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MATHEMATICAL MODELS OF GROUP CHOICE AND NEGOTIATIONS

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1. Introduction

In the last decade increasing numbers of researchers in marketing have recognized the importance of group choice and negotiations in many kinds of consumer behavior, organizational purchasing, management decision-making, and marketing channel relationships. Families purchasing durables, buying committees selecting suppliers, and salespeople negotiating terms are the examples that come first to mind. However, intra- and inter-firm management teams are increasing in importance and broaden the relevance of group choice and negotiations to a wider variety of management tasks. For example, CalCom Inc. now uses integrated product development teams that include all of the key disciplines to cut development time (Sales & Marketing Management, March 1991). Procter & Gamble, Kraft General Foods, Quaker Oats, and Johnson & Johnson are just a few of the manufacturers who now assign sales teams to work closely with retailer teams on merchandising (New York Times, July 14, 1991). Digital Equipment Corp. and Ramtek have recently begun to engage in team selling to prospects who indicate an interest in the products of both manufacturers (Sales & Marketing Management, May 1990). The functioning of these groups requires joint decision-making and negotiations on issues important to all parties.

Despite the prevalence and importance of group decision making and negotiations in marketing, relatively few researchers are actively exploring this subject and even fewer concentrate on modeling group choice and negotiations. In this chapter, our focus is on models that attempt to describe the outcomes of actual group decisions, including negotiations. (We also review normative models as important background and foundation for descriptive modeling.) Descriptive modeling provides insight into the group decision-making process and the ability to predict the outcomes of those processes, which can be used to guide the actions of decision-makers and help them anticipate each other's actions.

The work that has appeared in marketing has drawn on research from a large number of areas outside marketing including social welfare theory, decision theory, game theory, labor economics, small group processes, social decision schemes, and work on power and influence in a variety of contexts.

Few, however, make use of more than one or two research streams in other disciplines. The underlying similarity of the problems being addressed in the many approaches to the study of group choice leads us to suspect that more advantage could be taken of work that has been done in other disciplines, in the development of group choice and bargaining models in marketing. Greater awareness of this work could begin this process and is the goal of the first major section of this paper. One important way in which this body of research can be used is to develop model classifications schemes or typologies to organize our knowledge and serve as guides to the modeling of group choice and negotiations in various contexts. In this paper we also begin this task.

The paper is organized as follows. In §2 we establish the scope of this paper. The goal of the first major section, §3, is to review the development and testing of mathematical models of group choice and bargaining in marketing and several other relevant disciplines. This review is designed to indicate the progress that has been made in the mathematical modeling of group choice and bargaining and to highlight convergence and inconsistencies in findings across studies. Specifically, §3.1 reviews normative models, §3.2 reviews models from social psychology with emphasis on social decision schemes, and §3.3 reviews modeling in marketing applied to families, organizations, other groups of consumers, and buyers and sellers. In §4 we suggest some dimensions that should prove useful in developing model classification schemes or typologies. The goal of the final section, §5, is to demonstrate how specific situations can be described in terms of these classifying dimensions and how classification can serve as a guide to modeling. We provide three examples of areas in which modeling decisions as group choices is important for the accurate representation of the decisions involved and suggest how this modeling might proceed in light of the how each problem is positioned on important dimensions.

2. Focus and Definitions

The focus of this chapter is on descriptive mathematical models of choice or decision-making in groups. (We treat choice and decision-making as synonyms.) Although many conceptual models of group decision-making have been proposed and many hypotheses concerning group processes have been tested which have implications for the development of mathematical models, we concentrate on the mathematical models themselves.

We define a group as a collection of people who are united by bonds among members' interests and goals that are sufficiently strong to overcome disunifying influences (Deutsch 1973, p.49). These bonds may be complex and enduring, as they are in families, or they may be simpler and more transient, as in many business relationships. Note that our view of groups and group decisions encompasses negotiators and bargaining tasks. Although the contexts from which bargaining models have grown and those to which they are applied tend to differ from the origins and uses of group choice models, the similarities between the underlying problems are greater than the differences. We view negotiations as a subset of group decision-making, typically involving two parties and an allocation of resources.

Finally, we focus on decision-making as opposed to problem-solving. Laughlin (1980) refers to these as judgmental and intellective tasks, respectively, and suggests that a particular task may thought of as lying on a continuum anchored by these extremes. Because this is a continuum, the distinction between decision-making and problem-solving is not always clear, but generally problem-solving emphasizes the construction of response alternatives that are demonstrably correct, while decision-making is performed when there is no objectively correct answer and emphasizes the selection of an alternative based on member preferences or opinions.

3. Models of Group Choice

We begin by reviewing normative models of group choice and negotiations. Then we discuss modeling in social psychology, primarily that motivated by social decision scheme theory. Finally, we review work in marketing that has used group choice models in the contexts of family consumption, organizational purchasing, various other groups, and buyer-seller negotiations.

3.1 Normative Theories of Group Choice

In this section we review models of group choice based primarily on social welfare theory, decision theory, and game theory[1]. Although our primary interest is in models that explain or describe behavior, normative theories of group choice form an important foundation on which many descriptive models have been and could be built. Also, by making explicit the assumptions needed to support various functional forms, the normative models provide directions regarding the appropriate formulation and the information requirements for a given situation.

The normative approach to group choice deals with the specification of decision rules that map the preferences of individual group members into a collective choice, preference order, or utility function. There are, of course, many possible mappings. Normative research in group decision theory examines how *reasonable* these decision rules are. Some of the concerns in making a normative evaluation are how *equitable* a decision rule is in its distribution of the benefits or losses resulting from the group's actions, and the *efficiency* of the group's choice. For example, suppose option *A* provides equal expected benefits to all members, and option *B* does not. Also, suppose option *B* is preferred to option *A* by each individual in the group. How does the decision rule resolve the tradeoff between *efficiency* and *equity*? Further, how are the interests of the various group members weighed one against the other? Are the preferences of all members equally important in determining the group's choice? If not, is there a normative basis for determining the relative weights given to the various individuals? As can be seen even from this brief discussion, the choice of an appropriate group decision rule is non-trivial. Consequently, much of the normative research in group decision making

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[1] Due to space considerations we do not review coalition theory. See Murnighan (1978) and Shubik (1982) for reviews of the social psychological, political science and game theory approaches.

is aimed at *(i)* specifying some primitives that ought to be satisfied by any group decision rule, and *(ii)* examining the efficiency and equity characteristics of rules derived on the basis of these primitives.

The foundation for much of this research was laid by the fundamental results of Condorcet, reported in Arrow (1950) and Guillbaud (1966). Condorcet argued that the only reasonable way to choose one from among several feasible alternatives is to pick the item that is at least as preferred as any other by at least half of the group members. In a democratic society, basing collective decisions on the will of the majority has obvious efficiency and equity appeal. Attractive as this decision rule is, Condorcet also showed that in some situations a majority winner need not exist. For example, consider the following preference orders for a group of three individuals:

$$a \succ b \succ c$$
$$b \succ c \succ a$$
$$c \succ a \succ b$$

Applying Condorcet's principle, the group should prefer a to b, b to c, and c to a, leading to an intransitivity. There is no Condorcet winner. Arrow (1951) showed that this paradox of collective choice is impossible to eliminate.[2] His "General Possibility Theorem" showed that there is no procedure for obtaining a group ordering (ranking) of the various alternatives, based on individual members' rankings, that satisfies five reasonable and seemingly innocuous assumptions. Yet, groups must make defensible choices. The focus of much subsequent research has centered on specifying and examining various rules by which groups can make collective choices based on the wishes of their members.

Much of the work in social welfare theory has been aimed at examining ways in which one or more of Arrow's assumptions can be changed to avoid the impossibility result. For example, Sen (1982) examined the possibility of arriving at a group choice (the single best outcome) rather than an

[2]Subsequently, Gibbard (1977, 1973) and Satterthwaite (1975) have shown that any decision rule which satisfies Arrow's axioms is *manipulable* in the sense that it can be to the advantage of some group members to behave strategically (by controlling the agenda, or misrepresenting preferences). Manipulability is generally considered undesirable because it implies that some group members (those who can control the agenda) can exert unacceptable influence on the collective choice.

entire ordering. Black (1958) suggested a restriction on the individual preferences (single peakedness) which, if satisfied, would lead to a group ordering. Others have examined the ways in which commonly used choice procedures (*e.g.,* plurality, Borda sum-of-ranks, and approval voting) satisfy and violate Arrow's requirements. (See Fishburn 1986; Gupta and Kohli 1990; Plott 1976; Sen 1977 for reviews). Beginning with Kemeny and Snell (1962), several authors have proposed computational methods for deriving a *group consensus ranking* based on the individual preference orders. Recognizing that a transitive majority order may often not exist, these models attempt to find that ranking of the alternatives which minimizes the sum of its "distance" from the individual rank orders. (See Ali, Cook and Kress 1986; Cook and Kress 1985; Cook and Seiford 1978; Blin and Whinston 1974; Bowman and Colantoni 1973.) Finally, the decision theoretic approach, discussed next, shows that the impossibility can indeed be avoided, but the information requirements on individual preferences must be increased. These models start with cardinal utility functions (instead of Arrow's rank orders), and require interpersonal comparability.

Decision-Theoretic Aggregation Rules.[3] The decision theoretic approach is distinguished by its reliance on cardinal measures of individual preferences and the specification of rationality postulates for the group's behavior to derive preference aggregation rules. Individual preferences are assumed to be either risky utility functions (usually assessed through lottery procedures), or measurable value functions (usually assessed through conjoint measurement). We shall use $u_i(x)$ ($v_i(x)$) to denote the *i-th* individual's utility (value) for alternative x (for the need to distinguish between utility and value functions, see Dyer and Sarin 1982, Sarin 1982), and $G_u(x)$ ($G_v(x)$) to denote the corresponding group preference rules.

Group utilities under risk: Beginning with Harsanyi (1955), the basic concern of this stream of research has been to examine the implications of requiring the group's actions to be governed by postulates of rationality similar to those of expected utility theory for individuals. For example, starting

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[3]This discussion is based largely on Dyer and Sarin (1979), Keeney and Kirkwood (1975), and Keeney and Raiffa (1976).

with risky individual utility functions, Harsanyi (1955) showed that if the group is to behave in a Bayesian rational manner then G_u, the group utility function, must be additive.

$$G_u(x) = \sum_{i=1}^{n} k_i u_i(x) \qquad (1)$$

where $u_i(x)$ is the i-th individual's utility for the consequence, x, of some alternative (u_i's scaled from zero to 1), and the k_i's are constants which reflect the relative weight accorded to the i-th individual's preferences.

We can now introduce two important issues regarding group utility functions. First, use of group utility functions requires not only cardinal individual utilities, but also the exogenous specification of k_i's. This implies the need to make interpersonal comparisons among the members. More specifically, the individual utilities must be *cardinally comparable*. That is, if we apply any permissible affine transformation to the i-th individual's utility function, we must also apply the *same* transformation to every other individual utility function, or adjust the values of k_i's. Thus, the price paid for satisfying Arrow's assumptions has been an increase in the information requirements of individual utilities (interpersonal comparisons are not required in Arrow's formulation). Further, making such comparisons is especially difficult for risky utilities because such functions do not reveal strength of preference (Dyer and Sarin 1982; Sarin 1982). Scaling and weighting individual utility functions can be particularly troublesome in the absence of norms or clear power structures within the group.

The second issue is that of the distribution of benefits among group members. It is best illustrated with the following two-person example from Keeney and Raiffa (1976). Consider the following three alternatives:

Alternative *A*: $u_1 = 1$ and $u_2 = 0$ for certain

Alternative *B*: There is a 50-50 chance of either $u_1 = 0$ and $u_2 = 1$, or $u_1 = 1$ and $u_2 = 0$

Alternative *C*: There is a 50-50 chance of either $u_1 = 1$ and $u_2 = 1$, or $u_1 = 0$ and $u_2 = 0$

Using the additive function *(1)*, the group must be indifferent among the three alternatives, although this may not be considered equitable. With alternative *B* one of the two will be envious of the other. With alternative *C*, either both get their best outcome, or both get the worst outcome and may, therefore, be considered more equitable. The additive function only considers the sum of utilities, not its distribution.

An alternative to the additive function, relying on a weaker set of assumptions than Harsanyi's (see Keeney 1976; Keeney and Kirkwood 1975; Keeney and Raiffa, ch. 10, 1976), is given by:

$$G_u(x) = \sum_{i=1}^{n} k_i u_i(x) + k \sum_{\substack{i=1;\\ j>i}}^{n} k_i k_j u_i(x) u_j(x) + \cdots + k^{n-1} k_1 k_2 \cdots k_n u_1(x) u_2(x) \cdots u_n(x) \tag{2}$$

where $0 < k_i < 1$, $k > -1$, and $\Sigma k_i \neq 1$. In this formulation, *k* provides information about the group's attitude toward risk, and k_i denotes the importance assigned to the preferences of the *i*-th individual. To see the effect of *k*, let us return to the three alternatives example. Suppose $k_1 = k_2 = 0.4$, $k = 1.25$. Then, $G_u(x) = 0.4\ u_1(x) + 0.4\ u_2(x) + 0.2\ u_1(x)u_2(x)$. As the group's utilities are specified for consequences of alternatives, the group's expected utility for some *alternative,* say *B*, is calculated as follows. $G_u(B) = 0.5[(0.4)(1) + (0.4)(0) + (0.2)(1)(0)] + 0.5[0.4(0) + 0.4(1) + 0.2(0)(1)] = 0.4$. This should be contrasted with $0.4(Eu_1(B) = 0.5) + 0.4(Eu_2(B) = 0.5) + 0.2(Eu_1(B) \cdot Eu_2(B) = 0.25) = 0.45$, where $Eu_i(B)$ is the *i*-th individual's expected utility from *B*. In the latter calculation individuals (not the group) assess the expected utility. Thus, it fails to model the group's behavior under uncertainty. This distinction will be important in discussing the cooperative game-theoretic models of bargaining.

So, the group's utilities for alternatives *A, B,* and *C* are 0.4, 0.4, and 0.5, respectively. The group prefers to take the chance that both people will get the worst outcome (alternative *C*) rather than assuring the best outcome to one of its members (alternative *B*). This preference for alternative *C* has often been interpreted as reflecting the group's concern for posterior equity, represented by the multiplicative term in *(2)*. But there is a problem with this interpretation (Dyer and Sarin 1979). Because risky utility functions do not reflect strength of preferences, it is difficult to say whether the preference for alternative *C* reflects equity proneness or risk proneness. Dyer and Sarin (1979) suggest that the value of *k* in *(2)* is better interpreted as a representation of the group's attitude towards risk.

When $k=0$, *(2)* reduces to the simple additive form proposed by Harsanyi (1955), *(1)*, and the group's risk attitude is completely determined by the risk attitudes of its members. When $k<0$ ($k>0$), the group will be more (less) risk averse than its members. A substantial body of experimental literature has noted "shifts" in the group's risk preferences. That is, the group's preferences are often observed to be more, or less, risk averse than those of the average member. Thus, the normative model *(2)* has considerable descriptive validity.

The ability to reflect "shifts" in a group's risk attitude comes at a price. It is no longer possible to guarantee that the chosen outcome is Pareto-optimal (*i.e.*, there is no other alternative that is preferred to the chosen one by all group members). To see this, suppose there is a fourth alternative, D, with $u_1=u_2=0.48$ for certain. Note that both individuals prefer B ($Eu_1=Eu_2=0.5$) to D. Using *(2)*, the group's expected utility for alternative D is 0.43, which is greater than the group's 0.4 expected utility for alternative B. Thus, the group's choice is not Pareto-optimal. Kirkwood (1979) has shown that if $k>0$, this possibility is inescapable.

Group values under certainty: In the preceding section we saw that the multiplicative part of *(2)* could not properly be interpreted as a reflection of the group's concern for equity. Yet, the ability to model such concerns is both theoretically and empirically important. Dyer and Sarin (1979) have postulated several assumptions, based on measurable value functions, to obtain the following result.

$$G_v(x) = \sum_{i=1}^{n}\lambda_i v_i(x) + \lambda\sum_{\substack{i=1;\\ j>i}}^{n}\lambda_i\lambda_j v_i(x)\, v_j(x) + \cdots + \lambda^{n-1}\lambda_1\lambda_2\cdots\lambda_n v_1(x)\, v_2(x)\cdots v_n(x) \qquad (3)$$

where $0 < \lambda_i < 1$, $\lambda > -1$, and $\Sigma\lambda_i \neq 1$. Note the similarity in the basic forms of equations *(2)* and *(3)*. In both cases, individual preferences are measured on a cardinal scale, and the group preference function requires cardinal comparability. But because the preferences are expressed for riskless alternatives, the value functions in *(3)* reflect strength of preference. Consequently, the important difference lies in the interpretation of λ. When $\lambda=0$, *(3)* reduces to a simple additive form and the group is inequity neutral. Any distribution of v_i's that results in the same sum of values is equally preferred. When $\lambda>0$ the group is inequity averse, preferring a balanced distribution of individual values. Finally, when $\lambda < 0$, the group is inequity prone, preferring variations in individual values. To see this consider the following two-alternative, two-person example:

Alternative *A*: $v_1 = 0.5$ and $v_2 = 0.5$

Alternative *B*: $v_1 = 1$ and $v_2 = 0$

If $\lambda = 0$, $\lambda_1 = \lambda_2 = 0.5$, then $G_v(x) = 0.5\ v_1(x) + 0.5\ v_2(x)$, and the group is indifferent between the two alternatives. If $\lambda = 1.25$ and $\lambda_1 = \lambda_2 = 0.4$, then the equal distribution of alternative *A* is preferred. Finally, if λ is negative, the unequal distribution of alternative *B* would be most preferred. Thus, this formulation permits us to model the group's attitude towards equity. But, we are restricted to riskless alternatives.

Together, equations *(2)* and *(3)* provide different preference-based approaches to modeling the experimentally observed phenomena of group polarization, e.g., and concern for equity. However, neither formulation alone can deal adequately with both. Clearly, shifts in risk attitudes and concerns for equity can arise in the same decision making situation. One obvious possibility would be to separately assess individual values and utilities, and to calibrate both of these models. Then, by comparing the values of λ and k, shifts in a group's preferences from a simple average could be attributed to the appropriate phenomenon. For example, if $\lambda > k = 0$, shifts in the group's preferences could be attributed to concerns for equity alone. Of course, the cost of this added ability to separate the phenomena is the need for added information about individual preferences.

Eliashberg and Winkler (1981) have proposed an alternative model that can simultaneously incorporate individual and group attitudes toward risk and concern for equity. In their model, each individual's preferences are expressed as cardinal utilities for entire vectors of payoffs to every group member. (By contrast, in equations *(2)* and *(3)*, the individual utility functions express only that person's own preference for a consequence). Thus, the utility functions represent "an individual's preferences concerning equitable and inequitable distributions" as well as attitudes toward risk. The individual utility functions are then combined (using linear or multilinear aggregation functions) to yield a group utility function. The analysis shows that large groups of risk averse members might be expected to be approximately risk neutral (a risky shift).

Bargaining[4]: Game-Theory. Two principal traditions characterize the game theoretic analysis of bargaining. Nash's (1950) bargaining problem is the basic framework adopted by cooperative game theorists for the study of two-person bargaining situations. The approach adopted in this literature is to codify as axioms certain basic negotiation principles and to derive a unique final outcome. The process by which the final outcome is negotiated is not modeled. The noncooperative game theoretic tradition (Osborne and Rubinstein 1990; Rubinstein 1982; Sutton 1986), by contrast, attempts to model the bargaining process with detailed rules of play and then determine the optimal behavior of the bargainers, as well as the final outcome. The principal models and results of each tradition are discussed next.

Cooperative game theory: In the framework of Nash's bargaining problem, payoffs from each feasible outcome are represented in a two-dimensional space where each axis represents the cardinal utility to one of the parties. Each point (b_1, b_2) in the feasible region (B) represents the utilities that would accrue simultaneously to the two bargainers from some resolution of the conflict. The feasible region is bounded by the *Pareto boundary*, i.e., the set of outcomes that is not dominated by any other feasible outcome. An important assumption made in this approach is that there is a *threat* or *conflict* point, $c=(c_1, c_2)$, that explicates the utilities for each bargainer in the event of no agreement. Given the bargaining problem (B,c), and without regard to the actual physical outcomes involved, cooperative game theorists specify axioms that lead to one of the points on the Pareto boundary as the unique, logical solution concept, $f(B,c)$.

Before proceeding with descriptions of the various solution concepts it is useful to point out the differences between the decision theoretic models of the previous section and the bargaining problem

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[4]Closely related to bargaining is the topic of arbitration, where a third party resolves the conflict between two principals. For a model of arbitration that reflects both the game theoretic and the decision theoretic approaches, see Eliashberg (1986).

discussed here. Clearly, in both cases individual cardinal utilities determine the outcome of a group process. However, there are important distinctions. First, the primary emphasis of the decision theoretic models is to specify how a group should behave so that its actions are consistent with some postulates of rationality. The axiomatic models of bargaining, by contrast, are more concerned with the allocation of payoffs among the individuals and the rationality postulates are imposed on the individuals, not the dyad. Consequently, the decision theoretic approach for risky alternatives seeks to determine the expected value of the group utility function. The bargaining approach, by contrast, seeks a solution *after* each individual's expected utility has been determined for each possible alternative. Second, the conflict point plays a prominent role in bargaining models. Either negotiator has the opportunity to break off negotiations and receive the conflict payoff. Further, the focus of the models, typically, is on the gains of each individual (relative to the conflict payoff). Such threat points are not of immediate relevance to a theory which seeks to model how a group should behave as a single entity in a rational manner. Consequently, no such outcome is singled out in the group decision models. An important consequence of modeling conflict points, together with a transformation invariance assumption (see *A1* below), is that the game-theoretic models typically do not have to rely on interpersonally comparable utility functions. In fact, as discussed below, we can examine directly the impact of different degrees of interpersonal comparability on the resulting solution concept. A final distinction is that some of the game-theoretic solutions are limited to two-person groups. Extensions to *n*-person games can lead to indeterminacy in the final outcome.

The most prominent of the solution concepts in cooperative game theory is the **Nash Solution** (Nash 1950, 1953). Nash postulated four properties (axioms) that it would be reasonable for a bargaining solution to satisfy:

A1: *Invariance under affine transformations*: If a bargaining problem (B', c') is obtained from (B, c) by the transformation $b_i \rightarrow \alpha_i b_i + \beta_i$, where $\alpha_i > 0$, for $i = 1,2$. Then, $f_i(B', c') = \alpha_i f_i(B, c) + \beta_i$ for $i=1,2$.

Because cardinal utility functions are unique only up to affine transformations, this axiom requires that the particular (arbitrary) scaling chosen ought not to change the outcome of the game (only the utility numbers associated with the outcomes may be different). Because separate transformations may be applied to the utilities of the two negotiators, this condition implies that interpersonal comparisons are neither required, nor even permitted.

A2: *Symmetry:* If the bargaining game (B, c) is symmetric, *i.e.*, $c_1 = c_2$, and (b_1, b_2) in B implies (b_2, b_1) in B, then $f_1(B, c) = f_2(B, c)$.

This axiom states that only the utilities associated with the feasible outcomes and the conflict point determine the relative payoffs to the negotiators. No other information is needed or permitted to determine the outcome.

A3: *Independence of Irrelevant Alternatives:* If (B, c) and (D, c) are bargaining problems with $B \subseteq D$ and $f(D, c) \in B$, then $f(B, c) = f(D, c)$.

That is, suppose that the outcome agreed upon by the two negotiators, with all the alternatives of T available to choose from, also belongs to the smaller set B. Then, if the same negotiators (endowed with the same utility functions, and having the same conflict payoffs) were to negotiate over the smaller set B, they ought still to agree on the same outcome as before, T. The unavailable alternatives of D should be regarded as irrelevant. This has proven to be one of the most problematic of the axioms.

A4: *Pareto Optimality:* Suppose (B, c) is a bargaining problem, $b \in B$, $d \in B$, and $d_i > b_i$ for $i = 1,2$. Then $f(B,c) \neq b$.

This implies that the negotiators never agree on an outcome b whenever another outcome d, which is better for both, is available. It also implies that some agreement should always be reached, because c is not Pareto optimal. It is worth noting that Pareto optimality is required in bargaining models, whereas it can not be guaranteed in any group utility function that is not additive. The difference, once again, arises from Nash's requirement of a weaker form of collective rationality than that demanded by the decision theoretic models of group choice. In fact, Roth (1979) showed that if *A4* is replaced with individual rationality *(f(B,c) ≥ c)* only the Nash solution, or the conflict point itself can satisfy the axioms. Further, Roth (1977) showed that replacing this axiom with *strong* individual rationality, *(f(B, c) > c)*, results uniquely in the Nash solution.

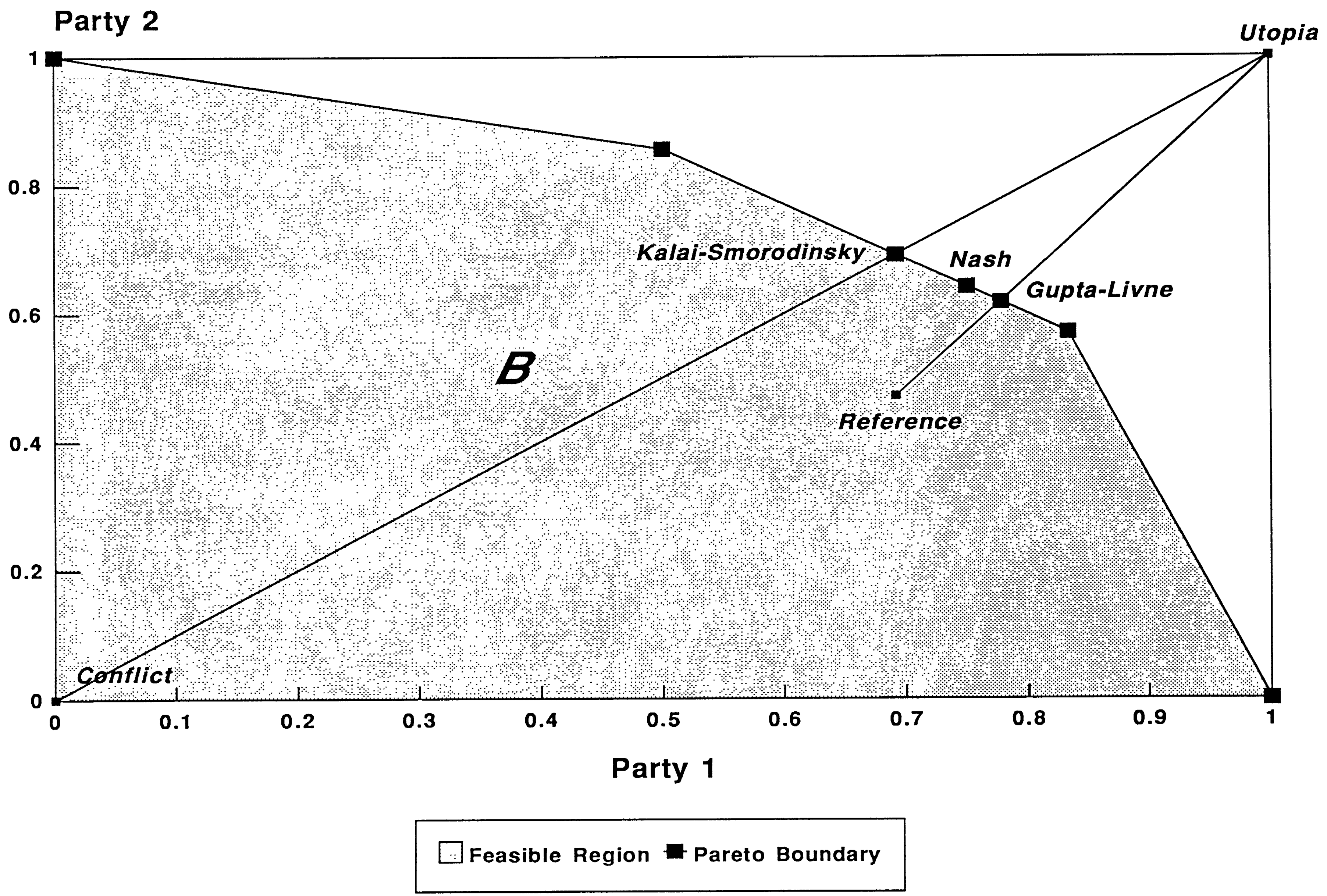
Nash (1950) showed that the four axioms **A1-A4** are *uniquely* satisfied by the point on the Pareto boundary that maximizes the product of the gains, over the conflict point, to each of the parties, *i.e.* $n(B,c)=x$ such that $x \geq c$ and $(x_1 - c_1)(x_2 - c_2) \geq (b_1 - c_1)(b_2 - c_2)$ for all b in B, $y \geq c$ and $b \neq x$. (See Figure 1.)

Insert Figure about here.

The multiplicative form of the Nash solution is particularly noteworthy. When the individual utilities are assessed on riskless alternatives (value functions), the Nash solution may be seen as emphasizing the concern for equity we noted in the multiplicative group value function *(3)*. For any given sum of gains over conflict payoffs, the product of value gains will be maximized for more equal individual gains. Thus, one interpretation of the Nash solution is that if each negotiator agrees that the axioms are reasonable, then in the terminology of MacCrimmon and Messick (1976), each is socially motivated by *proportionate cooperation.* That is, each negotiator wants to increase the joint gains, but only to the extent that their own share in it also increases. Consequently, if two negotiators, endowed with their personal preferences and the social motive of proportionate cooperation were faced with the bargaining problem *(B,c)*, they would be expected to agree upon the Nash solution *n(B,c)*. Finally, we note that the Nash solution can be extended to *n*-person groups (Kaneko and Nakamura 1979). Note, however, that even if only one individual prefers his or her status-quo to an alternative that is everyone else's first choice, the latter alternative will not be the group's choice. Effectively, each individual has the power to veto the desires of all others (there is no such veto power in the decision-theoretic models).

Perhaps the most significant impact of Nash's seminal work has been in demonstrating that the seemingly indeterminate bargaining problem could indeed be structured and unique outcomes could determined on the basis of simple, reasonable assumptions. Consequently, much of the subsequent

Figure 1
Some Solution Concepts



game-theoretic research in bargaining has dealt with examining the effects of changing the axioms. In addition to a desire for fuller understanding of the problem, some of this research has also been motivated by a wish to represent the results of actual negotiations more accurately.

Kalai and Smorodinsky (1975) motivated their solution concept with the example shown in Figure 2. In the figure, note that compared to the payoffs in the smaller game *(B,c)* (defined by the convex hull of (0,0), (0,1), (3/4, 3/4), and (1,0)), player 2 always gets a larger payoff in the game *(D,c)* (defined by the convex hull of (0,0), (1,0.7) and (1,1)). Yet the Nash solution payoff $n_2(D,c)=0.7$ is smaller than $n_2(B,c)=3/4$. That is, player 2's payoff actually decreases even though the possibilities have improved. To resolve this, Kalai and Smorodinsky suggested replacing the independence to irrelevant alternatives axiom *(A3)* with:

A5: *Individual Monotonicity:* If the set B of feasible outcomes is enlarged to the set D in such a way that, for every payoff to player 1, the range of feasible payoffs to player 2 is increased, then player 2's final payoff in the enlarged game, $f_2(D,c)$, should be at least as large as the payoff $f_2(B,c)$ in the smaller game.

They then proved that this monotonicity requirement, along with Nash's other axioms, provides a unique solution $ks(B,c)=x$ such that $x \in B$, $(x_1-c_1)/(x_2-c_2)=(U_1-c_1)/(U_2-c_2)$ and $x \geq b$ for all b in B such that $(b_1-c_1)/(b_2-c_2)=(U_1-c_1)/(U_2-c_2)$. U_i is the highest utility provided by any feasible outcome to the i-th player. The *ks* solution lies at the intersection of the Pareto boundary with the line joining the conflict point to the "utopia point" (U_1, U_2). In figure 2, the *ks* solution is (10/3, 10/3).

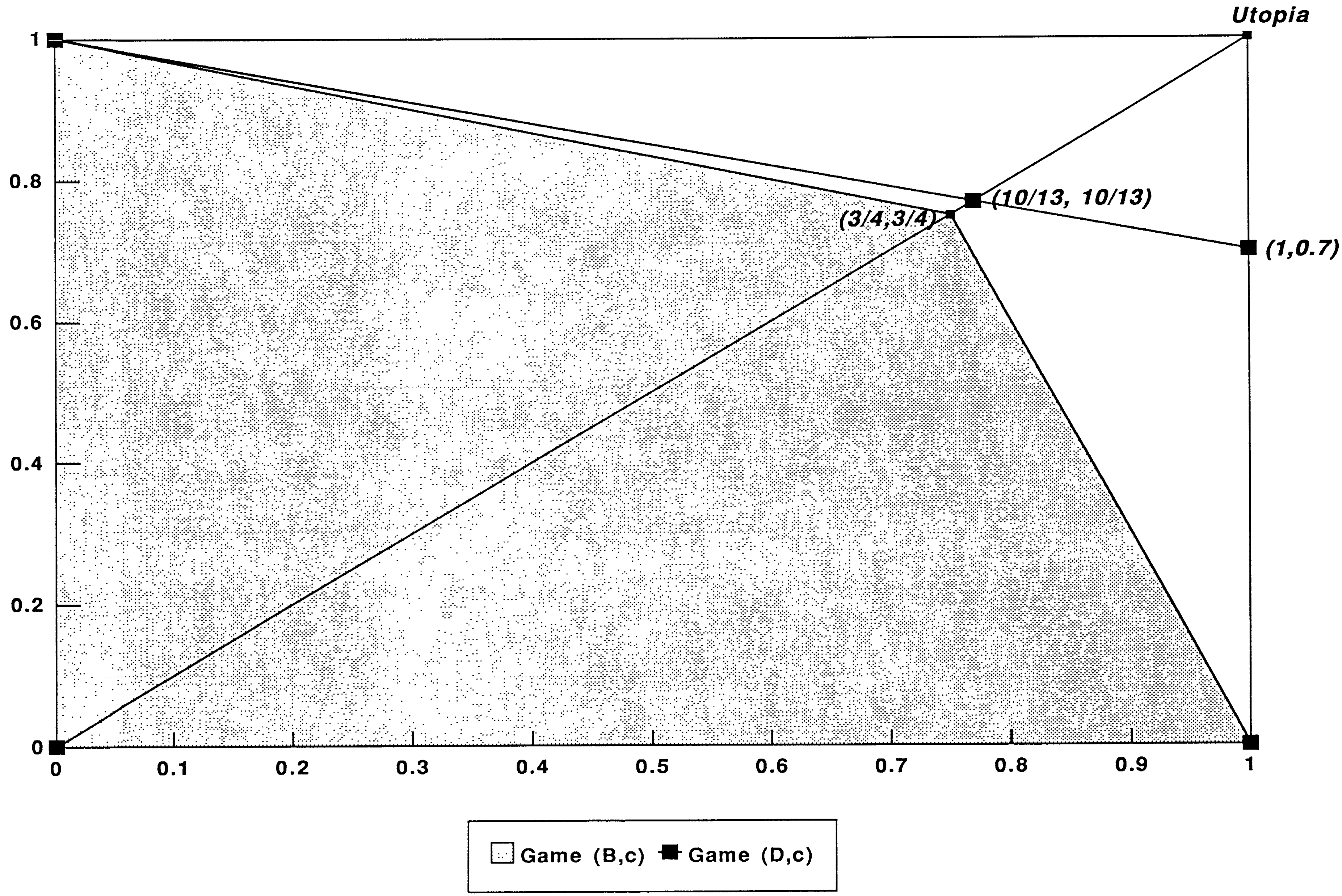
The *ks* solution can also be expressed in the following manner

$$\text{maximize} \qquad k_1 b_1 + k_2 b_2$$

$$\text{such that} \qquad \frac{k_1}{k_2} = \frac{U_1 - c_1}{U_2 - c_2}$$

We can now examine the relation between the *ks* solution and the decision theoretic models. As in *(3)* with $\lambda=0$, the *ks* solution attempts to maximize the joint payoffs. But, instead of expressing the concern for equity with a multiplicative term, the desirability of an outcome is determined by the

Figure 2
Individual Monotonicity



ratios in the constraint. If $U_i - c_i$ is interpreted as the i-th player's "need" (Brock 1980), then the scaling constants k_i can be interpreted as providing more weight to the needier player. Notice that here the weights are determined endogenously as a ratio of differences in cardinal utilities, rather than exogenously as they are in the decision-theoretic models. Because ratios of differences are invariant under affine transformations, interpersonal comparisons of utilities are not needed. Finally, the constraint shows that the *ks* solution corresponds with MacCrimmon and Messick's (1976) social motive of proportionate equalitarianism; in a symmetric game, choices which decrease the ratio between the larger payoff to the smaller payoff are preferred.

In the bargaining solutions considered so far interpersonal comparisons have not been permitted. We now consider a solution, due to Roth (1979), which relies on ordinal interpersonal comparisons. That is, statements such as, "You gain more than me if *A* is chosen," are permissible. Specifically, Roth proposed the following axiom:

A5: *Independence of Ordinal Transformations Preserving Interpersonal Comparisons:* If B′ is derived from *B* via a transformation $t=(t_1, t_2)$ (of feasible payoff vectors in *B* to feasible payoff vectors in *B′*) which: *(i)* preserves each player's ordinal preferences, and *(ii)* preserves information about which player makes the larger gain at any given outcome, then $f_i(B',c') = t_i(f(B,c))$.

Note that a transformation *t* which preserves interpersonal comparisons divides the payoff space into the three regions $\{b|b_1 - c_1 = b_2 - c_2\}$, $\{b|b_1 - c_1 > b_2 - c_2\}$, and $\{b|b_1 - c_1 < b_2 - c_2\}$ (see figure 3), and it ensures that each point (after transformation) stays in its original region. Roth showed that $r(B,c) = x$ such that $\min\{x_1 - c_1, x_2 - c_2\} > \min\{b_1 - c_1, b_2 - c_2\}$ for all y in the strong Pareto set of *B* and $y \neq x$, is the unique solution which satisfies strong individual rationality, strong Pareto optimality, independence of irrelevant alternatives, and independence of ordinal transformations preserving interpersonal comparisons. That is, the solution *r* picks the strongly Pareto optimal point which comes closest to giving the players equal gains. And this is true even in a game with an asymmetric payoff space. (See figure 3). The function *r* can also be described as a lexical maximin function which seeks to maximize the minimum gains available to either player. This interpretation provides a useful link to

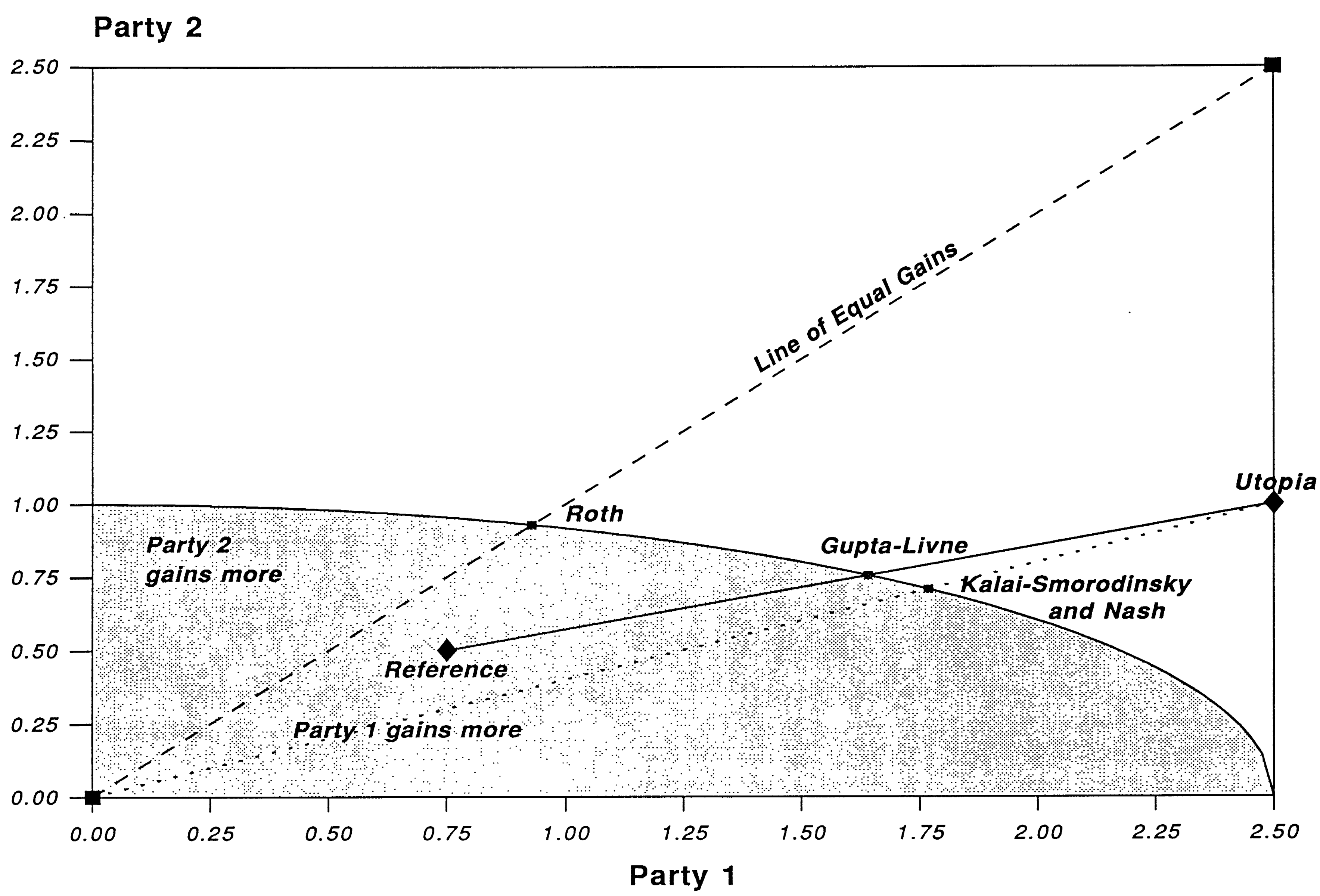
Rawls' theory of justice. Rawls (1971) argued that such a function is fair because of its concern for the least advantaged person. Although in being solicitous of the disadvantaged, the preferences of a majority can be overridden by those of a small minority. MacCrimmon and Messick (1976) suggest that lexical maximin is a conditional social motive such that one is altruistic when ahead and self-interested when behind. It involves a desire to improve the position of the person who is behind but does not necessarily force the person who is ahead to make a sacrifice. Finally, this solution concept corresponds well with experimental findings that suggest that bargainers do make interpersonal comparisons and outcomes tend toward equality.

Insert Figure about here.

The solution concepts described above have the following property in common. When the payoff space is symmetric and the conflict points equal, the final outcome will also be equal. We now discuss two additional solution concepts which need not satisfy this property. Roth (1979) introduced the **Asymmetric Nash Solution**. He showed that if the symmetry requirement is replaced by the requirement that bargainers receive positive (but not necessarily equal) increments of utility in a symmetric game, the unique resulting solution maximizes $\prod_{i=1}^{n} U_i^{\gamma_i}$, where U_i is the *i-th* person's utility normalized so that the conflict outcome yields zero utility and the γ_i's are some measure of the asymmetry in the negotiations. Svejnar (1986), derived the same result for a bargaining game in which the asymmetry results from exogenously measured differences in the bargaining power of the union compared to that of management. Neslin and Greenhalgh (1983) estimated the γ_i's in an experimental setting and interpret them as the relative bargaining abilities of the two parties.

The asymmetric solution adds an exogenously determined measure of power as an additional parameter of the bargaining problem. **Gupta and Livne** (1988) proposed an axiomatic model that adds some prominent outcome, termed the "reference" outcome, that can also be expected to have an important bearing on the outcome of negotiations. Raiffa (1982) argued that in integrative bargaining

Figure 3
Roth's Solution Concept



situations the effort should be to find agreements that are better for both parties. Following Raiffa's argument, Gupta and Livne suggested shifting the focus from the conflict outcome to another outcome over which both parties should attempt to improve jointly. The importance of such a reference point, whether it is a commonly agreed upon starting point, an especially credible bargaining position, or simply a focal point (Schelling 1960) in the minds of the negotiators, has been noted in several empirical studies (Ashenfelter and Bloom 1984; Bazerman 1985; Roth 1985a). For example, in multiple issue negotiations the middle-point on each issue often becomes focal (Pruitt 1981; Raiffa 1982) and the bargainers then try, together, to find other agreements that are better for both. The solution derived by Gupta and Livne selects the point that is Pareto optimal and lies on the line connecting the reference and the utopia points. (See Figure 1.) Note the similarity of this function to that of Kalai and Smorodinsky. If the only reference outcome available to the bargainers is the conflict outcome the two solutions will be identical. If some other outcome can be expected to serve as the reference point the solutions may diverge. (This is the case in figure 1.)

Building on Coleman's (1973) and Wilson's (1969) models of logrolling, Gupta (1989) has shown that the reference outcome model is especially appropriate for integrative, multiple-issue negotiations where the two parties do not have equal power. Gupta also shows that any solution which is independent of irrelevant alternatives is particularly inappropriate for multiple issue negotiations.

Our discussion so far raises an important problem in the cooperative theory of bargaining. There is a multiplicity of solution concepts and each is supported by a set of reasonable axioms. (For more complete reviews of this literature, see Kalai 1985 and Roth 1979.) How, then, is one to choose from among them? What would happen if each party favored a *different* solution concept (had a different social motive)? Is it possible that negotiators may fail to reach an agreement because each of them insists that his or her favored solution concept is the appropriate one to use?

Van Damme's (1986) analysis addresses some of these issues and rests on two postulates: *(i)* if a player advocates a particular solution concept in one bargaining situation, then the player should adhere to this solution concept in any other bargaining situation; *(ii)* if the demands of the players are not jointly feasible, the players should continue bargaining over the set of payoffs not exceeding their previous demands. He then showed that if the concession-making process follows these two postulates then the equilibrium strategy for either bargainer is to demand the payoff implied by the Nash solution. Thus, van Damme provides useful insights into an important indeterminacy. However, as van Damme has noted, the optimality of the Nash solution is crucially dependent on the specific concession making process modeled (postulate *ii*). Further, the social psychology literature suggests that the same individual can favor different social motives (solution concepts) in different bargaining situations, violating postulate *(i)*.

Noncooperative Game Theory: The bargaining problem paradigm and the associated axiomatic approach have been criticized for ignoring some important details of the negotiation process. (See, for example, Svejnar 1986.) In addition to being unable to provide any insight into the negotiation process, inability to predict the time of agreement and Pareto-inefficient breakdowns has been identified as an important shortcoming of the static axiomatic approach. Starting with Harsanyi (1956) and especially since Rubinstein (1982), the new approach in the economics literature is to model bargaining as a *noncooperative game* with detailed rules of play[5]. The typical game has multiple Nash equilibria (pairs of strategies that are best responses to each other). But in many cases, the strategies involve making empty threats off the equilibrium path. Consequently, the objective of the analysis is to identify subgame perfect or sequential equilibrium strategies for the players. Within this tradition, a principal distinction lies between models that assume bargainers have complete information and those that allow incomplete information.

The **complete information** game analyzed by Rubinstein (1982) has the following characteristics. Two players seek to divide a cake of size 1. If they agree, each receives her share. If they fail to agree, both receive zero. The bargaining process involves the players alternating offers: player 1

[5]Bartos (1974), Bishop (1964), Coddington (1968) and Cross (1965) have proposed models of the concession-making process. However, the objective of these models is not to identify equilibria. Rather, the effort is aimed at specifying constructs, processes, and parameter values that can reproduce commonly observed concession making patterns.

makes the initial proposal to receive some share x at stage 0. Player 2 immediately accepts or rejects the offer. In the event of a rejection player 2 makes a counteroffer at stage 1. Player 1 immediately responds and so on. The payoff to player 1 (player 2) equals her share of the cake agreed to at stage t, multiplied by δ_t^1 (resp. δ_t^2), where δ_1, $\delta_2 < 1$ represent the discount factors which provide incentive to reach an early agreement. A strategy for either player specifies her proposal/reply at each stage, as a function of the history to that point. Clearly, any division of the cake is a Nash equilibrium (not to be confused with the Nash solution concept). So, a *subgame-perfect* equilibrium is sought. That is, neither bargainer may commit to a contingent course of action that it would not be in her best interest to take if the contingency actually arose, even if the contingency lies off the equilibrium path. For example, a threat by player 1 to walk away if she does not receive 80% of the cake is not credible. This is because, if player 2 does offer only 10%, it is not in player 1's best interest to enforce the threat. Subgame-perfect equilibria rule out such empty threats. Remarkably, Rubinstein showed that the subgame-perfect equilibrium is unique and agreement is immediate with player 1 receiving share $\frac{1-\delta_2}{1-\delta_1\delta_2}$ while player 2 receives share $\frac{\delta_2(1-\delta_1)}{1-\delta_1\delta_2}$. This outcome is intuitively appealing because the more impatient one is (i.e., the greater the discount factor), the smaller the final payoff is. It has also been shown (Binmore, Rubinstein and Wolinsky 1986) that this game is equivalent to one in which there is an exogenous risk that the bargaining process will end without an agreement (e.g., the possibility that a third party will exploit the opportunity before the two negotiators can come to terms). Further, when the length of a single bargaining period or the risk of exogenous breakdown is sufficiently small, or the discount factors approach one, the noncooperative equilibrium approaches the corresponding axiomatic Nash solution. Rubinstein (1982) also showed that if there is fixed, per stage cost of bargaining, c_i, (the *i-th* player's payoff in the t-th stage is $1\text{-}tc_i$), the player with lower costs will receive the entire pie. If $c_1 < c_2$, the agreement is reached at stage 0, if $c_2 < c_1$ the agreement is reached at stage 1, and if $c_1 = c_2$, multiple equilibria arise.

Binmore, Shaked and Sutton (1989) have applied Rubinstein's formulation to examine the precise role of the conflict outcome in the cooperative framework. Suppose initially that the conflict outcome is $(0,0)$ and the resulting cooperative solution is $(n_1(B,0),\ n_2(B,0))$. Now, suppose each bargainer has some *outside option* (x_1 for party 1 and x_2 for party 2): an option that the party can achieve if negotiations are not completed. How should the availability of such options affect the final outcome? The typical response in the cooperative framework suggests that the availability of such options potentially changes the relative *power* of the two parties and this should be reflected by changing the conflict outcome (originally at $0,0$) to $x=(x_1,\ x_2)$. A new solution $b=(b_1,b_2)$ should then be computed such that $b \in B,\ b \geq x$, and $(b_1-x_1)(b_2-x_2) \geq (y_1-x_1)(y_2-x_2)$ for all $y \in B,\ y \geq x$. The noncooperative result, assuming a discount factor approximately equal to 1, suggests that each party should continue to get its payoff according to the original game (n_1,n_2), unless the party's outside option provides a higher payoff. In the latter contingency, the party with the more favorable outside option (say $x_2 > n_2$) should receive the value of the outside option, (x_2), while the other party gets the rest $(1-x_2)$. The basic argument supporting this result is that as long as the outside option is inferior to the solution of the original game $(x_i < n_i)$, the threat to leave the negotiations in favor of the outside option is not credible and should therefore not affect *power* at all. If the outside option of one of the parties is superior, the other party need only offer some epsilon in addition to keep the negotiations going. Thus, the noncooperative analysis helps provide useful insight into the cooperative formulation.

The principal drawbacks of this approach are that there is a first mover advantage even when the game is perfectly symmetric in all other ways and that agreement is immediate. Ochs and Roth (1989) have reported experimental data inconsistent with first mover advantage and Roth, Murnighan and Schoumaker (1988) have reported experimental results showing that many agreements are reached only at the "eleventh hour" (labeled the deadline effect). Further, results on the ability of these models to predict the final outcome are inconclusive. Whereas several studies report results largely in agreement with the equilibrium prediction (Binmore, Shaked and Sutton 1985, 1989; Weg and Zwick 1991) others report systematic deviations in the direction of equal division of payoffs (Güth, Schmittberger and Schwarz 1982; Güth and Tietz 1987; Neelin, Sonnenschein and Spiegel 1988; Ochs and Roth 1989; Zwick, Rapoport and Howard 1991).

In contrast to the complete information models which suggest that all agreements will be efficient and immediate, models of **incomplete information** bargaining suggest the possibilities of inefficient delays and impasse. The typical imperfect information model posits a buyer who owns a single indivisible unit of an object and a potential buyer. The seller's (buyer's) valuation *s(b)* represents the lowest (highest) price for which the seller (buyer) is willing to sell (buy). Typically, each player knows her own valuation, but may be uncertain about the other's valuation. In games of *one-sided uncertainty* one of the valuations, say *s*, is common knowledge. The seller, however, is uncertain about *b*. In games of *two-sided uncertainty* both players are uncertain about the other's valuation. The uncertainty is typically represented by either a continuous distribution (e.g., uniform) or by discrete probabilities on the other's type (e.g., a buyer with a high or low valuation). These distributions or probabilities are common knowledge.

Because games with imperfect information do not have proper subgames, subgame-perfect equilibria are inapplicable. Instead, the attempt is to identify *sequential equilibria*. For a sequential equilibrium, in addition to specifying the offer strategy, each uncertain player's beliefs given every possible history must also be specified. A sequential equilibrium is a set of strategies and beliefs such that for every possible history each player's strategy is optimal given the other's strategy and her current beliefs about the other's valuation, and the beliefs are consistent with Bayes' rules. Typically, weaker players (e.g., buyers with high valuations) attempt to conceal their types by making non-revealing offers and strong players attempt to take actions (such as rejecting offers) aimed at convincing the other player of their strength. Because information can be transmitted credibly only through offers and counteroffers, the process of learning typically requires multiple stages, leading to delays in reaching agreements.

The precise results differ depending on the particular assumptions made about the negotiations (see Table 1, adapted from Srinivasan and Wei 1991). We summarize the principal results:

Table 1: Features of Some Bargaining Models with Incomplete Information
(adapted from Srinivasan and Wei 1991)

Features / Authors	Fudenberg & Tirole		Sobel & Takahashi	Cramton	Rubinstein	Grossman & Perry	Perry	Admati & Perry	Chatterjee & Smauelson	Cramton
Year	83		83	84	85	86	86	87	88	91
Incomplete Information[1]	2	1	1	2	1*	1	2	1	2	1
Distribution of Uncertainty[2]	D	D	C	C	C	C	D	D	D	C
Offer Pattern[3]	U	U	U	U	A	A	A	A	A	A
Time Horizon of Bargaining[4]	2	2	∞	∞	∞	∞	∞	∞	∞	∞
Bargaining Cost[5]	δ	δ	δ	δ	δ	δ	c	δ	δ	δ & c
Strategic Delay	N	N	N	N	N	N	N	Y	N	Y
Unique Equilibrium	N	Y	Y	N	Y	Y	Y	Y	N	N
Delays	2	2	Y	Y	2	Y	N	Y	Y	Y
Pareto Inefficient Breakdown	Y	Y	N	N	N	N	Y	N	N	Y

Notes:

1. 1: One-sided uncertainty
   2: Two-sided uncertainty

2. D: Discrete
   C: Continuous

3. U: Uninformed player makes all offers
   A: Alternating offers

4. 2: Two period game
   ∞: Infinite horizon game

5. δ: Discount factor
   c: Fixed cost per stage

i) Length of delay in reaching an agreement: In games where the two players alternate offers at fixed times (Rubinstein's game), negotiations are expected to require more stages when information is imperfect. However, if the time delay between successive offers is sufficiently short, agreements could be reached almost immediately and the uninformed party loses all her bargaining power (Gul and Sonnenschein 1988; Gul, Sonnenschein and Wilson 1986). Thus, delays would merely occur due to the mechanics of the time required to make offers, not because they serve some strategic purpose. To better account for the significant delays often observed in negotiations, some authors have considered the effects of allowing *strategic delays* in making offers and responses (Admati and Perry 1987; Cramton 1991). That is, players are permitted to respond or make counteroffers *at any time* after some minimum time between offers has passed. This leads to a signalling equilibrium in which the informed bargainer chooses to signal her position with strategic delays in responding. The minimum time between offers is now shown not to be critical in determining the equilibrium.

Sharp differences in the length of delay also result from assumptions about bargaining costs. In fixed stage cost models agreements or failures to reach agreement occur immediately (Perry 1986). By contrast, in discounting models bargaining continues until the players are convinced that gains are not possible. Cramton (1991) models both types of costs and finds that if one of the players has a significant cost advantage or if the informed party's valuation is sufficiently low, negotiations end immediately (sometimes at an impasse). Otherwise, the delays resemble those in discounting models.

ii) Pareto-inefficient breakdowns: If only discounting costs apply, Pareto-inefficient breakdowns can occur only if the bargaining horizon is finite or there is an exogenous risk of breakdown. In fixed cost models, breakdown can occur even in infinite horizon models (Perry 1986, Cramton 1991). For example, in Cramton's model, if the buyer's valuation is sufficiently low the buyer terminates negotiations immediately because the gross profits from negotiations are smaller than the transaction costs. Bargaining impasse is also possible in Vincent's (1989) common value model where the buyer, in addition to being uncertain about the seller's valuation, is also uncertain about her own. Finally, in each of these models, breakdowns either occur right at the start of negotiations, or not at all.

iii) Uniqueness of equilibrium: Without uniqueness some coordination among the players would be required. And coordination is not a part of the types of models being considered here. (See Crawford 1991 for a discussion of the coordination issues raised by multiplicity of equilibria.) Recall that in sequential equilibria uninformed players update their beliefs using Bayes' rule. But, if the other's behavior is off the equilibrium path (an event assigned zero likelihood) Bayesian updating is not possible. This may provide incentives to the players to deviate from the equilibrium path, increasing the number of possible outcomes. Two approaches have been adopted to deal with this problem. First, in games of one-sided uncertainty, if the informed player is permitted merely to accept or reject offers, uniqueness is possible (e.g., Fudenberg and Tirole 1983, Vincent 1989). For example, if the buyer makes all the offers and the buyer's valuation is known, the offers reveal no information. Along the equilibrium path there is always a positive probability that the buyer will either accept or reject. Thus, there is no out-of-equilibrium behavior for the informed player or updating complications. But once both players can make offers or both are uncertain, out-of-equilibrium behavior cannot be ruled out. In such cases the approach is to pre-specify conjectures regarding off-equilibrium actions such that it is no longer in the best interests of the deviating player to do so. For example, under optimistic conjectures, if an off-equilibrium offer is made by player 1, then player 2 updates her beliefs to assume that player 1 is of the weakest type (Rubinstein 1985). Thus, with optimistic conjectures, only pairs of strategies that are sequentially rational along the equilibrium path have to be identified. However, even with conjectures (optimistic or otherwise), multiple equilibria can be supported in games of two-sided uncertainty (Chatterjee and Samuelson 1988; Fudenberg *et. al.* 1985). Further, because the choice of conjectures is arbitrary, there is some difficulty in singling out a particular equilibrium.

iv) The final outcomes: There is general agreement that impatient players, (*i.e.*, buyers (sellers) with high (low) valuations or higher costs) obtain less favorable outcomes. However, in the models of Cramton (1991, 1984), Fudenberg and Tirole (1983), and Perry (1986), a very impatient (high cost) player can benefit because her threat to walk away if her offer is rejected is credible. Finally, the less uncertain player stands to gain more from the negotiations (see, for example, Grossman and Perry 1986).

The preceding summary of results and Table 1 show that the noncooperative approach adds much to our understanding of the bargaining process. Many of the implications listed above are intuitively plausible, but the precise results are also very sensitive to the particular assumptions made about the bargaining process, as Sutton (1986) points out. Because details of the bargaining process can differ so widely from one case to another, it becomes imperative for future research to examine other scenarios. We suggest two of particular interest to marketers: multiple issue negotiations and negotiations in which the players are permitted communication in addition to just offers and responses.

In multiple issue negotiations one player's gain is not necessarily the other player's loss. Especially, in imperfect information situations it may be the case that passage of time does not necessarily reduce the payoffs, but actually increases them as the bargainers "discover" opportunities for joint gains. This could provide an explanation not yet considered in the noncooperative models for delays in reaching agreements.

Many of the groups of interest to marketers have long histories of joint decisions (e.g., families and manufacturer-retailer dyads). Two characteristics of these groups are particularly noteworthy. First, negotiations among the individuals are often strongly influenced by past agreements (Corfman and Lehmann 1987; Corfman, Lehmann and Steckel (1990). (See the earlier discussion of reference and focal points.) Second, because they can often communicate freely, it is possible for them to reach agreements that are not necessarily self-enforcing. Rather, their agreements may be rendered stable through external factors such as the force of law or pressure from public opinion, internal factors such as prestige and credibility considerations, or because the individuals are simply unwilling to renege on moral grounds. Harsanyi and Selten (1989) and Harsanyi (1977) suggest that cooperative models are more appropriate under these circumstances. While the use of cooperative models may be reasonable, they still do not provide insight into the process itself. Issues such as why or when the individuals may feel obliged to be socially motivated or accede to the external or internal factors mentioned above are not examined.

Examining these and other scenarios in a non-cooperative framework should help distill some principles which will hold over a wide range of possible processes and identify the critical factors that render one bargaining situation different from another. Such insights could help in reformulating or creating new sets of axioms within the largely context-free framework of cooperative models and guide us in their application.

Conclusion. The motivation of normative models is to determine the best group decision given certain explicit and "reasonable" assumptions about the characteristics the decision or the decision process should have. Given that these assumptions hold or are adopted by the decision-makers, normative theories "predict" outcomes. The literature on empirical tests (primarily experimental) of the predictive ability of these models is very large and we will not attempt to review it here (though the marketing literature is examined in some detail in the next section). Here we shall discuss an important regularity in deviations of actual outcomes from those predicted by these models and its consequences for future research in model development.

A common finding is that subjects tend to choose outcomes with payoffs to group members that are closer to equal than is predicted by the models (see, especially, Ochs and Roth 1989 for an insightful analysis of several bargaining experiments). In fact, examining the results of several experiments designed to test noncooperative game theoretic models, Güth and Tietz (1987) concluded that even those results that did correspond to model predictions did so "solely due to the moderate predictions of equilibrium payoffs which makes the game theoretic solutions more acceptable." Because the noncooperative models make their predictions solely on the basis of maximizing individual utilities, one may hypothesize that if the objectives of the decision makers explicitly include some preference for equity, as suggested in work by Corfman and Lehmann (1991), such deviations from model predictions may be eliminated.

In most of the models discussed above individual utilities are idiosyncratic. (With the exception of Eliashberg and Winkler (1981), others' utilities are not a part of one's own utility.) These individual utilities are then combined and the combination function attempts to reflect the relevant concerns for equity and fairness. Yet, when these models make predictions of unequal payoffs a common result is that the mean tends to be closer to equal division than predicted. It should, however, be noted that in many of these same instances simple equal division is also a poor predictor of the final outcome (e.g., Gupta and Livne 1990; Miller and Komorita 1986; Ochs and Roth 1989; Selten 1972). Some have suggested that it might be appropriate to develop models in which distributional concerns are incorporated in the individual utility functions themselves (e.g. Camerer 1990; Ochs and Roth 1989). The suggestion is supported by behavioral decision theory results tat imply that people have clear notions about what is fair (Kahneman, Knetsch and Thaler 1986a, 1986b). Eliashberg and Winkler's (1981) approach explicitly incorporates each individual's equity concerns in the utility functions and could prove to be particularly useful in future research. However, there are some important modeling difficulties with this suggestion. Suppose individual preference functions were adjusted for distributional concerns. Once these adjusted preferences were known to the other decision makers, would there be a need be to readjust for the "new" preferences? If individuals continued to make such adjustments to individual utility functions in what cases would conflict remain to be resolved by the group decision rule or the bargaining function? And, is it not the express purpose of these models to suggest how *conflicting individual* preferences ought to be resolved? Gupta and Livne's (1988) approach to escaping these questions is to leave the individual utility functions unadjusted and add a reference outcome as an additional parameter of the model, to reflect the equity concerns. The modeling effort is to specify how conflicting preferences can be reconciled in the presence of a prominent, perhaps equity induced, candidate outcome. Alternately an approach similar to that of van Damme's (1986) could be adopted. First, each individual's favored notion of fairness could be determined. Then the process of resolving conflicts in both the notions of fairness and individual preferences for outcomes could be modeled.

The behavioral decision theory literature suggests that people's notions of fairness are quite context specific. This may mean that small differences in group or task characteristics can have significant impacts on the favored notions of fairness and on the final outcomes. This would render the task of developing useful models especially difficult. Yet, the success of the approaches that have already been proposed, and the advances made in modeling situation specific effects in other areas of marketing suggest that this is a fruitful direction for both theoretical and empirical research. To this end, a model classification scheme that can serve as a guide to context specific modeling of group choice and negotiations is suggested in §4.

3.2 Social Psychology

Most mathematical modeling of group choice in social psychology has been motivated by Davis's (1973) theory of social decision schemes. This is a very useful general theory into which a wide variety of group choice models can be incorporated. A social decision scheme is a model that dictates how individual preferences will be combined to create a group choice. A well-defined social decision scheme for n alternatives can be represented as an mxn stochastic matrix, $\boldsymbol{D}$, which operates on Π, a vector of probabilities that each of m possible distributions of member preferences will occur (where m is determined by the numbers of members and alternatives). Together Π and $\boldsymbol{D}$ determine the probabilities that various alternatives will be chosen by a group, $(P_1, P_2, \ldots, P_n)$.

$$(P_1, P_2, \ldots, P_n) = \Pi\ D = (\pi_1, \pi_2, \pi_3, \ldots, \pi_m) \times \begin{bmatrix} d_{11} & d_{12} & \ldots & d_1n \\ d_{21} & d_{22} & \ldots & d_2n \\ \cdot & & \ldots & \\ \cdot & & \ldots & \\ \cdot & & \ldots & \\ d_{ml} & d_{m2} & \ldots & d_{mn} \end{bmatrix} \tag{4}$$

Given the ith distribution of member preferences, d_{ij} is the probability that the group will choose alternative j. As observed by Davis (1973, p.102), the d_{ij}'s reflect "tradition, norms, task features, interpersonal tendencies, local conditions, or some combination of these within which the group is embedded."

For the most part, social decision scheme theory has been used by researchers in one of two ways: model-fitting and model-testing. With the model-fitting approach the $\boldsymbol{D}$ is estimated that best fits the data (cf. Kerr et al. 1976). The results of the model-fitting exercise may serve as input to the model-testing approach which has its basis in theory. With the model-testing approach, the $\boldsymbol{D}$ matrix is fully specified and its predictions are compared to the outcomes of group decisions to determine whether the specified decision scheme can be rejected as having produced the data (cf. Laughlin and Earley 1982).

A variety of social decision schemes have been proposed and tested. These include "weight of opinion" schemes such as:

Proportionality: The probability of selecting an alternative, A, is the proportion of members for whom A is the first choice,

Equiprobability: All alternatives that are the first choice of one or more members are equally likely to be chosen,

Majority: If A is the first choice of at least half (or 2/3) of the group it will be chosen, and

Plurality: The first choice alternative of the largest number of members will be chosen.

Other schemes depend upon characteristics of the alternatives as well as individual preferences and are only appropriate when the alternatives possess the requisite properties. For example:

Compromise or arithmetic mean or averaging: If the alternatives are located on an interval scale, the arithmetic mean of alternatives that are the first choice of one or more members will be chosen, and

Truth wins: If one or more members prefer A to all other alternatives and it is the correct choice, it will be chosen.

(The latter scheme is more appropriate for problem-solving than decision-making or choice tasks.)

In the context of decisions involving risk additional schemes have been considered:

Highest expected value: The alternative with the highest expected value among those that are the first choice of one or more members will be chosen,

Extreme member or most (least) risky wins: The most (least) risky of the set of alternatives that are the first choice of one or more members will be chosen,

Neutral member: The most neutral of the set of alternatives that are the first choice of one or more members will be chosen, and

Risk-supported (conservative-supported or caution-supported) wins: The most (least) risky of the set of alternatives that are the first choice of two (or some other number) or more members will be chosen.

Variations on these schemes incorporating secondary schemes for contingencies (e.g., no majority and ties) have also been considered.

Research investigating the appropriateness of different social decision schemes in a wide variety of decision-making contexts has found that the equiprobability model performs well in tasks with high uncertainty or ambiguity (members do not know the probabilities with which various outcomes will occur if an alternative is selected) while moderately uncertain tasks are associated more with plurality schemes and low uncertainty tasks, with majority schemes (Davis 1982). Examples of low uncertainty tasks are provided by mock juries (Davis et al. 1975; Davis et al. 1977; Gelfand and Solomon 1975, 1977) and many kinds of risky choice (Castore, Peterson and Goodrich 1971; Davis et al. 1974; Laughlin and Earley 1982) in which majority schemes appear to be better predictors than others. The difference between the schemes used in high versus low uncertainty tasks may be because when there is certainty members are more confident in their opinions and, thus, hold their preferences more firmly. When many members are not easily swayed, a more decisive scheme is required. There is also some evidence that tasks in which members become personally involved require more decisive schemes for resolution (Kerr et al. 1975), perhaps for the same reason.

Laughlin and Earley (1982) suggested that the distinction made earlier between intellective (problem-solving) and judgmental (decision-making) tasks relates to the decision scheme used. Because people engaged in tasks that are closer to the intellective end of the continuum are likely to produce more arguments and more compelling arguments favoring the "best" answer, they expect truth wins and truth-supported wins schemes to be used. In risky choice, risk-supported and caution-supported schemes and stronger choice shifts are expected. More judgmental tasks, on the other hand, should result in a balance of arguments less strongly favoring risk or caution, making majority schemes more appropriate and producing weaker shifts. Although they did not evaluate the arguments used in group discussion, they did find that the risk- and caution-supported models predicted best where there were strong shifts and that the majority rule predicted best where shifts were weaker.

McGuire, Kiesler and Siegel (1987) hypothesized that the risky choices examined in prospect theory research fall under Laughlin's definition of intellective tasks because they offer defensible or compelling alternatives (which may or may not be correct). Thus, they expected and found that when these tasks involved face-to-face discussions they were characterized by "norm wins" schemes rather than majority schemes. In this case the norm was the prospect theory finding: risk aversion for gains and risk seeking for losses.

In their studies of risky group choice, Crott, Zuber and Schermer (1986; Crott and Zuber 1983) compared the performance of several Condorcet functions (those that consider the members' entire preference rankings of the alternatives) to the proportionality, equiprobability, majority, and arithmetic mean decision schemes (which consider only each group member's first choice). They found that the Condorcet models performed consistently better than the social decision schemes, suggesting that when examining risky choice it is important to use models that consider more than just the members' most preferred alternatives.

Another model that takes into account members' rankings of all alternatives was proposed by Harnett (1967) and based on Siegel's (1957) work on aspiration levels. He incorporated not only each member's preference ranking, but also a ranking of the relative distances between adjacently ranked alternatives. Harnett defined "level of aspiration" as that represented by the more preferred of the two alternatives between which there is the largest difference in preference (of all adjacent pairs). An individual is said to be "satisfied" by alternatives at or above this aspiration level and dissatisfied by those below it. This model predicts that the group will choose the alternative that satisfies the largest number of members. (This is equivalent in spirit to approval voting, discussed earlier.) Harnett's tests of this model provided some support, especially when unanimity was the formal rule and when there was no clear majority.

Castore, Peterson and Goodrich (1971) compared Harnett's model, Luce and Raiffa's (1957) modified majority rule model, and a mean rule in the risky choice setting. The modified majority rule model also uses the members' rankings of the alternatives. The magnitude of an index, $V(x)$, determines the expected group ranking and is the number of times x "wins" in paired comparisons with each other alternative across all group members, less the number of times x "loses." Harnett's model and the modified majority rule model performed equally well, always predicting the direction of choice shifts, although the models did not always predict as extreme a shift as was made by the groups. Castore et al. hypothesized that the models' ability to predict shifts was due to an interesting characteristic of the data; in most cases, subjects' second choices were riskier than their first choices. If a majority was not found on first choices the groups may have moved to the second, riskier, choices.

A variety of other models have been used to predict jury decisions. (Work using social decision schemes was mentioned earlier.) Many of these models depend on the binary characteristic of jury decisions (the two alternatives being guilt or acquittal) and the formal rule for the decision (consensus or a quorum), making them less useful for modeling less structured and formal decisions. An example is a binomial model that predicts the probability that a jury will convict (or acquit) based on the required quorum and the probabilities that individual jurors will vote to convict (Penrod and Hastie 1979):

$$P_G(j)=\sum_{i=Q}^{n}\binom{n}{i}P_i(j)\,(1-P_i(j))^{n-1} \tag{5}$$

Here, $P_G(j)$ is the probability that the n-member jury (G for group) will vote for alternative j (either conviction or acquittal), $P_i(j)$ is the probability that member i will vote for j, and Q is the minimum number of votes required to convict.

3.3 Marketing

Although considerable research has been conducted on decisions made by families, organizations, and buyers and sellers, few models have been proposed and fewer tested. In this section we will review models that have been applied to these problems in research on families, organizations, buyers and sellers, and others.

Families. Many choices made by families are made with the influence of more than one family member. In particular, decisions involving larger financial commitments and products or services that will be used by more than one member tend to be the result of a group choice. Sheth's (1974) theory of family buying decisions is a comprehensive conceptual model of family choice based on the Howard-Sheth (1969) theory of buyer behavior. Although not testable in this form, it has provided a useful framework for research on family purchasing by suggesting relationships that should be considered in the creation of empirical models.

The empirical models that have been proposed for modeling family choice have been primarily variations on a weighted linear model in which the weights reflect the influence of the family members (usually spouses) and determine how strongly each member's preferences are reflected in the group preference or choice.

$$P_G(j) = \sum_i W_i P_i(j) \tag{6}$$

In this representation, $P_G(j)$ is the group's preference for alternative j or the probability that the group will choose that alternative. W_i is member i's influence in the group or the probability that member i will end up making the choice for the group, and $P_i(j)$ is member i's preference for alternative j or the probability that member i would choose j acting independently.

In their examination of post-choice satisfaction, Curry and Menasco (1979; Curry and Woodworth 1984) suggested an equal weight variation on this model, equivalent to social decision scheme's compromise model, in which spouses' preferences are averaged. In their other model one spouse capitulates, effectively allowing his or her influence weight to go to zero.

Kriewall (1980) also used a weighted linear model. She estimated influence weights for parents and a child in each family using data on preferences for colleges (for the child to attend) and then modeled influence weights as a function of several family member characteristics. Results indicated that the following factors contributed to influence: sex role, education, volunteer status, occupational status, academic status, and income. Krishnamurthi (1981) used a similar approach in his investigation of M.B.A. job choices. He estimated an equal weight model, a model in which the weights reflect the relative influence of the individuals, and models that weight each individual's preferences not only by general relative influence, but also by preference intensity at the attribute level (Conflict Resolution Models). These three types of models had some predictive ability, although there were no significant differences among them in their ability to predict actual choices.

A model suggested by Steckel, Lehmann and Corfman (1988) combines Kriewall's two-stage estimation procedure into one (omitting the estimation of influence weights), by using influence-related characteristics as instrumental variables and assuming systematic variation of the weights across individuals and groups.

$$W_i = \frac{1}{n} + \sum \alpha_k T_{ki} \tag{7}$$

In this model the weights are functions of individual characteristics. The variable T_{ki} is the amount of characteristic k possessed by member i (in an n-member group) relative to that possessed by the remaining member(s) of the group and α_k is the impact of characteristic k on influence. (As this study was not conducted using families, discussion of the results appears in a later section.)

Models estimated by Corfman and Lehmann (1987) continued in this spirit, but they incorporated preference intensity as one of a large number of determinants of relative influence:

$$P_G^*(j) = \beta_0 + \sum_k \beta_k T_{ki} \tag{8}$$

Here, $P_G^*(j)$ is the probability that the group will choose alternative *j*, which is member *i*'s preferred alternative. Corfman and Lehmann proposed that T_{ki} be either the difference between the amounts of a trait spouses possess or the proportion (although there was no significant difference in predictive ability between these representations). Of the seventeen traits they examined, the following had significant main effects on influence: preference intensity, decision history (whose choices had been favored in preceding decisions), expertise, sex role, bargaining skill, sociability, desire to support the relationship, and desire to win and control. As preference intensity had by far the strongest effect, situations in which spouses had equally intense preferences and those in which one had stronger preferences than the other were examined separately. Results indicated that when one spouse had more intense preferences than the other, his or her preferences drove the decision. When preferences were equally intense, who had "won" in the preceding decision was used to determine whose preferred alternative would be chosen the next time. Corfman and Lehmann also estimated the weighted probability model using Steckel et al.'s (1988) instrumental variable approach and found that its predictive ability was markedly worse. (When $P_G(j)$ and $P_i(j)$ are viewed as probabilities, Equation *(6)* is the weighted probability model.)

Taking a different approach, Curry, Menasco and Van Ark (1991) have applied three of the normative solutions discussed earlier to the multiattribute dyadic choice problem (Harsanyi's additive model, Nash's cooperative solution concept, and Gupta and Livne's (1988) reference point solution). The conflict they model is due to differential weighting of attributes by two parties selecting a single, multiattribute option from a set of feasible alternatives. A unique feature of their analysis is that they provide the solution in terms of the attribute values and not just in terms of the utilities as is usual in the literature. The authors argue that "solution points are therefore one step closer to management activities such as product positioning and new product design." Further, they derive solutions for the cases where the conflict outcome is at *(0,0)* and where there is an exogenously specified cut-off value for each attribute (e.g., any alternative rated less than 3 on attribute one and 5 on attribute two is unacceptable). Results of a husband-wife dyadic choice experiment show successful prediction of the attribute values of the dyad's first choice and rankings of the alternatives. They also found that the "reference outcome plays a distinct role from that of *(0,0)* conflict point and a 'locally shifted' conflict point (a point in which a previous agreement serves as a conflict outcome)."

Organizations. The structure of organizations and the relationships among their members ensures that most purchase decisions are made with the influence of more than one person. The group of people who influence the outcome of a purchase decision has come to be known as the "buying center" (Wind 1967, 1978). The degree of involvement of the members of this group varies, however, from the case in which a purchasing agent takes the preferences of other buying center members into account to true group decision-making. Although several conceptual models of organizational buying have been proposed (e.g., Robinson and Stidsen 1967; Robinson, Faris and Wind 1967; Webster and Wind 1972; Sheth 1973; Bonoma, Zaltman and Johnston 1977; Thomas 1980) little empirical modeling of the group choice phase of the process has been published.

Choffray and Lilien's (1978, 1980) industrial market response model was the first attempt to develop an operational model of organizational buying. In their model the decision process incorporates four submodels, the last of which concerns the group decision. They proposed four classes of models for this stage: weighted probability, voting, minimum endorsement, and preference perturbation models. Special cases of the weighted probability model are the autocracy model in which $W_i = 0$ for all but one member of the group and the equiprobability model in which all W_i's are equal. The voting model is the same as the plurality social decision scheme:

Let x_{ij} = 1 if *i* prefers *j* to the other alternatives, 0 otherwise.

$$\begin{gathered} \textit{If } z_j = \sum_i x_{ij}, \textit{ then} \\ P_G(0) = Pr\,[z_0 = \max_j (z_j)] \end{gathered} \tag{9}$$

The minimum endorsement model says that to be chosen by the group, an alternative must be preferred by at least a prespecified number of group members. Thus, the majority social decision scheme is a special case of this model. If that minimum equals the number of group members, it becomes the unanimity model. Given that the group has reached consensus, the following is the probability that the group chooses alternative 0:

$$P_G(0) = \frac{\prod_i P_i(0)}{\sum_j \prod_i P_i(j)} \tag{10}$$

The preference perturbation model is the only one of these models that incorporates more preference information than just the members' first choices. This model is essentially a truncated version of the city-block distance approach to producing consensus rankings (Kemeny and Snell 1962). As it is not concerned with producing an entire group ranking of the alternatives, it stops once it has found the single alternative that "perturbs" individual preferences the least, i.e. would require the smallest number of preference shifts in the members' rankings to make the alternative the first choice of all members. If γ_w is the group's pattern of preference structures (*w* indicates a particular set of n individual preference rankings) and $Q(j|\gamma_w)$ is the perturbation required under pattern γ_w to make alternative *j* preferred by all members, then the following assumption is made:

$$\frac{P_G(j|\gamma_w)}{P_G(l|\gamma_w)} = \frac{Q(l|\gamma_w)}{Q(j|\gamma_w)} \tag{11}$$

$If\ Q(l|\gamma_w) = 0$, then $P_G(l|\gamma_w) = 1\ and\ P_G(j|\gamma_w) = 0$.

The group choice is, thus, determined by:

$$P_G(0) = \sum_w P_G(0|\gamma_w)\, Pr(\gamma_w) \tag{12}$$

The only test of the models suggested by Choffray and Lilien in an organizational purchasing context was performed by Wilson, Lilien and Wilson (1991). They proposed and tested a contingency paradigm which suggested that different models from Choffray and Lilien's set are appropriate in different buying situations as defined by the type of task (new task versus modified rebuy) and degree of "perceived risk" (a combination of technical uncertainty and financial commitment required). They concluded that situational factors do have an impact on the types of choice schemes used by buying centers. Specifically, they found that decisions with low perceived risk were characterized well by the autocracy model, compromise decisions schemes (especially the voting model) were better predictors of decision outcomes with moderate perceived risk, and the majority model was the best predictor of the decisions with high perceived risk. This is counter to results of studies discussed earlier which found that majority schemes were most associated with low uncertainty tasks. This difference may be because the subjects responded more to the expenditure component of Wilson et al.'s perceived risk manipulation than to the technical uncertainty component.

Steckel (1990) tested two group decision-making models in a different kind of organizational task--the choice of a corporate acquisition. The models he tested were the Borda solution and the Core. Two aspects of the task were manipulated: the incentive structure (individual or group) and whether the consequences of choosing particular alternatives were certain or uncertain. Groups with individual incentives more often reported used a majority rule, while those with a group incentive reported searching for consensus. The Core predicted better than the Borda solution under all conditions.

Rao and Steckel (1989) compared three group choice models using experiments involving an organizational task (the choice of faculty employment candidates) and a task involving ad hoc groups (a restaurant choice). Two of the models were from decision theory, additive and multilinear (Keeney and Kirkwood 1975; Keeney and Raiffa 1976; Raiffa 1968), and the third was a variation on the weighted linear model which incorporates a polarization term.

$$P_G = \sum_i W_i P_i(j) + \phi(\overline{P}_i(j) - K) \tag{13}$$

The polarization term is the difference between the mean preference of the individuals in the group and a neutral value, *K*, multiplied by the shift parameter, Φ. $P_G(j)$ shifts upward for values of the group mean higher than *K*. In their experiments the polarization model slightly outperformed the additive model which outperformed the multilinear model.

Steckel and O'Shaughnessey (1989) proposed a weighted linear model for choice between two alternatives which splits an individual's power into two components, constructive and destructive power.

$$P_G(j) = \frac{1}{2} + \sum_i \delta_i X_i + \sum_i \sigma_i Y_i \tag{14}$$

Here, δ_i and σ_i represent *i*'s constructive and destructive power, respectively, and X_i and Y_i represent *i*'s liking of the first alternative relative to the second and *i*'s dislike for the first alternative relative to the second.[4] They do not test this model, but assume that the form is appropriate and use it as a basis for the estimation of power.

Other groups. While one tends to think of families and organizations when group choice in marketing is mentioned, other groups also make joint consumption decisions. In the following studies ad hoc groups and groups of friends were studied. (See also Rao and Steckel, 1989, discussed earlier.)

Three such problems were examined by Corfman, Lehmann and Steckel (1990; Steckel, Lehmann and Corfman 1988): the choice of a musical selection, a stock market investment, and a restaurant. In the 1988 study on music and stocks (mentioned earlier) they estimated a weighted probability model in which the weights were functions of opinion leadership and decision history, both of which were significant. In the 1990 study they estimated the model in equation *(8)*, and

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[4] $X_i = 1$ if *i* prefers the first alternative to the second, 0 if *i* is indifferent, and -1 if *i* prefers the second. $Y_i = -X_i$.

found that for all three decision tasks, preference intensity was the most important predictor. Expertise and decision history were also important. Further analysis of the differences between decisions in which preference intensities were either equal or unequal supports the process suggested by Corfman and Lehmann's (1987) study.

Buss (1981) also used variations on the weighted probability model in his study using groups of friends and strangers, and compared them with equal weight models: Borda-up, Borda-down, plurality, and random models. (The nature of the task is not clear from the paper.) Rather than estimate or model the weights, Buss used member perceptions of influence, communication effectiveness, and preference intensity. The Borda models and the weighted probability model using preference intensity outperformed the others in predicting first choices. This suggests that it may be important to consider both the members' entire preference rankings and their preference intensities.

Buyer-seller negotiations. Negotiations between buyers and sellers can occur over a large variety of issues including price, quantity, delivery, terms, promotional support, product configuration, materials, warranties, and service. Much modeling of buyer-seller negotiations has made use of various cooperative game-theoretic models of bargaining, especially the Nash solution. Although these models were developed as normative models, because they choose "fair" solutions they are reasonable starting points for modeling outcomes of actual negotiations in which fairness is an important motivation. Researchers in fields other than marketing have conducted experiments to test the axioms and resulting solution concepts (e.g., Bartos 1974; Heckathorn 1978; Nydegger 1977; Nydegger and Owen 1974; Rapoport and Perner 1974; Roth and Malouf 1979; Svejnar 1986). Generally, experimental tests indicate that, although the cooperative bargaining models have some predictive ability, they have a number of shortcomings when such factors as imperfect information, information sharing strategies, multiple issues, reference points, interpersonal comparisons of utility, future negotiations, profit goals and compensation, asymmetric power structures, bargaining skill, and experience are introduced. As discussed in §*3.1*, some of these factors have been added to axiomatic models.

In their investigation of the strategic use of preference information in bargaining, Chatterjee and Ulvila (1982) addressed the situation in which bargainers have agreed to the principles that yield a Nash solution. Although they did not examine whether bargainers naturally gravitate toward an "optimal" solution, they observed that even when this kind of solution is the goal, information sharing strategies can prevent bargainers from arriving at it.

A study by Bazerman, Magliozzi and Neale (1985) demonstrated that experience in a market results in settlements that are increasingly close to the Nash solution. This generally occurred because the profits of the less profitable party increased with experience, while the other party's profits remained relatively constant. This "learning" process appeared to occur even faster for negotiators who had been given difficult profit constraints.

Eliashberg et al. (1986) simulated negotiations in a price-setting context and tested the predictive ability of Nash's theory and two group utility functions from group decision theory: additive and multilinear functions. They found that all three models predicted well, even when power was unequal and only partial information on utilities was initially available, and that Nash's theory performed better than the decision theory models.

The Nash model has also been used to predict outcomes of multiple issue negotiations. Neslin and Greenhalgh (1983, 1986) tested the Nash model's predictive ability in three issue negotiations between buyers and sellers. They found that this solution had some predictive ability (42% of the settlements represented the Nash solution in the 1986 study). Further, situational power (dependence) and bargaining skill had no effect on the settlements, Nash solutions were reached less often when both bargainers viewed the interaction as a one shot deal, and Nash solutions were reached more often when the seller was on commission.

Noting the ambiguous predictive ability of Nash in these studies Gupta (1989) observed that although multiple issues can be represented in the Nash framework, some essential characteristics of multiple issue bargaining are not captured (e.g., logrolling and the role of reference points) and Independence of Irrelevant Alternatives is assumed, which is often not appropriate. Two proposed solutions to this bargaining problem that take reference points into account are the Gupta-Livne (1988) model and Curry, Menasco and Van Ark's (1991) attribute based formulation, both described earlier. The Gupta-Livne prediction was tested in manufacturer-retailer bargaining experiments (Gupta 1989; Gupta and Livne 1989). In Gupta (1989), only in cases in which the negotiators had equal power did their model predict the agreements accurately, but in those cases it clearly outperformed both the local Nash and Kalai-Smorodinsky models. In the unequal power conditions, using an empirically derived reference point resulted in significantly improved predictions. In Gupta and Livne (1989), where the reference point was provided by the previous agreement between the two parties, substantial support for the model's predictions was reported. As discussed earlier, support for the special role of the reference outcome has also been reported by Curry, Menasco and Van Ark (1991) in the context of husband-wife dyadic choice.

4. Dimensions of Choice Situations

Existing research on mathematical models of group choice and negotiations suggests numerous directions for future research. There are two basic levels at which this work might proceed. The first is a micro-level focus on the specification of group choice models for specific applications. Some suggestions for future research at this level have been made in the preceding discussion. The second is a macro-level focus on integrating what we know and creating classifications of models appropriate in different contexts. The latter approach involves the development of typologies or contingency paradigms for models that classify them according to the situations in which they are the best reflections of group choice behavior (cf. Frazier and Rody 1991, Kohli 1989; Wilson et al. 1990). Enough modeling of group choice behavior has been done to allow us to begin to make generalizations about the contexts in which particular models will be the best predictors of group choice. Further, research on determinants of influence in a variety of settings suggests when certain variables should be incorporated into group choice models. In order to take advantage of what we have learned and structure future research in a useful way, this broader focus is essential. We organize this section around this classification goal and discuss dimensions that should prove useful in developing model classification schemes.

The dimensions that might be considered in developing typologies fall into two categories: task characteristics and group characteristics. Although others could be proposed, those listed below seem particularly promising candidates for inclusion in model typologies.

TASK CHARACTERISTICS:

Intellective vs. judgmental
Risky vs. non-risky alternatives
Low vs. high uncertainty
Feedback on previous decisions available vs. not available
High vs. low importance
Low vs. high perceived risk
Information on members' valuations complete vs. incomplete
New vs. familiar
Low vs. high time pressure
Group choice dependent on vs. independent of members' outside alternatives
Resource allocation vs. discrete choice vs. ranking vs. rating
Single vs. multiple issues or attributes

GROUP CHARACTERISTICS:

Two-person vs. small vs. large
Primary vs. institutional
Individual vs. group vs. task goals dominant
Decision rule imposed vs. flexible
Cohesive vs. non-cohesive
Existing vs. newly-formed
Ongoing vs. terminal
Members have equal vs. unequal power

One of the problems with attempting to integrate theory concerning the distinctions implied by these dimensions with results of empirical studies, is that few studies have examined variations along these dimensions while controlling for other factors. Thus, for example, models that predict well under high levels of uncertainty in one study may perform poorly in another study due to differences in the tasks and groups examined. The discussion that follows draws heavily on theory and intuition, uses empirical evidence for support when it exists, and makes some observations about findings that appear to contradict theory and each other. Examples of models that may be appropriate in different choice contexts appear in Tables 2 and 3.

Insert Tables 2 and 3 about here.

4.1 Task Characteristics

Intellective vs. judgmental tasks. As specified in the Introduction, our concern is with decision-making rather than problem-solving tasks. However, decision-making tasks can fall on the intellective/judgmental dimension anywhere short of being completely intellective. The location of a task on this dimension will depend on 1) whether there is an objectively correct answer, 2) if there is, when this answer can be confirmed as correct (either upon its discovery accompanied by an "aha" reaction or following implementation of the group decision), and 3) if there is not a correct answer, whether the group, nevertheless, perceives that there is one. As an example of 2), if the goal of an investment group is to select the set of investments that will yield the highest returns, there is a correct answer. However, this answer will not be available until after (perhaps, long after) the decision is made, making the task more judgmental. As an example of 3), the choice rule used by a management team is likely to depend upon whether they believe there is a

Table 2

Choice Context Dimensions and Examples of Models: Task Characteristics

Task Characteristic	Requirements	Examples of Models
Intellective task	Allow "best" choice.	Truth wins models WLM[6] with expertise
Judgmental task	Incorporate member preferences.	Normative models, majority, plurality, WLM
	Incorporate influence.	Asymmetric Nash, Gupta-Livne, WLM with power sources
Risky choice	Spread responsibility.	Majority, plurality, approval voting
	Avoid individuals' less preferred alternatives.	Lexical maximin, preference perturbation
	Incorporate risk attitude.	Keeney & Kirkwood (75)
	Limit degree of risk.	Extreme member wins
	Incorporate more than just most preferred alternative.	Any model using rankings or ratings (e.g., not proportionality, plurality, majority)
Risky choice with high uncertainty	Reflect lack of confidence in ability to choose well.	Normative models, equiprobability and other decision schemes
Incomplete information on members' valuation of alternatives	Reflect uncertainty about valuations.	Noncooperative bargaining models with incomplete information.
Important task	Incorporate preference strengths.[7]	Group value function, Nash, WLM
	Incorporate concern about other's preferences.	Eliashberg & Winkler (81), WLM with preference intensity
	Incorporate influence.	Asymmetric Nash, Gupta-Livne, WLM with power sources
	Spread responsibility.	Group utility function, majority, plurality, approval voting
	Avoid individuals' less preferred alternatives.	Lexical maximin, preference perturbation
Unimportant task	Simple rule.	Social welfare rules, majority, plurality
	Indiscriminate or assign responsibility.	Equiprobability Autocracy

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[6]Weighted linear models

[7]Input to group decision is a rating versus a ranking.

High task familiarity	Incorporate influence.	Gupta-Livne, asymmetric Nash, WLM with power sources
Low task familiarity	Incorporate member ability. Incorporate characteristics of risky decisions.	WLM with expertise (See risky decisions.)
High time pressure	Simple and decisive rules. Assign responsibility. Reflect use of faster acting power sources. Incorporate cost of time.	Social welfare rules, majority, plurality, compromise Autocracy WLM with coercion and rewards (vs. those requiring persuasion) Noncooperative bargaining models, process theories of bargaining
Group choice dependent on members' outside alternatives	Predicted outcome a function of conflict outcome.	Bargaining models
Group choice independent of members' outside alternatives	Predicted outcome independent of conflict outcome.	Social welfare rules, decision-theoretic models, social decision schemes, WLM
Allocate resource	Divide resource among members or alternatives.	Bargaining models, any model with probability of choice as dependent variable (interpret as proportion)
Choose from among discrete alternatives	Select single alternative. Allow "compromise" over series of decisions.	Most models WLM with decision history, logrolling models
Provide ranking of alternatives	Rank alternatives in set.	Borda, consensus ranking, modified majority model
Provide ratings of alternatives	Rate alternatives.	Nash, Rawls, decision-theoretic models, compromise decision scheme, any model with probability of choice as dependent variable(interpret as rating)
Multiple issues or attributes	Reflect interdependence of decisions. Allow logrolling.	Curry, Menasco & Van Ark (91), Gupta-Livne, Nash Logrolling models

Table 3

Choice Context Dimensions and Examples of Models: Group Characteristics

Group Characteristic	Requirements	Examples of Models
2-person groups groups.	Need not be able to accommodate models, unanimity model, WLM,	Most models, including bargaining larger (but not proportionality, plurality, majority)
Large groups	Accommodate coalitions.	Coalition models
	Decisive.	Majority, plurality
	Incorporate impersonal power sources.	WLM with legitimate and expert power
Primary groups	Reflect importance of equity and allow compromise.	Group value functions, consensus ranking models, cooperative bargaining models, consensus decision scheme, compromise decision scheme, WLM with decision history
	Incorporate preference strengths.	Group value functions, Nash, WLM
	Incorporate concern about other's preferences.	WLM with preference intensity, Eliashberg & Winkler (81)
	Incorporate personal power sources.	WLM with referent, reward, coercive, etc. power
	Incorporate personal and relationship goals.	WLM with conflict attitude, importance of winning, desire to support relationship, sex roles, status, etc.
Institutional groups	Incorporate impersonal power sources.	WLM with legitimate and expert power
	Incorporate personal goals.	WLM with importance of winning and control, etc.
Individual goals dominate	Reflect member concern with impact of decision on self.	Noncooperative bargaining models, process models of bargaining, WLM with bargaining skill, coercion, rewards, and personal goals (e.g., competitiveness, conflict attitude, etc.)
Group goals dominate	Reflect member concern with impact of decision on group, e.g.,	
	Equity, compromise, risk attitude, and efficiency	Social welfare rules, decision-theoretic models, consensus ranking models, cooperative bargaining models, consensus decision scheme, compromise decision scheme, WLM with decision history

	Preference strengths	Group value functions, cooperative bargaining models, WLM
	Relationship goals	WLM with desire to support relationship, sex roles, etc.
Decision quality dominates	Reflect member concern with quality of decision, e.g.:	
	Find "best" answer.	Truth wins, WLM with expertise
	Choose fair answer.	(See equity and compromise models above.)
Cohesive groups	Reflect importance of equity and allow compromise.	Social welfare rules, group value function, consensus ranking models, cooperative bargaining models, compromise decision scheme, WLM with decision history
	Incorporate preference strengths.	Group value function, Nash, WLM
	Incorporate concern about other's preferences.	Eliashberg & Winkler (81), WLM with preference intensity
	Incorporate "nice" power sources.	WLM with expert, reward, referent, etc. power
	Predict decisions that do not need to be self-enforcing.	Models other than noncooperative bargaining models
Existing groups	Incorporate outcomes of prior decisions.	Logrolling models, Gupta-Livne, WLM with decision history
	Incorporate more characteristics of primary groups.	(See primary groups.)
New groups	Incorporate impersonal power sources.	WLM with legitimate and expert power
Ongoing groups	Reflect importance of equity and allow compromise.	Cooperative bargaining models, compromise decision scheme, WLM with decision history
	Incorporate concern about other's preferences.	Eliashberg & Winkler (81), WLM with preference intensity
	Incorporate "nice" power sources.	WLM with expert, reward, referent, etc. power
Terminal groups	Incorporate more aggressive power sources.	WLM with coercive power, bargaining skill, etc.
Members have equal power	Weight members equally.	Normative models, social decision schemes
Members do not have equal power	Reflect relative power of members.	Gupta-Livne model, asymmetric Nash model, WLM with power sources

correct strategy. If they believe there is, they are more likely to use or try to use group choice rules appropriate for intellective tasks.

In tasks closer to the intellective end of the spectrum, choice rules in the spirit of truth wins or truth-supported wins may be used and the judgments of experts are likely to be given more weight in the identification of "truth". Feedback on past decisions will also be important for learning about task performance and for providing evidence concerning member expertise and ability. (See Bordley, 1983, for a model based on historical accuracy.) In judgmental tasks, preferences are the issue. When alternatives are discrete, majority and plurality rules and turn-taking may be used as non-judgmental ways of reaching solutions. When alternatives are continuous, compromises may be reached. When power is unequal and exercised, strength of preference will probably play a role and, although expertise may still be an important issue, other sources of power, such as bargaining skill and reward power, are likely to become relatively more important.

Risk and uncertainty. The definitions of decisions under uncertainty and risk vary in literature on group decision-making. To facilitate discussion and comparisons among studies, the following definitions will be used here. Both decisions under uncertainty (ambiguity) and risk are those for which members do not know with certainty what will happen if they select particular alternatives. The difference is that in the case of decisions under risk, the members know the probabilities associated with the possible consequences of selecting an alternative, while with decisions under uncertainty, they are not aware of these probabilities (Davis 1982). A task's degree of uncertainty is associated with whether these probabilities are known or not. (If a decision is not perceived as a decision under risk, it cannot be a decision under uncertainty.) Two ways of interpreting an alternative's degree of risk have been used. In one, an alternative is considered riskier when it has higher variance (e.g., $p=.5$ of either \$100 or \$10 would be considered more risky than either $p=.1$ of \$10 and $p=.9$ of \$100, or $p=.1$ of \$100 and $p=.9$ of \$10, which have equal variance). The other interpretation adds a distinction, saying that an alternative is more risky when a more preferred consequence has a lower probability of occurring than a less preferred consequence, even when the alternatives have equal variance (e.g., $p=.1$ of \$100 and $p=.9$ of \$10 would be considered more risky than $p=.1$ of \$10 and $p=.9$ of \$100). We will adopt this additional distinction.

In the realm of decisions under risk, tasks can vary in their uncertainty. Very high uncertainty tasks should tend to make group members equal because ability to select a best alternative is uniformly low and there are no bases for intense preferences. This is probably why equiprobability rules have often performed well in these situations. Sources of influence not associated with the quality or nature of the choice itself may, however, be important in these situations because winning or getting one's way can become the more important goal for individuals. Feedback on the consequences of past decisions is valuable in uncertain tasks. When feedback is available, a task may become more certain or groups may perceive it as being more certain and behave accordingly, even though the conclusions they draw may be spurious. In tasks with moderate to high levels of uncertainty, expertise and information power that might improve predictions of probabilities should play important roles. Groups making decisions under risk with no uncertainty have been observed to use majority schemes (Davis 1982). One reason for this may be that these schemes spread responsibility across the members of the majority, which may be desirable in high risk contexts. It also appears that models that incorporate more than just the members' most preferred alternative predict better than "first choice' models (Crott, Zuber and Schermer 1986). Models that avoid least preferred (most or least risky?) alternatives (lexical maximin, preference perturbation), incorporate risk attitude (Keeney and Kirkwood 1975), and limit the degree of risk (extreme members decision scheme) should also perform better in risky choices.

Task importance and perceived risk. When decisions are perceived as being unimportant, simple rules that give members equal weight are more likely to be used, one individual may be permitted or assigned to make the decision for the group, and there is less motivation to exercise power (unless, as suggested above, being powerful takes over as the goal for one or more members). When decisions are important, especially in judgmental tasks, individual preference intensities and power sources tend to play an important role. On the other hand, when tasks are more intellective, there may be a desire to use choice rules that spread responsibility, such as majority and plurality rules. If one can assume task importance leads to involvement, similar observations may be made about the involvement dimension.

Perceived risk is generally thought of as having two components: uncertainty (implying, therefore, some degree of risk) and the importance of the decision (more important decisions having higher variances because the differences between the possible outcomes are larger). The little work that has been done on modeling group decisions in situations that vary in their perceived risk is difficult to integrate with some of the observations about uncertainty and task importance just made. This is partly because other task factors may have strong effects and because the effects of uncertainty and task importance cannot be separated in these studies. In situations with high perceived risk, Wilson et al. (1990) found that majority and voting rules were the best predictors and Kohli (1989) found that expert and information power were important, while reward and coercive power and status were not. One partial explanation is that the highest levels of uncertainty encountered in the cited studies were probably not extreme, hence, the importance of expertise and information. As suggested earlier, the motivation for the use of majority and voting rules may have been a desire to spread risk when making important decisions. In Wilson et al.'s study, compromise models (voting, equiprobability, preference perturbation and weighted probability models) performed best at moderate levels of perceived risk and the autocracy model, at the lowest level of perceived risk. The latter finding is consistent with the observation made about unimportant decisions that are delegated to a single member. In an organizational setting, especially, this kind of efficiency makes sense.

Information on valuation of alternatives. Models of group decision-making typically assume that the groups members know which alternatives each prefers and how much. However, there are cases in which members may not have this information and may choose not to reveal it to each other explicitly. Noncooperative bargaining models that allow incomplete information (either one-sided or two-sided uncertainty) are designed to model this situation.

Task familiarity. Two aspects of task familiarity are important to consider. When tasks are new to the group they may or may not be new to the members. As recognized by Wilson et al. (1990), task familiarity and perceived risk are not independent. This is probably why their subjects used the responsibility-spreading models (majority models spreading responsibility the most and voting models, to a lesser degree) in tasks that were new to both the members and the group, and models representing compromise, in familiar tasks. In new tasks it may also be appropriate to model the choice at the attribute level because this is where the group's attention will usually be focussed (Krishnamurthi 1981, and Curry and Menasco 1979). When the task is new to the group, but familiar to one or more members, the expertise and information they possess are likely to be important. When the task is familiar to the group, members are likely to possess the same amounts of expertise, and other power sources will play larger roles. Exceptions occur when the relevant expertise is very specialized and not easily transferred. In these cases, the more expert member(s) will be given greater influence, or even complete responsibility (Wilson et al. 1990), even when the task is familiar.

Time pressure. When a group is facing pressure to make a decision quickly, simple and decisive choice rules that give members equal weight, such as plurality and pure compromise, may be used. Alternatively, one or a small number of members may quickly emerge as leaders based on expertise or seniority (legitimate power) and be given responsibility for the decision, suggesting the appropriateness of the autocracy model and consideration of expert and information power (Isenberg 1981; Spekman and Moriarty 1986). When preferences are intense or the decision is important, time pressure may result in the use of more forceful power sources, like coercion and the offering of rewards, rather than persuasion which often takes more time (Kohli 1989). Further, the noncooperative modeling approach, which explicitly accounts for time costs, may prove especially useful.

Dependence of group choice on members' outside alternatives. Typically, group choice models account for the possibility of breakdown in decision-making or the decision not to decide as one of the set of alternatives the group considers choosing. The value of that alternative is assumed to be the same for all members. However, there are cases (more often negotiations) in which members have alternatives to a group choice or negotiation that are not equal, giving more power to those with more valuable alternatives. The effects of this kind of power should be reflected in the outcome predicted by the chosen model. Bargaining models are designed to accommodate this situation and model the final outcome as a function of the conflict outcome. Any model that weights members to reflect their influence over a decision can also be modified to reflect power derived from the existence of outside alternatives.

Nature of choice. Bargaining models are designed for *resource allocation* among group members and any model whose dependent variable is a probability may also be useful here, because the probability can be interpreted as the proportion of a resource to be awarded to an alternative or member. When alternatives are *discrete* they may still be ordered in a way that allows some kind of compromise to be made (e.g., a low vs. moderate vs. high-priced product), but when they are not compromise must occur over a series of decisions in the form of turn-taking or equalizing gains over a period of time. Thus, models that incorporate the outcomes of preceding decisions (Corfman and Lehmann 1987) and logrolling (Coleman 1973) are appropriate.

A ranking or rating of all alternatives may be required when a group is making an analysis of a set of alternatives for informational purposes, such as expert ratings of appliance brands or suppliers, or when the alternatives are one or few of a kind and group's preferred alternative(s) may end up being unavailable, as in home purchases and hiring decisions. Consensus ranking approaches (Kemeny and Snell 1962; Bowman and Colantoni 1973) are designed for cases in which group *rankings* are required. However, most other models can be adapted to provide not only the group's first choice, but its second and third, etc., by eliminating each alternative from the set after it is chosen. Models that have probability of choice as the dependent variable may also be used to create rankings. Decision-theoretic models, the compromise decision scheme, and any model with probability of choice as the dependent variable (e.g., many weighted linear models) can be used to provide joint *ratings* of alternatives in a set.

In some situations groups will need to develop a group preference function. This occurs when a group wishes to establish its priorities and develop a rule for use in future choices, possibly before it is known exactly which alternatives will be available. Decision theoretic models may be useful here, as will models that allow the estimation of group part worths (e.g., Krishnamurthi 1981).

Number of issues. While some collections of decisions to be made by a group can be treated sequentially and somewhat independently, some issues are so closely related that they cannot be modeled separately. For example, for a two-career couple with children, the choice of a town to move to, each job, a house to buy, and school(s) for the children cannot be decided independently. Buyers and seller also often negotiate multiple, related issues including price, quantity, terms, and delivery. Examples of appropriate models for multiple issues include Coleman's (1973), Curry, Menasco and Van Ark's (1991), and Gupta's (1989) models. Most noncooperative bargaining models as currently formulated are *inappropriate* for studying multiple issue negotiations.

4.2 Group Characteristics

Group size. Bargaining models are designed for the smallest groups--those with two members (although some can be generalized for larger groups, e.g., the Nash and Gupta-Livne models). Many of the social decisions schemes are not helpful for two-member decisions because they require that an alternative be advocated by more than one member (those based on proportionality, majority, or plurality). Most of the models we have reviewed are appropriate for small groups. It is the larger groups that often require special models due to the role of coalitions in their joint decisions. Decisive rules (e.g., majority) may be used in larger groups because consensus is more difficult to reach. When members have differential power in a large group's decision, legitimate and expert power are likely to play important roles because they are effective across a wider range of other members. Also, because behavior in a large group is visible to a larger number of people, these groups are often characterized by preference for socially acceptable, impersonal power use over such approaches as the use of coercion or rewards (Kohli 1989).

Primary vs. institutional groups. Primary groups are "held together by common traits and sentiments and tends to be spontaneous" (Faris 1953, p.160). Families, cultural groups, and cliques are examples. Institutional groups, such as purchasing departments and juries, exist to perform functions other than those that bind the members to the group and, thus, tend to be externally formed, externally controlled, and more heterogeneous. In primary groups the people are often more important than the task and there tends to be greater trust, liking, and empathy. Therefore, preference intensity can be an important power source, equity is more important, and compromise or effort to equalize gains over time is often visible (Corfman and Lehmann 1987). Institutional groups may have some of these characteristics, but because the task is usually more important than the people, at least initially they tend to rely more on impersonal power sources, such as legitimate power and expertise. Informal power may evolve as well, perhaps based on expertise or referent power, resulting in increasing power for influential members (in contrast to the preference-based turn-taking observed in families).

Dominant goal or reward structure. Whether the members of a group are more concerned about the benefits of a group choice to themselves as individuals, or care more about the welfare of other members and the group itself, or whether the performance of the task is paramount, will affect the appropriateness of models chosen. The choices of groups whose members are more focussed on their own personal rewards, as often happens in resource allocation tasks, may be better modeled by noncooperative bargaining models (which are less concerned than other normative models with a decision's equity), and weighted linear models that incorporate power sources that are not necessarily relationship preserving (e.g., rewards and coercion) and personal goals like winning and controlling. On the other hand, groups whose members care about each other and the health of the group will be more interested in equity (making many normative models potentially useful) and relationship goals (suggesting that they be incorporated into weighted linear models). Groups whose members care most about the quality of the decision must establish criteria for making this judgment. Intellective tasks may be evaluated externally, making truth wins and WLM models with expertise useful. Judgmental tasks may be deemed of high quality if they are fair, suggesting the use of many normative models and models that incorporate decision history.

Decision rule imposed vs. flexible. In some situations a formal group decision rule may be imposed on a group either from the outside or by agreement among members that one will be adopted. Possibilities include majority, plurality, approval voting, consensus, and autocracy rules. Rather than simplify the problem, as it would appear on the surface, this adds interesting complexity to the modeling problem. In these situations, unless group interaction is prohibited or impossible, the choice becomes at least a two stage process. In the first stage members discuss alternatives and their preferences, and a variety of influence processes may operate to change opinions and positions on the issue (perhaps without opinion or preference change). Thus, the input to the decision rule is not as it would have been before the group's interaction. If preferences or opinions are sufficiently strongly held, a model that reflects the formal decision rule will still predict well.

Cohesiveness. Groups may become cohesive or viscid (Hemphill and Westie 1950) through experience together, communication, liking, and respect which build stronger bonds. Members of cohesive groups are characterized by the ability to work well together, concern about each other's preferences, and trust. Their concern for equity in judgmental tasks, suggests the use of models that incorporate the outcomes of past decisions. In task-oriented groups, expertise and legitimate power should be incorporated and are likely to play more important roles than less socially acceptable power sources (Kohli 1989). Trust among group members makes the self-enforcing (equilibrium) solutions of the noncooperative bargaining models unnecessary. In these groups, the members are motivated to implement the decision even when it does not represent their individual preferences.

Group history. The less familiar members are with each other, the more likely they are to behave in socially acceptable ways (Bormann 1975). As it is more acceptable to use impersonal power sources, expertise and legitimate power are more likely to be used than coercion and aggression, for example (Kohli 1989). For groups that are not newly formed, the processes and outcomes of past decisions are often important and should be included in models. Turn-taking, equalizing of gains, and logrolling may be motivators for the outcome of the current decisions. Time also allows institutional groups to evolve informal power structures and develop characteristics of primary groups, which would alter their decision-making processes. For example, levels of trust and loyalty are determined and may affect willingness to influence other members and use particular sources of power.

Group future. When a group will continue to exist and make decisions following the decision being studied, awareness of future dependence is likely to make the group more cooperative and concerned with equity. In these cases, models with cooperative normative foundations may be useful (Neslin and Greenhalgh 1986) and "destructive" power sources are less likely to be used. Terminal groups have far less incentive to nurture the relationship any more than necessary to resolve the current issue. In these cases models of noncooperative bargaining and coalition formation may be appropriate.

Power allocation. In groups whose members have roughly equivalent amounts of influence over the decision, normative models and social decision schemes may be very useful. However, when members have differential amounts of influence this must be reflected in the chosen model. As discussed earlier, the Gupta-Livne and the asymmetric Nash models may be adapted to reflect unequal power and many have formulated weighted linear models that incorporate sources of influence.

4.3 Conclusions

The potential applications of this classification approach are many, although few appear in the marketing literature. As suggested by Wilson et al.'s (1991) and Kohli's (1989) work, the decision-making process in buying centers can vary considerably due to differences in aspects of the task and group. Between them they explored the nature of the buying task, perceived risk, time pressure, buying center size, familiarity and viscidity, and individual power bases and influence attempts. Further investigation of these and other dimensions will provide us with better understanding of how the buying center operates in different contexts and should lead to both better decision-making in buying centers and more effective selling to them. A similar set of dimensions should be used to structure the investigation of buyer-seller negotiations and will result in similar benefits. When this approach is taken to the study of family choice the relevant dimensions will change and the group characteristics will vary less. The benefits of greater understanding of family choice in different settings are better family decisions and improved strategies for advertising, promotion, distribution, and design of products and services typically selected with the input of more than one family member.

In this section we have described and discussed a set of dimensions that should prove useful in the development of model typologies. While our discussion has been largely one-dimensional, there are undoubtedly many interesting interactions to explore. For example, the relationships between group cohesiveness and task riskiness, group size and time pressure, and task importance and uncertainty, may imply different models for different combinations of characteristics. Existing research provides some guidance for the development of these typologies, but careful observation and experimentation are necessary before we can begin to complete the picture. We recommend that future studies focus on exploring small numbers of task and group dimensions, developing theory concerning model performance in each context, and testing the models. Rather than focussing on developing models for single situations, we hope that researchers will concentrate on work that generalizes from one context to another due to understanding of the factors or dimensions that are driving the ability of models to predict.

5. Applications

The examples in §5.1, §5.2, and §5.3 are provided to show how specific group tasks might be positioned on relevant dimensions and how this information can be used as a guide to modeling the group choice or negotiation. The three examples were chosen because they are areas in which further work would be interesting and valuable and, more importantly, because they are situations that are not typically modeled as group decisions. We hope also to illustrate the dangers of assuming that many common choices are made by individuals when they are, in fact, made by multiple decision-makers. These examples represent different stages of development. They progress in terms of existing work and remaining questions from fairly formal to very open and in need of greater definition.

5.1 Repeat Buying in Households

Repeat purchase behavior has often been studied in the context of a household's purchases of non-durable goods. Based on theories of consumer behavior, households are often expected to display *variety seeking* or *reinforcement* (last brand loyal) behavior. Empirically, several authors (e.g., Givon 1984; Kahn, Kalwani and Morrison 1986; Bawa 1988) have relied on household purchase panel data to propose and test models of variety seeking and reinforcement behaviors. Typically, based on each household's string of purchases the household's preferences are summarized by a transition matrix:

$$P_{HH} = \begin{bmatrix} p_{AA} & p_{AB} \\ p_{BA} & p_{BB} \end{bmatrix}$$

where p_{qr} is the probability of buying brand *r* if brand *q* was bought in the previous occasion. The general first order transition matrix, ***P***, can reflect a variety of preference structures:

i)	Strict preference for A(B)	$p_{AA}(p_{BB}) > 0.5$ and $p_{BA}(p_{AB}) > 0.5$
ii)	Variety seeking	$p_{AA} < 0.5$ and $p_{BB} < 0.5$, and
iii)	Reinforcement	$p_{AA} > 0.5$ and $p_{BB} > 0.5$.

Furthermore, if $p_{AA} = p_{BA}$ and $p_{AB} = p_{BB}$, the behavior is zero order.

Such empirical analyses, contrary to theoretical expectations, have generally concluded that households are "indifferent to variety" (Givon 1984). However, Kahn, Morrison and Wright (1986) and Kahn, Kalwani and Morrison (1986) have suggested that these findings may be due to errors in model specification. Households usually have more than one member and the resulting purchases may reflect some resolution of the conflict among individual preferences. For example, it may be that individual members of the household are indeed variety seeking or last brand loyal; however, the process of resolving conflicts among the individuals may lead to a household transition matrix that *appears* to represent a zero-order process. Thus, a completely specified model must account for individual preferences ($P_i(j)$ in equation *6*) *and* the conflict resolution mechanism adopted by the household (analogous to $\boldsymbol{w}_i$ in equation *6)*.

Because the focus of such studies is on detecting first order preferences (e.g., variety seeking or reinforcement), Gupta and Steckel represent individual preferences with separate transition matrices P_H and P_W, for two-member households. But, as we have seen in the preceding discussion, there are many possible conflict resolution mechanisms. The possibilities can be narrowed, however, on the basis of the task and group characteristics:

Task characteristics: Repeat purchases of non-durables can be classified as primarily judgmental, low uncertainty, familiar tasks in which one or more of several discrete alternatives must be chosen.

Group Characteristics: The family is an ongoing, small, primary, and cohesive group.

Given these characteristics, and the relative importance that each individual places on the choice, the household may be expected to resolve the conflict in preferences in the following ways. First, suppose the decision is considered important by both members. Given the group characteristics listed above, and because the alternatives available are typically discrete, we may expect an effort to equalize gains over time through a *turn-taking* mechanism (i.e., $P_{HH} = 1 \cdot P_H + 0 \cdot P_W$ on occasion 1, $P_{HH} = 0 \cdot P_H + 1 \cdot P_W$ on occasion 2, and so on). In terms of equation *6*, the w_i's change predictably from purchase occasion to purchase occasion. A second possibility arises if the weights are constrained to be the same on each purchase occasion (i.e., $P_{HH} = w_H \cdot P_H + w_W \cdot P_W$ on every purchase occasion). The household compromises on each purchase occasion, and the relative preference intensities are reflected through the w's. Gupta and Steckel examine the case $w_H = w_W = 0.5$, labeled *strict compromise*. Note that if one of the family members places low importance on the choice, then the household's preferences will coincide with those of the member with higher preference intensity (e.g., if H places low importance on the choice $P_{HH} = 0 \cdot P_H + 1 \cdot P_W$). In this case the household essentially behaves as a single individual and the current approach for analyzing purchases should be sufficient. Finally, Gupta and Steckel examine a conflict resolution mechanism suggested by Kahn, Morrison and Wright (1986), labeled *decoupled superpositioning*. Consider the case in which both members place moderate to high importance on the choice (ruling out capitulation). However, instead of being constrained to make a single joint choice (as in turn-taking or strict compromise), each individual purchases according to his or her own preferences, as and when the need arises. This may be the case for a product category such as toothpaste, where consumption is not joint and the family budget permits each member to satisfy his/her own preferences. For such a household, the observed purchase string is a superpositioning of decoupled, individual buying processes. Following Kahn, Morrison and Wright (1986) assume that the individual interpurchase times are independent, identical, and exponentially distributed. Then, in terms of equation *6*, the household behaves as if a given individual's weight (say w_H) is 1 on some purchase occasions and 0 on others. However, unlike the predictable weight changes under turn-taking, w_H is 1 or 0 depending on the exponentially distributed interpurchase times. That is, decoupled superpositioning is a more "random" form of turn-taking.

Having enumerated these possibilities, the central question addressed by Gupta and Steckel is, "given only the household level purchase string, is it possible to infer the conflict resolution mechanism (turn-taking, strict compromise, or decoupled superpositioning), and the individual transition matrices?" The following example briefly demonstrates their approach.

EXAMPLE: Suppose the two individual transition matrices are:

$$P_H = \begin{array}{c|cc} & A & B \\ \hline A & 1 & 0 \\ B & 0 & 1 \end{array} \qquad P_W = \begin{array}{c|cc} & A & B \\ \hline A & 0 & 1 \\ B & 1 & 0 \end{array}$$

Next, suppose the conflict resolution mechanism is turn-taking and the first purchase is A. Then, because H always chooses the last brand purchased and W always switches, the resulting purchase string will be:

Household's Choice	*A A B B A A B B A A B B A A...*
Chosen by	*H W H W H W H W H W H W H*

The resulting transition matrix is:

$$P_{HH} = \begin{array}{cc} 0.5 & 0.5 \\ 0.5 & 0.5 \end{array}$$

First, note that the same household transition matrix would result under any of the other conflict resolution mechanisms, or even under simple coin-tossing. But, also note that each run in the household's string is of length 2! Such a pattern of run lengths could not have resulted under any of the other conflict resolution mechanisms. Recognizing this, Gupta and Steckel suggest an approach based on analyzing the *distribution of run lengths*. For 2-member households, their results show that:

i) it is possible to distinguish strict turn taking from other conflict resolution processes. Further, under strict turn taking, it is possible to accurately infer the underlying individual transition matrices.

ii) when household purchases are decoupled, it is possible to recover the individual transition matrices if both members seek variety or are reinforcers, or one family member is variety seeking and the other is a reinforcer. This result also implies that it is possible to recover the individual transition matrices whenever the family uses any conflict resolution process that is less random than decoupling.[5]

iii) classification is more accurate for longer purchase lengths

Gupta and Steckel's work shows that careful modeling of the conflict resolution mechanism can enable us to gain more precise insights than are possible when the group processes are ignored. Future research should:

i) develop more powerful tests and statistical theory for discriminating among the various processes.

ii) empirically apply this approach to various product categories and examine the results against those expected on the basis of the theory of variety seeking behavior.

iii) attempt to classify products according to conflict resolution processes. This should be especially helpful in validating and enriching the contingency approach discussed earlier in this section.

5.2 Interdependent Decisions in Marketing Channels

In many markets it is common to observe that members of the distribution channel are independently owned. In such conventional channel systems, typically, each party attempts to maximize it own profits (that is, each member behaves in a *locally* optimal manner). Also, it is often the case that if mechanisms for coordination among channel members could be found, system-wide efficiency could be increased. Further, such efficiency gains usually require that at

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[5] Recall that under decoupling, knowing who made the previous purchase provides no information about who will make the current purchase. Under strict turn taking the identity of the current purchaser is known with certainty. Other conflict resolution processes can, therefore, be represented paramorphically by varying the probability of who makes the next purchase. The ability to distinguish the least informative case (decoupling) from it equivalent zero order process suggests, therefore, the ability to also distinguish any other more systematic conflict resolution process.

least some members behave in a *locally sub-optimal* manner. For such situations, the marketing channels literature has largely focussed on the following questions:

a) what mechanisms, initiated by which channel member, are likely to lead greater channel efficiency?

b) under what conditions is coordination likely to increase the channel's efficiency?

However, important and related questions about the process by which channel members choose how to increase efficiency, and how to share in the joint gains are less well studied. To understand how the group choice literature can add to the marketing literature by providing models suitable for examining these questions, consider the manufacturer's problem of choosing quantity discounts. That is, how will the buyer-seller dyad arrive upon a quantity discount schedule that helps increase the profits of both?

Most previous research on quantity discounts focuses on a single decision maker, the seller, attempting to maximize his own profits (e.g. Lal and Staelin 1984, Dada and Srikanth 1987). The normative results of such an approach are appropriate if the underlying process is one where the seller presents a discount schedule to the buyer who then decides whether or not to avail himself of it. However, as Buffa (1984), Banerjee (1986) and Kohli and Park (1989) observe, questions of pricing and lot sizing are often settled through negotiations between the two parties. Consequently, models of cooperative or noncooperative bargaining, discussed above, should be useful in developing more realistic descriptions. Kohli and Park's (KP) modeling approach makes an important contribution in this regard.

In KP's formulation, the buyer and the seller jointly attempt to agree upon the quantity discount schedule. First, based upon the costs and profits of the two parties, the set of Pareto-optimal discount policies is determined. Then, the joint agreement is predicted by applying cooperative game-theoretic solution concepts to choose one policy from this set.

In terms of our classification scheme, KP's model treats the quantity discount problem as an intellective, single-issue, one-time decision between two parties who are certain about heir own payoffs and those of the other. On the basis of our classification scheme, the following extensions of their research seems promising:

i) KP assume bargaining over a single product. Often, however, the same buyer may purchase multiple products from the seller. KP's results could be used to make separate predictions of the discount schedule for each of the products. However, it will often be possible to increase the efficiency of the system even further by coordinating the purchases. In such cases the profits earned from uncoordinated purchasing would no longer be Pareto-optimal, and other discount schemes that help increase the gains of both parties could be found. Models of integrative, multiple-issue bargaining (e.g. Gupta 1989) could be used to predict how the parties would share in the additional gains from coordination. The no- discount and uncoordinated purchasing policies would represent the conflict and reference outcomes respectively. The final outcome would be expected to be Pareto-superior to both of these outcomes.

ii) For simplicity, KP assumed that the payoffs are known with certainty and that the game is played in a single period. Often, however, buyers and sellers transact business year after year with uncertainty about own and others' future costs. Also, this uncertainty is often resolved at the end of each year, making the relative gains from the current year's discount policy more apparent to both sides. For such situations, models in which past outcomes can be expected to influence future negotiations, and in which equity has an important role to play may provide a better description (e.g. Corfman and Lehmann 1987).

iii) In deriving the expected discount policy, KP assumed only that it be Pareto-optimal. KP then answer the question of which among the several Pareto-optimal policies will be chosen by separately applying various solution concepts. But, is it possible to rule out some of these solution concepts and get a more definite prediction?

It may be possible to do so by making other assumptions, in addition to Pareto-optimality, to which bargainers can reasonably be expected to adhere. For example, suppose that the buyer is able to reduce her costs and thus increase her profitability for every possible discount policy.

Then, it may be reasonable to expect that the chosen discount policy will be such that the buyer's profits do not decrease. But this assumption is the same as Kalai and Smorodinsky's *individual monotonicity* axiom (*A5*). Consequently, we should not expect the dyad to agree upon the discount policy represented by the Nash outcome. Or, in times of increasing sales and profits, it may be reasonable to assume that the previous agreement among the same dyad would play the role of a reference point over which both parties can Pareto-improve. In this case, the Gupta-Livne solution might be appropriate.

iv) KP's analysis assumes a single buyer and seller. Suppose, now, that there are multiple buyers, each with an idiosyncratic utility function. Further, suppose it is possible for the buyers to form a cooperative and bargain with the seller as a coalition. What quantity discount structure can be expected to emerge? How will the gains from coalition formation be shared among its members? Models of coalition formation could provide useful insights. Alternately, by representing the agreements that *might* be reached with other buyers as the seller's outside options, Binmore, Shaked and Sutton's (1989) noncooperative bargaining model with outside options may also prove useful.

5.3 Strategic Decision-Making

While many models for strategic decision-making have been developed, most of these are normative models, and very few treat the selection of a strategy as a group decision. Typically, models of organizational decisions (other than purchasing decisions) have assumed, at least implicitly, that there is a single decision-maker, although it is clear that many of these decisions are made by groups. For example, many studies of competitive behavior have used the prisoner's dilemma to represent the situation (e.g., Burke 1988; Corfman and Lehmann 1990). Investigations using this game invariably involve only two people, each one representing a competitor. Conclusions have been drawn about how individuals behave in these games and what individual and situational characteristics affect behavior. However, we must not assume that the outcomes would be the same if groups were playing the competitive roles. The polarization literature alone implies that group behavior may be quite different from individual behavior in these settings.

Let us take the example of a small group of managers setting advertising budgets in a competitive environment, framed as a prisoner's dilemma with two alternatives--a high budget and a low budget. This task is probably viewed by management as a fairly intellective task. Although the success of a strategy cannot be known before its implementation, a "correct" answer is a strategy that results in acceptable profits. This suggests that "truth wins" rules may be used, with truth revealed through the input of experts and those in possession of pertinent information. If the task is familiar, expertise may be shared and, thus, play a less important role. This is also a risky and moderately uncertain task, because the probabilities associated with possible competitor actions are not known. Further, the decision is important. Thus, majority and voting rules may be used to spread responsibility and, again, expertise and information are likely to be important power sources when not possessed by all members. Because feedback on the outcomes of similar past decisions is likely to be available, the group can learn about the competition and improve their predictions, making the task less uncertain and encouraging the use of compromise models (e.g., weighted probability). Feedback is also likely to reinforce the influence of true experts. The fact that this is an ongoing institutional group makes the use of impersonal power sources (e.g., legitimate and expert power) more likely, although primary ties and informal power structures may have evolved resulting in the use of personal and, perhaps, destructive power sources.

Given that we want a model for a small group to make a single choice between discrete alternatives and that it is important to examine the roles of expertise, legitimacy, and, perhaps, other sources of power, a variation on the weighted linear model would be a good place to start. If expertise is shared by the members or competitor actions are extremely uncertain (resulting in high perceived risk), majority and plurality models may do an adequate job of predicting. Although the prisoner's dilemma structures the alternatives as discrete, in reality they lie on a continuous dimension making compromise a possibility if judgments conflict. If we allow continuous alternatives, the weighted Nash model might be appropriate.

6. Conclusions

It should be clear from this review that there is a wealth of research on group choice in other disciplines that researchers in marketing have only begun to tap. Opportunity to benefit from and extend this work exists in a number of areas. Normative models of group choice and bargaining could be further explored for their appropriateness as foundations for descriptive modeling. Examples are modeling the weights in the asymmetric Nash model (Neslin and Greenhalgh 1983) or the reference alternative in the Gupta-Livne solution (1988) to reflect relative imbalances in power, information, and relationship factors. Social decision schemes should be further explored for their usefulness in describing management decision-making behavior in common contexts. Wilson et al. (1991) have begun this task in the buying center. This paper does not review the wealth of research in social psychology on determinants of influence in group decision-making and bargaining (e.g., French and Raven 1959; Kelley and Thibaut 1978; Rubin and Brown 1975; Tedeschi, Schlender and Bonoma 1973). However, this work would be valuable input to the development of models and model typologies.

Above all, we feel research on group choice and negotiations should proceed with a focus on what makes different models better descriptors of choice behavior than others in different contexts. This approach is a good way to take advantage of existing research and provides structure for further investigation.

A final goal of this paper was to illustrate the importance and prevalence of group choice in marketing. Few choices are made by individuals truly independent of others and many are explicitly joint. Our examples in §5, were designed to illustrate the hazards of assuming that most choices are independent. As we discussed, purchase histories can be misinterpreted if they are assumed to belong to a single individual and actually reflect the preferences of multiple family members, buyers and sellers often negotiate issues that have been assumed to be set by one party and either accepted or rejected by the other, and strategy is more often formulated by formal or informal management teams than by individuals, which may result in inaccurate choice predictions for them.

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