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THE CHICAGO BOARD OPTIONS EXCHANGE AND MARKET EFFICIENCY

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Introduction

Since call option trading started on the Chicago Board Options Exchange (CBOE) in April 1973, interest in both the investment and the academic communities has grown as rapidly as the volume of contracts traded. In May 1973, the first full month of trading on the CBOE, a total of 34,599 contracts were traded; during May 1975, the monthly volume reached 1.2 million contracts on the CBOE, and 250,000 contracts on the American Stock Exchange. At present the New York Stock Exchange and certain regional exchanges are evaluating the feasibility of adapting option trading for their respective exchanges.

The option contract as traded on the CBOE is a right which allows the buyer of the option to exercise the privilege of buying or calling from the CBOE Clearing Corporation 100 shares of a designated security at a predetermined "striking" price within a specific time interval. Thus the CBOE option contract differs from the traditional over-the-counter call option in that the CBOE option does not link a specific option writer to an option buyer. This intermediary function of bringing together option writers and option buyers which is performed by the CBOE Clearing Corporation permits a continuous secondary market in which the option buyer can sell his option and the option writer can close his position by purchasing an option contract identical to the one he had written.

The existence of the continuous secondary market enables an empirical investigation to be made of the CBOE and its relationship to the secondary security market. This study will review the theoretical underpinnings of an option valuation model, present the data and methodology used in evaluating the efficiency of the market, and the results along with conclusions of the study.

The Theory of Option Pricing

To date, much work has been done to derive a formulation for the theoretical value of an option. Merton [5] rigorously develops a theory of option pricing which covers not only put and call options, but also warrants. Additionally he develops the Black and Scholes (B-S) valuation theory of option pricing under weaker assumptions and presents several extensions of the B-S theory. Merton indicates that the Black and Scholes development is attractive because, "it is a complete general equilibrium formulation of the problem and because the final formula is a function of 'observable' variables, making the model subject to direct empirical tests."

The Black and Scholes [2] model establishes a relationship between the expected return on the option, the return on the underlying security, and the riskless rate. Using B-S notation:

$$W = XN(d_1) - Ce^{-rt^*} N(d_2) \qquad (1)$$

where

W = the theoretical price of a call option for a single share of stock;

X = the current price of the stock;

C = the striking price of the option;

r = the short-term rate of interest;

t^* = the duration of the option;

$$d_1 = \frac{\ln (X/C) + (r + 1/2\sigma^2)t^*}{\sigma\sqrt{t^*}};$$

$$d_2 = d_1 - \sigma\sqrt{t^*};$$

$N(d)$ = the value of the cumulative normal density function;

σ^2 = the variance of the return on the underlying stock.

This option valuation model for a given option is only a function of two variables: the underlying stock price and the time until the expiration of the option. All other elements of equation 1 are fixed for a given option contract and remain unchanged until the option expires. The striking price and the number of shares (100 shares per option) could change under the regulations of the CBOE in the event that stock dividends, distributions, stock splits, and recapitalizations, or reorganizations with respect to the underlying stock, took place during the life of the option. An adjustment to the data was made for those options where this occurred. The risk free rate of interest was estimated by using the short-term Treasury Bill Rate which matched the duration of the option and remained the same for the life of the option. The variance in the return of the underlying security was assumed to remain constant over the entire time period of the study. This assumption is necessary, since it would be difficult to estimate the future variance in any other manner then the past predicts the future.

In essence B-S have postulated a relationship between the value of an option and the value of its underlying security which is always in equilibrium, that is to say that the change in the option price will be perfectly correlated with the change in the underlying stock price. If this were not true, then it would be possible to create a position by hedging the option and the underlying security which had zero risk and would yield a return greater than the risk-free rate of interest for a given time period. By the assumption of the capital asset pricing model, any portfolio (position) with zero risk must have an expected return equal to the risk-free rate. The purpose of this study is to evaluate the underlying pricing mechanism of the CBOE subject to the B-S model.

Data and Methodology

A computer file was generated from the raw option data contained in the Wall Street Journal and Barrons from April 1973 to December 1974. This file contains a company ID number for each of the thirty-two firms for which options were traded on the CBOE. Each Friday during the time span covered, the closing market price of the underlying stock was recorded, along with the striking price of a particular option for a given maturity. Each firm had three different maturity ranges for a given striking price so that there were three option market prices: short maturities (1-3 months), intermediate maturities (3-6 months), and long maturities (6-9 months). Since the option market price, striking price, and underlying security market price are affected by stock splits, an adjustment was made to the data for those firms which had capitalization changes during the time period.

The time to maturity of each individual option measured in weeks was also recorded. A proxy for the risk-free rate was choosen as the yield to maturity of T-Bills for the three maturity ranges corresponding to the maturity of the respective options. The variance of the rate of return for the underlying securities was computed over the 1965 to 1972 time span. The values of the cumulative normal density function were then calculated for each individual option as shown in equation 1.

This data was used to calculate the returns earned from a riskless hedge constructed by owning a call option and "shorting" the underlying stock. The opportunity cost of foregoing a risk-free investment was calculated and incorporated into the determination of the differential return in the following manner: An amount of money equal to the cost of buying the option was invested at the risk-free rate determined by the maturity of the option for one week, the return earned from this investment was subtracted from the gain or loss experienced by owning the call option during the same week. The funds generated by the short sale of the underlying stock were likewise invested at the same risk free rate for a week. In actual practice, because of the institutional structure and practices of short selling, the funds from the short sale would not be made available for reinvestment at the riskless rate. The impact of these restrictions reduces the profitability of creating riskless hedges for the investor, however it has no effect on the empirical testing of this study. Possibly one solution to this problem would be the creation of riskless hedges solely through the writing and buying of options in various butterfly and spread strategies. The return earned from this portion of the hedge was added to the loss or gain experienced from the short position in the underlying stock for that particular week. Using B-S [1] notation, the excess dollar returns can be defined as

$$\delta = \Delta W - N(d_1)\Delta X - Wr\Delta t + N(d_1)\, Xr\Delta t \qquad (2)$$

where

ΔW = the change in the model value of the option;

$N(d_1)$ = number of shares of underlying stock needed to establish the hedge position, which is a function of the relationship between the striking price and the stock price the time until the option expires, the risk-free rate, and the variance of the return of the stock;

ΔX = the change in the stock price;

W = the model value of the option for the beginning of the week under consideration;

X = the stock price at the beginning of the week;

r = the risk-free rate;

Δt = one week.

A differential return was then calculated for each maturity range of a given option. The analysis that was used considered each maturity range as independent and used the following notation: S for the short-term, I for the intermediate-term, and L for the long-term. For a given week, the differential returns were aggregated for a given maturity range.

The entire sample of firms whose options were traded during this time period were also categorized into groups by the size of the variance of their returns. Hence the thirty-two firms were broken up into four groups of eight firms consisting of those eight firms with the largest variances, these eight firms with the next largest variances, those with the third largest variances, and those with the smallest variances.

Each of these groups had an aggregate weekly differential return for the eighty-eight weeks of the study. These risk-adjusted hedged returns where postulated by B-S and Merton to be uncorrelated with the stock market in general. This hypothesis was tested by evaluating the excess profits (losses) of option trading versus returns on the market portfolio of common stocks using the following regression model:

$$R_{opt,t} = a_i + B_i \, Rm_i t + u_t$$

where

a_i = the estimated intercept and a measure of the abnormal performance of the option market-making mechanism;

B_i = the estimated slope, theoretically predicted to be zero;

$R_{opt,t}$ = the differential returns between the riskless hedge and an equivalent risk-free return;

$R_{m,t}$ = the weekly rate of return on the Standard and Poor's 500 market index.

In Merton's discussion of the B-S model he cites certain problems with their construction of the model. These problems were considered in generating the data base for this study and include the following:

1. "Because the original B-S derivation assumed constant interest rates in forming their hedge positions, it did not matter whether they borrowed or lent long or short maturities. The derivation here clearly demonstrates that the correct maturity to use in the hedge is the one which matches the maturity date of the option." [5]
2. "The continuous-trading assumption is necessary to establish perfect correlation among nonlinear functions which is required to form the

perfect hedge portfolio mix. The Samuelson and Merton model is an immediate counter-example to the validity of the formula for discrete-trading intervals." [5]

The first problem was handled by selecting various risk-free rates and matching the maturities of these rates with the maturities of the options. The second problem, although still present, has been alleviated by the continuous nature of the CBOE market as compared to the over-the-counter Put-and-Call market originally studied by B-S.

Empirical Findings

Table 1 contains the regression estimates for the a and B terms for the various categories of options. These categories are based on a classification scheme which segmented the various options by length of time to maturity as well as by magnitude of the variation in returns of the underlying stock.

The regression parameters estimated from all of the maturities and all of the options lumped together indicate that there is a significant relationship between the movement of the market and the excess differential returns. The intercept term indicates that if we were to follow B-S's model in constructing a riskless hedge we would have lost a substantial amount of money. Clearly these results seem to indicate a rejection of the B-S model; however, as was indicated by Merton, the various maturities of the options must be considered. The aggregation of maturities across all firms leads to results that are misspecified as well as misleading and therefore should be disregarded. They are presented merely as a confirmation of Merton's maturity criticism.

The cells of Table 1, which contain the regression parameters for the firms classed by maturity of the option as well as variances of the returns, indicate that there is no significant relationship between the returns on the market portfolio and the differential excess returns on

TABLE 1
Regression Results of Market Efficiency

	Short-term	Intermediate-term	Long-term	Equality of Regressions for Identical Variance
p	?16967	-.029415	-.013493	.5512
	?622	.0027456 (.3362)	.0014383 (.3688)	
		-.081784	-.034627	.9404
		.0079120 (.2712)	.0036581 (.3799)	
		-.068997	-.031659	.9790
		.0067239 (.2460)	.0034227 (.2759)	
		-.097593	-.041985	.9785
		.0090136 (.3216)	.0044331 (.3822)	
		-.027167	-.023397	
		.0031637 (.4158)	.0015848 (.7321)	
		.9952	.9983	

the riskless hedge positions. This result is consistent with the expected theoretical relationships espoused by B-S and Merton.

The intercept term (a) is negative for all combinations. This indicates that the returns earned by establishing a riskless hedge using the B-S formulation was not enough to compensate the investor. Two factors may be used to explain that shortcoming. Since the variance of returns of the underlying stock had to be estimated using historical data, there may be a bias introduced because the actual market pricing mechanism of the option may have relied on future estimates of the variability of stock returns. A measurement error in estimating the historical measure of risk is also possible and would have some impact on the results. The second problem deals more with the unsettled macro conditions during the particular time period of the study. Being a new market, the CBOE was suffering from growing pains (Finnerty and Oben [3]) and was subject to all the start-up problems new ventures face. The risk-free rate of interest had reached historical proportions because of the inflationary conditions and expectations of this time period. It is very difficult to determine the impact this had on the financial markets, and even harder to incorporate these factors into a theoretical model.

An analysis of covariance to test the quality of the individual regressions was performed and the results are presented in Table 1. The high range of the significance levels for identical maturity (.9548 to .9983) indicate that the model formulation is affected in generally the same way by various variances of return. That is to say, for a given maturity the estimating procedure for the slope and intercept terms is uneffected by the differences in the variance of returns of the underlying security.

For the middle, middle low, and low variance options a similar statement can be made. There is no difference in the estimating procedure for a given variance class across different maturities. The significance level of .5512 for the high variance options indicates that the procedure for estimating the intercept and the slope for high variance firms may not be the same.

In general this study lends support to the validity of the B-S theoretical valuation model for options. However, some measurement problems have been identified. These included a better method for incorporating the risk of the underlying security into the model, and the existence of unaccounted for institutional factors which may affect the market pricing mechanism.

More effort is required in identifying and evaluating the various market factors which will lead to a more robust specification of the option valuation model. This in turn will lead to a clearer understanding of the impact that the CBOE has had on financial markets in general and the stock exchanges in particular.

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