Division of Research
Graduate School of Business Administration
The University of Michigan

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THE EFFECT OF MARKET STRUCTURE ON BANK PERFORMANCE

Working Paper 152

by

David Brophy
Christopher James

The University of Michigan

In "A Theory of Risk Averse Bank Behavior,"[1] we presented a theory of risk-averse bank behavior and analyzed its implications for bank asset and liability management. It was concluded that the proportion of loans held in the portfolio and, therefore, the equilibrium of expected return on loans were jointly dependent upon the composition of deposit liabilities. This result followed directly from two assumptions: (1) the expected loss due to net cash disbursements is a function of the volume of time and demand deposits, and (2) loans, unlike investments in government securities, carry a risk of default and therefore have a probability distribution of returns.

Assuming that demand deposits are more volatile in terms of unexpected withdrawals than are time deposits, it is seen that the composition of bank liabilities has a direct impact on the standard deviation of returns to stockholders and, therefore, on the risk incurred by investors. Loans also contribute to the variation of total returns and thus to the risk incurred by the banks. Given these two assumptions it was shown that decisions as to the proportion of loans held in the portfolio and therefore the rate charged on loans are not independent of deposit market characteristics.

The purpose of this second paper is to extend the theory of bank behavior to analyze the effects of market structure on the asset and liability portfolio. To analyze the effects of market structure one must realize that banks simultaneously participate in two markets: the market for loans and securities and the market for input: demand and time deposits. Previous theories of the banking firm have implied that each market was isolated from competitive characteristics of the other. This conclusion conflicted with previous empirical work which indicated, in particular, that entry into the deposit market effected performance in the loan market.[2,3] It will be shown in this paper that the theory we presented in the earlier paper is consistent with the belief that structure in both markets is important in analyzing bank behavior.

The effect of changes in market structure on bank performance will be analyzed through an analysis of the effects of entry by competing banks in both the loan and deposit markets. Following the approach of Alfred Broaduss,[4] we shall analyze changes in market structure through the use of shift parameters in the loan demand and deposit supply functions.

Part I

Let a_L represent an index which varies directly with the intensity of loan market competition as measured here by the number of lending institutions. The analysis which follows will focus only on the number of competitors, but other structural changes can be presented by the index a_L as well. The loan demand function will be written as a function of both the proportion of loans in the portfolio, X_L, and the shift parameter, a_L. Therefore, we have

(1) $$E_L = E_L(X_L, a_L)$$

and

(2) $$\frac{\partial E_L}{\partial a_L} < 0, \text{ and } \frac{\partial E_L}{\partial X_L} < 0.$$

Equation (2) indicates that new entry into the loan market will shift the demand-for-loan curve faced by existing banks to the left. Further, it will be assumed that new entry makes the demand curve more elastic at the old optimum rate of interest.

In the deposit market let a_d represent an index that varies directly with the intensity of deposit market competition as measured by the number of institutions issuing demand deposits. An increase in a_d, therefore, will represent a leftward shift and downward pivot in the demand deposit supply function at the old equilibrium rate of interest. The demand deposit supply function can be written, therefore, as:

(3) $$DD = DD(r_D, r_{TD}, a_D)$$

and

(4) $$\frac{\partial DD}{\partial a_d} < 0, \quad \frac{\partial DD}{\partial r_D} > 0, \quad \frac{\partial DD}{\partial r_{TD}} < 0,$$

where r_D is the rate of interest paid on demand deposits and r_{TD} is the rate paid on time deposits.

The method by which comparative static results for changes in market structure can be derived is to totally differentiate the first order conditions for a maximum, as described in equations (20) through (25) of Working Paper 151, with respect to all endogenous variables and the two shift parameters a_D and a_L. The system could then be solved, using Cramer's rule, for

$$\frac{dX_L}{da_L}, \frac{dX_L}{da_D}, \frac{dr_T}{da_D}, \frac{dr_D}{da_D}, \frac{dr_D}{da_L}, \text{etc.}$$

Working with the complete system as expressed in equations (20) through (25) quickly renders the problem unmanageable and unnecessarily complex. To find the effect of each shift parameter on all of the endogenous variables would require working with and solving a five-equation system. The complexity imposed in working with all five equations is unnecessary, since the main concern is how deposit market changes affect loan allocation; and since changes in the deposit market influence the loan market only through the cash withdrawal loss function, a model that contains only one asset market variable and one deposit market variable will yield the comparative static results described.

The model that will be used for the comparative statics will be based on several simplifying assumptions. First, it will be assumed that the proportion of the portfolio held in cash is exogenously given. In other words, the bank will be assumed to keep a constant percentage of deposits in the form of cash, the percentage being in excess of their reserve requirements. Second, assume that the rate paid on, and the level of, time deposits is given. This assumption is made to reduce the problem to manageable proportions, but, as will be seen below, it still incorporates the significant features of the model. Third, assume that the risk on loans is exogenously given and therefore not a function of the proportion of loans granted. Thus, the expected return on loans will be less than the explicit rate of interest r_L whenever $\sigma_L^2 > 0$, but, since $\sigma_L^2 = \bar{\sigma}_L^2$, E_L, will be less than r_L by a constant proportion. This assumption is made to simplify the analysis but it will not alter the results qualitatively. Finally, assume that expected losses due to net cash disbursements is a function of $\bar{X}_c$ and the ratio of demand deposits to time deposits, $\frac{DD}{TD}$. The same assumptions as above will be made about the probability distribution of net withdrawals, so that together with the assumptions just made, N can be written as

$$N = n \int_{X_c}^{c} (Z - X_c) k(Z) dZ \equiv \frac{n(c-X_c)}{2(c-b)} \tag{5}$$

or

$$N = N(DD).$$

Given the above assumptions, the maximization problem can be formulated as follows

$$(6)\quad \max_{X_L, r_D} = \frac{1}{\rho}\{\frac{F}{w}(X_L E_L + (M - X_L)\bar{E}_g - N) - \frac{1}{w}(\overline{TD}\ r_{TD} + DDr_D) - R\beta\sigma_w\}$$

where $M - X_L = X_g$

and $M = 1 - \bar{X}_c$.

Since the proportion of assets held in cash is fixed, given the constraint that $X_L + X_g + \bar{X}_c = 1$, then

$$X_g = 1 - \bar{X}_c - X_L$$

or

$$X_g = (M - X_L)$$

and the problem can be solved as an unconstrained maximization.

The first order conditions for the unconstrained maximization are

$$(7)\quad \frac{\partial V}{\partial X_L} = \frac{F}{w}(E_L + X_L \frac{\partial E_L}{\partial X_L} - \bar{E}_g) - R\beta \frac{\partial \sigma_w}{\partial X_L} = 0$$

$$(8)\quad \frac{\partial V}{\partial r_D} = \frac{1}{w}\frac{\partial DD}{\partial r_D} E_a - \frac{F}{w}\frac{\partial N}{\partial r_D} - \frac{1}{w}(D + \frac{\partial D}{\partial r_D} r_D) - R\beta \frac{\partial \sigma_w}{\partial r_D} = 0 .$$

Recalling that

$$\frac{\partial \sigma_w}{\partial X_L} = \frac{F}{w}\frac{\partial (X_L^2\sigma_L^2 + \sigma_n^2)^{1/2}}{\partial X_L} ,$$

then equation (7) can be rewritten to show the marginal revenue equalities for value maximization

(9) $$E_L + X_L \frac{\partial E_L}{\partial X_L} - \beta R \frac{\partial \sigma_{\Pi}}{\partial X_L} = \overline{E}_g \ .$$

Equation (9) yields the same result as equation (26) of the first paper, that the proportion held in loans is a function of the supply function of deposits, since σ_{Π} is a function of the ratio of demand to time deposits.

Equation (8) indicates that the implicit return on demand deposits will be a function of the return on assets, E_a, the deposit supply function, and the contribution of demand deposits to the risk of the firm.

Given equations (7) and (8), changes in market structure can be introduced by incorporating the indices a_L and a_D into the supply function for deposits and the loan demand functions. Define as described above

$$E_L = E_L(X_L, a_L)$$

$$DD = DD(r_D, a_L)$$

and totally differentiate the first order conditions with respect to X_L, r_D, a_L, and a_D. Placing the results in matrix form we have

(10) $$\begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{matrix} dX_L = p\ da_L + s\ da_D \\ dr_D = q\ da_L + u\ da_D \end{matrix}$$

where $$F_{11} = \frac{F}{w} \frac{\partial MR_L}{\partial X_L} - R\beta \frac{\partial^2 \sigma_w}{\partial X_L^2}$$

$$F_{12} = \frac{1}{w} \frac{\partial DD}{\partial r_D} (MR_L - \bar{E}_L) - R\beta \frac{\partial^2 \sigma_w}{\partial X_L \partial r_D}$$

$$F_{21} = \frac{1}{w} \frac{\partial DD}{\partial r_D}(MR_L - \bar{E}_L) - R\beta \frac{\partial^2 \sigma_w}{\partial X_L \partial r_D}$$

$$F_{22} = \frac{1}{w} \frac{\partial^2 DD}{\partial^2 r_D} E_a + \frac{F}{w} \frac{\partial^2 N}{\partial r_D^2} - \frac{1}{w} \frac{\partial MC_D}{\partial r_D} - R\beta \frac{\partial^2 \sigma_w}{\partial r_D^2}$$

$$p = - (\frac{F}{w} \frac{\partial MR}{\partial a_L}) \; da_L$$

$$q = - (\frac{1}{w} \frac{\partial D}{\partial r_D} \frac{\partial E_L}{\partial a_L}) \; da_L$$

$$s = (\frac{1}{w} \frac{\partial DD}{\partial a_D} (MR_L - \bar{E}_g) + R\beta \frac{\partial^2 \sigma_w}{\partial X_L \partial a_D}) \; da_D$$

$$u = - \frac{1}{w}(\frac{\partial DD}{\partial r_D} \frac{\partial N}{\partial a_D} + \frac{\partial^2 DD}{\partial r_D \partial a_D} E_a - \frac{\partial DD}{\partial a_D} \frac{\partial N}{\partial r_D} - \frac{F}{w} \frac{\partial^2 N}{\partial r_D \partial a_D}$$

$$- \frac{1}{w} \frac{\partial MC_D}{\partial a_D} - R\beta \frac{\partial^2 \sigma_w}{\partial r_D \partial a_D} \quad .$$

Let the above matrix be denoted as Δ, and, if the second order conditions for a maximum are satisfied, then Δ is negative semidefinite. For notational ease we have defined

$$MR_L = E_L + \frac{\partial E_L}{\partial X_L} X_L$$

and

$$MC_D = DD + \frac{\partial DD}{\partial r_D} r_D .$$

Using Cramer's rule, the relationship between the proportion of loans held in the portfolio and loan market and deposit market structure can be solved for. As is shown in the appendix, the proportion of loans held in the portfolio varies directly with the degree of competition in the demand deposit market, i.e.,

$$\frac{dX_L}{da_D} > 0 .$$

As entry occurs in the demand deposit market, banks, given our assumptions, face a more elastic supply of deposits at the previous equilibrium interest rate. The ratio of demand to time deposits in the more competitive environment will fall, reducing both the expected loss due to net disbursements and reducing the variance of losses, thus reducing the level of risk ($R\beta\sigma_W$). This result is the significant different between previous models of banks and the risk-averse models. Part II will discuss the implications of this result for bank regulation and interest rate ceilings.

Solving the model for $\frac{dr_D}{da_L}$ yields the interesting result that entry into the loan market will lower the equilibrium implicit yield on demand deposits. This result stems from the fact that as entry occurs in the loan market, the marginal returns on loans fall. Since, as the first order conditions indicate, the return paid on deposits varies directly with the return on earning assets, and since the return on earning assets has fallen, the rate banks are willing to pay on demand deposits will also fall.

Finally, as expected, $\frac{dX_L}{da_L}$ is less than zero. As entry occurs, shifting the demand for loans curve to the left, banks substitute government securities for loans in their portfolio. In other words, entry has increased the attractiveness of government securities, yielding a lower expected return but having less risk than loans.

The results of solving for the effects of entry on both portfolio decisions and on interest rates paid are significantly different than the results found using a model with risk neutrality. By incorporating risk into the analysis, and by assuming that investors are risk averse, it has been shown that performance in both the loan and deposit market can be analyzed only if the competitive environment in both markets is investigated.

The use of comparative statics, through the shift parameters can be extended to analyze other structural changes; for example, a change in the degree of concentration or changes in entry barriers. An analysis of such changes is beyond the scope of this paper, but should be pursued as the basis of future empirical work.

Part II

In this section the results of Part I will be analyzed in light of current restrictions on interest rate payments and bank entry. Currently, banks are forbidden to pay explicit interest on demand deposits. The rationale for this prohibition is based on the argument that, if banks are allowed to compete for deposits by offering higher rates on deposits, then they will be forced to seek higher yielding assets which are, simultaneously, riskier, and more risk in the portfolio can lead to instability and the increased probability of bank failure. Opponents to the prohibition have argued that the allocation of assets in the portfolio is independent of the rate of interest paid on deposits and it is the return on earning assets which determines the return paid on deposits.

The model of bank behavior presented in Working Paper 151 indicates that the ratio of demand deposits to time deposits and, therefore, the deposit supply functions do affect portfolio decisions. The analysis does not indicate, however, that banks will bid wildly for deposits but rather recognizes that banks are conscious of the risk return trade off. In fact, the capital asset pricing model, which is the basis of valuing the equity of the bank, is characterized by increasing risk aversion. Accepting risk in excessive amounts will, the model indicates, lead to a fall in the value of the firm. In short, the model indicates that financial markets will monitor bank decisions and will insure that improper portfolio decisions are not made.

The case for entry restrictions in commercial banking is, again, based on the assumption that competition will lead to instability through increasing risk in both assets and through higher yields on deposits. Here the results of the model are ambiguous. Entry in the loan market implies a lower proportion of assets held in loans and a lower implicit yield on demand deposits. This result would indicate less risk and more stability. New entry by commercial banks also means entry in the deposit markets. As was shown in Part I, entry in deposit markets will lead to an increase in the proportion of assets held in loans and a reduction in the rate paid on deposits. The result, therefore, is ambiguous, although it does suggest that no a priori statement about stability can be made and that empirical analysis is necessary.

A general comment can be made on the argument that by protecting banks from competition society assures itself of stability in banking. If investors are, in general, risk averse, and if capital markets are reasonably efficient, then there does not appear to be any a priori reason to expect that bankers in their attempt to maximize returns will subject themselves to increasing amounts of risk. It would appear to be far more efficient to replace the regulator with competitive markets and let stockholders and investors in general oversee the performance of banks.

Appendix

In this appendix, the method for solving for change in the endogenous variables (X_L, r_D) with respect to the shift parameters a_L, a_D will be discussed. To begin the analysis, it should be noted that if the second order conditions for a maximum are satisfied, then the matrix in equation (10) is positive. If we define the elements in the matrix as F_{11}, F_{12}, F_{21}, F_{22}, respectively, then the second order conditions for an unconstrained maximization require that

$$F_{11} < 0$$

$$F_{22} < 0$$

and

$$F_{11}F_{22} - F_{12}^2 > 0$$

since $F_{12} = F_{21}$ by Young's Theorem.

The sign of the comparative static terms will be based, therefore, on the sign of the coefficients of the shift parameters da_L and da_D.

Define

$$p = - \frac{F}{w} \frac{\partial MR_L}{\partial a_L}$$

$$q = - \frac{1}{w} \frac{\partial D}{\partial r_D} \frac{\partial E_L}{\partial a_L}$$

$$s = \frac{1}{w} \frac{\partial DD}{\partial a_D} (MR_L - \overline{E}_g) + R\beta \frac{\partial^2 \sigma_w}{\partial X_L \partial a_D}$$

$$u = - (\frac{1}{w} \frac{\partial DD}{\partial r_D} \frac{\partial N}{\partial a_D} + \frac{\partial^2 DD}{\partial r_D \partial a_D} E_a - \frac{\partial D}{\partial a_D} \frac{\partial N}{\partial r_D} + \frac{F}{w} \frac{\partial^2 N}{\partial r_D \partial a_D}$$

$$- \frac{1}{w} \frac{\partial MC_D}{\partial a_D} - R\beta \frac{\partial^2 \sigma_w}{\partial r_D \partial a_D}) .$$

Then using Cramer's rule

$$\frac{dX_L}{da_L} = \frac{\begin{vmatrix} p & F_{12} \\ q & F_{22} \end{vmatrix}}{|\Delta|}$$

$$\frac{dX_L}{da_D} = \frac{\begin{vmatrix} s & F_{12} \\ u & F_{22} \end{vmatrix}}{|\Delta|}$$

$$\frac{dr_D}{da_L} = \frac{\begin{vmatrix} F_{11} & p \\ F_{21} & q \end{vmatrix}}{|\Delta|}$$

$$\frac{dr_D}{da_D} = \frac{\begin{vmatrix} F_{11} & s \\ F_{21} & u \end{vmatrix}}{|\Delta|} .$$

P is greater than zero since

$$\frac{\partial MR_L}{\partial a_L} = \frac{\partial E_L}{\partial a_L} + \frac{\partial^2 E_L}{\partial X_L \partial a_L} < 0 .$$

By the assumption that entry shifts the demand curve to the left and increases the elasticity at the initial condition, q is greater than zero since

$$\frac{\partial D}{\partial r_D} > 0 \text{ by assumption,}$$

and $$\frac{\partial E_L}{\partial a_L} < 0 \text{ by definition of entry.}$$

s is less than zero since

$$\frac{\partial DD}{\partial a_D} < 0 \text{ by definition.}$$

$$MR_L - \bar{E}_g > 0 \text{ by the first order conditions, and}$$

$$\frac{\partial^2 \sigma_w}{\partial X_L \partial a_D} < 0$$

since $$\frac{\partial \sigma_w}{\partial X_L} = \frac{F}{w} (\sigma_{\Pi}^2)^{-1/2} X_L \sigma_L^2$$

and $$\frac{\partial^2 \sigma_w}{\partial X_L \partial a_D} = \frac{\partial F}{\partial a_D} (\sigma_{\Pi}^2)^{-1/2} X_L \sigma_L^2 - \frac{F}{w} \frac{(\sigma_{\Pi}^2)^{-3/2}}{2} 2\sigma_n \frac{\partial \sigma_n}{\partial a_D} < 0 .$$

u is less than zero since

$$\frac{\partial DD}{\partial r_D} < 0 \quad \text{by definition.}$$

$\frac{\partial N}{\partial a_D} < 0$ since an increase in a_D leads to a decline in $\frac{DD}{TD}$.

$\frac{\partial^2 DD}{\partial r_D \partial a_D} < 0$ since entry pivots the supply curve downward.

$\frac{\partial DD}{\partial a_D} > 0$ by assumption.

$\frac{\partial N}{\partial r_D} > 0$ by definition.

$\frac{\partial^2 N}{\partial r_D \partial a_D} < 0$ since an increase in a_D leads to a more elastic deposit supply function.

$\frac{\partial MC_D}{\partial a_D} > 0$ by definition of an increase in a_D;

$\frac{\partial^2 \sigma_w}{\partial r_D \partial a_D} > 0$ by definition of an increase in a_D .

Thus, since

$p > 0$

$q > 0$

$s < 0$

$u < 0$

$$\frac{dX_L}{da_L} \quad \frac{\begin{vmatrix} p & F_{12} \\ q & F_{22} \end{vmatrix}}{|\Delta|} = \frac{p\, F_{22} - q\, F_{12}}{|\Delta|} < 0 \, .$$

Since $F_{12} > 0$ because $\frac{\partial DD}{\partial r_D} > 0$, $(MR - \bar{E}_g) > 0$

and $R\beta \frac{\partial^2 \sigma_w}{\partial X_L \partial r_D} < 0$,

therefore

$$\frac{dX_L}{da_D} = \frac{s F_{22} - u F_{12}}{|\Delta|} > 0$$

$$\frac{dr_D}{da_L} = \frac{F_{11} q - F_{21} p}{|\Delta|} < 0 \quad .$$

Footnotes

[1]James, Christopher M., and Brophy, David, "A Theory of Risk Averse Bank Behavior," Working Paper 151, University of Michigan, August 1977.

[2]Edwards, Franklin R., "Concentration and Competition in Commercial Banking," Research Report No. 26, Federal Reserve Bank of Boston.

[3]Jacobs, Donald P., "Business Loan Costs and Bank Market Structure," National Bureau of Economic Research, Paper 115, 1971.

[4]Broaduss, Alfred, "Banking Market Structure and Bank Performance -- A Theoretical Analysis," Proceedings of a Conference on Bank Structure and Competition, Federal Reserve Bank of Chicago (October 1972), pp. 134-165.