Real Investment, Capital Intensity and Interest Rates

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In a growing economy the cash flows from investment projects can be expected to be rising over time. In this paper we explore the interactions of growth and uncertainty of cash flows with variable capital intensity in the decision to invest. We derive simple rules for the optimal timing of investment projects based on IRR and NPV. We show that the ability to vary capital intensity raises the specter of perverse responses of investment to interest rates. Variable capital intensity is a sufficient condition for the perverse responses that can occur when growth rates are high or uncertainty is high. An empirical analysis of a panel data set on residential investment in the 1980s confirms the predictions of the model.

It is now widely recognized that the decision to invest is a decision to exercise a real option and that many insights from the theory of financial options apply to real investment decisions. One important difference between real and financial options is the ability to vary the capital intensity of the investment, i.e., the capacity or output level. The capital intensity of a project (as opposed to the scale of a project) is important when there is a fixed factor like land or labor. Analytically the ability to vary capital intensity means that the exercise price of the option is endogenous rather than fixed. This paper pursues the implications of this endogenous exercise price for investment decisions.

Irreversibility causes projects to be optimally delayed until the net present value (NPV) and the internal rate of return (IRR) exceed critical values far larger than their traditional break-even levels. One consequence of this delay is that our usual intuition may no longer hold. For example, standard investment theory argues that an increase in the interest rate will reduce investment spending because some projects whose IRR earlier exceeded the cost of capital will now have IRRs below the necessary hurdle rate and will not be undertaken. This reasoning may not work when projects are optimally delayed because at optimal exercise, the NPV is positive and the hurdle rate exceeds the cost of capital. The increase in interest rates and cost of capital can be offset by a decrease in the option value that accelerates investment projects.

In a growing economy the NPV and IRR on delayed investments are rising over time. An increase in the interest rate may discourage investment by increasing the cost of capital but speed optimally delayed investment because the option value of waiting has fallen. On balance the acceleration of projects causes increased investment. This does not mean that negative NPV projects are being undertaken. It does imply that positive NPV

projects are undertaken earlier. Eventually with a growing economy the same projects are commenced. However, during a given period, if interest rates rise, investment may increase over what it would have been with steady interest rates.

High levels of uncertainty about future cash flows are a sufficient but not a necessary condition for this counterintuitive behavior. Similar perverse reactions to interest rates can occur under certainty when capital intensity is variable. Optimal delay beyond the naive hurdle (IRR=cost of capital) occurs in certainty models when the intensity of investment is variable and the cash flows from the investment are rising. For example, consider the decision to construct a building on a plot of land. If the land is in a growing urban area, rents will be rising over time. Both the expected waiting time to invest and the optimal capital intensity are positively related to the expected growth rate of rents. If the owner commits today to the currently optimal intensity, he sacrifices some revenue in the future compared with what he would obtain if he waited and constructed the larger building that becomes optimal next period.

When interest rates rise, there are two offsetting effects. The first is the increase in the hurdle IRR because the cost of capital is higher. This first effect always occurs. The second is the increase in the project IRR because investment is made with less capital. The second occurs only when capital intensity is variable. When the interest rate rises, the optimal intensity falls since investors will substitute other factors for capital. With a production function exhibiting decreasing returns to scale, the decline in capital intensity will result in an increase in the output/capital ratio, a decrease in the cost of investment per unit of output, and a corresponding increase in the IRR from the project. If the project IRR increases more than the cost of capital, the net effect accelerates some projects. We show that the critical parameters are the elasticity of substitution in production and the ratio of the growth rate to the interest rate. The positive interest rate response occurs when the ratio of the growth rate to the interest rate is large relative to one minus the elasticity of substitution.

High levels of uncertainty are also a sufficient condition for positive interest rate responses to occur in our model. Any level of uncertainty, however, increases the likelihood of positive responses relative to the certainty case because it reduces the critical ratio of the growth rate to the interest rate.

Notice that our argument is not a hysteresis argument as in Dixit (1989) that investment persists because of sunk costs of entry and exit. Instead investment is displaced in time by the effects of interest rates on capital intensity. Pindyck (1988) addresses capacity choice and expansion by focusing on "operating options" to produce output if it is justified by the stochastic demand once capacity is in place. By contrast, our model is motivated by land development decisions. Net revenue is stochastic and the investor chooses the optimal capital intensity given a fixed factor like land or labor[1]. Our model does not address the option to expand which reduces the incentive to invest and which is explored extensively in Abel et al. (1996). Like Abel et al., however, we link our contingent claims approach to the q-theory of investment. While we do not consider strategic option exercise games, our results are complementary to Grenadier's (1996) in providing a rational foundation for puzzling investment behavior in real asset markets.

Some authors have discussed the possibility of positive interest rate responses in real options models. The possibility was first raised by Heaney and Jones (1986). In their model, the interest rate response balances the negative effect of interest rates on the present value of the cash flows with the negative effect on the value of waiting. For short-lived projects the first effect on the PV of the cash flows is small so that the second effect on the value of waiting dominates. Capozza and Li (1988) find positive responses in an urban context with infinitely lived projects. In their model the exercise price includes the opportunity cost of the land, which falls when interest rates rise. The positive interest rate effect arises when the opportunity cost of the land is high relative to the capital needed for the project and uncertainty is high. Amin and Capozza (1991) develop a two-factor model and find that the Capozza and Li result holds when both interest rates and rents are stochastic. Ingersoll and Ross (1992) develop a model of real investment that focuses on stochastic interest rates and bullet projects. In their model, as in Heaney and Jones the positive effect occurs for short duration projects.

Our model addresses the production issues and differs from most earlier work by incorporating variable intensity in the decision to invest. The positive response occurs

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[1]The appendix in Pindyck (1988) briefly characterizes a capacity choice problem similar to ours. McDonald and Siegel (1986) also illustrate their model of the value of waiting to invest with a related model where labor input can vary.

when growth rates and/or volatility are high if the production function incorporates substitution between capital and other factors of production. The model provides readily testable implications for empirical work and we contribute the first empirical tests that verify the existence of positive interest rate responses. Our empirical tests use a powerful panel data set on residential investment in 56 metropolitan areas during the 1980s. We show that both growth rates and the volatility of growth rates are significant determinants of the interest rate elasticity. More than 25% of the observations fall in the positive interest rate response region.

In addition to the new theoretical and empirical findings on interest rates,we derive simple rules for the optimal timing of investment. We show the relationship among cash flow yield, IRR, NPV, and Tobin's q under both certainty and uncertainty. The results reveal that growing cash flows are a sufficient condition for projects to be optimally delayed

In the next section we outline the structure of the general model of real investment with variable intensity . The following two sections discuss two illustrative cases--with and without uncertainty. The optimal decision rules for investment are summarized in these sections. The fifth section provides the link between individual projects and aggregate investment which we analyze in the empirical work. The sixth section describes the data for our empirical tests and discusses the empirical results. The final section concludes and discusses policy implications.

Problem Structure

Our point of departure is the general model of the option to replace capital in Capozza and Li (1994). Here we simplify by allowing a single state variable, $X(t)$, the net cash flow per unit of output from the project, and by assuming that projects are new investments, i.e., no durable capital committed in the past will be lost if the project is undertaken. There is at least one fixed factor, e.g., land, and at the time of investment, the investor can choose the capital intensity, K, and output or capacity level, $Q(K)$.[2]

[2]Capital is assumed to be infinitely durable and does not depreciate.

The production function, $Q(K)$, is assumed to be increasing and concave. Without loss of generality, a unit of capital is assumed to cost one dollar so that K is also the cost of the investment in the project.[3] Once the capital cost of the investment is committed to the project, it is assumed to be irreversible.

The net cash flow, $X(t)$, at time, t, may evolve stochastically over time following geometric Brownian motion of the form

$$dX/X = gdt + \sigma dz, \tag{2}$$

where g is the mean growth rate, σ is the standard deviation of the growth rate, and dz is the increment of a standard Wiener process. Decision makers know the parameters of the process so that there is either perfect foresight (when $\sigma=0$) or rational expectations (when $\sigma>0$).

At any time t, the price of a unit of capacity is the present value of expected future cash flows:

$$P(X) = E_t\left[\int_t^{\infty} X(s)e^{-r(s-t)}ds\right] \tag{4}$$

where E_t is the expectation conditional on the information about the risk-adjusted cash flow process as of time t and r is the discount rate[4]. The value of the annual output level $Q(K)$, is $Q(K)P(X)$.

If the project is undertaken at time t, the net present value (NPV) of the project is

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[3]While this assumption simplifies the notation, there is no loss of generality. The results also apply to the more general case in which the unit cost of capital is stochastic as long as the state variable $X(t)$ is viewed as the ratio of unit cash flow to the cost of a unit of capital (McDonald and Siegel, 1986).

[4]We do not model the interest rate process so that our results should be viewed as comparative statics with respect to the interest rate. In effect, we treat interest changes as permanent. This is similar to assuming that interest rates follow a random walk in a stochastic setting. If interest rates mean revert, i.e. changes have a temporary and a permanent component, the rational reactions to the rate changes will be smaller and will depend on the rate of reversion. The relevant interest rate is one matched both to the horizon and the risk characteristics of the investment. Since the projects are long-lived the interest rate should be a long term one. Evidence of mean reversion is much weaker in the long rate than in the short rate.

$$V(X) = Q(K)P(X) - K \tag{8}$$

which is the present value of future cash flows minus the cost of the investment. The investment problem is to choose the number of units of capital, K, and the time of investment, $T \geq t$, to maximize the value of the investment opportunity,

$$W(X) = \max_{T,K} E_t[V(X(T))e^{-r(T-t)}] \tag{11}$$

where E_t is the conditional expectation defined above, T is a random first stopping time adapted to the cash flow process and K is chosen to maximize the NPV at time T. $W(X)$ represents the value of a perpetual warrant or option to invest in the project at any future date. It is also the present value at time t of the NPV at the optimally chosen time of investment.

The Deterministic Model

In this section we derive the optimal investment rules under certainty (i.e., perfect foresight) and show that delaying investment relative to the traditional *IRR*=cost of capital or $NPV \geq 0$ rules, is optimal even under certainty. Let the state variable in (1) increase at a constant rate g $(0 \leq g < r)$ with a standard deviation $\sigma = 0$. Then from (2), the present value of the net cash flow is

$$P(X) = \frac{X}{r-g}. \tag{18}$$

Note that the cash flow at time T is $X(T)=X(t)e^{g(T-t)}$. The value of investment opportunity in (4) can be rewritten as

$$W(X) = V(X^*)\left[\frac{X}{X^*}\right]^{r/g} \quad for \quad X \leq X^* \tag{20}$$

where $X^*=X(T)$ is the threshold or hurdle cash flow for an optimal investment to take

place.

Optimal Decision Rules

From (3), the first-order condition with respect to capital K implies that when the optimal level of capital K^* is chosen, the marginal benefit of capital equals the marginal cost of capital:

$$Q'(K^*)\, P(X^*) = 1 \tag{22}$$

where $Q' = dQ(K)/dK$. Equation (7) defines the optimal K as a function of X and implies that $\frac{\partial K}{\partial X} > 0$.

Similarly, from (6), the first-order condition with respect to X^* implies that at time T, the current cash flow yield $Q(K)X/K$ satisfies:

$$\frac{Q^*X^*}{K^*} = r. \tag{26}$$

This equation is similar to the Jorgensonian (1964) rule to invest when the cash flow, Q^*X^*, equals the user cost of capital, rK^*. Equations (7) and (8) determine the optimal threshold level, X^*, and the optimal level of capital K^*.

Note that from (3) and (5), for an optimally chosen level of capital, K^*, the internal rate of return (IRR) for the project is

$$IRR = \frac{QX}{K} + g,. \tag{29}$$

From (2), g is also the rate of appreciation of the price of output. The two terms in (9) are the current yield and the capital gains yield. These components are analogous to the dividend yield and capital gains from investing in an asset that pays steadily increasing dividends.

From (8) and (9), investment takes place when the IRR reaches the hurdle rate

given by

$$IRR^* = r + g. \tag{30}$$

In capital budgeting theory in finance and neoclassical investment theory in economics, the traditional rule is to invest if the *IRR* of the project exceeds the cost of capital, r. For irreversible projects with the possibility to delay, if cash flows are rising over time, the optimal rule is to delay the project until the current yield, rather than the IRR, equals the cost of capital. Thus, the hurdle IRR^* must include the expected growth rate of cash flows. That is, even under certainty the appropriate hurdle IRR is not equal to the cost of capital.

Substituting (5) into (3) and using (8) yields the critical net present value, V^*,

$$V^* = \frac{g}{r-g}K^*, \tag{32}$$

which is proportional to the optimal level of capital K^*. (11) indicates that the critical value must reflect the present value of growth in cash flows.[5]

The critical value of Tobin's q, defined as the present value of cash flows per unit of the cost of investment, $q = \frac{Q(K)P(X)}{K}$, is

$$q^* = \frac{r}{r-g}. \tag{35}$$

The decision rules are summarized in Table 1. Note that these decision rules are independent of the specification of the production function.

The decision rules based on IRR and NPV are illustrated in Figure 1.[6] With an option to delay the project, the *IRR* and the NPV of the project rise over time as the cash

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[5]Note that, from (12)-(14), when the production function is Cobb-Douglas ($\rho \to 0$), investment takes place immediately or is delayed forever. The optimal level of capital is either zero or infinity.

[6]Since the current yield and Tobin's q are linear transformations of *IRR*, their graphical illustrations are similar to that of *IRR*.

flow, X, rises. When $X<X^*$, IRR is less than IRR^* and the NPV from investing today is less than the value of the investment opportunity, W. In this situation, the value of the investment opportunity includes the NPV from investing today and a positive value of waiting. Optimal investment takes place when $X=X^*$; at that time the $IRR=IRR^*$ and the $NPV=V^*$.

An Explicit Solution

Dividing (7) by (8) and using (5) implies that the elasticity of output, given by $\gamma(K) = \dfrac{K\,Q'(K)}{Q(K)}$, at the optimal level of capital is

$$\gamma(K^*) = 1 - \frac{g}{r}, \tag{37}$$

which satisfies $0<\gamma(K)<1$ for $0<g<r$. Since the elasticity of output is decreasing, (13) determines a unique optimal level of capital K^*.

To obtain an explicit solution, we specify the production function to be CES,

$$Q(K)=[a+(1-a)K^{-\rho}]^{-1/\rho},$$

where a = the asset's distribution coefficient with $0<a<1$ and $\rho=(1-\pi)/\pi>0$ with $0<\pi<1$ being the coefficient of the elasticity of substitution between capital and other factors of production such as labor or land. The elasticity of output is

$$\gamma(K) = \frac{1}{1+[a/(1-a)]K^{\rho}}$$

which satisfies $0<\gamma(K)<1$ and is decreasing in K. As $\pi\to 1$, $\rho\to 0$, and the production function reduces to the Cobb-Douglas, $Q(K) = K^{1-a}$.

From (13) and using the production function, the optimal level of capital K^* then is

$$K^* = \left(\frac{1/a - 1}{r/g - 1} \right)^{1/\rho}. \tag{50}$$

From (8), the threshold level that triggers investment is given by

$$X^* = r \left[\frac{1 - a}{1 - g/r} \right]^{1/\rho}. \tag{51}$$

The optimal level of capital K^*, the critical net present value, V^*, and the value of the investment opportunity, W, are increasing functions of the growth rate, g, and decreasing functions of the level of the interest rate, r. These comparative statics results are consistent with those in Capozza and Li (1994) where cash flows are assumed to increase linearly. While the effect of the growth rate on the threshold level, X^*, is ambiguous in Capozza and Li (1994), here a higher growth rate always implies a higher threshold level and more delay before investment takes place. This occurs because, when cash flows rise exponentially rather than linearly, an increase in the growth rate has a greater impact on the option value to invest in the future than the value of investing today.

A higher level of the interest rate implies a higher threshold level and a longer time until the investment takes place in models without a variable level of capital (e.g., McDonald and Siegel[7], 1986; Capozza and Helsley, 1990). However, an increase in the interest rate is found to lower the threshold level and hasten investment in Capozza and Li (1994) where output is determined by a Cobb-Douglas production function. Here the interest rate effect is ambiguous, as we demonstrate in the next sub-subsection.

Interest Rate Effects

The impact of interest rate changes on the threshold level and on investment is more complicated in this model. To transform the multiplicative relationship in (15) into an additive one, we study *the interest rate elasticity of the threshold*: the ratio of the percentage change in the threshold level, X^*, to the percentage change in the level of the interest rate.

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[7]In the final extension of their model, McDonald and Siegel do allow the capital/labor ratio can vary.

From (15), the interest rate elasticity of the threshold is

$$\frac{\partial X^*/X^*}{\partial r/r} = \frac{1}{(1-\pi)(1-\ g/r)}\left((1-\pi)\ -\ \frac{g}{r}\right), \tag{52}$$

which is positive (negative) if $g/r<(>)1-\pi$. This leads to the following proposition:

Proposition 1. Let cash flows, X, increase at a constant rate, g. Assume that the production function is CES with an elasticity of substitution, π. Let CY be the current yield, IRR the internal rate of return, V net present value and q be Tobin's q. Then for cash flows X near the threshold X^*, if $g/r\leq(>)1-\pi$,

$$\begin{aligned} CY_r(X) &\leq(>) CY_r^*, \\ IRR_r(X) &\leq(>) IRR_r^*, \\ V_r(X) &\leq(>) V_r^*, \\ q_r(X) &\leq(>) q_r^*, \end{aligned}$$

where the partial derivatives with respect to the interest rate are positive in the first two inequalities and are negative elsewhere.

The proof appears in the appendix. The positive effects of interest rate changes on the critical values CY^* and IRR^* are expected since they include the opportunity cost of capital. If the capital intensity is given exogenously, CY and IRR are independent of the interest rate. In this model, they are determined endogenously and hence vary with the interest rate. A higher interest rate reduces the optimal levels of capital and output. For production functions that exhibit decreasing returns to the variable factor, a lower level of output implies a higher output/capital ratio and a higher CY and IRR.

As the interest rate increases, the present value of the cash flows decreases and thus the NPV and Tobin's q decrease. An increase in the interest rate also reduces the present value of investing in the future and thus lowers the critical values for NPV and q.

The response of investment to interest rates is determined by the effect of the interest rate change on any of the decision variables relative to the effect on its critical value. A positive response of investment to interest rate changes arises when an increase in

the interest rate results in a larger increase in the IRR than in its critical value *IRR**.[8]

Figures 2-3 illustrate the effect of a change in the interest rate on *IRR*, *IRR**, *NPV*, *V(X)*, *V** and the value of investment opportunity *W(X)*. Suppose that the interest rate rises from r_1 to r_2. Consider the case where $g/r<1-\pi$. In this case (illustrated in Figure 2), the hurdle rate rises from X_1^* to X_2^*. The increase in the interest rate raises *IRR* and *IRR** but lowers *V(X)*, *V** and *W(X)*. Indeed, for projects with cash flows between X_1^* to X_2^*, *IRR** rises more than *IRR* and the NPV falls more than the value of the investment opportunity. As a result, these projects that would have been undertaken are delayed.

Let us now turn to the case where $g/r>1-\pi$ (shown in Figure 3). The hurdle rate falls from X_1^* to X_2^* as the interest rate rises from r_1 to r_2. While the increase in the interest rate still raises *IRR* and *IRR**, but lowers V(*X*) and *W(X)*, *IRR* rises more than *IRR** and *W(X)* falls more than V(*X*) for projects with cash flows between X_2^* and X_1^*. Therefore, these projects, which would not have been undertaken when *IRR* was less than *IRR** or NPV was less than the value of the investment opportunity, are now acceptable since *IRR* exceeds *IRR** and NPV exceeds the value of the investment opportunity.

The General Problem

We now turn to the stochastic case in which $\sigma > 0$ and where expectations are rational. When the cash flow evolves stochastically over time, the present value of cash flows and the value of the investment opportunity can be priced as contingent claims (options). To use the contingent claims approach, we assume that the stochastic changes in the state variable are spanned by existing assets in the economy. In this way, an equilibrium model such as the continuous-time version of the Capital Asset Pricing Model can be used to determine the risk-adjusted returns on the present value of cash flows and the option value to invest.

The present value of cash flows per unit in (2) is given by

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[8]For Cobb-Douglas production function ($\pi = 1$), the positive effect always occurs. This result is consistent with the finding in Capozza and Li (1994).

$$P(X) = \frac{X}{r-\hat{g}}, \tag{76}$$

where $\hat{g}$ is the risk-neutral growth rate, $\hat{g}=g-\lambda$ and λ is the risk premium for the portfolio replicating the stochastic evolution of the cash flow, X. Since $r-\hat{g}=(r+\lambda)-g$, the term $r+\lambda$ is the risk-adjusted discount rate for cash flows. The option value to invest satisfies the following ordinary differential equation:[9]

$$\frac{\sigma^2}{2}X^2 W_{XX} + \hat{g}XW_X - rW = 0. \tag{82}$$

The boundary conditions are

$$\begin{aligned} W(0) &= 0, \\ W(X^*) &= V(X^*), \\ W_X(X^*) &= V_X(X^*). \end{aligned} \tag{83}$$

The conditions have the usual interpretations. The first condition follows from the observation that the project is worthless if the current and future cash flows are zero. The second is the high-contact or continuity condition, stating that at the time of investment, the value to invest in the future equals the project's current NPV. The third is the "smooth-pasting condition" or first order condition, which ensures that the threshold, X^*, is chosen optimally.

The solution to the problem (18) subject to the first and second boundary conditions in (19) is (compare (6))

$$W(X) = V(X^*)\left[\frac{X}{X^*}\right]^{\alpha}, \tag{84}$$

[9]Note that $d \equiv r-\hat{g}$ is the dividend yield, X/P, from investing in an asset with price P. $\hat{g}=r-d$ does not reduce to the risk free rate since the dividend yield is not zero.

where $\alpha = \frac{1}{\sigma^2}\left[-(\hat{g}-\frac{\sigma^2}{2}) + \sqrt{(\hat{g}-\frac{\sigma^2}{2})^2 + 2\sigma^2 r}\right] > 1$. As $\sigma \to 0$, $\hat{g} \to g$, $\alpha \to r/g$. As $\sigma \to \infty, \alpha \to 1$.

Optimal Decision Rules in the Stochastic Case

The smooth-pasting condition in (19) is analogous to a first-order condition of (20) with respect to the threshold level X^* (see Merton (1973)). This condition implies the critical current yield:

$$\frac{Q^*X^*}{K^*} = r + \frac{\sigma^2}{2}\alpha. \tag{90}$$

From (21), optimal investment occurs when the current yield equals the cost of capital, r, plus an uncertainty premium, $(\sigma^2/2)\alpha$.

From (9), the *IRR* of the project is the current yield plus the expected growth rate, g. (21) implies that investment is made when the *IRR* reaches the hurdle rate given by

$$IRR^* = r + g + \frac{\sigma^2}{2}\alpha. \tag{92}$$

Compared with the hurdle rate in the deterministic case given by (10), the hurdle rate, IRR^*, in the stochastic case contains an uncertainty premium since the value of waiting is higher under uncertainty. This hurdle rate differs from the required rate of return, $IRR^* = r + \lambda$, in the traditional theory, where λ is the risk premium defined earlier.

The critical value of NPV is (from (21))

$$V^* = \frac{\hat{g} + (\sigma^2/2)\alpha}{r-\hat{g}} K^*, \tag{95}$$

which implies that at the optimal time the NPV yield, V^*/K^*, must equal the value of the growth, $\frac{\hat{g}}{r-\hat{g}}$, plus the value of the volatility of the cash flows, $\frac{(\sigma^2/2)\alpha}{r-\hat{g}}$.

From (21), the critical value for Tobin's q, is

$$q^* = \frac{r + (\sigma^2/2)\alpha}{r - \hat{g}}. \tag{98}$$

The decision rules in the stochastic case are also summarized in Table 1. The investment policy for the case with stochastic cash flows is similar to that in the deterministic case (illustrated in Figure 1) except that the optimal level of capital and the threshold level are higher.

The condition for the optimal level of capital is the same as (7). Dividing (7) by (21) and rearranging yields

$$\gamma(K^*) = \frac{r - \hat{g}}{r+(\sigma^2/2)\alpha}, \tag{99}$$

which is a stochastic generalization of (13).

An Explicit Solution

For the CES production function, the optimal level of capital and the threshold level of cash flows are

$$K^* = \left(\frac{1/a - 1}{\alpha - 1}\right)^{1/\rho}, \tag{100}$$

$$X^* = \left(r + \frac{\sigma^2}{2}\alpha\right)\left(\frac{1- a}{1- 1/\alpha}\right)^{1/\rho}. \tag{101}$$

The comparative statics results with respect to the growth rate are the same as in the deterministic case. As in other option models, the option value to invest increases in the volatility of cash flows. Thus the higher the volatility, the longer will be the time until the investment is undertaken. In addition, the higher the volatility, the higher is the level of capital that will be committed when the investment takes place. These results are similar to

those in Capozza and Li (1994).

Interest Rate Effects

The interest rate elasticity of the threshold level is

$$\frac{\partial X^*/X^*}{\partial r/r} = \frac{1}{(1-\pi)(1-\hat{g}/r)}\left[(1-\pi)-\eta\right]. \tag{102}$$

where

$$\eta = \frac{(r-\hat{g})}{\alpha(\alpha-1)\sqrt{(\hat{g}-\sigma^2/2)^2+2\sigma^2 r}}$$

which is increasing in $\hat{g}$ and σ and approaches g/r as $\sigma \to 0$ and one as $\sigma \to \infty$. Thus, for any g, if the volatility is sufficient large, $\eta > 1-\pi$, and $X_r^* < 0$.

Proposition 2. Assume that cash flows, X, evolve according to a geometric Brownian motion with drift g and standard deviation, σ. Also assume that the production function is CES with elasticity of substitution π. Let CY be the current yield, IRR the internal rate of return, V the net present value and q be Tobin's q. Then for cash flows X near the threshold X^*, if $\eta \lesseqgtr 1-\pi$,

$$\begin{aligned} CY_r(X) &\lesseqgtr CY_r^*, \\ IRR_r(X) &\lesseqgtr IRR_r^*, \\ V_r(X) &\lesseqgtr V_r^*, \\ q_r(X) &\lesseqgtr q_r^*, \end{aligned}$$

where the partial derivatives with respect to the interest rate are positive in the first two inequalities and are negative elsewhere.

The proof is also given in the appendix. As in the deterministic case, an increase in the interest rate raises the IRR and IRR^* of the project, but lowers the NPV and option value to invest. If the volatility is low, IRR^* rises more than IRR, and the NPV falls more than the option value to invest. Thus some projects that would have been undertaken are delayed as the interest rate rises. However, if volatility is high, IRR rises more than IRR^*,

and the option value falls more than the NPV. Consequently, investments that would not have been made are hastened (see Figures 2-3).

Note that the hurdle rate *IRR** and the option value of the project are positively related to the volatility. If the volatility is high, there is more delay before investment is made. The positive interest rate effect arises when projects awaiting development are undertaken when the interest rate rises.

Figure 4 summarizes the effect of a change in the interest rate on the timing of investment for various levels of the expected growth rate and volatility. Let $(\bar{g},\bar{\sigma})$ be any point located on *the interest rate-neutral curve*, i.e., the collection of (g,σ) at which the interest rate changes have no impact on investment. For a given level of the interest rate and a given level of the elasticity of substitution, the positive effect of interest rate changes on investment arises if $g>\bar{g}$ or $\sigma>\bar{\sigma}$. The interest rate-neutral curve shifts downward as the elasticity of substitution rises. This implies that the larger the elasticity of substitution, the more likely is the perverse effect.

Aggregation

To analyze the effect of changes in the interest rate on investment directly, we assume that the rate of arrival of new projects is a constant, *N,* at any point in time[10]. These projects are *ex ante* identical with the initial cash flow *X* following a cumulative distribution function *F(X)* with *F'>0.* The number of projects that will be undertaken at time *t* is

$$n(t) = \sum_{i=1}^{N} 1_{(X^i \geq X^*)} \qquad (119)$$

where $1_{(X^i \geq X^*)}$ is an index function that takes the value of one if the initial cash flow from asset *i* satisfies $X^i \geq X^*$ and zero otherwise. The level of investment is defined to be the

[10]This assumption is motivated by the empirical analysis of housing permits in the next section. If the number of households in a metropolitan area is growing, then at each moment in time the number of new feasible projects is approximately equal to the number of new households. See Capozza and Sick (1993) for a more detailed accounting of the relationship between household growth and the arrival of residential investment projects.

expected number of projects undertaken[11] at time t:

$$I(t) \equiv E[n(t)] = N[1-F(X^*)]. \tag{122}$$

Since $F' > 0$, investment rises as X^* falls and vice versa.

The interest rate elasticity of investment is then:

$$I_r/I = - b(X^*)\, x_r^* \tag{124}$$

where $b(X)=XF'(X)/[1-F(X)]>0$. We have shown in equation (28) that the interest rate elasticity of the threshold level, x_r^*, is positive if the expected growth rate and volatility are low and is negative otherwise. From (28), rising interest rates adversely affect investment only if the expected growth rate and volatility are low. The positive effect of interest rates on investment arises if the expected growth rate or volatility is high.

The Empirical Tests

The Sources of Data

To test the model's implications for interest rate effects we analyze residential investment which is recognized to be sensitive to interest rates. Our sample includes annual data on single family building permits in 64 metropolitan areas during the 1980s. The source for these data is the Building Permits Branch of the Bureau of the Census of the U.S. Department of Commerce.

In metropolitan areas, a major determinant of the growth rate of the real rents is the growth rate of population in that area (Capozza and Helsley, 1989, 1990). Therefore, we use population growth as a proxy for real rental growth because data on the implicit rents of homeowners are not available. The population data by metro area are obtained from the Bureau of Economic Analysis of the U.S. Department of Commerce.

Our sample spans the period from 1980 to 1989. This period is selected for two reasons. First, the data on building permits were not available before 1980. Second, the

[11]This definition is motivated by the housing permit data we use in the next section. Permit data register the number of housing units but not the gross change in the capital stock.

standards for defining metropolitan areas were modified in 1980 and 1990 by the Office of Management and Budget. Even during this sample period, the number and composition of metropolitan areas are not constant. We can construct consistent data for 64 large metropolitan areas during the sample period. The average annual growth rates of population are calculated for each of the 64 metropolitan areas. After eliminating those with negative average growth rates, 56 metropolitan areas remain in the sample.

The relevant interest rate is one matched to the horizon and risk of the investment. Since we are studying residential investment we choose the primary conventional home mortgage rate to represent the nominal interest rate. Mortgage rates reflect the maturity and default risk of the residential assets and therefore are a suitable discount rate for net rental income (cash flow). The time-series data for the mortgage rate were retrieved from CITIBASE data.

The real interest rate is calculated in two ways--first as the nominal mortgage rate minus the CPI inflation rate. The CPI is obtained from the CRSP tape from the University of Chicago. An alternative real interest rate is calculated using the average of the current and preceding two years' inflation rates as a proxy for the expected inflation rate. Other time series methods for estimating expected inflation rates were also tested.

Description of the Variables

HP_{it}: the number of building permits for single-family homes issued in metropolitan area i for year t.

$GHP_{it} = \log(HP_{it}/HP_{i,t-1})$: the annual growth rate of building permits in percent for single-family homes issued in metropolitan area i for year t.

POP_{it}: the population in metropolitan area i in year t.

$GPOP_{it} = \log(POP_{it}/POP_{i,t-1})$: the annual growth rate of population in percent in metropolitan area i for year t.

$GHPC_{it} = GHP_{it} - GPOP_{it}$: the annual growth rate of building permits per capita in percent for single-family homes issued in metropolitan area i for year t.

RM (RMA): the annualized real mortgage rate in percent defined as the beginning-of-year yield on the primary conventional mortgage minus the CPI inflation rate (minus a 3 year average of current and the preceding two years of CPI inflation)..

$GRM_t = \log(RM_t/RM_{t-1})$: the annual percentage change in the real mortgage rate for year t. (*GRMA* uses *RMA*)

$RATIO_{it}=GHPC_{it}/GRMt_t$: the interest rate elasticity of building permits per capital issued in metropolitan area i for year t (*RATIOA* uses *GRMA*)

$AGPOP_i$: the average annual growth rate of population in metropolitan area i.

$SGPOP_i$: the standard deviation or volatility of the growth rate of population in metropolitan area i.

Summary Statistics

Table 2 reports the summary statistics for the growth rates of building permits, population and other relevant variables for the sample period, 1981-1989. The average annual growth rates of building permits (GHP) range from -12.0 percent to 15.5 percent across the 56 metropolitan areas, with a mean of 2.8 percent and a standard deviation of 5.9 percent. The average annual growth rates of building permits per capita (GHPC) vary in a similar range, with a lower mean of 1.4 percent but a similar standard deviation of 6.1 percent. The small change in the variability indicates that the cross-sectional variance of the growth rates of building permits is much greater than that for the population growth rates.

The real mortgage rate (RM) is significantly positive during the sample period, with a mean of 7.7 percent and standard deviation of 3.4 percent. Figure 5 plots the real mortgage rate at the beginning of each year. *RM* rose from 1% to 13% during 1980-1983 and then declined for most of the rest of the sample period. It fell to nearly 6 percent at the beginning of 1989. Not surprisingly, the percentage change in the real mortgage rate (GRM) also shows high variability, with a mean of 18 percent and standard deviation of 67 percent. The t-ratio, implies that the mean of GRM is indistinguishable from zero.

When averaged over the sample period and across the metropolitan areas, the elasticity of residential investment as measured by RATIO = GHPC / GRM is close to zero. This weak relationship between aggregate changes in the construction activity and changes in the interest rate is consistent with the findings of earlier empirical tests of neoclassical models of the investment demand (see Hall (1977)). The time-series means of RATIO, however, vary substantially across metropolitan areas, with a minimum of -6.7

and maximum of 13.5.

The average population growth rate (AGPOP) and the standard deviation of the population growth rate (SGPOP) also exhibit large variability among the metropolitan areas. AGPOP varies from 0.1 percent to 5.1 percent and SGPOP fluctuates from 0.1 percent to 2.2 percent across the metropolitan areas.

The lower panel of Table 2 reports the cross-correlations between the variables. As expected, GHP is highly correlated with GHPC with a coefficient of 0.999. The variable RATIO is more correlated with SGPOP (with a correlation coefficient of 0.24) than AGPOP (with a correlation coefficient of only 0.08). The correlation coefficient between AGPOP and SGPOP is moderate at 0.24. The correlation between the two variables suggests that the expected population growth rate and the volatility of the growth rate are not close proxies for each other.

Subgroup Analysis

To examine what determines the effect of changes in the interest rate on residential investment we first rank the metropolitan areas by their average population growth rates and their volatilities. We then assign each metropolitan area to quartiles based on the rankings. Panel A of Table 3 reports the subgroup means of the interest rate elasticity of building permits per capita, i.e., RATIO = GHPC/GRM. The standard errors are given in parentheses below the means. The shaded region in the table highlights the cells where the perverse response occurs.

The last row in panel A of Table 3 reports the means of the variable RATIO for four quartiles formed from the average population growth rate AGPOP. The mean for the lowest average growth quartile is -2.40 with a standard error of 0.84. This mean is significantly negative at 1 percent level. The means for the other quartiles are positive. The mean of the third quartile is significant at the 10 percent level.

The first row in panel B of Table 3 reports the differences in the means of each quartile from the lowest average growth quartile and the χ^2 statistics for testing the joint hypothesis that the means are all equal. To adjust for heteroskedasticity, the standard errors for the differences in means and the χ^2 statistics are calculated using the heteroskedasticity-consistent covariance matrix estimator of White (1980). These

differences are all significantly positive. The joint hypothesis that the subgroup means are all equal is rejected at 5 percent level. These results are consistent with the prediction of the model that the interest rate effect on investment is negative if the expected growth rate is low and is positive otherwise.

The last column in panel A of Table 3 presents the means of RATIO for the four quartiles formed from the volatility of the growth rates. The interest rate elasticities reflected in the subgroup means increase in volatility. The interest rate effect is negative and statistically significant for the first two volatility quartiles, while the effect is almost zero for the third volatility quartile and significantly positive for the highest volatility quartile. The last row in panel B of Table 3 reports the differences in means between each volatility quartile and the lowest volatility quartile. The differences are significantly positive for the third and highest volatility quartiles. From the χ^2 statistic in that row, the hypothesis that all volatility subgroup means are equal is strongly rejected at the 1 percent level. This provides further evidence that the interest rate effect on investment is not independent of the volatility of the growth rates. The positive relationship between the interest rate elasticity of investment and the volatility of the growth rates is again consistent with the prediction of the model.

To investigate the roles of the mean population growth rate and volatility of population growth rate simultaneously, we divide each volatility quartile into four sub-quartiles from ranked average population growth rates of metropolitan areas within the quartile. While the sub-quartile means are negative and mostly significant within the lowest volatility quartile, they are positive and largely significant within the highest volatility quartile. Similar to the results in the last row of panel A of Table 3, the means in the subquartiles are not strictly monotonic. Nevertheless, they tend to increase in the average population growth rates. This implies that the effects of interest rate changes on investment are related to both the mean and the standard deviation of the growth rates.

Regression Results

To further investigate the effects of changes in the interest rate on investment, we conduct pooled time-series and cross-sectional regressions of the variable RATIO on the average population growth rate, the volatility of the population growth rate and other

related variables. Since the interest rate elasticity of investment may also be related to the level of interest rates and other exogenous factors like government regulations, tax rates and monetary policies, we include dummy variables for each calendar year except the first year to capture the time series variation of any excluded variables. Some year dummy variables are significant.

Table 4 reports the regression results. The standard errors appear in parentheses below the regression coefficients. The first χ^2 statistic tests the joint hypothesis that all coefficients are zero. The second χ^2 statistic tests the joint significance of all coefficients except the intercept and year dummies. Since specification tests reject the hypothesis that the residuals in the regression models are homoskedastic, the heteroskedasticity-consistent covariance matrix estimator of White (1980) is used to calculate the standard errors and χ^2 statistics.

The regression intercepts are significantly negative at the 1 percent level in each regression. This indicates that the average effect of interest rate changes on investment is negative.

In the first regression, the ratio of the average population growth to the real mortgage rate, AGPOP/RM, is used as the explanatory variable. The regression coefficient for this ratio is 0.86, with a standard error of 0.41, which implies that this variable is significantly positive at the 5 percent level. This result is consistent with the prediction of the deterministic model that the effect of interest changes on investment is positively related to the ratio of the growth rate of cash flows to the real interest rate. For the perverse effect of interest rate changes on investment to occur, the fitted value of the dependent variable must be positive. This regression implies that the ratio, AGPOP/RM, must exceed 1.42 (=1.22/0.86) with an asymptotic standard error (see White (1984)) of 1.22 for the positive interest rate effect to occur. According to the deterministic model, the critical value is $0 \leq 1-\pi \leq 1$ where π is the elasticity of substitution. Since the standard error is large, the estimated critical value is not distinguishable from either zero or one.

In the second regression, the average population growth rate, AGPOP, is the explanatory variable. The coefficient estimate (0.74) is also more than two standard errors away from zero. Since the variation in the real interest rate over time is captured by the year dummies, the positive coefficient for the variable AGPOP is consistent with the

model's prediction that the effect of interest rate changes on investment is positively related to the growth rate of cash flows for a given level of the interest rate. The positive effect of interest rate changes on investment occurs if the average growth rate exceeds 1.73 (=1.28/0.74) percent with a standard error of 1.31 percent. The estimated critical value from this second equation is plotted in Figure 6. This critical value lies between the median (1.19 percent) and the third quartile (1.95 percent) of the population growth rates. This implies that the positive response is estimated to occur in more than 25% of the observations. Alternatively, the ratio of the average growth rate to the average real mortgage rate (7.7 percent) must exceed 0.22. The implied value of the elasticity of substitution, $\pi = 0.78$.

Both the model and the data in Table 3 suggest that the relationship between the effect of interest rate changes on investment and the average growth rate is not monotonic. To account for a possible nonlinear relationship, the third regression introduces a quadratic function for the average population growth rate. The coefficient estimates for the average population growth rate and its squared value are 2.57 and -0.42, respectively. They are both significant at the 5 percent level. For the range of the average population growth rates reported in Table 2, the critical value of the average growth rate that must be met for the positive effect of interest rate changes to arise is 1.20 percent (the median is 1.19 percent) with a standard error of 0.47 percent. Equivalently, the positive response to interest rate changes occur if AGPOP/RM exceeds 0.16 (=1.20/7.7). The implied elasticity of substitution is $\pi = 0.85$.

In the fourth regression, we examine the significance of the volatility of population growth rates in explaining the effect of interest rate changes on investment. The regression coefficient is 6.08 with a standard error of 2.40. This is significant at the 1% level and is consistent with the prediction of the stochastic case of the model. From the coefficient estimate, the positive effect of interest rate changes arises if the volatility is greater than 0.63 percent (the standard error is 0.43 percent). This critical value is between the median 0.46 percent and the third quartile 0.71 percent implying that the perverse interest rate effect would occur in between 25 and 50% of the observations. The R^2 of this regression is 5.6 percent, while the R^2 of regressions using average population growth rate as explanatory variables are less than 1 percent. These R^2 suggest that the volatility of

growth has more explanatory power than the average growth rate.

In the last regression, the explanatory variables include the average growth rate, the squared value of the average growth rate and the volatility of the growth rate. The coefficient for the volatility remains significant at the 5 percent level. The coefficients for the other variables lose their significance although they retain the expected sign. The χ^2 test shows that the three explanatory variables are jointly significant at 5 percent level.

The results in tables 3 and 4 use the realized real mortgage rate. To examine whether these results are robust to this proxy for the real interest rate, we also tried an alternative calculation for the real interest rate. This alternative for expected inflation is a moving average of current and the preceding two years' inflation rates. The results (not reported) are similar to those in Table 4. Not surprisingly, the overall fit improves with this variable since the moving average reduces the noise in the dependent variable. The coefficients are smaller since the variance of the alternative real interest rate is lower.

Conclusions

In this paper we have analyzed real investment with variable capital intensity both theoretically and empirically. These investments include situations where capital can be varied relative to other inputs to the investment. This, of course, is a wide class of investments since it includes any investment decision that involves fixed amounts of either land or human capital and would include decisions to develop an ore deposit or an oil field, all land development decisions whether residential or commercial, decisions to renovate or redevelop existing industrial or commercial facilities (e.g., factories), and decisions to invest in human capital.

Two types of theoretical results are derived. First, simple decision rules for optimal investment timing are derived and elaborated in terms of current yield, internal rate of return, net present value yield, and q ratio. In a growing economy the simplest optimal timing rule is for current yield where optimal investment occurs when the current yield equals the cost of capital in the certainty case and equals the cost of capital plus an uncertainty premium ($r + \frac{\sigma^2 \alpha}{2}$). The IRR equals the cost of capital plus the growth rate of cash flows $(r+g)$ under certainty. With uncertainty the uncertainty premium must be

included $(r + g + \frac{\sigma^2 \alpha}{2})$. The net present value and q rules are only slightly more complicated (see Table 1). Note that investments are optimally delayed relative to traditional investment criteria even under certainty if cash flows are growing.

Secondly, we derive the conditions under which positive responses of investment to interest rates can occur. In the model uncertainty is not a necessary condition for positive responses of investment to interest rates to occur. High growth rates are sufficient to cause the positive response. High volatility increases the likelihood of positive responses when growth rates are positive and is sufficient if growth is zero. Intuitively an increase in the interest rate raises both the hurdle IRR and the project IRR. The project IRR rises because higher interest rates reduce the optimal capital intensity and increase the corresponding output/capital ratio. If the project IRR rises more than the hurdle IRR, projects are accelerated in time rather than delayed by the interest rate increase. When the elasticity of substitution between capital and other factors is high, acceleration is more likely.

The empirical analysis of residential investment during the 1980s in 56 metropolitan areas confirms the predictions of the model. Both growth rates and volatility of growth rates are significant explanatory variables in regressions with the interest rate elasticity of investment as a dependent variable. In the data sample the estimated interest rate neutral curve places between 25 and 50% of the observation in the perverse region.

The model could be extended in a number of ways. First, we have assumed that output is sold in a competitive market so that there is no interaction between the capital intensity and the price of output. For investments in monopolistic settings, including infrastructure investments like roads, bridges, etc., demand is downward sloping. A less than perfectly elastic demand should weaken the effect of capital intensity on investment decisions. Second, as with most models of real investment decisions, our model is a one factor model and we have modeled only the stochastic process for project revenue. As indicated above, if the process for long-term interest rates is included and it has a temporary as well as a permanent component, the optimal reactions to interest rate changes will be smaller. We conjecture that this should not, however, affect the sign of the optimal

reaction[12].

The policy implications of these results are profound. If increases in the interest rate can accelerate investment spending then extreme care must be taken when monetary decisions are designed to transmit restraint to the economy through their effect on investment. It becomes extremely important to understand the conditions under which the perverse results are obtained.

Since it is the ratio of the growth rate to the interest rate that is critical, positive interest rate responses can occur either when growth rates are high or when interest rates are very low. Therefore, policy is most at risk at the extreme points--near the peak when growth rates can be high and at the trough if interest rates are very low. That is, when monetary policy is most needed may also be the times when it is least or even perversely effective.

There are also regional implications to the results. Since some parts of the country grow faster than others, interest rate policy can have very different effects regionally. Indeed, the regions most in need of restraint from interest rate policy are also the areas most likely to respond perversely.

Appendix

Proof of proposition 2 (proof of Proposition 1 is a special case in which $\sigma \to 0$.).

We first prove the inequality for *CY*. Since *CY(X)* is a continuous function of *X*, it is sufficient to consider the case in which $X=X^*$. In this case, $CY(X^*)=CY^*$ and thus $CY_r(X^*)<(>)CY_r^*$ is equivalent to $cy_l<(>)cy_l^*$ where $cy=\log(CY)$ and $l=\log(r)$. Since $cy_l - cy_l^* = -x_l^*$, the first inequality follows from the equation before the proposition.

Note that α is the root of the quadratic equation: $(\sigma^2/2)\alpha(\alpha-1)+\hat{g}\alpha - r=0$. The right hand side of (21) can also be written as ϕr where $\phi=(1-\hat{g}/r)/(1-1/\alpha)>1$, which approaches unity as $\sigma \to 0$. To show $CY_r>0$, note that $cy_l = -[(\phi-1)+(\hat{g}/r)]k_l^*/\phi>0$

[12]Amin and Capozza do provide a two factor model with stochastic interest rates. Analytical results cannot be obtained, but qualitative results are similar to the one factor model.

where $\phi>1$ and $k_l^*=-\alpha_r/[\rho(\alpha-1)]<0$. To show $CY_r^*>0$, note that

$cy_r^*=1+\phi_r/\phi=(1-\eta)/(1-\hat{g}/r)>0$ where $\eta<1$.

Note that $IRR(X)=CY(X)+g$, $V(X)+K^*=CY(X)K^*/(r-\hat{g})$, and $q(X)=CY(X)/(r-\hat{g})$. Thus

$$\begin{aligned} & \log[IRR(X)-g] - \log(IRR^*-g) \\ = & \log[V(X)+K^*] - \log(V^*+K^*) \\ = & \log[q(X)] - \log(q^*) \\ = & \log[CY(X)] - \log(CY^*). \end{aligned}$$

The inequalities for *IRR*, *V* and Tobin's *q* can be shown analogously.

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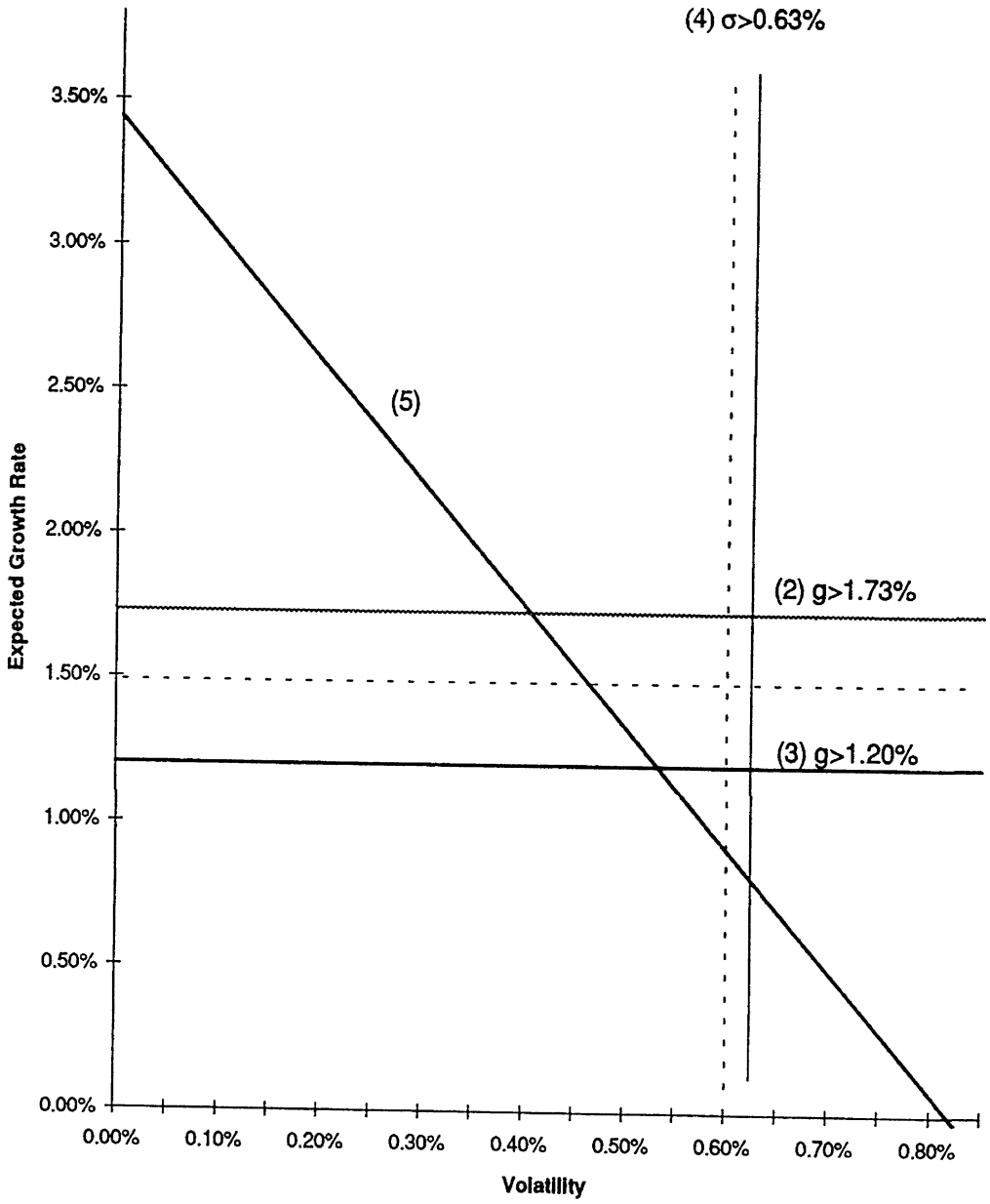


Fig. 6. Estimates of Interest Rate Effects.

Each curve is drawn using estimated regression coefficients in Table 4. The regression number is next to the curve. The dashes indicate sample means.

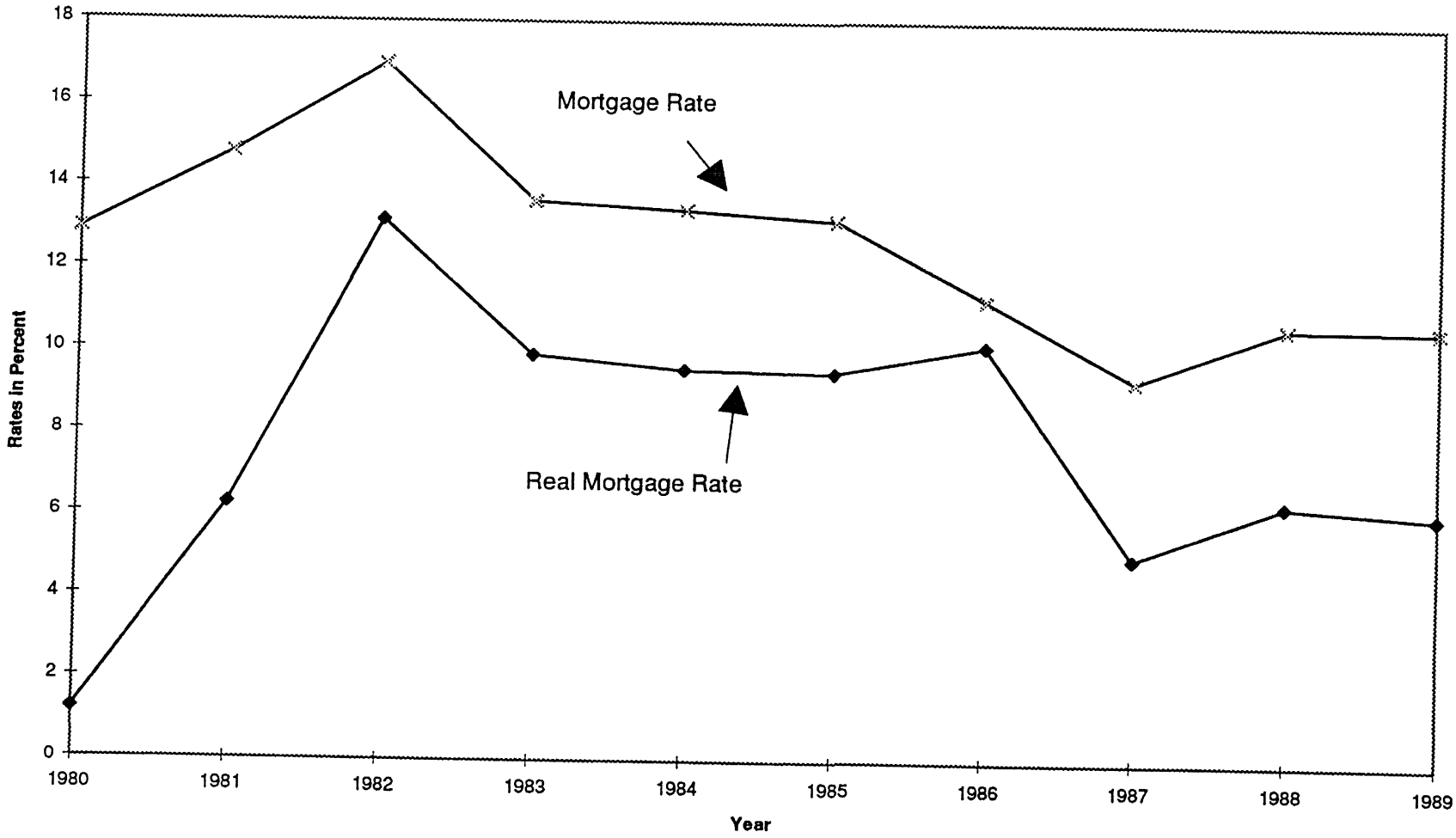


Fig. 5. The beginning-of-year Mortage Rate and the Real Mortgage Rate as the Difference between the Mortgage Rate and the Annual CPI Inflation Rate.

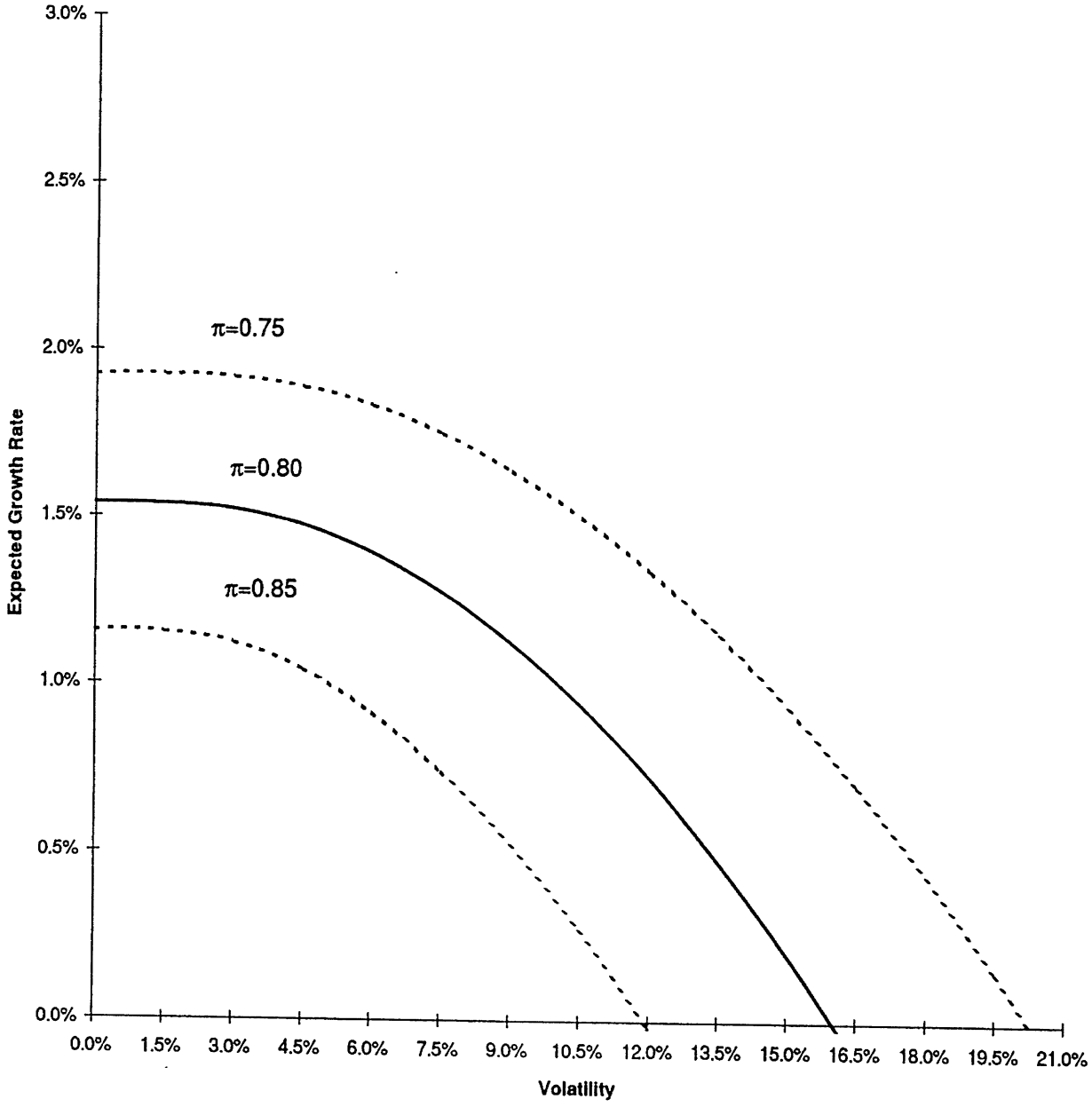


Fig.4. Effects of Interest Rate Changes

The curves plot combinations of growth and volatility for which the interest rate has a neutral effect. *pi* is the elasticity of substitution.

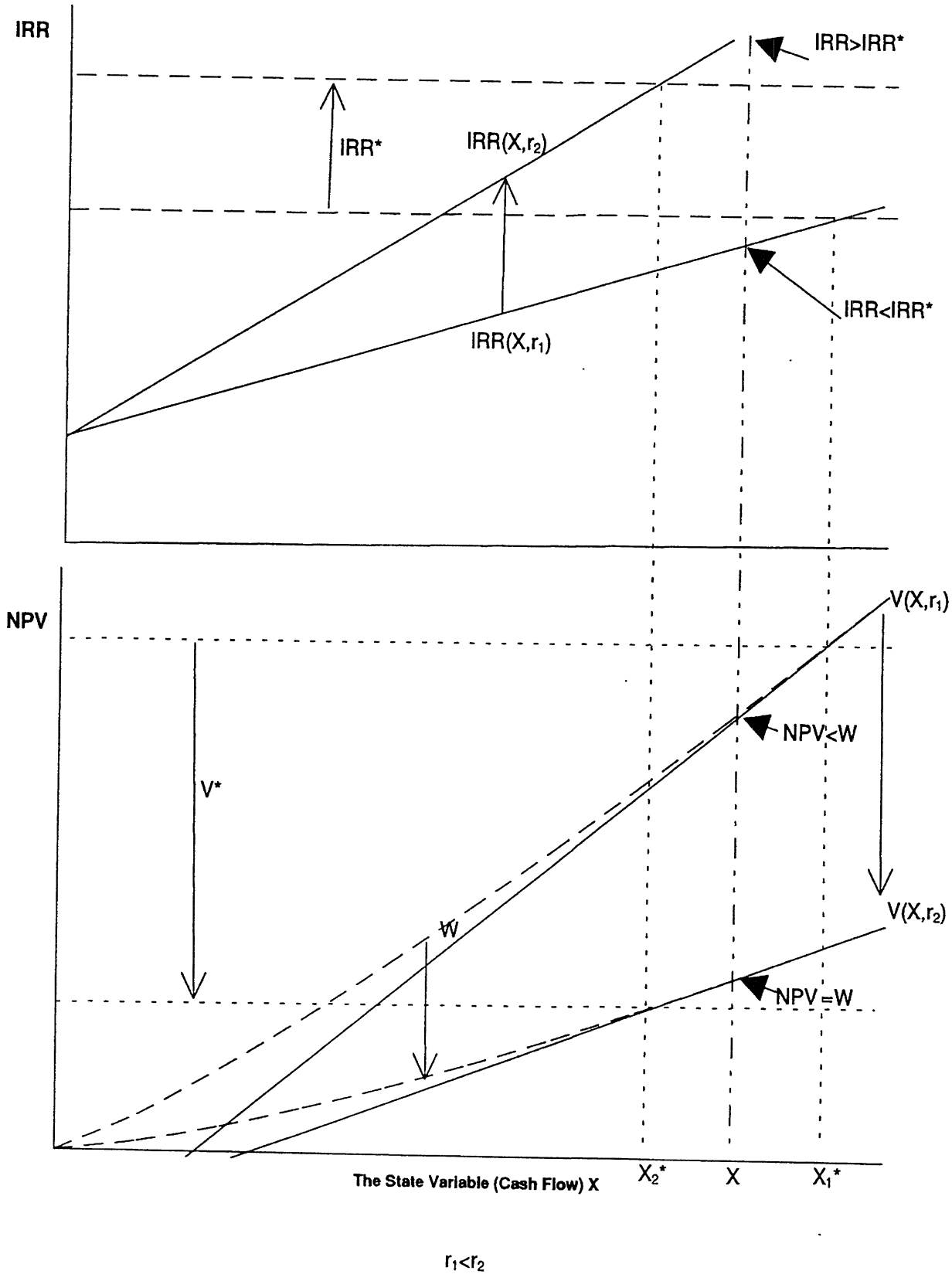
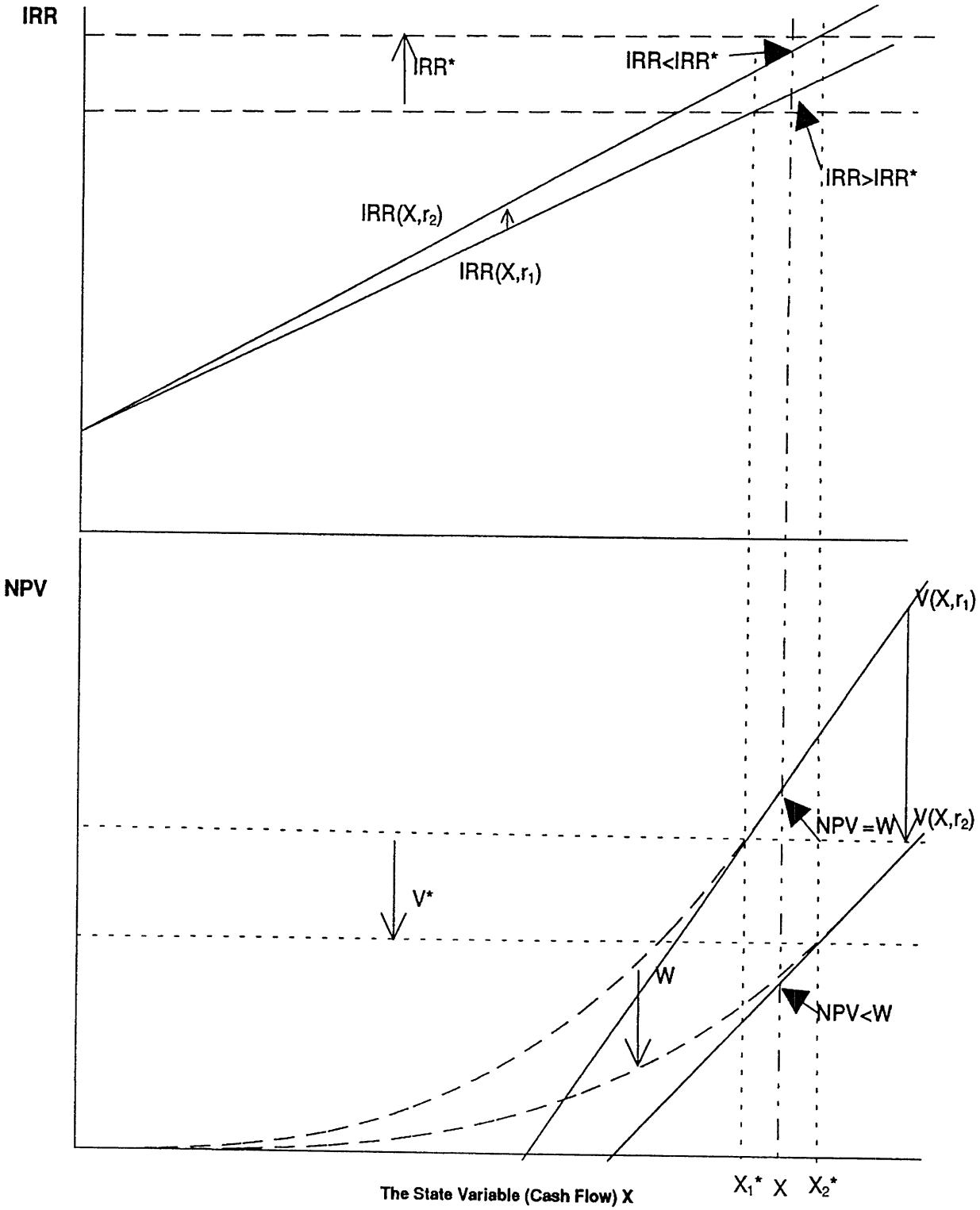


Fig. 3. Shifts in NPV and IRR as the Interest Rate Rises: High Growth or Volatility. V is NPV. W is the option value. r is the interest rate.



$r_1 < r_2$

Fig. 2. Shifts in NPV and IRR as the Interest Rate Rises: Low Growth and Volatility. V is NPV. W is the option value to invest. r is the interest rate.

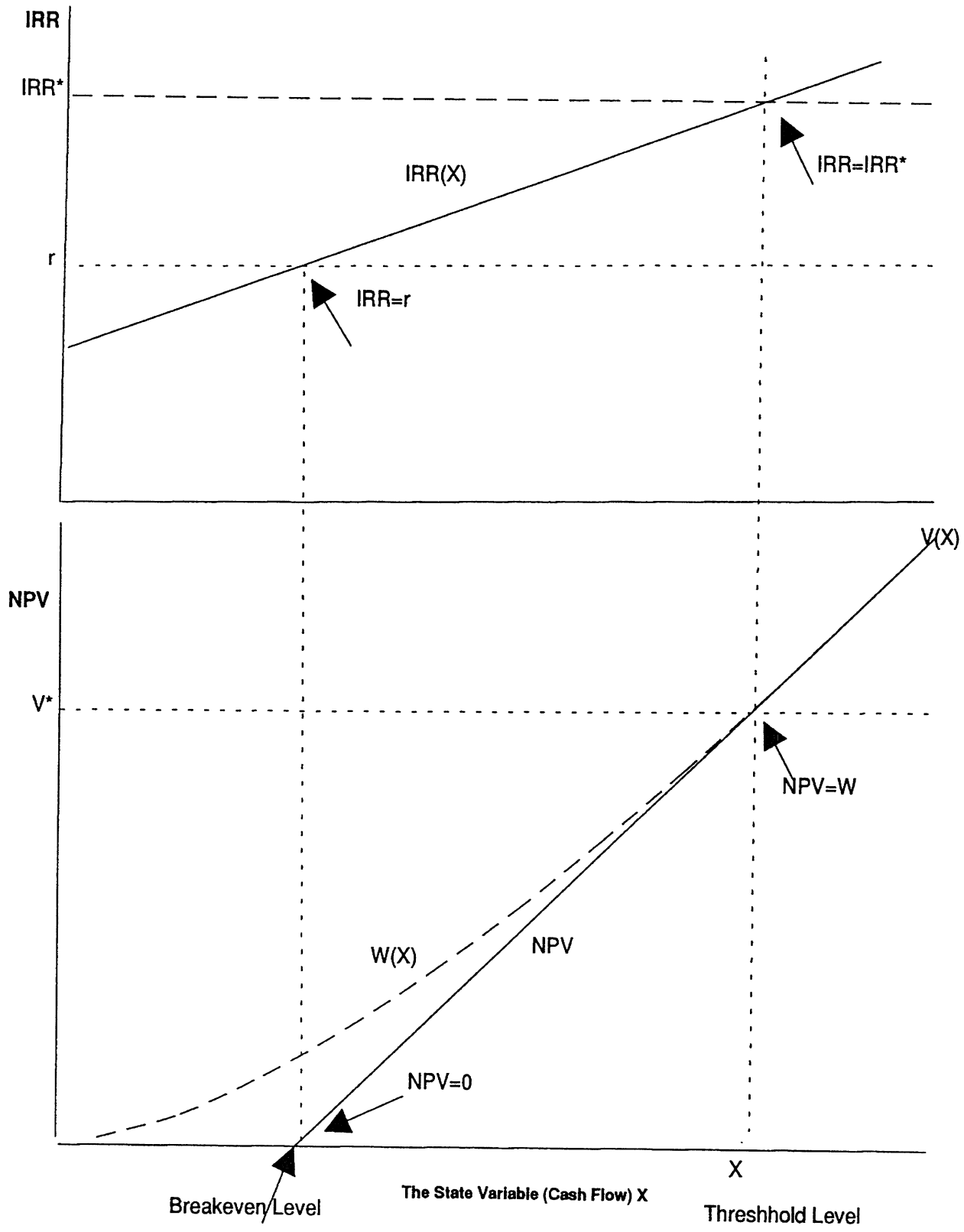


Figure 1. Optimal NPV and IRR Rules.
V(X) is the NPV. *W(X)* is the option vlaue to invest. *r* is the interest rate.