$ is decreasing in t. Now,

$$\begin{aligned}
&\frac{\partial}{\partial t}\left[U_t(t) + \log(\delta)U(t)\right] < 0 \\
&\Leftrightarrow U_{tt}(t) + \log(\delta)U_t(t) < 0 \\
&\Leftarrow U_{tt}(t) < 0 \qquad \text{[As } U_t(t) > 0 \text{ and } \log(\delta) < 0]
\end{aligned}$$

Thus, a sufficient condition for the FOC to obtain a global maximum is $U_{tt}(t) < 0$. Now,

$$\begin{aligned}
U(t) &= \sum_{n=2}^{N} V(n)p(n|t) \\
&= P(2|t)V(2) + \sum_{n=3}^{N-1}\left[V(n)\left(P(n|t) - P(n-1|t)\right)\right] + (1 - P(N-1|t))V(N) \\
&= \sum_{n=2}^{N-1} P(n|t)\left(V(n) - V(n+1)\right) + V(N) \\
U_{tt}(t) &= \sum_{n=2}^{N-1} P_{tt}(n|t)\left(V(n) - V(n+1)\right) \\
&< 0 \qquad \text{[As CDFC implies that } P_{tt}(\cdot) \geq 0]
\end{aligned}$$

Proof of Proposition 2

Case 1: The Ascending-Bid Auction: Let $g(v, n)$ be the density of the second-highest valuation bidder given n total bidders. The debtholders' expected revenue for a specific n is:

[42] The problem has solution with $t^* < \infty$ as U(t) is bounded from above by H while δ^t approaches 0 as t increases.

$$D(F,n) \equiv \int_0^F vg(v,n)dv + \int_F^H Fg(v,n)dv \tag{4}$$

Let $\Delta(F,t)$ be the expected utility for the debtholders for a given t, where

$$\Delta(F,t) \equiv \sum_{n=2}^{N} p(n|t)D(F,n) \tag{5}$$

The debtholders' objective is to choose t to maximize $\delta^t\Delta(F,t)$. From the FOC[43] we obtain

$$\begin{aligned}\frac{\partial}{\partial t}\delta^t\Delta(F,t) &= \delta^t\Delta_t(F,t) + \delta^t \log\delta\Delta(F,n)\\ &= \delta^t\left(\Delta_t + \log\delta\Delta\right)\end{aligned}$$

We would like to show that at $t = t^*$ (argmax for the all equity firm) $\frac{\partial}{\partial t}\delta^t\Delta(F,t) < 0$. Thus, we need to show,

$$\left[\frac{\partial}{\partial t}\delta^t\Delta(F,t)\right]_{t=t^*} < \left[\frac{\partial}{\partial t}\delta^t\Delta(H,t)\right]_{t=t^*} = 0 \qquad \forall F$$

where the RHS of the above is from the optimization problem of the all equity firm. Thus, we need to show:

$$\begin{aligned}&\Delta_t(F,t^*) + \log\delta\Delta(F,t^*) < 0\\ \Leftrightarrow\ &\frac{\Delta_t(F,t^*)}{\Delta(F,t^*)} < -\log\delta\\ \Leftrightarrow\ &\frac{\Delta_t(F,t^*)}{\Delta(F,t^*)} < \frac{\Delta_t(H,t^*)}{\Delta(H,t^*)}\end{aligned}$$

The RHS of the last inequality again comes from the optimization problem of the all equity firm, where $\Delta(F = H,t) = V(t)$. However, from the mean value theorem, we have

$$\frac{\Delta_t(F,t^*)}{\Delta(F,t^*)} = \frac{\Delta_t(H,t^*)}{\Delta(H,t^*)} + (\hat{F} - H)\frac{\partial}{\partial F}\frac{\Delta_t(\hat{F},t^*)}{\Delta(\hat{F},t^*)}$$

[43] The proof provided in lemma 1 can be replicated to show that the FOC provides a global maximum for the debtholders' problem. The first derivative of $(\Delta_t + \Delta\log\delta)$ is negative for $t > t_D$ and positive for $t < t_D$ as long as $[D(n-1) - D(n)] < 0$, which is obviously true from first order stochastic dominance.

Given that $(\hat{F} - H) < 0$, it is sufficient to show that

$$\frac{\partial}{\partial F}\frac{\Delta_t(F,t)}{\Delta(F,t)} > 0$$

$$\text{or } \Delta_{tF}(F,t)\Delta(F,t) - \Delta_t(F,t)\Delta_F(F,t) > 0 \quad \forall F, t \tag{6}$$

Now from equation (5):

$$\Delta_F(F,t) = \sum_{n=2}^{N} p(n|t) D_F(F,n)$$

$$\Delta_t(F,t) = \sum_{n=2}^{N} p_t(n|t) D(F,n)$$

$$\Delta_{tF}(F,t) = \sum_{n=2}^{N} p_t(n|t) D_F(F,n)$$

From equation (6) we need to show,

$$\left(\sum_{n=2}^{N} p_t(n|t) D_F(F,n)\right)\left(\sum_{n=2}^{N} p(n|t) D(F,n)\right) > \left(\sum_{n=2}^{N} p_t(n|t) D(F,n)\right)\left(\sum_{n=2}^{N} p(n|t) D_F(F,n)\right)$$

Expanding the above we obtain:

$$\begin{aligned}
&[p_t(2|t)D_F(F,2) + p_t(3|t)D_F(F,3) + ... + p_t(N|t)D_F(F,N)] \\
&\quad \times [p(2|t)D(F,2) + p(3|t)D(F,3) + ... + p(N|t)D(F,N)] \quad > \\
&[p_t(2|t)D(F,2) + p_t(3|t)D(F,3) + ... + p_t(N|t)D(F,N)] \\
&\quad \times [p(2|t)D_F(F,2) + p(3|t)D_F(F,3) + ... + p(N|t)D_F(F,N)] \\
&\iff \sum_{i=2}^{N}\sum_{j=i}^{N} \Gamma(i,j) > 0 \qquad \text{where a generic term, say, } \Gamma(3,N) \text{ is:}
\end{aligned}$$

$$\Gamma(3,N) = \begin{bmatrix} p_t(3|t)D_F(F,3)p(3|t)D(F,3) + p_t(3|t)D_F(F,3)p(N|t)D(F,N)+ \\ p_t(N|t)D_F(F,N)p(3|t)D(F,3) + p_t(N|t)D_F(F,N)p(N|t)D(F,N) \end{bmatrix}$$

$$- \begin{bmatrix} p(3|t)D_F(F,3)p_t(3|t)D(F,3) + p(3|t)D_F(F,3)p_t(N|t)D(F,N) \\ +p(N|t)D_F(F,N)p_t(3|t)D(F,3) + p(N|t)D_F(F,N)p_t(N|t)D(F,N) \end{bmatrix}$$

$$= [p_t(3|t)D_F(F,3)p(N|t)D(F,N) + p_t(N|t)D_F(F,N)p(3|t)D(F,3)]$$

$$- [p(3|t)D_F(F,3)p_t(N|t)D(F,N) + p(N|t)D_F(F,N)p_t(3|t)D(F,3)]$$

$$= p_t(3|t)p(N|t)\,[D_F(F,3)D(F,N) - D_F(F,N)D(F,3)]$$

$$+ p_t(N|t)p(3|t)\,[D_F(F,N)D(F,3) - D_F(F,3)D(F,N)]$$

$$= [D_F(F,3)D(F,N) - D_F(F,N)D(F,3)]\,[p_t(3|t)p(N|t) - p_t(N|t)p(3|t)] \tag{7}$$

It is sufficient to show that $\Gamma(i,j) > 0\ \forall i,j$. This is true if, in the generic term above, both components are negative. Examining the second component, from MLRP of $p(\cdot)$,we have

$$\frac{p_t(3|t)}{p(3|t)} < \frac{p_t(N|t)}{p(N|t)} \Rightarrow [p_t(3|t)p(N|t) - p_t(N|t)p(3|t)] < 0$$

For the first component to be negative, we must have, similarly,

$$\frac{D_F(F,3)}{D(F,3)} < \frac{D_F(F,N)}{D(F,N)}$$

To demonstrate the validity of the last inequality we establish the following:

$$\frac{D_F(F,n_1)}{D(F,n_1)} < \frac{D_F(F,n_2)}{D(F,n_2)} \quad \text{for } n_1 < n_2$$

$$\Longleftrightarrow \frac{\int_F^H g(v,n_1)dv}{\int_0^F vg(v,n_1)dv + \int_F^H Fg(v,n_1)dv} < \frac{\int_F^H g(v,n_2)dv}{\int_0^F vg(v,n_2)dv + \int_F^H Fg(v,n_2)dv}$$

$$\Longleftrightarrow \int_F^H g(v,n_1)dv \int_0^F vg(v,n_2)dv < \int_F^H g(v,n_2)dv \int_0^F vg(v,n_1)dv$$

$$\Longleftrightarrow \frac{\int_0^F vg(v,n_2)dv}{\int_0^F vg(v,n_1)dv} < \frac{\int_F^H g(v,n_2)dv}{\int_F^H g(v,n_1)dv} \tag{8}$$

Examining the LHS of the expression (8),

$$
\begin{aligned}
\frac{\int_0^F vg(v,n_2)dv}{\int_0^F vg(v,n_1)dv} &= \frac{\int_0^F v\frac{g(v,n_2)}{g(v,n_1)}g(v,n_1)dv}{\int_0^F vg(v,n_1)dv} \\
&< \frac{\int_0^F v\frac{g(F,n_2)}{g(F,n_1)}g(v,n_1)dv}{\int_0^F vg(v,n_1)dv} \quad \text{(from the MLRP of } g(\cdot)) \\
&= \frac{g(F,n_2)}{g(F,n_1)} \\
&= \frac{\int_F^H \frac{g(F,n_2)}{g(F,n_1)}g(v,n_1)dv}{\int_F^H g(v,n_1)dv} \\
&< \frac{\int_F^H \frac{g(v,n_2)}{g(v,n_1)}g(v,n_1)dv}{\int_F^H g(v,n_1)dv} \quad \text{(from the MLRP of } g(\cdot)) \\
&= \frac{\int_F^H g(v,n_2)dv}{\int_F^H g(v,n_1)dv}
\end{aligned}
$$

To show that $t_D(F)$ is increasing in F, all we need to show is that $\frac{\Delta_t(F,t)}{\Delta(F,t)}$ is increasing in F, which has already been established.

Case 2: First-price sealed-bid auction Let the symmetric bidding strategy of a bidder be $\psi(v_i, n)$ and now let $h(\psi, n)$ be the density function for the highest bid. The debtholders' objective is to choose t to maximize $\Delta(F,t)$, where

$$
\Delta(F,t) \equiv \sum_{n=2}^{N} p(n|t)D(F,n)
$$

$$
D(F,n) \equiv \begin{cases} \int_0^F \psi h(\psi,n)d\psi + \int_F^{\psi(H,n)} Fh(\psi,n)dv & \text{if } \psi(H,n) > F \\ \int_0^{\psi(H,n)} \psi h(\psi,n)d\psi & \text{if } \psi(H,n) \leq F \end{cases}
$$

Now define $\widehat{F}$ such that $\psi(H,N) = \widehat{F}$. Now $\psi(v,n)$ is increasing in both arguments. Therefore, for all $F > \widehat{F}$, the debtholders get all the revenue from a first price auction $\forall v \leq H$ and $\forall n \leq N$. Thus, the debtholders are the sole claimants of the cash flows and will pick $t_D = t^*$.

For all $F < \widehat{F}$, the exact same proof strategy as that for the ascending-bid auction yields t_D strictly less than t^*. Thus, t_D is at least weakly less than t^*.

Proof of Proposition 3 Follows from the proof of Proposition 2 case 2 (First-price sealed bid auction).

Proof of Proposition 4 Since equity always prefers an ascending-bid auction, we need to provide a proof only for this case. Let $E(F,n)$ be the expected revenues to equity for a given face value of debt F and let $g(v,n)$ again be the density function of the second highest value bidders. Equityholders choose $t = t_E$ to maximize $\delta^t \Omega(F,t)$, where

$$\Omega(F,t) \equiv \sum_{n=2}^{N} p(n|t)E(F,n)$$
$$E(F,n) = \int_F^H (v-F)g(v,n)dv$$

The FOC[44] of the equityholders' problem implies

$$\frac{\Omega_t(F,t_E)}{\Omega(F,t_E)} = -\log\delta$$

To prove that equity always chooses $t_E > t^*$, we need to show that $\frac{\Omega_t(F,t^*)}{\Omega(F,t^*)} > -\log\delta = \frac{U_t(t^*)}{U(t^*)}$. Now, conservation of value implies the following identity,

$$U(t) \equiv \Delta(F,t) + \Omega(F,t) \qquad \forall t, \forall F$$

Thus,

$$U_t(t) \equiv \Delta_t(F,t) + \Omega_t(F,t)$$
$$\frac{U_t(t)}{U(t)} = \frac{\Delta_t(F,t)}{U(t)} + \frac{\Omega_t(F,t)}{U(t)}$$
$$\frac{U_t(t)}{U(t)} = \frac{\Delta_t(F,t)}{\Delta(F,t)}\frac{\Delta(F,t)}{U(t)} + \frac{\Omega_t(F,t)}{\Omega(F,t)}\frac{\Omega(F,t)}{U(t)}$$
$$\frac{U_t(t)}{U(t)} = \frac{\Delta_t(F,t)}{\Delta(F,t)}\alpha(F,t) + \frac{\Omega_t(F,t)}{\Omega(F,t)}(1-\alpha(F,t)) \tag{9}$$
$$\text{where } \alpha(F,t) = \frac{\Delta(F,t)}{U(t)} < 1.$$

Since, by Proposition 2, $\frac{\Delta_t(F,t^*)}{\Delta(F,t^*)} < \frac{U_t(t^*)}{U(t^*)}$, we have that $\frac{\Omega_t(F,t^*)}{\Omega(F,t^*)} > \frac{U_t(t^*)}{U(t^*)} = -\log\delta$.
Similarly, to show that t_E is increasing in F, all we need to show is that $\frac{\Omega_t(F,t)}{\Omega(F,t)}$ is increasing in F. Following the exact same proof strategy as in proposition 2, we get an analogue to equation (7) as

[44]The proof provided in lemma 1 can be replicated to show that the FOC provides a global maximum for the equityholders' problem. The first derivative of $(\Omega_t + \Omega \log\delta)$ is negative for $t > t_E$ and positive for $t < t_E$ as long as $[E(n-1) - E(n)] < 0$, which is obviously true from first order stochastic dominance.

follows:

$$[E_F(F,3)E(F,N) - E_F(F,N)E(F,3)]\,[p_t(3|t)p(N|t) - p_t(N|t)p(3|t)] \tag{10}$$

To prove $\frac{\Omega_t(F,t)}{\Omega(F,t)}$ is increasing in F, it is sufficient to show that equation (10) is positive. We have already shown the second term of equation (10) to be negative. Thus, it is sufficient to show that the first term is also negative. To obtain that we next show the following:

$$\begin{aligned}
&[E_F(F,n_1)E(F,n_2) - E_F(F,n_2)E(F,n_1)] < 0 \quad \forall\, n_1 < n_2 \\
&\Leftrightarrow -\int_F^H g(v,n_1)dv \int_F^H (v-F)g(v,n_2)dv + \int_F^H g(v,n_2)dv \int_F^H (v-F)g(v,n_1)dv < 0 \\
&\Leftrightarrow \int_F^H g(v,n_2)dv \int_F^H (v-F)g(v,n_1)dv < \int_F^H g(v,n_1)dv \int_F^H (v-F)g(v,n_2)dv \\
&\Leftrightarrow \frac{\int_F^H (v-F)g(v,n_1)dv}{\int_F^H g(v,n_1)dv} < \frac{\int_F^H (v-F)g(v,n_2)dv}{\int_F^H g(v,n_2)dv} \\
&\Leftrightarrow \frac{\int_F^H vg(v,n_1)dv}{\int_F^H g(v,n_1)dv} < \frac{\int_F^H vg(v,n_2)dv}{\int_F^H g(v,n_2)dv} \quad \text{(rearranging terms)} \\
&\Leftrightarrow \int_F^H v\frac{g(v,n_1)}{\int_F^H g(v,n_1)dv}dv < \int_F^H v\frac{g(v,n_2)}{\int_F^H g(v,n_2)dv}dv \\
&\Leftrightarrow \int_F^H v\,q(v,n_1)dv < \int_F^H vq(v,n_2)dv \qquad \text{where } q(\cdot) \text{ is the conditional distribution.}
\end{aligned}$$

It is well known that a conditional distribution inherits the FOSD property from its parent. Hence, $\int_F^H v\,q(v,n)dv$ is increasing in n. Thus, the last inequality is true. ∎

Proof of Proposition 5: Consider the equityholders' problem. At $F = L$ the firm is just an all equity firm and, thus, $t_E(F = L) = t^*$. Hence the value obtained ($\delta^{t^*}U(t^*)$) is the maximum possible. Now $t_E(F)$ is increasing in F (proposition 4) and $\frac{\partial}{\partial t}\delta^t U(t) < 0$ for all $t > t^*$(lemma 1). Thus, conditional on the equityholders choosing t, the value of the firm is globally decreasing in F.

Now consider the debtholders' problem. For $F > \widehat{F}$, $t_D = t^*$ (proposition 3) and $t_D(F)$ is increasing in F (proposition 2). Using the fact that $\frac{\partial}{\partial t}\delta^t U(t) > 0$ for all $t < t^*$(lemma 1), the value of the firm is lower than the maximum possible value ($\delta^{t^*}U(t^*)$) for $F < \widehat{F}$ under the debtholders' choice.

From the continuity of $\delta^t U(t)$, the value of the firm crosses under the debtholders' choice with that under the equityholders' choice at some $F^* \in (L, \widehat{F})$. The optimality of the allocation rule stated in the proposition follows. ∎

Proof of Proposition 6: We restrict attention to the ascending-bid auction for purpose of this result. Assume that for $n = 2$ each bidders' valuation is distributed over $[L, H(2)]$. For $n \geq 3$ any increase in the number of bidders occurs by addition of a bidder with valuation distributed over $[L, H(n)]$, where $H(n-1) > H(n)$.

Since, all we need to show is existence, we can confine our attention to a specific parameterization. Suppose $N = 3$, $H(2) = 1$, $H(3) = 0.5$, and $F = 0.5$. Thus, with two bidders, both have values distributed on $[0, 1]$. However, if the third bidder comes in he has a valuation distributed over $[0, 0.5]$.

In this case, the equityholders' choice is quite clear. Since the highest possible value of the third bidder is equal to the face value of debt, any benefit from the existence of the third bidder is obtained just by the debtholders. Thus, equityholders can only be worse off by waiting and increasing the probability of there being three bidders. Consequently, $\frac{\partial}{\partial t}\delta^t \psi(t) < 0 \, \forall t$ and $t_E = 0$.

On the other hand, at the firm level there is always some benefit from this competition. Therefore an all equity firm will choose to wait longer than the equityholders if $\frac{\partial}{\partial t}\delta^t U(t) > 0$ at $t = 0$, which is true if $p_t(2, t = 0)$ is low enough (a sufficient condition for which is $p_t(2, t = 0) < \frac{\log(\delta) U(0)}{V(3) - V(2)}$). Thus, $t^* > 0 = t_E$.

Similarly, it can be shown that in this case $t_D > t_E$. ∎

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