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A STOCHASTIC MODEL OF THE NATIONAL POLITICAL SYSTEM IN THE UNITED STATES

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by

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BACKGROUND OF THIS PAPER

This paper was submitted as a part of the requirement of Statistics 576 in the fall term, 1970. It has since been revised and will be submitted for publication in Public Opinion Quarterly.

Political History As a Stochastic Process

The history of the interaction of political parties in national politics has been analyzed from many points of view and with many different tools. Political scientists and political historians have considered the various elections, their results, and their portrayal of the characters, times, issues, the uniqueness of each situation, and the short-term trends which ultimately cause the particular result. While historians are likely to draw parallels and note similarities between events and periods, they are quick to disclaim broad generalizations about explicit relationships between events more than a few years apart, or the possibility of cycles in the pattern of events. Moreover, historians are quick to note that political parties, even in the best of situations, really have very little continuity in terms of issues, dominant figures, and interest groups, and had even less prior to the emergence of the Democratic and Republican parties in the 1840s. For example, historians would discount generalizations about the explicit continuity of the Republican party of Coolidge, Lodge, and Hoover with the Republican party of Eisenhower, Taft, and McCarthy, and even more with the Republican party of Nixon, Rockefeller, Percy, and Regan. It is hard to say more than that each is a largely heterogeneous group responding

to its view of the demands of the times. It is still more dubious to make statements about the ideological continuity of the Federalists, the Independent Republicans, the Whigs, and the Republicans of 1860, and to associate these parties with contemporary Republicans is historically naive.

If we approach party interactions from a macroscopic point of view, however, it is interesting to examine some of these possibilities anew. When the first 180 years of American political history, which were characterized by essentially continuous two-party rule, are considered the control of the presidency, the House of Representatives, and the Senate represents a succession of discrete occurrences. Can inferences be made from information about this succession of occurrences without reference to the specific circumstances of each? Is it possible to make generalizations between elections in the nineteenth and twentieth centuries and use the results to predict the outcome of elections in the future? Is it possible to develop a theory of American political parties based on an "in-party-out-party" assumption regardless of the changes in ideologies and personalities involved? What can be said about the continuing characteristics of the parties and their power?

The succession of discrete occurrences suggests Markov analysis, particularly when the probabilities of observations and the correlations between events are of interest. With this in mind, data were collected from Historical Statistics of the United States[1/] for elections through

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1/ U.S., Department of Commerce, Bureau of the Census, 1960.

1958. The resulting data are presented in a "total" form including data from 1789 through 1958, and in an abbreviated form including only the elections from 1860 to 1958. This division was made to allow examination of the results of grouping the several parties existing prior to 1860 under the titles of Republican and Democratic which are used in the total data. It should be noted that, for purposes of the total data, Federalists, Independent Republicans, National Republicans, Whigs, and Republicans are considered Republicans, while Anti-Federalists, Democratic-Republicans, Jacksonian Democrats, and the Democrats beginning with Martin VanBuren are considered Democrats. This is not defended on historical grounds, and there is no assumption of ideological continuity or comparability on any basis other than that two major parties existed consistently in American history, and these are concatenated to produce a continuous set of data points. As the Whig party became less significant, the Republican party emerged, and similar transitions can be noted for the other parties mentioned. What is presented is a pattern of "in-party" versus "out-party" confrontations which have continued throughout our history, and it is the dynamics of this relationship which are to be examined.

The Raw Data

For the purposes of examination, the elections were coded as transitions from just after one election to just after the next. The three institutions considered were the House, Senate, and presidency. Each institution was recorded as Republican or Democratic after each election,

and then the transition which has taken place as a result of the election is entered sequentially.

The House of Representatives

Figure 1 shows the transitions for the House for the total period and the period since 1860. The entries are the numbers of transitions. Here, as with the Senate, control of the House has been designated as either strong or weak to give a finer breakdown of the transitions involved. The controlling party has strong control of the body when it controls at least 55 per cent of the seats. This is an arbitrary rule based on the lack of strict party discipline at the national level. Hence, although the current Democratic majority is more than enough to qualify the Senate as strongly controlled, it is clear that the presence of the Southern Democrats and divisions within the Democratic party make its actual control precarious.

There is a considerable difference in the data of the two matrices in Figure 1. This is due to the so-called Era of Good Feeling (another broad generalization not supported by objective historians) following the Jefferson administration. Jefferson, Madison, and Monroe and the Democratic-Republican party ruled America for twenty-four years. While this might be ample grounds for eliminating the early data from the data used in modeling, it should be noted that this period was not much different statistically from the period of Republican control during and after the Civil War (Presidents Lincoln, Johnson, Grant, Hayes, Garfield, and Arthur) or the Democratic control under Presidents F. Roosevelt and Truman.

TOTAL, 1789-1958

---TO---

---FROM---

	Strong Republican	Weak Republican	Weak Democratic	Strong Democratic
Strong Republican	11	5	2	5
Weak Republican	5	1	4	2
Weak Democratic	3	4	4	5
Strong Democratic	3	2	6	23

⟹

	Republican	Democratic
Republican	22	13
Democratic	12	38

SINCE 1860

---TO---

---FROM---

	Strong Republican	Weak Republican	Weak Democratic	Strong Democratic
Strong Republican	11	4	1	3
Weak Republican	4	0	1	2
Weak Democratic	1	3	3	3
Strong Democratic	2	0	5	7

⟹

	Republican	Democratic
Republican	19	7
Democratic	6	18

Fig. 1. Matrices showing control of the House of Representatives in terms of two-year transitions.

The Senate

Figure 2 shows the same breakdown for the Senate. Again the effects of the Era of Good Feeling are evident except that in this case the heavily Democratic figures tend to portray the two parties as about even over the total period, while in the House the early data broke the seeming balance in the post-1860 data and showed the House more strongly Democratic over the total period. Another obvious observation is that the House is considerably more volatile than the Senate, which has a much stronger tendency to remain in the control of the controlling party. This is to be expected since only one-third of the Senate is elected in any one election.

The House and the Senate

Figure 3 shows the relation between control of the House and Senate. This figure also points out the relationship, which will be seen again later, between the Republican party and the Senate and the Democratic party and the House. Only once in the post-1860 data has the Republican party held the House and not the Senate, and that was after the midterm election under a Democratic president just prior to the election of Lincoln. As will be noted later, at no time while there was a Republican president in office has the Republican party controlled the House and not the Senate. Similarly, only once since 1860 has the Democratic party held the Senate and not the House under a Democratic president. The matrices in Figure 3 show that it is much more likely that the Republicans will control the Senate while the Democrats control the House than vice versa. It is also interesting to note that the Republicans have been

TOTAL, 1789-1958

---FROM--- \ ---TO---	Strong Republican	Weak Republican	Weak Democratic	Strong Democratic
Strong Republican	19	6	1	2
Weak Republican	5	3	3	3
Weak Democratic	0	2	4	4
Strong Democratic	3	3	2	25

$\Rightarrow$

	Republican	Democratic
Republican	33	9
Democratic	8	35

SINCE 1860

---FROM--- \ ---TO---	Strong Republican	Weak Republican	Weak Democratic	Strong Democratic
Strong Republican	14	5	1	0
Weak Republican	5	3	2	3
Weak Democratic	0	2	1	2
Strong Democratic	1	3	1	7

$\Rightarrow$

	Republican	Democratic
Republican	27	6
Democratic	6	11

Fig. 2. Matrices showing control of the Senate in terms of two-year transitions.

TOTAL, 1789-1958

---TO---

---FROM---	Republican control of House and Senate	Republican control of House but not Senate	Democratic control of House but not Senate	Democratic control of House and Senate
Republican control of House and Senate	22	0	7	4
Republican control of House but not Senate	1	0	0	2
Democratic control of House but not Senate	2	0	3	5
Democratic control of House and Senate	7	3	0	31

⟹

	Republican Senate	Neither party controls both houses	Democratic Senate
Republican Senate	22	7	4
Neither party controls both houses	3	3	7
Democratic Senate	7	3	31

SINCE 1860

---TO---

---FROM---	Republican control of House and Senate	Republican control of House but not Senate	Democratic control of House but not Senate	Democratic control of House and Senate
Republican control of House and Senate	18	0	5	2
Republican control of House but not Senate	1	0	0	0
Democratic control of House but not Senate	1	0	3	4
Democratic control of House and Senate	5	0	0	11

⟹

	Republican Senate	Neither party controls both houses	Democratic Senate
Republican Senate	18	5	2
Neither party controls both houses	2	3	4
Democratic Senate	5	0	11

Fig. 3. Matrices showing control of the House and Senate in terms of two-year transitions.

slower to lose control of the Congress than have the Democrats. The Republicans have gone from control of both houses to the control of only one seven times and have lost both at once only four times, while for the Democrats the figures are three and seven respectively. This would again confirm the notion that the Republicans are more likely to control the Senate, which changes hands more slowly.

The presidency

The two small matrices on the top of Figure 4 show the distribution of transitions after presidential elections for the same periods as discussed before. Again, as with the Senate, the total matrix shows a more balanced party situation than does the post-1860 matrix, since the Republicans had a number of strong presidents in the latter period. These figures do not indicate, however, the relative strength of the presidents. To take a crude measure, the presidents were classified as either one- or two-term presidents in the larger matrix in Figure 4. This raised a number of questions of interpretation which were resolved arbitrarily on the basis of the number of elections won. Hence, succeeding vice-presidents were ignored, which resulted in the classification of Truman and T. Roosevelt as one-term presidents. Franklin Roosevelt, elected four times, was considered as two two-term presidents.

There are a number of interesting observations which can be made from these data. Only once has a two-term Republican president followed a two-term Republican president--U.S. Grant; and only three times have two-term Democratic presidents followed in succession--Madison

TOTAL
PRESIDENTIAL TRANSITIONS

	Republican	Democratic
Republican	13	8
Democratic	8	13

NUMBER OF PRESIDENTIAL
TRANSITIONS SINCE 1860

	Republican	Democratic
Republican	11	4
Democratic	5	5

NUMBER OF PRESIDENTIAL TRANSITIONS
BY TERMS OF OFFICE

	2-term Republican	1-term Republican	1-term Democratic	2-term Democratic
2-term Republican	1	2	0	0
1-term Republican	0	6	4	4
1-term Democratic	3	3	1	0
2-term Democratic	0	2	2	3

Fig. 4. Matrices showing control of the presidency in terms of four-year transitions.

and Monroe during the Era of Good Feeling and F. Roosevelt. Never has a two-term president followed a two-term president of the other party. Never has a one-term Democratic president followed a two-term Republican president. (The first to do so was Kennedy, who falls after the sample taken here.) Never has a two-term president followed a two-term president of the same party, in spite of numerous historical opportunities for this to happen.

With this as background, the control of the federal government as a whole was considered. The first approach was to consider eight states[2/] which ranged from complete Republican control, designated by a 1, to complete Democratic control, designated by an 8. The data were collected and are shown in Figure 5. It was obvious that the resulting matrix was too sparse for good estimations of transition probabilities by relative frequencies. Even in the total matrix forty-one of the sixty-four entries are zero. Consequently, the matrix was condensed to the five-state version shown in Figure 6. Here the data range from Republican control of the presidency, House, and Senate,designated by a 1, to complete Democratic control, designated by a 5. Again the data indicate that the control of the presidency, House, and Senate are highly correlated and that, in fact, the states most likely to occur are those in which complete control remains with the same party.

2/ We use this term throughout the paper to designate a state of being or situation, not a state of the union.

KEY

	President	House	Senate
State 1	Republican	Republican	Republican
State 2	Republican	Democratic	Republican
State 3	Republican	Republican	Democratic
State 4	Republican	Democratic	Democratic
State 5	Democratic	Republican	Republican
State 6	Democratic	Democratic	Republican
State 7	Democratic	Republican	Democratic
State 8	Democratic	Democratic	Democratic

TOTAL, 1789-1958

--From-- \ ---To---	1	2	3	4	5	6	7	8
1	19	7	0	2	0	0	0	1
2	1	1	0	1	0	1	0	4
3	0	0	0	0	0	0	0	0
4	1	0	0	3	0	0	0	2
5	2	0	0	0	0	0	0	1
6	1	0	0	0	0	1	0	0
7	1	0	0	1	0	0	0	1
8	3	0	0	0	3	0	3	27

SINCE 1860

--From-- \ ---To---	1	2	3	4	5	6	7	8
1	16	5	0	1	0	0	0	0
2	0	1	0	1	0	1	0	3
3	0	0	0	0	0	0	0	0
4	1	0	0	2	0	0	0	0
5	2	0	0	0	0	0	0	1
6	1	0	0	0	0	1	0	0
7	1	0	0	0	0	0	0	0
8	1	0	0	0	3	0	0	9

Fig. 5. Matrices showing eight-state transitions.

KEY

1 = Republicans control President, House, and Senate
2 = Republicans control President and one house
3 = One party controls President and other controls both houses
4 = Democrats control President and one house
5 = Democrats control President, House, and Senate

TOTAL, 1789-1958

--From-- \ --To--	1	2	3	4	5
1	19	7	2	0	1
2	1	1	1	1	4
3	3	0	3	0	3
4	2	0	1	1	1
5	3	0	3	3	27

⇒

	1 and 2	3	4 and 5
1 and 2	28	3	6
3	3	3	3
4 and 5	5	4	32

SINCE 1860

--From-- \ --To--	1	2	3	4	5
1	16	5	1	0	0
2	0	1	1	1	3
3	3	0	2	0	1
4	2	0	0	1	0
5	2	0	3	0	9

⇒

	1 and 2	3	4 and 5
1 and 2	22	2	4
3	3	2	1
4 and 5	3	3	10

Fig. 6. Matrices showing five-state transitions.

If these data are to be useful in looking at the dynamics of party interactions, the sequence of events should also be considered. This sequence is shown in Figure 7 and will be considered in more detail later. Figure 8 shows the sequence of states for the House, the Senate, the presidency, and the sequence of states from the eight-state transitions shown in Figure 5.

The question at this point regards the validity of our assumption that the data collected and presented thus far can be used together with the theory of stochastic processes to produce a model of the interactions of the parties which will give probabilities of future events. If it is, how good are these estimates likely to be?

Estimation and Statistical Analysis

In any problem of statistical estimation, there are usually several alternative methods that can be used to estimate the same parameters. Maximum likelihood, method of moments, and relative frequency estimations are the methods most frequently used.

Method of maximum likelihood was perhaps the most elegant candidate, but it was frought with algebraic difficulties because it called for knowledge of the frequency distribution function which contained the parameters to be estimated. (In a more extensive study, the algebraic difficulties would have to be overcome.) The problem under consideration suggested a rather complex multinomial structure with quite laborious algebra. This suggested the method of moments or the method of relative frequency estimation. Since the relative frequency approach was by far

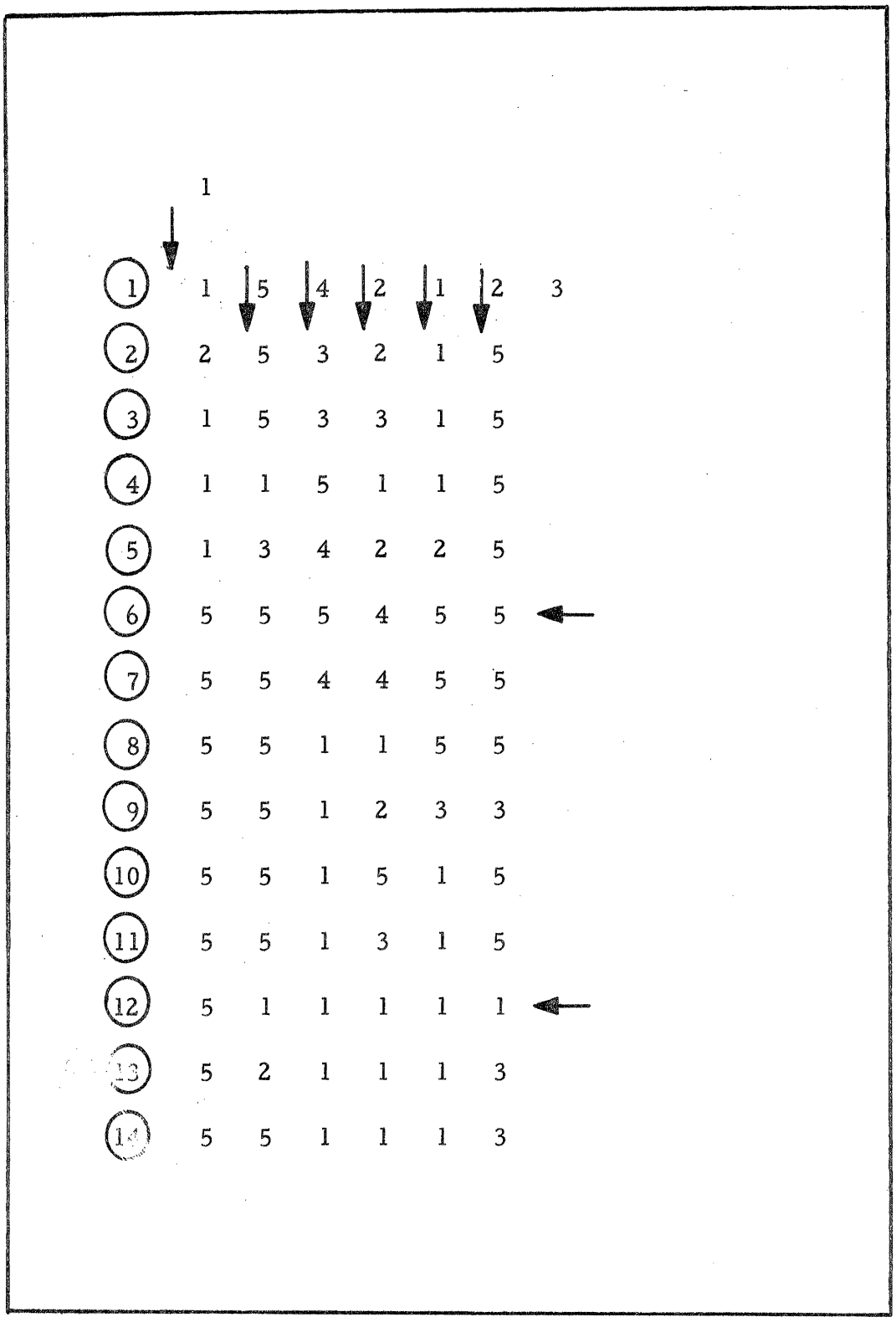

Fig. 7. Sequence of states in five-state process. (Read down successive columns; see Figure 6 for key to states.)

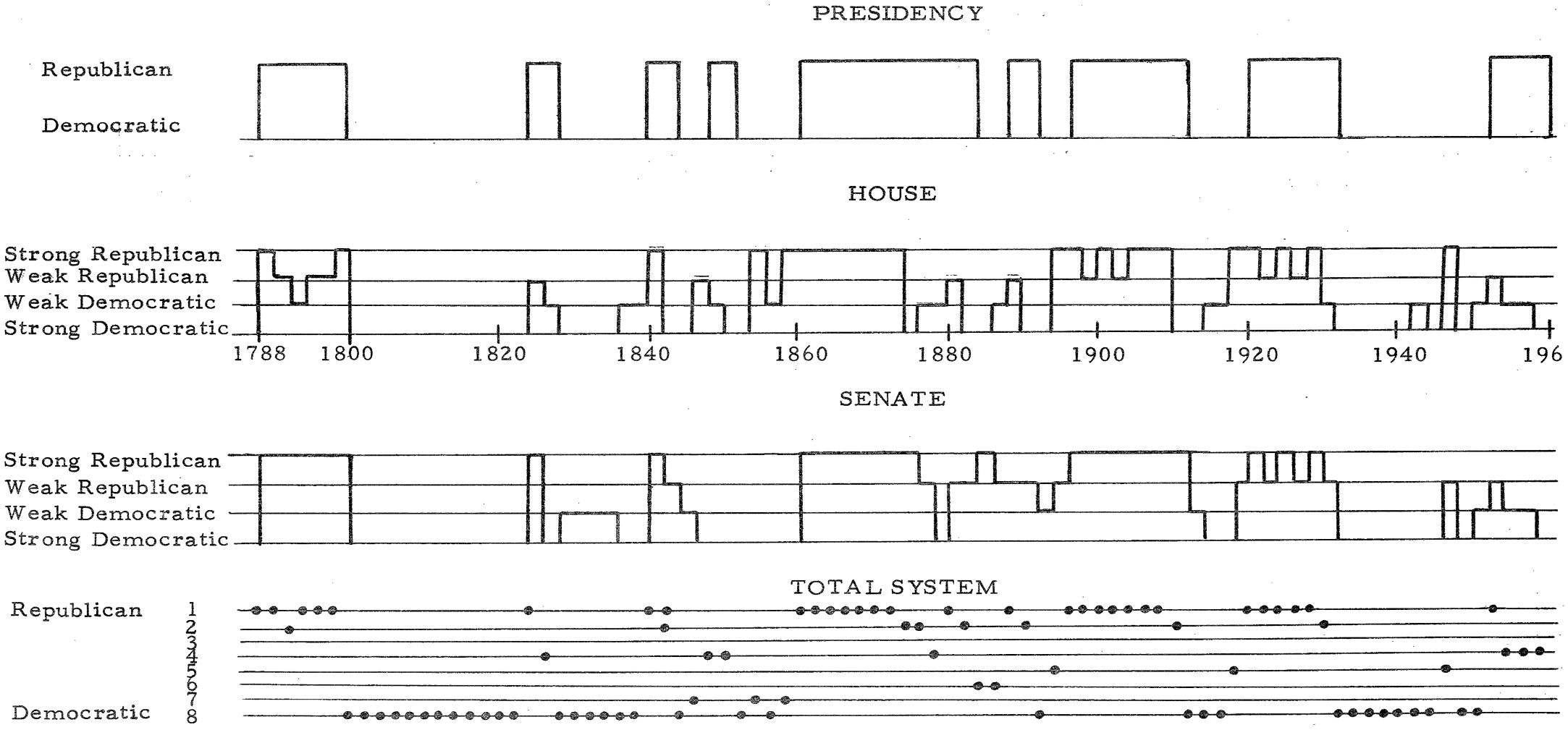

Fig. 8. Party affiliation and strength in political institutions, 1788 to 1958. (See Figure 5 for key to states in "Total System.")

the simpler of the two approaches and facilitated numerical calculations, it was chosen for use in our subsequent estimations.

The first matrix to be estimated was the eight-state transition matrix. The sparseness of this matrix suggested that more data were needed for estimation. A five-state version was adopted as the compromise between the level of detail desired and the ability to obtain estimates for most transition probabilities.

The one-step (two years) five-state transition matrix $\hat{P}_5'$ together with $\hat{P}_5' \times \hat{P}_5'$ and the relative frequencies implied by $\hat{P}_5' \times \hat{P}_5'$ are shown in Figure 9.

No real rigorous testing was conducted for the Markovness of the process. The Markov assumption, however, was supported by the following intuition:

> It is likely that the political state which the public chooses depends upon how satisfied or dissatisfied the public is with the previous administration. Voters seldom go back further than one term in history to choose leadership for the future. We indeed admit that in some cases, especially in cases of senators, the reputation of candidates does influence their election to office, and that our system of electing presidents through the Electoral College does not necessarily reflect the choice of the majority. We nevertheless

ONE-STEP TRANSITION MATRIX = $\hat{P}^1_5$

	1	2	3	4	5
1	.655	.241	.007	.000	.003
2	.125	.125	.125	.125	.500
3	.333	.000	.333	.000	.333
4	.250	.000	.250	.250	.250
5	.108	.000	.008	.008	.730

	1	2	3	4	5
1	.486	.188	.101	.003	.191
2	.224	.005	.138	.009	.505
3	.366	.008	.161	.027	.366
4	.337	.006	.183	.008	.337
5	.197	.003	.114	.008	.584

$\Rightarrow$

	1	2	3	4	5
1	14	5	3	0	6
2	2	0	1	0	4
3	3	0	1	0	3
4	1	0	1	0	1
5	7	0	4	0	22

IMPLIED
RELATIVE FREQUENCIES

$\hat{P}^1_5 \times \hat{P}^1_5$

Fig. 9. The five-state transition matrix and the relative frequencies implied by $\hat{P}^1_5 \times \hat{P}^1_5$. (See Figure 6 for key to states.)

submit that the performance of the current administration exerts the greatest influence in the choice of the next administration. We further make another heroic abstraction that party affiliation alone is the most important variable because it implies a preconceived performance characteristic. (In an extended study this would be one of the first abstractions to be eliminated.)

An equally important question was whether the political process under consideration was stationary. (A process will be called stationary if the transition matrix of the process remains unchanged with the passing of time.) A Chi Square test was carried out in the following manner:

1. The one-step (two years) transition matrix was further collapsed to three states $\hat{P}^1_3$ so that there would no longer be any zero elements in the matrix and the cells would have large enough values (usually five is considered about the minimum size) for the Chi Square test.
2. It was hypothesized that if, in fact, the process was stationary, we would expect the transition matrix to remain the same through time such that $\hat{P}^1_3 \times \hat{P}^1_3$ would be, statistically speaking, the same as $\hat{P}^2_3$, or alternatively, we would expect $\hat{P}^1_3 \times \hat{P}^1_3$ and $\hat{P}^2_3$ to imply the same relative frequency matrix.

Matrices of relative frequencies implied by $\hat{P}_3^1 \times \hat{P}_3^1$ and $\hat{P}_3^2$ are included in Figure 10 together with the Chi Square test. The test indicated a non-stationary process.

In the next section we remedy this situation.

Off-Year Elections--Source of Nonstationarity

Further study of the data revealed that we must indeed have a non-stationary process since we have estimated the transition matrix using two-year steps and since it is certain that the presidency of the the United States remains with the same political party on off-years. This implies that entry is permitted only to certain states in an off-year. This prompted us to hypothesize that perhaps the two-step transition consisting of the product of the off-year and the election year transition matrices should be stationary. The problem arose whether (off-year $\hat{P}_3^1$) X (election year $\hat{P}_3^1$) or (election year $\hat{P}_3^1$) X (off-year $\hat{P}_3^1$) should be tested against $\hat{P}_3^2$. Inasmuch as the two products were slightly different and we had no obvious reason to choose one or the other, the average of the two was taken to represent the two-step (four-year) transitions that were to be tested against directly estimated $\hat{P}_3^2$. The relevant data for the test are shown in Figure 10. The test indicated that the average of (off-year $\hat{P}_3^1$) X (election year $\hat{P}_3^1$) and (election year $\hat{P}_3^1$) X (off-year $\hat{P}_3^1$) was statistically the same (.025 level) as $\hat{P}_3^2$ estimated directly from the raw data.

$\hat{P}_3^2$ Frequency Table

	1 and 2	3	4 and 5
1 and 2	22	3	12
3	3	1	4
4 and 5	10	6	23

$\hat{P}_3^1 \times \hat{P}_3^1$ Implied Frequency Table

	1 and 2	3	4 and 5
1 and 2	22.9	3.84	10.2
3	3.23	1.37	3.4
4 and 5	8.58	4.64	25.8

$$S = \frac{(.9)^2}{22.9} + \frac{(.84)^2}{3.84} + \frac{(1.8)^2}{10.2} + \frac{(.23)^2}{3.23} + \frac{(.37)^2}{1.37} + \frac{(.6)^2}{3.4} + \frac{(1.42)^2}{8.58} + \frac{(1.36)^2}{4.64} + \frac{(2.8)^2}{25.8} = 1.70$$

$$\chi^2_{9-1-3,\ .05} = \chi^2_{5,\ .05} = 1.15 < S$$

$\Longrightarrow$ The two frequency tables are different

$\Longrightarrow$ Process is nonstationary

Frequency table implied by:

$1/2[\ (\text{Even-year } \hat{P}_3^1) \times (\text{Odd-year } \hat{P}_3^1) + (\text{Odd-year } \hat{P}_3^1) \times (\text{Even-year } \hat{P}_3^1)\ ] =$

	1 and 2	3	4 and 5
1 and 2	21.6	3.04	12.4
3	3.18	1.63	3.18
4 and 5	10.5	5.06	23.4

$$S = \frac{(.4)^2}{21.6} + \frac{(.04)^2}{3.04} + \frac{(.4)^2}{12.4} + \frac{(.18)^2}{3.18} + \frac{(.63)^2}{1.63} + \frac{(.82)^2}{3.18} + \frac{(.5)^2}{10.5} + \frac{(.94)^2}{5.06} + \frac{(.4)^2}{23.4} = .78$$

$\chi^2_{(5),\ .025} = .831 > .78 \Longrightarrow$ The above matrix is the same as $\hat{P}_3^2$

Fig. 10. Goodness of fit test, $\hat{P}_3^2$ versus $\hat{P}_3^1 \times \hat{P}_3^1$. (See Figure 6 for key to states.)

Once it was shown that the four-year transitions were stationary, a computer program was written to raise transition matrices to higher and higher powers to reach the equilibrium solution π. Equilibrium solutions obtained for four-year transitions (whether starting from an off-year or election year) indicate that virtual equilibrium is reached after thirty-two years, with states one and five being the most probable and state five having a slight advantage. The detail supporting this is shown in Figure 11.

Naturally, the one most important reason why we model is to be able to make predictions about the future performance of the data-generating system. We shall, however, postpone the predictions to look at the political system from a somewhat different point of view, namely, to describe the system via a queuing model.

The Political System As a Queuing Model

It would be interesting to view the process as a queuing system. In fact, this can be done with the five-state chain used thus far. If a value of two is assigned to a state when the Republicans control the presidency, and one is added for each house of Congress which the Republican party controls, the result is a scale from zero, or complete Democratic control, to four, or complete Republican control, with a value of two representing a split between the presidency and the Congress. If this index is considered as the number in the queue at any time, this number can be visualized as the result of entries, such as winning control of the Senate by the Republicans, and departures, such as

OFF-YEAR TRANSITIONS

$\hat{P}^1_{5\,(off)}$ (frequency)

	1	2	3	4	5
1	10	6	2	0	0
2	2	0	1	0	0
3	0	0	2	0	0
4	0	0	0	1	0
5	0	0	3	3	14

ELECTION YEAR TRANSITIONS

$\hat{P}^1_{5\,(on)}$ (frequency)

	1	2	3	4	5
1	9	1	0	0	1
2	0	1	0	1	4
3	3	0	1	0	3
4	2	0	1	0	1
5	3	0	0	0	11

$\hat{P}^1_{5\,(off)} \times \hat{P}^1_{5\,(on)}$

	1	2	3	4	5
1	.502	.106	.016	.056	.320
2	.688	.061	.048	.000	.203
3	.429	.000	.143	.000	.429
4	.500	.000	.250	.000	.250
5	.289	.000	.059	.000	.652

$\Downarrow$

π = (.406, .046, .049, .023, .477)

$\hat{P}^1_{5\,(on)} \times \hat{P}^1_{5\,(off)}$

	1	2	3	4	5
1	.515	.273	.135	.014	.064
2	.111	.000	.156	.267	.467
3	.238	.143	.255	.064	.300
4	.278	.167	.343	.038	.175
5	.119	.071	.142	.118	.550

$\Downarrow$

π = (.256, .135, .181, .094, .334)

Fig. 11. Equilibrium solutions (π) and their derivation. (See Figure 6 for key to states.)

losing control of the House. The number of the state occupied at any time is then the number of the state occupied after the previous election plus the value of bodies gained by the Republicans minus the value of bodies lost to the Democrats.

Having shown in the previous section that the process given by Z(A, B), where A is the state, as above, of the system in the previous period and B is an indicator of whether the transition is from odd to even or, vice versa, is Markov and stationary, it is now possible to write equations in the following form:

$$P^{n}_{i,(1\text{-}5)} = \sum_{k=0}^{4} P^{n-1}_{k,s}\, p^{(1)}_{ki,S} \qquad \begin{array}{l} i = 0, 1, 2, 3, 4 \\ n = 1, 2, 3, \ldots \\ S = \begin{cases} 0 & \text{odd to even} \\ 1 & \text{event to odd} \end{cases} \end{array}$$

The transition matrix from odd to even shows from experience that a number of transition probability estimates are zero and the equation above collapses to the following:

e. g. : i = 3, 5 = 0 $\qquad P^{n}_{3,1} = P^{n-1}_{3,0}\left(\frac{1}{6}\right) + P^{n-1}_{4,0}\left(\frac{1}{11}\right) \qquad n = 1, 2, 3, \ldots.$

Given this format, the state probabilities may be calculated as far back as is of interest, and a given initial condition may be specified. The result is a set of N-step transition probabilities, and the system is Markov. Note, however, that the system described above admits multiple arrivals or departures in a single time interval and is not the standard type where only single arrivals or departures are admitted in a single time interval. Nevertheless, we can derive an expression for the probability of first return to any given state as follows:

$$p_{ii}^{N} = \sum_{k=0}^{4} p_{ik}^{N-1} p_{ki} \qquad i = 0, 1, 2, 3, 4 \qquad N = 1, 2, 3.....$$

N = 2-step (4-year) transition

Upon successive substitutions, we obtain:

$$p_{ii}^{(N)} = \sum_{k_1=0}^{4} \sum_{k_2=0}^{4} \sum_{k_3=0}^{4} \cdots\cdots \sum_{k_{N-1}=0}^{4} p_{ik_1}^{(1)} p_{k_1 k_2}^{(1)} p_{k_2 k_3}^{(1)} \cdots\cdots p_{k_{N-1}}^{(1)} \quad i = f(N)$$

$i = 0, 1, 2, 3, 4 \qquad N = 1, 2, 3, \ldots.$

Note that different values of N would result in different $p_{ii}^{(N)}$ which, for any given state i, would enable us to calculate an expected time to return to state i which would indicate a "political cycle." Of particular importance would be the expected time of return to states one and five. Another approach would be to obtain the most likely (mode of) N rather than the expected value of N. No analytical solutions became immediately obvious to the writers, and because of the timing involved any numerical solutions proved to be beyond the scope of this paper. As an alternative, the study of political cycles was carried out by using a graphical method coupled with a test of correlation among the states of the political system k periods apart.

Political Cycles

Looking at the graphs in Figure 8 and at the states and their sequences, as in Figure 7, there seems to be reason to expect a cycle of occurrences. Indeed, Figure 7 is presented so that elections occurring twenty-eight years apart are on common rows. Arrows mark rows six

and twelve, for each is remarkably consistent, suggesting perhaps some positive relationship.

Figure 12 reinforces this observation. To produce this graph, the values for the eight-state process were summed in groups of three for states which were twenty-eight years apart. For example, the value of the state after the first election (1789), or one, was added to the value after the fifteenth election (twenty-eight years later), or eight, and the value after the twenty-ninth election (another twenty-eight years later), or eight, was added to give a value of seventeen. This is the first value plotted in Figure 12. The second point represents the values of the eight-state chain after the second, sixteenth, and thirtieth elections, for a total of sixteen. This was done sequentially until the data were exhausted, and the results are shown in Figure 12. Since the minimum value which each state can have is one and the maximum is eight, the values range from three to twenty-four. The results are strikingly regular with the peak values being twenty-eight years apart and the Democratic peaks followed by Republican troughs twelve years later. The heavy influence of the Era of Good Feeling is shown in the high values of the first group of points, but the basic cycle remains constant.

With this in mind, the next step was to test for significant correlation to examine the nature of this cyclic observation. In Figure 13, the sums of squared differences for points a given number of years apart were tabulated, and the results were tested for significance. The variance of the values, S^2, was estimated as 3.07. A plot of the resulting values is

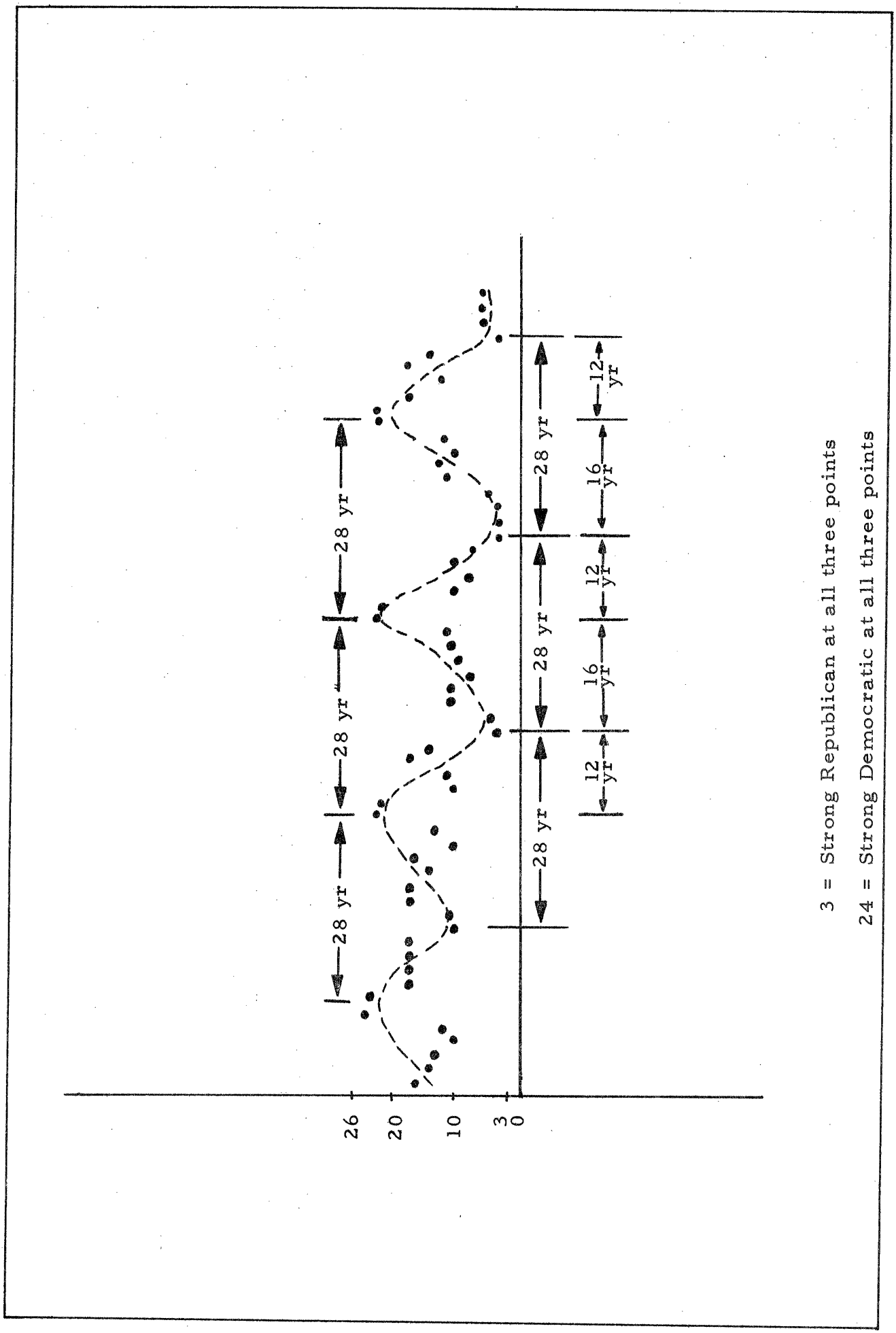

Fig. 12. Political cycles in the United States.

Number of Years Apart	Number in Sample (N)	Sum of Squared Difference (Σd_i^2)	Σd_i^2 /N-1	$\frac{\Sigma d_i^2 / N-1*}{S^2}$
6	83	345	4.22	1.38
10	81	480	6.00	1.96
14	79	527	6.77	2.21
18	77	443	5.73	1.87
20	76	419	5.59	1.82
22	75	405	5.47	1.78
24	74	409	5.59	1.82
26	73	381	5.29	1.72
28	72	385	5.42	1.76
30	71	399	5.70	1.86
32	70	434	6.29	2.05
34	68	436	6.42	2.10

* = Von Neuman ratio (where S^2 is an estimate of the sample variance)

Fig. 13. Analysis of cycles.

shown in Figure 14. Although the values fall short of significance at the 10 per cent level, the repetition of the cycle and the regularity of extreme points, as well as the fact that no functional relationship has been postulated for this period, tend to lend weight to the significance of the results. Although this does not imply a strict relationship, its historical consistency tends to make it a factor which an experimenter would be likely to consider in building a predictive model on the basis of statistical information and probabilities of occurrences. In the section on prediction, we shall make some use of the concept of the political cycles.

It should also be observed that some of the large values obtained in the sums of squared differences were in part due to the integer state values (1, 2, 3, ...). There is good reason to believe that application of a continuity correction would make the positive correlation significant, at least at the 10 per cent level, for the twenty-six to twenty-eight year periods. This would at the same time reduce the significance level of the negative correlation at twelve to fourteen years as would be expected by the assymetry of the pattern shown in Figure 14 where peaks follow alternate peaks by twelve to sixteen years.

The important question at this point is whether, by showing possible positive serial correlation of states twenty-eight years apart, we have refuted the Markov assumption. We can note that twenty-eight years is suspiciously close to thirty-two years, which we previously calculated as the length of time to equilibrium. It is very possible that a better unit of analysis would be seven presidential terms taken together.

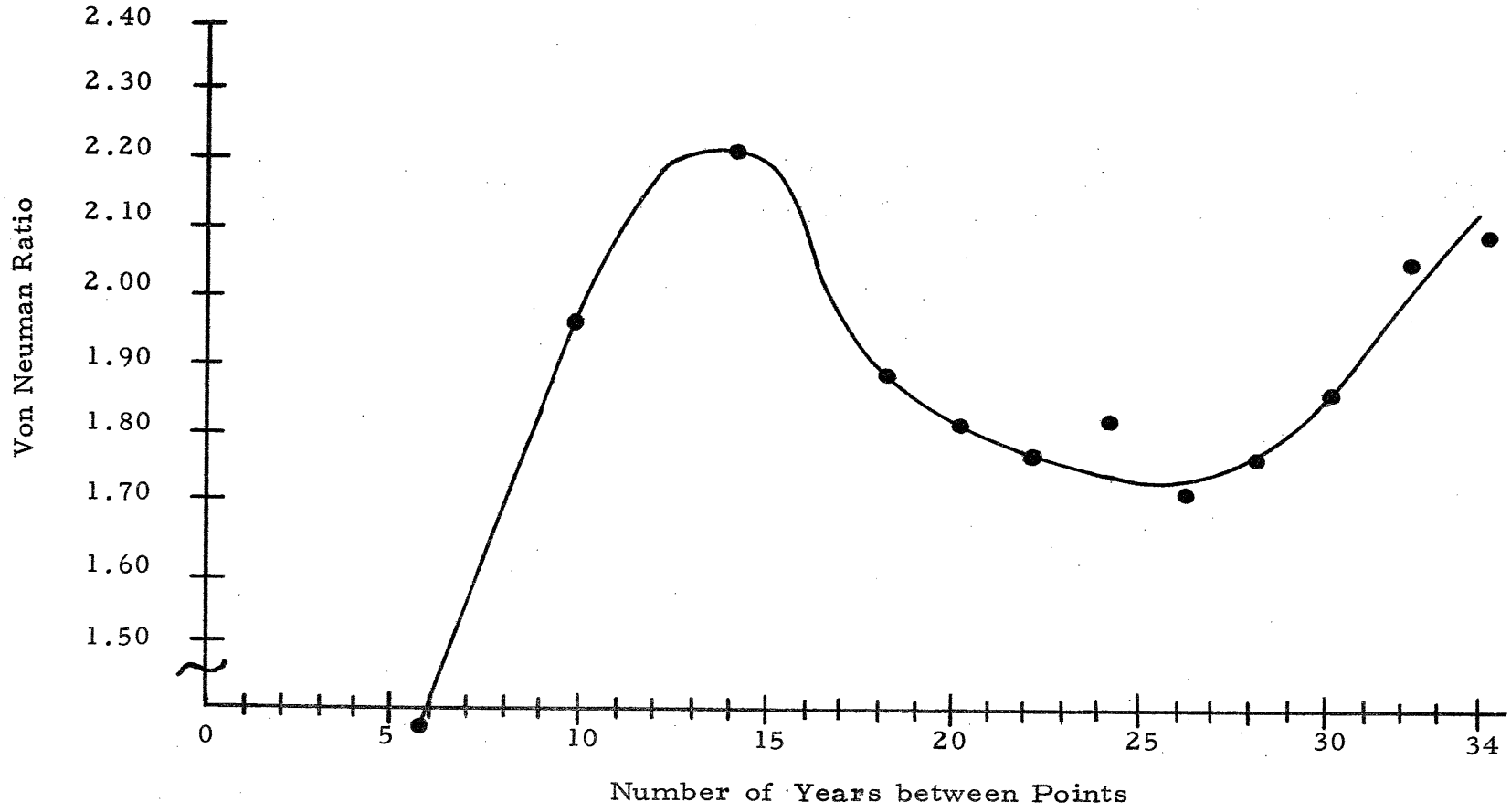

Fig. 14. Test for positive and negative serial correlation.

It should also be noted that pausity of data would become a potential problem if the unit of analysis is enlarged to twenty-eight years.

Having considered the data collected and examined to this point, the question which comes to mind is the usefulness of the statistical results to predict the outcome of future elections. The data used thus far in the development of the model run from the first election to the congressional election of 1958. This end date was chosen to allow the elections of the 1960s to be used for test points for the model developed. The final section of the paper will be devoted to applying the model to the events of the 1960s and examining the results and their possible extension into the 1970s.

Predictions

1960

Using the transition matrix estimated for transitions from off-years (OFF) and given that the political system was initially in state three, we obtain the following probabilities:

States:	One, three, five
Probability:	.428 .143 .428

This suggests a very tight race in which neither Republicans nor Democrats are favored. The political cycle, however, suggests a Democratic peak in close proximity to 1964, which would suggest that overall odds would be slightly in favor of the Democrats $\Longrightarrow$ predict Kennedy victory and state five. (Actual--state five.)

1962

Given state five and using the appropriate transition matrix (ON):

States: Three, four, five
Probability: .150 .150 .700

$\Longrightarrow$ predict state five (Actual--state five)

1964

Given state five and using matrix (OFF):

States: One five
Probability: .214 .786

$\Longrightarrow$ predict state five (Actual--state five)

1966

Given state five and using the matrix (ON):

States: Three, four, five
Probability: .15 .15 .70

$\Longrightarrow$ predict state five (Actual--state five)

1968

Given state five and using the matrix (OFF):

States: One, three, five
Probability: .289 .059 .652

Without consideration of the political cycle, the choice would be state five--in fact, the choice from state five would always be to state five.

The historical probabilities indicate that state five, complete Democratic control, should be the result. The effects of the cycle are unpredictable since the 1968 election falls only two to four years after

the predicted peak of Democratic control. The actual election result--a Republican president and a Democratic Congress--is a rare event. The last election in which the party controlling the presidency did not control at least one house of Congress was the election of Whig Zachory Taylor in 1848, and this was a three-way race resulting in a minority president, as was the 1968 Nixon election. (Taylor received only 47 per cent of the popular vote.) This explanation is not to be taken as an apology for the model, however, since we do not expect perfect predictive power from the model on all elections.

1970

Given state three and using the matrix (ON):

State:	Three
Probability:	1.00

$\Longrightarrow$ predict state three (Actual--state three)

1972

Given state three and using matrix (OFF):

States:	One,	three,	five
Probability:	.428	.143	.428

This indicates a close race very much like the Nixon-Kennedy race of 1960. Again, without consideration of the political cycle, there would be no reason to give any greater chance of winning to either party. The Republicans, however, are expected to reach their maximum strength in the vicinity of 1976; hence, this would suggest that in the 1972 race the Republicans are favored to become the victors.

Concluding Remarks

We have barely scratched the surface of a subject which shows quite a large potential for further research. It is important to note that although election prediction has become lifetime work for some individuals, stochastic processes have not been one of their frequently used tools. The results shown in this paper indicate that even a preliminary analysis such as this one results in admirable predictive power and suggests that stochastic processes may be the tool most appropriate for modeling political systems.

As a first extension, we propose that the performance of the administration in power be somehow introduced into the model, that various estimation procedures be used and compared, and that the queuing model and its usefulness and contribution above and beyond that of a Markov chain model be more fully investigated.

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