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Depreciation and the "Ability-to-Replace"

Concept of Corporate Taxation

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I. THE NATURE OF THE PROBLEM

The question of what is an appropriate depreciation charge for corporate taxation has been the subject of much debate.1/ Even before the very high rates of inflation of the past few years, the adequacy of a depreciation charge based on historical cost was often questioned. Consider, for example, this 1959 comment by Pierre F. Goodrich, then president of the Indiana Telephone Corporation:

> Since the Internal Revenue Code does not recognize operating costs measured in current dollars, they are not deductible for computing federal income tax payments, and the corporation in fact pays taxes on alleged earnings which do not exist in true purchasing power. If they were deductible, as they should be, reductions in federal income taxes,...of $266,000 in 1971 and $252,000 in 1970 would result. By requiring the use of the Uniform System of Accounts for utility accounting and by virtue of the Internal Revenue Code, the Government has condemned and confiscated during the last seven years over $1 million (in terms of dollars of the years in which they were paid) of the assets of this corporation through taxation of overstated earnings.2/

With the rising rates of inflation of the past few years, the belief that depreciation charges currently allowed are inadequate and ought to be significantly increased to reflect price-level changes has become even more widespread. Reginald H. Jones, chairman and chief executive officer of General Electric Company, writes:

> The money for capital investment comes from four sources: internally from depreciation and reinvested earnings, and externally from debt and equity. Business is in trouble on all four counts.
>
> In a period of inflation, depreciation allowances do not provide sufficient capital recovery to cover the cost of replacement at inflated prices. And these capital-cost recoveries this year are going to be four to six times as significant as retained earnings in the internal flow of funds.3/

Jones goes on to suggest increasing depreciation rates to keep up with the rising cost of replacement.[4/] A more rational approach, although more radical in terms of the existing structure of the Internal Revenue Code, is to base the depreciation charge for tax purposes on the price-level adjusted cost of the asset rather than on the historical cost. This suggestion has been made by a number of accountants and economists (as well as businessmen), including Edgar O. Edwards and William A. Paton. The depreciation charge to be applied for tax purposes would be that obtained by price-level restatement (as called for by APB Statement 3) of the depreciation charge based on historical cost. For example, the annual depreciation charge for both ordinary and price-level adjusted straight-line (S-L) depreciation for a $500 asset with a five-year service life and no salvage value is shown in Table 1. This example assumes a 10 percent rate of inflation during the second year, with a constant price level thereafter.

TABLE 1

Annual Depreciation Charges
for a $500 Asset with a Five-Year Service Life
Obtained Using Ordinary Straight-Line and
Price-Level Straight-Line Methods

Year	GNP Deflator	Ordinary S-L	Price-Level S-L
1	100	100	100
2	110	100	110
3	110	100	110
4	110	100	110
5	110	100	110
Total depreciation (in nominal dollars)		500	540

The recovery of \$540 at the end of the fifth year using price-level adjusted, straight-line depreciation does not correspond to the recovery of the historical cost of the asset (\$500) expressed in dollars of equivalent purchasing power at the end of the service life (\$550). Despite the fact that Jones and others look to the depreciation allowance for "sufficient capital recovery to cover the cost of replacement at inflated prices," the difference between the total depreciation using price-level S-L and the full replacement cost (a difference of \$10 in this case) is dismissed by both Professor Edwards and Professor Paton. Referring to this deficiency, Professor Paton notes:

> It should be pointed out, however, that there is no strong case for permitting an additional deduction in the amount of such "deficiency" in situations where it can be assumed that each year a portion of the revenue funds received equal to the depreciation deduction for the year adjusted to a current cost basis is promptly invested in productive plant, and that all resources are being employed either on a free competitive market, or--in the area of public utilities--in a regulatory climate which takes account of changes in the price-level and accepts the conditions of the competitive market as the regulatory standard....5/

Professor Edwards likewise argues that the reasoning behind the recovery of the full replacement cost is faulty:

> This line of reasoning is quite correct so long as funds equal to the depreciation charges are held in the form of cash when not used for immediate replacement.
>
> This, I submit, assumes an unnecessary rigidity in the decision-making process of the individual firm. There seems little justification for assuming that the firm must invariably select a particular mode of holding funds. Holding funds in the form of cash is but one alternative open to the firm....

> In any event, given a certain amount of funds, there must be a variety of forms in which these funds can be held so that purchasing power will remain intact.... If this is acknowledged, the case for exempting the full cost of actual replacement from taxation is buried.[6]

A closer examination of these arguments reveals that the same view of depreciation--to allow for recovery of replacement cost--held by Goodrich and Jones is held also by Edwards and Paton. However, Edwards and Paton also look at the return earned on revenues shielded from taxation by the depreciation charge as an additional source of capital recovery for the one specific case where the returns just keep pace with the rate of inflation. Interestingly, this argument has not been applied to the case in which the returns either do not keep up with inflation or exceed the rate of inflation. If the logic is valid in the one special case these authors cite, it should also serve as the basis for a general approach to determine an appropriate depreciation charge for corporate taxation.

This paper's objective is to present such an approach to depreciation which, by virtue of its focus on the relationship between the rate of return and the rate of inflation, might provide an economic benchmark against which to measure proposed changes in the Code. Once the returns earned on the depreciation charges are introduced, a very different picture may well emerge. For example, Mr. Ernest Henderson, III, then president of the Sheraton Corporation of America, has given this view of depreciation:

> A new ten million dollar building, normally assigned a 50-year life, would require--if straight-line depreciation at 2 percent a year were used--$200,000 each year for depreciation. In fifty years the building, having then presumably reached the end of its economic life, could in theory be replaced by funded reserves which would amount to the ten million dollars originally invested in the building.
>
> However, the money set aside does not in practice lie idle. If reinvested at 4 percent after income taxes and compounded annually for fifty years, these cash reserves, instead of the original $10,000,000 could aggregate to $30,533,000, a very generous sum even after taking into account increased reproduction costs expected in an inflationary economy.
>
> Assuming the depreciation reserves are earned, as they have been in Sheraton's past twenty-four year experience, and that these reserves are not required for debt amortization (on the theory that when property values are maintained, debt amortization is customarily replaced with new debt), and assuming that the annual depreciation reserves can continue to be readily invested and reinvested profitably during the fifty-year life of a new property at a rate better than 10 percent annually, then the cash ultimately realized from these reserves would amount to more than $232,781,000. At a 15 percent rate, which corresponds more closely to Sheraton's past twenty-four year actual experience, the amount becomes much larger, taxing the capacity of company calculating machines.[7]

Thus apparently Henderson would answer differently the question of the adequacy of the conventional depreciation charge. Whereas Goodrich and Jones are concerned about the real transfer of wealth from the stockholders to the government during inflation, Henderson recognizes the simultaneous transfer of wealth from the general public and the creditors to the stockholders. Both aspects must, of course, be part of any determination of the damage resulting from the conventional method of depreciation which reflects neither changes in purchasing power nor gains earned by the corporation on the revenues shielded from taxation by the depreciation charges.

Because the method of depreciation presented in this paper is able to deal with both of these aspects, it has certain interesting properties:

1. Assuming the purchase of identical assets and an identical capital structure, a firm whose return on investment exceeds the rate of inflation will obtain a depreciation deduction less than that for a firm whose return on investment is less than the rate of inflation.
2. Assuming the purchase of identical assets and identical revenues and expenses, a firm with a greater amount of debt in its capital structure will obtain a lower depreciation deduction than a firm with less debt.

Application of this method would thus introduce an ability-to-replace concept in corporate taxation which would play the same role as the ability-to-pay concept in the individual income tax. With the exception of the surtax exemption, such a concept has not been incorporated into the corporate tax structure, in part perhaps because of the realization that magnitude of income alone is a poor measure of a corporation's ability to pay.

In effect, we are proposing that the definition of the depreciation allowance, as given for example in Treasury Regulation § 1.167-1(a), be modified to read "that amount which should be set aside for the taxable year in accordance with a reasonable plan (not necessarily at a uniform rate) so that the aggregate of the amounts set aside, including the gains arising from investment of the receipts which have been shielded from taxation as a result of such depreciation allowance, plus the salvage value, will, at the end of the estimated useful life of the depreciable property, equal the cost or other basis of the property in dollars of equivalent purchasing power...."

To return to the example of the $500 asset considered earlier, the depreciation charge obtained by application of this proposal would depend on the actual use to which the after-tax cash throw-off from the asset was put. Table 2 shows the depreciation charge obtained in each of three cases.

Case I. The depreciation charge is held in the form of cash (and thus a real monetary loss is experienced).

Case II. The funds are invested in a real asset at the beginning of each year, which is sold at the end of the year at a price which just keeps pace with the price-level change during the year (and thus no real gain or loss is experienced).

Case III. The funds are invested in a real asset at the beginning of each year, which is sold at the end of the year at a price which yields a 10 percent after-tax rate of return each year (and thus a real gain is experienced in the third through fifth years).

TABLE 2

Annual Depreciation Charges for a $500 Asset with a 5-Year Service Life, Obtained Using Proposed and Price-Level Straight-Line Methods

Description of Case		Method of Calculation: Proposed Method Ex-Post	Method of Calculation: Proposed Method Ex-Ante	Method of Calculation: Price-Level S-L
Case I: Hold Cash				
Year	GNP Deflator			
1	100	$100.00	$100.00	$100.00
2	110	112.50	158.01	110.00
3	110	112.50	97.33	110.00
4	110	112.50	97.33	110.00
5	110	112.50	97.33	110.00
Total depreciation		$550.00	$550.00	$540.00
Total after-tax gain		0	0	0
Sum of depreciation and gain		$550.00	$550.00	$540.00
Case II: No Real Gain or Loss				
Year	GNP Deflator			
1	100	$100.00	$100.00	$100.00
2	110	110.00	126.20	110.00
3	110	110.00	104.60	110.00
4	110	110.00	104.60	110.00
5	110	110.00	104.60	110.00
Total depreciation		$540.00	$540.00	$540.00
Total after-tax gain		10.00	10.00	10.00
Sum of depreciation and gain		$550.00	$550.00	$550.00
Case III: Constant 10 Percent Gain				
Year	GNP Deflator			
1	100	$100.00	$100.00	$100.00
2	110	86.96	126.20	110.00
3	110	86.96	71.18	110.00
4	110	86.96	71.18	110.00
5	110	86.96	71.18	110.00
Total depreciation		$447.84	$439.74	$540.00
Total after-tax gain		102.16	110.26	116.92
Sum of depreciation and gain		$550.00	$550.00	$656.92

Three charges are given for each case; the first being the depreciation charge which would result from the proposal made in this paper if all returns and price-level changes are known with certainty (the ex-post value); the second is the corresponding charge obtained by means of an estimation method (described at length in the paper) adopted to provide the annual depreciation charge in the face of uncertain future rates of return and rates of inflation (the ex-ante value), and the third being the corresponding price-level S-L charges and returns, which is shown for comparison. It should be noted that the replacement cost is recovered in each case by the method suggested in this paper, using either ex-post or ex-ante values. For Case I, where the rate of inflation exceeds the rate of return, the proposed method provides a greater total capital recovery (the sum of the total gain and total depreciation) than the price-level adjusted S-L method. The total capital recovery is the same for both methods in Case II where the rate of return just equals the rate of inflation (the situation assumed by Professors Paton and Edwards). Finally, the total capital recovery is less under this method than it is in the price-level adjusted S-L method in Case III, where for the last three years the rate of return exceeds the rate of inflation.

The question of the appropriate statistical measure of both the rate of inflation and the economic service life of various assets will not be considered in this paper, although these questions deserve further study. For the purpose of this paper adequate approximations are available--the ratio of the difference between two successive years' GNP Implicit Price Deflator to that of the earlier year, and the service-life times in the 1962 Guidelines, respectively.[8/]

The problems which would arise in interfacing this method with the existing provisions of the Code are also not considered. Incorporation of price-level changes for depreciation, whether by use of the price-level S-L method or the method suggested here, would likely raise difficult questions if similar treatment were not incorporated throughout. The objective of this paper is to explore the logical consequences of the view that an appropriate depreciation charge is one which would ensure a firm the ability to replace (a view which is held by very many, including the authors quoted above). In order to pursue this investigation we must develop an approximation method which will allow measurement of the gains a firm earns on the revenue shielded from taxation by the depreciation charges.

II. A SIMPLE EXAMPLE

We shall begin our analysis by considering an especially simple example--that of a firm with a single asset costing B_o dollars and having a service life of N_o years with negligible salvage value and no debt in the capital structure of the firm. We also suppose a constant annual rate of inflation, p, and a uniform stream of receipts of R_o dollars before depreciation and taxes, which are the only expenses. For convenience, suppose the receipts are received at the end of each year of the asset's service life and the after-tax sum invested externally in fixed interest securities yielding an after-tax interest rate, k.

In accordance with the proposal stated previously, the uniform depreciation charge D_o (in recognition of the uniform stream of receipts and rates of return and inflation) is given by

$$D_o F_{N_o}(k) = B_o(1+p)^{N_o} , \tag{1}$$

where

$$F_{N_o}(k) = \frac{(1+k)^{N_o} - 1}{k} , \tag{2}$$

is the future value of an annuity of \$1 at an interest rate k for N_o years. The left-hand side of Equation (1) represents the sum of the depreciation charges and the after-tax returns earned on these charges, while the right-hand side represents the original cost of the asset in equivalent purchasing power as of the end of the asset service life. From Equation (1), the charge D_o may be found:

$$D_o = \frac{B_o(1+p)^{N_o}}{F_{N_o}(k)} . \tag{3}$$

The depreciation charge D_o is, aside from the price-level adjustment, the premium called for in the sinking-fund method of depreciation.[9/] The accumulated depreciation at the end of n years, $\Delta_{o,n}$ is defined to be the sum of the depreciation charges plus the return earned on these charges:

$$\Delta_{o,n} = D_o F_n(k) . \tag{4}$$

The annual interest earned on the accumulated depreciation is not added to the depreciation premium given by Equation (3) to obtain the total depreciation charge (as would be done with the sinking-fund method). Instead, the asset value is restated to total $B_o(1+p)^n$, and the difference, $pB_o(1+p)^{n-1} - k\Delta_{o,n-1}$, is credited to earnings and profits at the end of the year n.

Asset $pB_o(1+p)^{n-1}$

Depreciation Expense D_o

[Accumulated Depreciation] $D_o + k\Delta_{o,n-1}$

[Earnings and Profits] $pB_o(1+p)^{n-1} - k\Delta_{o,n-1}$.

By construction, the accumulated depreciation at the end of the service life, $\Delta_{o,No}$, just equals the cost of the asset in dollars of constant purchasing power, $B_o(1+p)^{No}$, and if N_o is an accurate estimate of the economic service life, the asset would then be abandoned.

Further insight into the economic meaning of the depreciation charge D_o may be obtained by looking at the situation from a rather different viewpoint. It is possible to view the problem of depreciation as arising from the fact that the income tax is collected annually, rather than once every N_o years--a period commensurate with the service life of the asset. If we did not have to determine an annual income and did not have to pay an annual tax on it, an amount,

$R_oF_{No}(k)$, would be available at the end of N_o years. Measuring the cost of the asset in dollars of equivalent purchasing power, we would have a net gain as measured at the end of the service life of

$$\text{Gain} = R_oF_{No}(k) - B_o(1+p)^{No} , \tag{5}$$

with a resulting tax liability

$$T_o = t[R_oF_{No}(k) - B_o(1+p)^{No}] . \tag{6}$$

Although the corporation need not pay this liability until year N_o, it might earmark T dollars of the funds invested each year for the purpose of satisfying this obligation at the end of year N_o

$$T = T_o/F_{No}(k) , \tag{7}$$

or, on substituting the value of T_o from Equation (6) the required amount would be

$$T = t(R_o - D_o) , \tag{8}$$

where D_o is given by Equation (3). From this point of view D_o is simply that amount which must be deducted from R_o in order that the resulting amount thus set aside for investment, T, has the conventional form of an annual tax liability. From this derivation we see that use of Equation (3) is economically equivalent to subjecting the gains on investment to a double taxation, for, in addition to the tax liability imposed on the annual interest earned each year, the liability T_o is imposed on both the revenues and the total interest earned, although this liability is deferred until the end of the asset's service life. Whether the benefit of the effective deferral of the liability outweighs the cost of having the gains on the external investment subject to taxation a second time depends on the parameters.[10/] It is more meaningful to compare the total after-tax return obtained on use of Equation (3) with the often-suggested price-level adjusted S-L method in which D_i, the depreciation charge for year i, is given in the case of a uniform rate of inflation, by

$$D_i = \frac{B_o(1+p)^i}{N_o} \qquad i=1 \rightarrow N_o \ . \tag{9}$$

The total after-tax return obtained using D_i, TR'_{S-L}, is found to be

$$\begin{aligned} TR'_{S-L} &= (1-t)R_oF_{No}(k) + t\sum_{i=1}^{No} D_i(1+k)^{No-i} \\ &= (1-t)R_oF_{No}(k) + \frac{tB_o}{N_o}(1+k)^{No}(1+q)F_{No}(q) \ , \end{aligned} \tag{10}$$

where $q = (p-k)/(1+k)$. The total after-tax return using depreciation charge D_o, TR'_o, is likewise given by

$$\begin{aligned} TR'_o &= (1-t)R_oF_{No}(k) + tD_oF_{No}(k) \\ &= (1-t)R_oF_{No}(k) + tB_o(1+p)^{No} \ . \end{aligned} \tag{11}$$

Comparison of expressions (10) and (11) shows that TR'_{S-L} is less than TR'_o if F_{No} (q) is less than $N_o(1+q)^{No-1}$. Since

$$F_{No}(q) = \sum_{i=o}^{No-1} (1+q)^i \ , \tag{12}$$

we see that TR'_{S-L} is less than TR'_o for $q > 0$, or $p > k$, but that TR'_o is less than TR'_{S-L} for $q < 0$ or $k > p$. Thus, as already noted, our method yields a greater total after-tax return than the often suggested price-level adjusted, straight-line method if the after-tax return on investment is less than the annual rate of inflation, and it is less than the total return from the price-level adjusted, S-L method if the after-tax return exceeds the rate of inflation. A more striking comparison than the example of the previous section is shown in Table 3, where the total after-tax return, less the common portion $(1-t)R_oF_{No}(k)$, is calculated for the example discussed by Henderson--a \$10 million hotel with a service life of 50 years with $t = .5$ and an after-tax rate of return on investment $k = 10$ percent for various rates of inflation. The returns obtained using the usual S-L and S-Y-D methods are also given for comparison.

TABLE 3

Difference in Total After-Tax Return Obtained Using Different Methods of Depreciation (in Millions of Dollars)

Method of Depreciation Used	Percentage Rate of Inflation (p)				
	0	5	10	15	20
Ordinary straight-line	116	116	116	116	116
Sum-of-years digits	189	189	189	189	189
Price-level adjusted S-L	116	222	587	2,226	10,778
Proposed method	5	57	587	5,418	45,502

$k = 10$ percent, $t = .5$, $B_o = \$10$ million, $N_o = 50$ years.

III. THE MODEL EXTENDED

Although the example considered in the previous section was useful in illustrating our method of depreciation, the after-tax cash throw-off from the original investment is more likely to be invested internally, rather than externally. Let us therefore suppose that no external investment opportunity exists; instead the firm acquires a second asset at the end of year $M < N_o$, costing B_M dollars with a service lifetime N_M and negligible salvage value. For convenience we shall first suppose $M + N_M \geq N_o$, that uniform receipts, R_M, are obtained at the end of each year of the second asset's service life, and that any additional funds needed in addition to the after-tax cash throw-off available from the first asset, MR'_o, are obtained by stockholder investment.

From the previous discussion it is clear that the depreciation charge on the original asset cannot be uniform because of the non-uniformity of the investment opportunities, but the discussion did not indicate how the varying charges are to be obtained. The following procedure is chosen: the depreciation charge is calculated using only the data available from completed transactions--thus the depreciation charge for the original asset for each year, j, during the first M years is to be that based on the zero rate of return earned to date:

$$D_{o,j} = \frac{Bo\ (1+p)^{No}}{N_o} \qquad j=1\rightarrow M\ . \tag{13}$$

Assuming no investment of the cash throw-off from the second asset, the same argument leads to the depreciation charge for the second asset at the end of year j:

$$D_{M,j} = \frac{B_M(1+p)^{N_M}}{N_M} \qquad j=M+1\rightarrow M+N_M \ . \tag{14}$$

Associating that fraction of the after-tax return obtained from the second asset representing the ratio of funds $MD_{o,1}$ provided by depreciation of the original asset to the cost B_M, the same reasoning which led to Equation (1) now suggests that $D_{o,j}$ for $j=M+1\rightarrow N_o$ be given by:

$$(N_o-M)D_{o,M+1} + \left(\frac{M\ D_{o,1}}{B_M}\right)(N_o-M)R'_M = B_o(1+p)^{No} \ , \tag{15}$$

where we have set $D_{o,1}$ to be the constant value of the depreciation charge for years $j=1\rightarrow M$ given by Equation (13) and $D_{o,M+1}$ to be the constant value of the depreciation charge for years $j=M+1\rightarrow N_o$ being sought. From Equation (14) the after-tax cash throw-off from the second asset R'_M may be found to be

$$R'_M = (1-t)R_M + \frac{tB_M(1+p)^{N_M}}{N_M} \ . \tag{16}$$

If we define an after-tax rate of return, k_M, corresponding to the external rate of return, k, used in the previous section by

$$R'_M(N_o-M) = B_M(1+k_M)^{No-M} \ , \tag{17}$$

we may write Equation (15) as

$$D_{o,M+1} = \frac{Bo(1+p)^{No}-M\ Do,1(1+kM)^{No-M}}{N_o-M} . \quad (18)$$

Equations (13) and (18) are the desired expressions for the depreciation charge on the original asset in terms of the set of rates of return on investment applicable during the asset's service life. If $N_M + M < N_o$, Equation (18) would still apply, but Equation (17) would be replaced by

$$R'_M N_M = B_M(1+k_M)^{No-M} . \quad (19)$$

In this example the return on investment suddenly increases from zero to some value k_M. For positive k_M, the final return on the depreciation charges taken prior to the acquisition of the second asset, $D_{o,1}$, will thus be too large, and $D_{o,M+1}$ as given by Equation (18) will be negative. This is a rather extreme example, in that an opportunity to obtain a positive return on investment exists only in year M, and as such gives rise to rather anomalous results, as shall be discussed later. Nevertheless, the general approach taken does result in depreciation recapture when the rate of return is significantly increased. Conversely, an increased depreciation charge is obtained in the case of a firm whose high rate of return is no longer sustained. It would be more desirable to base the depreciation charge on a rate of return obtained by using a moving average of the past rates of return, perhaps over a time span equal to the asset's service life N_o;

except in unusual cases such an averaged rate of return would result in less fluctuation in the depreciation charge. Such modifications are left to a later paper in the belief that the features of this general approach will be best understood by examination of the unmodified results.

The above analysis is based on the assumption of a uniform stream of receipts, R_o and R_M, and a uniform rate of inflation, p, being known with certainty. We may remove this restriction by the following device: at the end of year n we shall estimate the future rate of inflation and future rates of return by the year n values. We define

$$P_{M,n} = \prod_{j=M+1}^{n} (1+p_j) , \tag{20}$$

where p_j is the known rate of inflation for years $j \leq n$. We also define the future value in year N_o of the accumulated depreciation by

$$\Delta_{o,n}^{No} = \sum_{j=M+1}^{n} \{D_{M,j} \prod_{\ell=j+1}^{n} (1+k_{j,\ell})(1+k_{j,n+1})^{No-n}\} , \tag{21}$$

where $D_{M,j}$ is the depreciation charge for year j on the asset acquired in year M and $K_{j,\ell}$ is the after-tax rate of return in year l on the investments made in year j ($k_{j,j}$=0). We may approximate Equation (18) using these quantities:

$$\Delta_{o,n}^{No} + D_{o,n+1}F_{No-n}(k_{n,n+1}) = B_o P_{o,n}(1+p_{n+1})^{No-n} . \tag{22}$$

Note that Equation (22) is in the form of a regression equation--$D_{o,M+1}$ being determined in terms of past values of $D_{o,j}$ for $j=1 \rightarrow n$.

The required rates of return $k_{j,n+1}$ for $j=0 \rightarrow n$ are, themselves, given by a similar set of regression equations.

Before examining these auxilliary equations, we note the basic assumptions made in obtaining Equations (21) and (22):

1. The future rates of return on the past year's investments are assumed to equal the current rates of return on such investments.
2. The future rates of return on future investment are assumed to equal the current rate of return on the current year's investment.
3. The future rate of inflation is assumed to equal the current year's rate of inflation.

The first two approximations are also incorporated in the equation for the after-tax rates $k_{i,j}$. We define

$$K_{M,n} = \prod_{j=M+1}^{n} (1+k_{M,j}) \quad , \tag{23}$$

and

$$\Omega_{M,n} = \sum_{j=M+1}^{n} R'_{M,j} \prod_{\ell=j+1}^{n} (1+k_{j,\ell}) \tag{24}$$

as counterparts to $P_{M,n}$ and $\Delta_{M,n}$ where $R'_{M,j}$ is the after-tax receipts in year j from operation of the asset acquired in year M. We likewise define the future value in year N_o of $\Omega_{M,n}$ by

$$\Omega^{No}_{M,n} = \sum_{j=M+1}^{n} \left\{ R'_{M,j} \prod_{\ell=j+1}^{n} (1+k_{j,\ell})(1+k_{j,n+1})^{No-n} \right\} \quad , \tag{25}$$

as the counterpart to $\Delta^{No}_{M,n}$. If $j > M+N_M$ in the sum on the right-hand side in Equations (24) and (25) (i.e., if the economic life of the asset acquired in year M has expired), we can understand $R'_{M,j} = 0$. In terms of these factors the recursion relations for $k_{j,n+1}$ in terms of the set $\{k_{j,n}\}$ for $j=1 \to n-1$, for the assumptions noted, are

$$\Omega^{No}_{M,n} + R'_{M,n+1}F_{NM'-n}(k_{n,n+1})(1+k_{n,n+1})^{No-N_M'} = B_M K_{M,n}(1+k_{M,n+1})^{No-n} , \tag{26}$$

where we have set $N'_M = N_M + M$ if $N_M + M < N_o$ and $= N_o$ otherwise. The time horizon N_o enters into the above equation for the rates of return, so that an appropriately modified set of equations would have to be used for each depreciable asset. Since the depreciation charge enters into the determination of the after-tax cash throw-off $R'_{M,j} = (1-t)R_{M,j} + tD_{M,j}$, the complete set of Equations (22) and (26) (as modified for each asset) must be solved recursively.

In order to apply Equations (22) and (26) to the example considered in this section, we must consider holding cash as equivalent to the purchase of fixed interest securities yielding zero interest and with a one-year maturity. For each year in which this cash is not applied to any productive investment, the calculated rate of return will vanish:

$$R'_{n,n+1}(1+k_{n,n+1})^{No-(n+1)} = B_n(1+k_{n,n+1})^{No-n} , \tag{27}$$

but $R'_{n,n+1} = B_n$ = amount of cash held, so $k_{n,n+1} = 0$. The depreciation charge for the asset acquired in year M will thus be the same as that given by Equation (14). The calculated rates of return $k_{j,\ell}$ for years $j=1 \rightarrow N_o$ for the example of internal investment obtained by application of Equation (26) are shown in Table 4. In this table, the lifetime of the second asset is chosen such that $N_M + M \geq N_o$.

TABLE 4

Rates of Return $k_{j,\ell}$ for Internal Investment Model

j \ l	$l \leq M$	$l = M + 1$	$l = M + 2 \rightarrow No$
$j = 1 \rightarrow M$	0	$\hat{k}_M$	k^*_M
$j = M+1 \rightarrow No$	0	0	0

The parameters $\hat{k}_M$ and k^*_M which appear in Table 4 are given by

$$R'_M F_{No-M}(\hat{k}_M) = B_M(1+\hat{k}_M)^{No-M} , \tag{28}$$

and

$$(N_o-M)R'_M = B_M(1+\hat{k}_M)(1+k^*_M)^{No-(M+1)} . \tag{29}$$

Using Equation (17) for the ex-post value k_M, we may write equations (28) and (29) as

$$P_{No-M}(\hat{k}_M) = (N_o-M)/(1+k_M)^{No-M} , \tag{30}$$

and

$$(1+k_M^*)^{No-M} = \left(\frac{1+kM}{1+\hat{k}_M}\right)(1+k_M)^{No-M} , \tag{31}$$

where $P_{No-M}(\hat{k}_M) = (1+\hat{k}_M)^{-(No-M)} F_{No-M}(\hat{k}_M)$ is the present value of an annuity of \$1 for N_o-M years at a discount rate $\hat{k}_M$. It should be noted that the ex-ante value $\hat{k}_M$ overestimates the ex-post value k_M because it is calculated on the assumption that the future cash throw-off will be reinvested at the same rate of return, when in fact the example provides no investment opportunities after year M; k_M^* underestimates k_M because the recursion relations (22) and (26) are compensatory--that is, errors of under-valuation are followed by errors of over-valuation, and vice-versa. For numerical illustration, the value of $\hat{k}_M$ and k_M^* are presented for various values of N_o-M if k_M = 10 percent; see Table 5.

TABLE 5

Comparison of the Ex-Ante Values $\hat{k}_M$ and k_M^* (in Percentage) with the Ex-Post Value k_M = 10 Percent

	Number of Years Remaining (N_o - M)			
	5	10	25	40
$\hat{k}_M$	18	22	43	133
k_M^*	8.5	8.8	8.8	8.2

The rather excessive values of $\hat{k}_M$ for large values of $N_o - M$, together with the rapid recovery (within one year) of the values K_M^* to the ex-post value K_M in response to the step-function increase in the rate of return (from zero to k_M) result from the method of estimation chosen. As in the corresponding problem of negative depreciation, some modifications to the approach presented here might be desirable; again we postpone such considerations to a later stage of this method's development. As we shall see, the calculated ex-ante values of the depreciation charge tracks the ex-post values quite well despite the rather extreme values of $\hat{k}_M$, and, insofar as the rates of return as obtained here are needed only for this purpose, this problem is less serious than would appear.

The application of Equation (22) to the depreciation charge for the original asset using the above results is easily seen to yield:

$$D_{o,j} = \frac{Bo(1+p)^{No}}{N_o} = D_{o,1} \qquad j=1\rightarrow M,$$

$$\hat{D}_{o,M} = \frac{Bo(1+p)^{No} - MDo,1(1+\hat{k}_M)^{No-M}}{F_{No-M}(\hat{k}_M)} \qquad j=M+1\ ,$$

$$D^*_{o,j} = D_{o,M+1} + \frac{(D_{o,M+1} - \hat{D}_{o,M+1})}{N_o - M - 1} \qquad j=M+2\rightarrow N_o$$

where $D_{o,M+1}$ is the ex-post value as given by Equation (18).

By way of numerical illustration, we exhibit in Table 6 the ex-ante values of the depreciation charges $\hat{D}_{o,M+1}$ and $D^*_{o,M+S}$

corresponding to the case B_o = \$10 million, N_o = 50 years, and k_M = 10 percent for various values of p and M. The ex-ante depreciation charges shown in Table 1 for the example in the first section were obtained in a similar manner.

The depreciation recapture as indicated in Table 6 is a natural consequence of our basing the depreciation charge in part on the rates of return, so that a significant increase in such rates of return, as in the example considered, triggers the subsequent recapture of excess prior years' depreciation. Although the reader may find this result disturbing, it should be noted that the converse situation, a significant decrease in rates of return, would result in a greater depreciation charge; it would be hard to justify accepting this later result without also accepting the former.

The results shown in Table 6 are as expected--the magnitude of the ex-ante value $\hat{D}_{o,M+1}$ over-shoots somewhat the magnitude of the ex-post value $D_{o,M+1}$, and the magnitudes of the subsequent ex-ante values $D^*_{o,j}$ are thus slightly less than the magnitude of $D_{o,M+1}$. It is remarkable that, despite the rather excessive values of the ex-ante rate of return $\hat{k}_M$ as compared to the ex-post value k_M, the ex-ante depreciation charge $\hat{D}_{o,M+1}$ is not very different from the ex-post value $D_{o,M+1}$. Since a sudden 10 percent jump in the rate of return is likely to be uncommon, we could anticipate that for most cases the ex-ante results would track the ex-post values even better.

TABLE 6

Comparison of Ex-Ante with Ex-Post Values of the Depreciation Charge (in Millions of Dollars)

Case	Rate of Inflation (p) (in Percentage)		
	5	10	15
I. $M = 10$, $N_o - M = 40$			
$D_{o,1}$	2.3	23.5	217
$D_{o,M+1}$	(23.2)	(236)	(2,184)
$\hat{D}_{o,M+1}$	(26)	(266)	(2,452)
$D^*_{o,j}$	(23.1)	(235)	(2,177)
II. $M = 25$, $N_o - M = 25$			
$D_{o,1}$	2.3	23.5	217
$D_{o,M+1}$	(22.3)	(208)	(1,910)
$\hat{D}_{o,M+1}$	(25)	(253)	(2,333)
$D^*_{o,j}$	(22.2)	(206)	(1,892)
III. $M = 40$, $N_o - M = 10$			
$D_{o,1}$	2.3	23.5	217
$D_{o,M+1}$	(12.3)	(126)	(1,152)
$\hat{D}_{o,M+1}$	(15)	(159)	(1,471)
$D^*_{o,j}$	(12)	(122)	(1,116)

k_M = 10 percent, B_o = \$10 million, N_o = 50 years.

IV. THE REVENUE ALLOCATION PROBLEM

In the previous section the problem of the uncertainty of future receipts, rates of return, and rates of inflation was faced, but the receipts received at the end of year n + 1 associated with the individual assets were treated as known. In practice, of course, the allocation of total receipts to the individual assets is not known. If we first subtract R^S_{n+1}, the total of those receipts arising from specific investments (such as the return on a fixed income security, the holding of cash, the gain on sale of a specific asset, etc.), from the total receipts, R^T_{n+1}, we are left with an amount $R^T_{n+1} - R^S_{n+1}$ which must be allocated in some approximate fashion to the individual assets. Thus, we adopt the following approximate method of allocating this balance to the individual assets (excepting, of course, those whose return is included in R^S_{n+1}):

Assume that the fraction of the total receipts $R^T_{n+1} - R^S_{n+1}$ to be allocated to asset α, acquired in year M, with service life N_M, is given by

$$\frac{R^{\alpha}_{M,n+1}}{R^T_{n+1}-R^S_{n+1}} = \frac{\hat{R}^{\alpha}_{M,n+1}}{\sum\limits_{M=0}^{n} \sum\limits_{\alpha} \hat{R}^{\alpha}_{M,n+1}} \qquad n \leq N^{\alpha}_M + M , \tag{32}$$

where the revenue $\hat{R}^{\alpha}_{M,\ n+1}$ is given, for $n + 1 \leq N^{\alpha}_M + M$, by

$$\hat{R}^{\alpha}_{M,n+1} = \frac{B^{\alpha}_M \prod_{j=M+1}^{n}(1+p_j) - \sum_{j=M+1}^{n}\hat{R}^{\alpha}_{M,j} \prod_{\ell=j+1}^{n}(1+p_{\ell})}{P_{NM^{\alpha}+M-n}(p_{n+1})} . \tag{33}$$

The values of $\hat{R}^{\alpha}_{M,j}$ for $j = M + 1 \rightarrow n$ which appear in the sum of the numerator on the right-hand side of the above equation must be understood to have been obtained from prior application of Equation (33). These weights were chosen so as to be exact in the case where the internal rate of return on each asset each year is independent of the year of acquisition and is equal to the rate of inflation for that year. Moreover, by rewriting Equation (33) in the form

$$\frac{\hat{R}^{\alpha}_{M,n+1}}{\hat{R}^{\alpha}_{M,n}} = \frac{P_{NM^{\alpha}+M-n}(p_n)}{P_{NM^{\alpha}+M-n}(p_{n+1})}, \tag{34}$$

we see that $\hat{R}^{\alpha}_{M,n+1}$ is always positive and greater than $\hat{R}^{\alpha}_{M,n}$ if $p_{n+1} > p_n$. With the above allocation of receipts to those assets whose specific return is uncertain, Equation (26) may be generalized to the case of the multi-asset firm:

$$\Omega^{No,T}_{M,n} + \sum_{\alpha} \left\{ R'^{\alpha}_{M,n+1} F_{NM'\alpha-n}(k_{n,n+1})(1+k_{n,n+1})^{No-NM'\alpha} \right\}$$

$$= B^{T}_{M} \; K_{M,n}(1+k_{M,n+1})^{No-n} , \tag{35}$$

where $N'^{\alpha}_{M} = N^{\alpha}_{M} + M$ if $N^{\alpha}_{M} + M < N_o$ and $= N_o$ otherwise, and

$$\Omega^{No,T}_{M,n} = \sum_{j=M+1}^{n} \left\{ R'^{T}_{M,j} \prod_{\ell=j+1}^{n} (1+k_{j,\ell})(1+k_{j,n+1})^{No-n} \right\} , \tag{36}$$

with $B^{T}_{M} = \sum_{\alpha} B^{\alpha}_{M}$ and $R'^{T}_{M,j} = \sum_{\alpha} R'^{\alpha}_{M,j}$.

V. THE TREATMENT OF DEBT

Equation (35), while formally providing a solution to the problem of determining the rates of return $k_{j,\ell}$ required for our method of depreciation, leaves unanswered several questions relating to the treatment of specific assets and liabilities.

1. In the examples considered, the depreciation charge and the tax liability were the only expenses encountered. In the general case the cost of goods sold and the operating and financial expenses must be subtracted from the total revenues received; R^T_{n+1} would then represent the net income before depreciation and taxes.

2. The acquisition of non-depreciable property held for productive use, such as land or intangibles with indefinite service life, could be treated as depreciable property with an infinite lifetime. The portion of revenues $R^T_{n+1} - R^S_{n+1}$ to be associated to these assets would then be proportional to

$$\hat{R}^{\alpha}_{M,n+1} = \left\{ B^{\alpha}_{M} \prod_{j=M+1}^{n} (1+p_j) - \sum_{j=M+1}^{n} \hat{R}^{\alpha}_{M,j} \prod_{\ell=j+1}^{n} (1+p_\ell) \right\} p_{n+1} , \tag{37}$$

where B^{α}_{M} is the original cost of such an asset α acquired at the end of year M.

3. In the examples considered so far, it was assumed that there was no debt in the capital structure of the firm. Although the existence of debt is ignored in the usual methods of depreciation, it is sometimes suggested that only the equity portion of the depreciable investments be depreciated.[11/] Although this treatment might be

defended on the grounds that the capital recovery corresponding to the debt portion should not accrue to the stockholders, these benefits would be lost unless one is prepared to allow the debt holders the tax benefits of the depreciation charge. It is more reasonable to suppose that the interest charges have been set such that the debt holders have in effect "sold" the portion of the depreciation charges which would otherwise accrue to them had they leased the asset to the firm rather than financed its acquisition by the firm. We therefore suggest the calculation of the depreciation charge on the entire cost of the asset; however, we also suggest that the financial leverage utilized by the firm be reflected in the calculation of the rates of return to be used in calculating the depreciation charge--these rates should be returns on equity, rather than on total assets.[12/]

To this end, Equation (35) should be modified so that D^T_M, the total value of the interest-bearing liabilities on record at the end of year M, be subtracted from B^T_M on the right-hand side of the equation; likewise, $D^{LT}_{M,j}$, the principal portion of these liabilities payable at the end of year j should be multiplied by $(1 + k_{n,n+1})^{No-j}$ and the sum of these products for $j = M + 1 \rightarrow N_o$ subtracted from the left-hand side.

4. We have already noted that surplus cash may be treated as an investment in a zero interest one-year security. It may be more reasonable to consider the totality of working capital, less investments in fixed-interest marketable securities and interest-bearing current liabilities, as giving rise to a revenue stream proportional to

$$\hat{R}^{WC}_{M,n+1} = (1+p_{n+1})(\hat{B}^{CA}_{M} - \hat{D}^{CL}_{M}) , \tag{38}$$

in year n + 1 = M + 1, where $(\hat{B}^{CA}_{M} - \hat{D}^{CL}_{M})$ represents that portion of working capital excluding fixed-interest securities and debt. In this manner working capital is recognized as a necessary factor of production; the subtraction of the current liabilities from B^{CA}_{M} in Equation (38) and in the corresponding term on the right-hand side of Equation (35) is in keeping with the treatment of debt as noted in Section 3 above. In addition, since holding cash would, in our method, result in a lower rate of return and thus a greater depreciation charge, this treatment of the current liabilities removes any advantage to increasing the cash by simultaneously increasing payables.

These four areas requiring special care are not an exhaustive list nor are the suggested treatments definitive. These and other questions would best be examined by application of the methods described to specific firms--we hope to present such an analysis in a later paper.

NOTES

1. U. S., Congress, House Committee on Ways and Means, Tax Revision Compendium, Vol. 2, Nov. 16, 1959 (Washington, D. C.: Government Printing Office, 1959) pp. 793-932.

2. Indiana Telephone Company, "Notes to the Financial Statements," 1971 Annual Report. Reproduced in Financial Accounting Theory I: Issues and Controversies, ed. by Steven A. Zeff and Thomas A. Keller, 2d. ed; (New York: McGraw-Hill Book Company, 1973), p. 580.

3. "Financial Management during Inflation," Financial Executive, Feb. 1975, p. 12.

4. National Association of Manufacturers, "Financing Our Future," address delivered before the Congress of Industry, Dec. 6, 1974, as excerpted in Financial Executive, Feb. 1975, pp. 58-59. Similar views are expressed in Business Taxation--The Report of the President's Task Force on Business Taxation, 1970 (Washington, D. C.: Government Printing Office, 1970.)

5. U. S., Congress, House Committee on Ways and Means, "The Depreciation Deduction--LIFO Principle Should Be Extended to Cover Depreciable Assets," Tax Revision Compendium, Vol. 2, p. 882.

6. "Depreciation Policy under Changing Price Levels," The Accounting Review 29, No. 2 (Apr. 1954): 267-80. (Reprinted in Financial Accounting Theory I: Issues and Controversies, ed. by Zeff and Keller; 2d. ed., p. 324.)

7. "Letter to Shareholders," 1961 Annual Report of the Sheraton Corporation of America. (Reproduced in Financial Accounting Theory I: Issues and Controversies, ed. by Zeff and Keller; 2d. ed., p. 351.)

8. The use of the GNP Implicit Price Deflator (or indeed any comparable index) would by itself tend to result in an overestimate of the depreciation charge due to the inability of the index to reflect improvements in quality. Since such quality improvements should also be reflected in increased rates of return on investment and thus give rise to a reduction in the depreciation charge under our method of depreciation, the problem of quality change would appear to be less severe with our method than with the price-level adjusted S-L method. Depreciation Guideline service lives may be found in the U. S. Department of the Treasury, Internal Revenue Service, Depreciation Guidelines and Rules, pub. no. 456(7-62) (Washington, D. C.: Government Printing Office, 1962.)

9. Walter B. Meigs, Charles E. Johnson, and Thomas F. Keller, Advanced Accounting (New York: McGraw-Hill Book Company, 1966), p. 641.

10. If the external investment is in municipal or state bonds, the investment would be subject to taxation only once. Note that the tax liability T vanishes in this method if the internal rate of return k_i defined by

$$R_o F_{N_o}(k_i) = B_o(1+k_i)^{N_o}$$

and the external rate of return k both equal the inflation rate p.

11. "What is the logic for taking deductions in excess of your equity in a property? I ask that of anybody." Statement by Rep. Martha W. Griffiths, "Panel Discussions before the Committee on Ways and Means, H. R., 93 Cong., first session, on the Subject of General Tax Reform," Part 4 (Feb. 8, 1973), p. 582.

12. In this manner the necessity of associating the debt outstanding with specific assets is avoided--this difficulty was cited in response to the question posed in references above. In addition, this method results in a partial taxation of the monetary gains arising from the sale of debt in a period of inflation--the degree to which these monetary gains are taxed indirectly through lower depreciation charges increases with increasing returns on investment.