RATIONALITY, OVERCONFIDENCE AND LEADERSHIP

by

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and

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ABSTRACT

This paper examines the process by which individuals get selected to be leaders and the attributes of leaders. It develops a model in which managers of a priori unknown ability are being judged relative to each other to determine who should be appointed the leader of the group. Managers are making unobservable choices about the payoff distributions of the projects they manage, and their abilities are being (noisily) inferred ex post from observed project outcomes. We have three main results. First, all managers choose higher levels of project risks when they are competing for leadership. Second, an overconfident manager – one who underestimates his project risk – has a higher probability of being chosen as the leader than an otherwise identical rational manager. Third, an overconfident leader may be better for the firm’s shareholders than a rational leader. Numerous implications of the analysis for real-world leadership behavior, new product development, relation of risk-taking to firm size and organization culture are discussed.
RATIONALITY, OVERCONFIDENCE AND LEADERSHIP

“One of the things about leadership is that you can not be a moderate, balanced, thoughtful, careful articulator of policy. You've got to be on the lunatic fringe.” Jack Welch, CEO, General Electric, quoted in Lowe, Janet, C., Jack Welch Speaks, John Wiley & Sons, New York, 1998.

1. INTRODUCTION

In a seminal contribution to the economic theory of leadership, Hermalin (1998) points out that leadership has been a topic long ignored by economists. Questions concerning what leaders do, how they motivate others to follow them, and the attributes of leaders that set them apart from the rest have not been addressed by economists. Yet, leadership has been the focal point of research in organization behavior, corporate strategy and political theory1. This research has viewed leadership as playing an essential role in corporate performance. For example, Bennis and O'Toole (2000) assert, “The right CEO can make or break a company....” and Tichy (1997, p.23) argues that leaders are important because “They determine direction. They move organizations from where they are to where they need to be.” This may be one reason why CEO (Chief Executive Officer) compensation, the subject of much study in financial economics, is often substantially in excess of that at levels below2.

Leadership has many facets worth studying and hence many questions have been raised about it. For example, there are numerous stories about managers who rapidly ascended to leadership positions in their organizations, making one wonder about the attributes of those who become leaders. What attributes did leaders like Corning's Robert Ackerman, Coca-Cola's Roberto Goizueta and TIAA-CREF's John Biggs possess that helped them get appointed as CEOs? This question seems central to understanding leadership since the problem of appointing the right person to lead the

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1 See, for example, Quinn (1996), Lowe (1998), Tichy (1997), House and Baetz (1979), and Yukl (1997).
2 Lowe (1998) quotes Jack Welch, "We CEOs also live in a market economy. A company can trade a capable CEO for $20 million, just like with a professional baseball player. A new word called a CEO market has arrived in the United States". (p.195).
organization has gained sharper focus in recent years. A related question has to do with how leaders affect their organizations. How do leaders like Jack Welch of GE motivate people below them and influence their behavior? This leads one to also ask whether the attributes that help someone get appointed to a leadership position actually serve the organization well after the appointment. This is a question of ex ante versus ex post efficiency. The answer is far from obvious in light of recent examples of CEOs of prominent organizations being fired shortly after their appointments, the most notable perhaps being Douglas Ivester who lasted about two years as Coca-Cola CEO. It is possible that these errors are due to the manner in which those vying for leadership positions distort their behavior in order to elevate perception of their abilities as leaders. This raises an obvious question about the nature of these distortions.

The question of how leaders affect their organizations has been studied in the literature. Hermelin (1998) focuses on what leaders do to make followers voluntarily follow them. His insight is that leaders have superior information that they credibly convey to followers by making personal sacrifices in the form of moving first with high effort. But he does not consider how the race to be appointed a leader influences behavior, how leaders are chosen, and whether the choice mechanism is efficient in an ex post sense. In this paper, we focus on these aspects of leadership. We take as a given that the leader has both formal and real authority in the organization and that followers will follow the leader. By construction, leadership is desirable to all, so our focus is on how someone becomes a leader and whether the attributes that get someone appointed leader are those that truly benefit the organization after the leader is at the helm. Specifically, the questions we address are:

- If leadership has private benefits, how does the quest for leadership affect the behavior of those who seek to be leaders?

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3 Bennis and O'Toole (2000) claim: "Boards often go about CEO selection all wrong. The problem is simple: they don’t understand what defines real leadership today – or if they do, it scares them."

4 Bennis and O’Toole (2000) cite a study by the Center for Executive Options that one-third of Fortune 100 companies have replaced their CEOs since 1995.
• What are the attributes of leaders? In particular, what role does overconfidence play in one’s chances of becoming a leader?

• How do leaders affect their organizations? Are overconfident leaders better or worse than rational leaders for the organization?

To address these questions, we develop a two-period leadership-selection model in which there is initially one leader (CEO) and many followers (managers), all of whom are risk averse. Shareholders, who are risk neutral, own the organization. In each period, the CEO chooses a strategy for the whole organization, which affects the payoff distributions of all projects. Each manager chooses one project whose payoff distribution is affected by a choice made by the manager as well as by the overall strategy chosen by the CEO. We assume that mean returns and project risks are positively related in the cross-section.

The key to the model is the assumption of unknown ability. Each manager’s ability is a priori unknown to all and is being inferred over time from the observed payoffs on individual projects. Higher-ability managers make better CEOs because their strategy choices are better for shareholders. Thus, if the first-period CEO is planning to step down at the end of the period and select one of the managers to succeed her, she would like to hand over the reins to the manager she thinks has the highest ability. Knowing this, each manager chooses a riskier project than he would in the absence of the (implicit) tournament.

The intuition is as follows. With \( n > 1 \) competing managers who are a priori identical, if all managers choose the level of project risk that maximizes the expected utility of (output-contingent) compensation in the absence of promotion concerns, each manager has a \( 1/n \) probability of getting promoted. If a manager deviates by choosing slightly higher risk, then he simultaneously increases the probability of failure as well as that of an extremely high payoff if he succeeds. Thus, conditional on success, he stands above the crowd in the race to be CEO in the sense that he experiences a higher

\[ ^3 \text{Besides Hermalin (1998), Aghion and Tirole (1997) also point out that there is a distinction between formal and real authority in organizations. They view formal authority as the “right to decide” and real authority as “the} \]
payoff than any competitor. But in doing so, he puts his output-contingent compensation to greater risk than other managers. This means each manager trades off compensation risk against the private benefit of being promoted to CEO, and when this benefit is positive, each manager chooses higher risk than in the absence of promotion concerns.\(^6\)

We then introduce an overconfident manager among those competing to be CEO, assuming that no one (including the overconfident manager) knows there is an overconfident manager in their midst. Overconfidence is defined as underestimating project risk. There is a vast literature in psychology that asserts that overconfidence is a common form of behavioral irrationality (see Russo and Schoemaker (1990), for example). We find that the overconfident manager has the highest probability of being promoted to CEO when he is competing with otherwise identical rational managers.\(^7\) This result suggests that overconfidence may be a more common trait than rationality among leaders.

Of course, winning the race to be CEO doesn’t mean the overconfident manager is the best CEO for the shareholders in the second period. To analyze this issue, we consider two settings. In one, managers are choosing individual project risks and the CEO is determining strategy that affects the payoff distributions of all projects, as in the basic model. In the other, the CEO is again choosing overall strategy, but each manager is choosing an effort input; there is no project choice for an individual manager in the second period. In both cases, we allow each manager to assign some probability that the CEO is overconfident, to capture the idea that they may suspect overconfidence in the CEO but they are not sure.

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\(^6\) The notion of risk-taking as a part of the path to leadership is a recurring theme in organization behavior theory. For example, Tichy (1977, p.151) writes, “Winning leaders never take the easy way out. Risk and pain don’t deter them.” And Quinn (1996, p.160) writes, “To survive, organizations need leaders who take risks and who care enough to die for the organization - which would kill them for caring.”

\(^7\) We also consider a scenario in which all the managers are ex ante identical, and it is common knowledge that each manager may be overconfident with some probability. Managers take this into account while choosing their projects. The project risk each manager believes he has chosen is identical across all managers, but the true project risk is higher for overconfident managers. Consequently, an overconfident manager is more likely to outperform others and get promoted to CEO.
In the first case we find that an overconfident CEO chooses higher risk for the firm than a rational CEO, whereas individual managers choose lower project risks than they would if they could be sure the CEO was rational and higher project risks than they would if they could be sure the CEO was overconfident. Managers behave like this because they adjust for the higher firm-level risk choice of an overconfident CEO. Thus, when a (second-period) CEO is overconfident but managers can only suspect such irrationality, the overall risk level of the firm is higher than when the CEO is rational and all managers are sure the CEO is rational. Because risk and mean return are positively linked and shareholders are risk neutral, an overconfident CEO is better for the shareholders than a rational CEO of equal ability. The CEO's overconfidence narrows the preference gap between risk-averse managers and risk-neutral shareholders.

In the second case the result is similar. When managers suspect that the CEO is irrational, they supply less effort than they would if they were sure that the CEO was rational. The resulting reduction in firm value is offset by the higher mean payoff that accompanies the choice of higher risk by the overconfident CEO. This tradeoff results in a higher firm value with an overconfident CEO who is currently not identified as such than with a rational CEO everyone knows is rational.

In both cases then the best outcome for the shareholders is to have an overconfident CEO with managers who can do no better than merely suspect this overconfidence. The worst outcome is to have a rational CEO with the managers suspecting overconfidence. Thus, the overarching message of our analysis is twofold. First, overconfidence is likely to be a more prevalent attribute among leaders than in the general population. Second, such irrationality among CEOs can actually benefit shareholders.

In the real world, leaders are viewed as affecting the organization not only because those below them are altering their behavior to increase the chances of becoming leaders themselves, but also through their impact on organization culture. This is a term lacking economic content, primarily because economists have not addressed what it means. We view organization culture as a collection of explicit rules and implicit norms that guide employee behavior. The aspect of organization culture we focus on is the efficiency of task assignments, i.e., how efficiently employee abilities are discovered in
order to decide who should do what, who should be promoted, and who should be fired. We find that an overconfident CEO produces an organization culture in which employee abilities are revealed faster (more efficiently). Our analysis of leadership also generates implications for capital budgeting and capital structure.

Apart from its obvious connection to the Hermelin (1998) paper, our paper is also related to two other literatures. One is the literature on tournaments among agents. Lazear and Rosen (1981) develop a model in which they compare the efficiency of rank-order tournaments among agents with that of paying agents based on output level. With a rank-order tournament, an agent is paid based on his ordinal rank in the organization. While the two schemes are equivalent when agents are risk neutral, they are not so when agents are risk averse. With sufficiently high risk aversion, payment based on output is preferred to that based on rank. In our model, we have the agent being rewarded both for output and for his ordinal ranking; he is compensated based on his output level and promoted based on his rank.\(^8\)

The other literature our work is related to is that on the role of behavioral irrationality in economics. Daniel, Hirshleifer and Subrahmanyam (1998) show how investor overconfidence can affect stock returns to produce patterns in the data that are consistent with numerous documented asset-pricing anomalies like price overreaction, momentum and return reversals. Kyle and Wong (1997) examine the question of whether overconfident stock traders can survive if they are trading with rational traders. Manove and Padilla (1999) assume that entrepreneurs seeking funding are overoptimistic and then examine the role of intermediaries in allocating capital.

While these papers examine issues very different from ours, the common thread is the observation that accommodating certain types of well-documented behavioral irrationalities into

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\(^8\) A different sort of tournament issue is considered in Ramakrishnan and Thakor (1991). They examine the benefits of letting agents compete with each other in a tournament versus the benefits of letting them cooperate as part of a team; in the latter case each agent is paid on the basis of the team's output. The cooperation setting seems unsuited to the issues we want to address.
economics may go a long way in explaining real-world data with relatively simple models that do not need to assume highly complex information structures or embed very elaborate extensive-form games.

The rest of the paper is organized as follows. Section 2 contains the basic model. Section 3 has the analysis of the first-best outcome as well as the second-best outcomes with and without promotion concerns. Section 4 examines how managerial overconfidence affects the race to be CEO and whether an overconfident CEO is better for the shareholders than a rational CEO. Section 5 discusses model extensions and implications of the analysis, including those for organization culture. Section 6 concludes. All proofs are in the Appendix. Table 1 lists all the mathematical symbols used in the paper.

2. THE BASIC MODEL

In this section we describe the basic framework of our analysis. We begin by describing the organization structure of the firm, state the preferences of the agents involved, explain the probability distribution of project payoffs and their dependence on managerial ability, and end with a specification of the compensation structure for agents.

A. Organization Structure

We consider an all-equity firm with two levels of hierarchy. At the top of the hierarchy is a Chief Executive Officer (CEO), and reporting to her are \textit{a priori} identical managers. These managers as well as the CEO choose actions that affect the firm’s payoffs that shareholders care about.

There are two periods. At the end of the first period, outputs of individual managers are observed, based on which abilities are inferred. The CEO chooses the highest-ability manager to succeed her in the second period and steps aside. The newly-appointed CEO then heads the hierarchy in the second period. In much of our formal analysis, we will focus on either the first or the second period, so the analysis is essentially static.

B. Preferences
The shareholders of the firm are risk neutral (with respect to the earnings of the firm). The CEO and the managers who report to the CEO are risk averse. The CEO and the managers have identical preferences that can be represented by the following utility function over compensation

\[ U(x) = -\alpha(\beta - x)^3, \]  

(1)

where \( x \) is the compensation of the manager and \( \alpha \) and \( \beta \) are positive, finite constants. We shall assume that \( \beta \) is sufficiently large so that the function is increasing and concave over the range of feasible compensation.

C. Projects and Managerial ability

Each manager manages one project in the first period, and can choose the kind of project to manage. The feasible set of projects to choose from is the same for every manager, and the projects available to one manager are independent of the projects available to others. All projects require the same initial investment, and each has a stochastic output. The probability distribution of output for a project depends on the ability of the manager managing it and on the risk of the project. We shall assume that two projects available to the same manager are different if and only if they differ in their risk. The probability density function \( f_i \) of output \( y_i \) of a project managed by manager \( i \) depends on the ability \( A_i \) of the manager managing it and on the risk \( R_i \) of the project in the following way:

\[ f_i(y_i) = \begin{cases} 
\frac{1}{2R_i} \{ 1 + k(y_i - \gamma R_i)A_i \} & \text{if } |y_i - \gamma R_i| \leq R_i \\
0 & \text{if } |y_i - \gamma R_i| > R_i
\end{cases}, \]  

(2)

where \( k \) and \( \gamma \) are positive exogenous constants.

There is a continuum of projects with risk ranging from 0 to \( R_{max} \), and managers can choose any project (uniquely defined by its risk) from this set. The manager's choice of risk is unobservable to all but the manager himself. The key to this probability density function is that it is affected both by \( R_i \), the project risk (which the manager chooses) and by \( A_i \), the managerial ability (that is an unalterable attribute he is endowed with). The density function has a support \( [(-1 + \gamma)R_i, (1 + \gamma)R_i] \)
that is determined by the project risk chosen by the manager. The measure of the support is proportional to the risk, and the mid-point of the support, \( \gamma R \), increases as risk increases. Thus, a project with a higher risk has a higher unconditional mean output \( \gamma R \), and is also more likely to have more extreme outputs. The exogenous parameter \( \gamma \) determines how sensitive the mean output is to risk.

Managerial ability has a different effect. The ability of the manager determines the probabilities of high and low outcomes. A manager with high ability has a higher probability of high project outputs than a manager with low ability, conditional on their projects having the same risk. More formally, the output distributions for projects managed by managers of different abilities follow the monotone likelihood ratio property\(^9\); for \( A_L < A_H \), \( f(y|A_L, R)/f(y|A_H, R) \) declines as \( y \) increases. Project risk and managerial ability interact so that the effect of managerial ability on project output is magnified by project risk. For instance, managerial ability has no effect on project outcome when the risk is 0; the output in this case is 0 with probability one. When project risk is high, a manager with higher ability is more likely to get higher output and less likely to get lower output than a manager with lower ability. Thus, managerial ability affects project output more for projects with higher risk.

We shall assume that the ability of a manager is not known to anyone, not even the manager himself. All managers are ex ante identical, and their abilities are independent and identically distributed. The ability of each manager is uniformly distributed between \(-1/2\) and \(1/2\). This distribution is common knowledge.

**D. Manager's Compensation**

It is natural to assume that the compensation of a manager in this period and in the future should depend on his perceived ability. A manager with a higher perceived ability is likely to have a higher project output and is therefore more valuable to the firm. Thus, in a competitive labor market, what each firm pays its managers will reflect the perceived abilities of these managers. At the outset,

however, all managers are identical and cannot be distinguished on the basis of ability. The output of the project at the end of the first period is a signal of managerial ability, based on which the CEO and shareholders revise their beliefs about managerial ability. We assume that each manager’s output can be separately observed, so each manager’s ability can be distinctly reassessed at the end of the first period.

In our two-period setting the key point in time at which managerial ability differentiation occurs is at the end of the first period. The manager’s decisions at the start of the first period are motivated to influence how his ability will be perceived at the end of the first period, if either his second-period compensation or his chances of promotion to CEO for the second period depend on how his ability is viewed after the first period. Ability perceptions at the end of the second period (and beyond in a multiperiod setting) will be important to the manager but don’t matter in our analysis because our focus is on the promotion decision at the end of the first period. We will thus focus on managerial compensation as a function of perceived ability at the end of the first period.

From the optimal dynamic contracting literature (e.g. Holmstrom and Ricart i Costa (1986), and Rogerson (1992)), we know that the dependence of compensation on perceived ability will be strongest if managers are risk-neutral. With risk-averse managers the dependence will be weaker, but positive nonetheless.\(^{10}\)

An optimal contract can be derived by taking into consideration the risk-aversion of managers, the value of perceived ability in the labor market in current and future periods (which itself derives from the dependence of future project outputs on the manager’s ability) and the time value of money. We shall not attempt to solve for the optimal compensation contract here and instead assume that the manager’s compensation (for current and future periods) is linearly increasing\(^{11}\) in the perceived ability of the manager at the end of the first period. The compensation received by manager \(i\) is given

\(^{10}\) This effectively means that managerial compensation will be increasing in first-period output.

\(^{11}\) We believe that the results of the paper will hold in spirit even if the compensation is merely monotonically increasing in the perceived managerial ability.
by \( c_1 + c_2 E[A_i | y_i] \) where \( y_i \) is the output of the project managed by manager \( i \) and \( c_1 \) and \( c_2 \) are positive constants. The sequence of events is summarized in Figure 1.

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E. The Role of the CEO

The role of the CEO is to act as a leader by determining corporate strategy. In doing so the CEO synthesizes all available information and sets an overall direction for the firm. Thus, while managers deal with their individual projects and make decisions that affect the prospects of those projects, the CEO formulates a strategy that affects the prospects of all the projects, i.e. the prospects of the firm as a whole.

We model this in a simple way. The probability distribution of the payoff of each project depends on the ability of the manager responsible for that project, the project risk chosen by the manager, and the corporate strategy chosen by the CEO. That is, the density function \( f \) in (2) can now be written as \( f(y_i | A_i, R_{i0}, S, A_0) \), where \( S \) represents strategy and \( A_0 \) the CEO's ability. When we get to the analysis in subsequent sections, we will specify the specific functional-form adaptation of (2) that accommodates \( S \). For now, it suffices to note that \( \partial F(y_i | \ldots, y_i) / \partial A_{i0} < 0 \; \forall \; i \), where \( F \) is the cumulative distribution function associated with density function \( f \). Thus, higher ability results in a stochastically higher output for each project. The total output of the firm can be written as \( \sum_{i=1}^{n} y_i \).

This specification ensures that it is subgame perfect to promote the manager with the highest perceived ability to CEO in the second period. We assume that there is a private benefit \( B \) to the manager from being appointed CEO.

3. PROJECT RISK CHOICES BY MANAGERS: THE FIRST BEST AND PROMOTION CONCERNS
In this section we suppress the second-period game in which a new CEO is at the helm. Our focus is on the project risk choices made by the managers in the first period. Thus, we suppress the dependence of $f$ on $S$ and $A_0$ for now and return to the functional form in (2). We are examining each manager’s risk choice taking the CEO’s strategy and skill as given. This provides the basic framework for the rest of the analysis.

A. The First-Best Outcome

Suppose the ability of each manager is known in advance. The compensation paid to each manager will still depend on perceived ability, but it will be independent of project outcome. The reason is that we have no moral hazard problem here and the project outcome provides no new information about managerial ability. Each manager will act in the shareholders’ interest by choosing the project risk to maximize expected project output. Of course, if the project choice risk is observable, the choice of risk desired by the shareholders can be dictated to the manager. The expected output of a project managed by manager $i$ can be computed to be $kA_i R_i^2 / 3 + \gamma R_i$ if the risk of the project is $R_i$. To maximize this, all managers with ability $A_i$ higher than $-3\gamma / 2k R_{max}$ choose the maximum risk $R_{max}$ and managers with lower ability choose risk of $-3\gamma / 2k A_i$.

B. Second-Best Risk Choices Without Promotion Concerns

Consider now the case in which managerial compensation is given by $c_1 + c_2 E[A_i | y_i]$, so that it is increasing in perceived ability at the end of the first period. The manager’s choice of project risk is also unobservable. For now we suppress the implicit tournament among managers that determines which manager will be chosen to be CEO at the end of the first period. Thus, each manager seeks only to maximize the expected utility of compensation.

The key to each manager’s decision is to recognize how the project payoff at the end of the first period influences the posterior beliefs of others about his ability. Suppose the CEO believes that manager $i$ chose risk $R_i$. Then the posterior belief about the manager’s ability depends on his project’s output $y$ in the following way:
\[ g(a | y) = \frac{f_y(y | a)\Phi_1}{\Phi_2} = \frac{1}{\sqrt{2\pi}R} \frac{\Phi_1 + k(y - \gamma R)a}{\Phi_2} = 1 + k(y - \gamma R)a. \]  

(3)

The posterior density of the manager’s ability at \(a\) is the ratio of the likelihood of the joint event that the manager’s ability is \(a\) and the output is \(y\) to the unconditional probability that the output is \(y\). The likelihood is the product of \(1\), the prior density of ability at \(a\), and \(f_y(y | a)\), the density of output conditional on ability \(a\). The unconditional probability that the output is \(y\) is obtained by integrating this likelihood with respect to ability. The mean of the posterior density of managerial ability is given by:

\[ E[A | y] = \frac{1}{\sqrt{2\pi}} \int_{-\sqrt{2\pi}}^{\sqrt{2\pi}} a g(a | y)da = \int_{-\sqrt{2\pi}}^{\sqrt{2\pi}} a \frac{1 + k(y - \gamma R)a}{12} da = \frac{k(y - \gamma R)}{12}. \]  

(4)

Thus, the (mean) perceived ability is linearly increasing in the project output.\(^\text{12}\)

**Lemma 1:** Each manager chooses risk of \(R^* = \frac{36\gamma(\beta - c_1)}{c_2}k > 0\) when his objective is to solely maximize expected utility from compensation.

The intuition is as follows. The compensation received by manager \(i\) is linearly increasing in his project’s output. The manager, being risk-averse, wants to reduce the uncertainty of the project output and increase the mean output. If the mean output were independent of project risk, the manager would reduce the uncertainty of output by choosing the lowest possible risk. However, the mean project output is increasing in risk, with \(\gamma\) being the exogenous parameter that measures the sensitivity of the mean to risk; the risk chosen by the manager is thus positive and increasing in \(\gamma\). The parameters \(k\) and \(c_2\) measure the sensitivity of the manager’s compensation to the output, so the risk chosen by a risk-averse manager is decreasing in these parameters.

\(^{12}\) When there is uncertainty about the risk chosen by the manager, the risk \(R\) in (4) is replaced by its expected value.
C. Promotion Concerns

We now assume that the managers compete with each other for promotion to CEO at the end of period one. The present CEO wants to promote the person with the highest ability. In the absence of perfect knowledge, the CEO promotes the manager with the highest perceived ability\(^{13}\) after period one. As mentioned earlier, being promoted to CEO yields a manager an additional net utility of \(B\). Each manager now wants to maximize the total expected utility from compensation and promotion.

**Lemma 2:** Suppose manager \(i\) chooses risk \(R\) greater than the risk \(\hat{R}\) chosen by all the other managers. Then the probability of promotion of manager \(i\) is

\[
p_i = \frac{1 + \gamma}{2} - \frac{\hat{R}}{R} \left( \frac{1 + \gamma}{2} - \frac{1}{n} \right) > \frac{1}{n}.
\]

(5)

Here \(1/n\) is the probability of promotion for any manager (including \(i\)) if manager \(i\) also chooses \(\hat{R}\), the risk chosen by all the others. This lemma shows that a manager can increase his chances of promotion by increasing the risk of his project. The intuition is as follows. If the manager chooses the same level of risk as everybody else, he is expected to have the same project output at the end of the first period. Thus, with \(n\) *a priori* identical managers, the probability of being promoted is \(1/n\). If manager \(i\) chooses a higher risk, he increases his chance of promotion for two reasons. First, the mean output of his project is increasing in its risk at the rate \(\gamma\), and \(\gamma > 0\); a higher outcome ex post connotes higher ability. Second, even if the mean output is independent of risk (\(\gamma = 0\), the choice of a riskier project can increase the manager’s promotion probability. The reason is that when manager \(i\) does have a high output, his output tends to be higher than that of all the competing managers. This is because higher risk makes more extreme outcomes more likely even when it doesn’t affect the mean outcome. Because the manager’s choice of project risk is unobservable, the CEO cannot disentangle the effect of the chosen risk from that of managerial ability on the project output. Thus, an output higher than those of competing managers makes manager \(i\) appear more able than competitors and gets him promoted.
To see this in a simple illustration, suppose we have 20 managers and four possible output states: very low (VL), low (L), high (H) and very high (VH). If one chooses moderate risk, the output probabilities are 0.49 each for L and H and 0.01 each for VL and VH. If one chooses high risk, the output is equally likely to be any one of the four values. Now, if all managers choose moderate risk, the probability of being promoted to CEO is 1/20 for each. A deviating manager who chooses high risk has a higher probability of achieving VH and thus a higher chance of being promoted to CEO even if the expected project payoff is higher with moderate risk. This suggests the following result.

**Proposition 1:** All managers choose projects with the same risk $R^*$, and this is greater than the minimum risk as well as $R^r$, the risk chosen when the managers have no promotion concerns and seek only to maximize expected utility of compensation. The chosen risk with promotion concerns is

$$
R^* = \frac{18\gamma(\beta - c_1) + 6\gamma^2(\beta - c_1)^2 + 6B}{c_2 k} \left(\frac{1 + 7}{2} - \frac{1}{n}\right),
$$

and it is increasing in the benefit from promotion, the number of managers and the sensitivity of mean output to risk. It is decreasing in the sensitivity of compensation to project output and the sensitivity of project output to managerial ability.

If managers had no promotion/career concerns and the mean project output was independent of risk, each risk-averse manager would choose the minimum risk. We saw in Lemma 1 that each manager chooses risk higher than this minimum even without promotion concerns because the mean output is increasing in risk. As Lemma 2 suggests, the introduction of promotion concerns makes risk even more personally attractive to the manager.

The personal cost to the manager of taking more risk is the reduced expected utility from compensation. This must be traded off against the benefit of an enhanced probability of being promoted. As long as $B > 0$, the introduction of promotion concerns increases the chosen risk. Since all managers are identical, they all choose projects with identical higher risk.

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13 To be precise, the CEO promotes the manager whose posterior distribution of ability has the highest mean.
4. THE EFFECT OF MANAGERIAL OVERCONFIDENCE ON THE RACE TO BE CEO

The purpose of this section is to examine how well an overconfident manager does in the race to be CEO when the managers he is competing with are rational. Are overconfident managers more or less likely to succeed in becoming CEO? If promoted, will an overconfident person be a better or worse CEO than a rational person?

A. Effect of Overconfidence on Promotion Outcome

Suppose one of the managers, say manager $i$, is overconfident. We define overconfidence as underestimating the risk of projects. If a project has risk $R$, the overconfident manager erroneously believes that the risk is $R/C$, where $C > 1$ is the degree of overconfidence. We assume that no one (not even manager $i$ himself) is aware of the fact that manager $i$ is overconfident. Thus, the managers cannot distinguish this situation from the one in which everyone is rational. They choose projects with risk $R^*$ as in Proposition 1. The overconfident manager chooses a project with true risk $CR^*$ even though he believes he is choosing a project with risk $R^*$. Using Lemma 2, the probability that the overconfident manager gets promoted is

$$\frac{1}{n} + \left(1 - \frac{1}{C}\right)\left(\frac{1 + \gamma}{2} - \frac{1}{n}\right) > \frac{1}{n}. \quad (7)$$

Thus, we have the following result:

**Proposition 2:** An overconfident manager is more likely to get promoted when no one realizes the problem of overconfidence. The relative increase in the probability of the overconfident manager's promotion is increasing in the degree of overconfidence and in the number of competing managers.

We saw in Proposition 1 that the risk aversion of managers and the prospects of promotion interact to determine the optimal level of risk chosen by each manager. An overconfident manager unknowingly chooses a project with a higher risk and ends up hurting himself in an expected utility sense. The higher risk, however, increases the probability of promotion for the overconfident manager. The strength of this effect is increasing in the degree of overconfidence. The relative promotion-probability advantage of an overconfident manager is also increasing in the number of competing
managers. The reason is that when the number of managers is large, the probability that any particular manager gets promoted is small, so an increase in the probability of promotion due to higher project risk has a larger marginal impact on the promotion probability.

We now assume that all managers are ex ante identical and equally likely to be overconfident. There is a probability $\theta$ that a manager is overconfident with a degree of overconfidence $C$. The occurrence of overconfidence is independent across managers. This information is common knowledge.

All managers face the same problem of maximizing their expected utility over compensation and promotion. This problem is more complicated now because each manager must consider the possibility that he may be rational or overconfident. At the same time, he must realize that others may also be rational or overconfident in a similar way. Solving this problem, we get the following result:

**Proposition 3:** When each manager is likely to be overconfident with probability $\theta$, all managers believe they have chosen a risk $R^*$. For a rational manager, the truly chosen project risk is also $R^*$ and is less than $R^*$. An overconfident manager’s project risk has risk $CR^*$, greater than $R^*$. The risk $R^*$ is increasing in the private managerial benefit to promotion. It is decreasing in the sensitivity of compensation to output, in the sensitivity of output to ability, and in the degree of overconfidence. An overconfident manager is more likely to get promoted than a rational manager. Here:

$$R^* = u + \sqrt{u^2 + \nu},$$

where

$$u = \frac{18 \gamma (1-\theta + C\theta) (\beta - c_1)}{c_2 k \left[ 1 - \theta + \delta C^2 \right] + 3 \gamma^2 \theta (1-\theta)(C-1)^2},$$

and

$$\nu = \frac{108 B}{\alpha c_2^2 k^2 \left[ 1 - \theta + \delta C^2 \right] + 3 \gamma^2 \theta (1-\theta)(C-1)^2} \times$$

$$\left\{ (1-\theta) \left[ \frac{\theta}{2} \left( 1 - \frac{1}{C} \right) (1-\gamma) \right]^{\sigma-1} (1-\gamma) + \left[ 1 - \frac{\theta}{2} \left( 1 - \frac{1}{C} \right) (1+\gamma) \right]^{\sigma-1} (1+\gamma) \right\}$$

$$+ \theta \left[ 1 + \gamma - \frac{2}{n \theta} \right]$$

Because everything is symmetric, it is not surprising that all the managers adopt the same strategy and choose those projects that they think have risk $R^*$. They realize that the true risk of their
project could be $R^*$ or $CR^*$, depending on whether they are rational or overconfident. The risk of a project managed by a rational manager is less than the risk $R^*$ of projects when everyone is rational and everyone knows this. But the risk of a project managed by an overconfident manager is higher than $R^*$. Thus, introducing overconfident managers results in cross-sectional variation in the risk of projects.

Since the projects of overconfident managers have higher true levels of risk, they are more likely to get promoted than rational managers. Thus, the posterior probability $\pi$ that the promoted manager is overconfident is higher than $\theta$, the expected fraction (ex ante probability) of overconfident managers. Surprisingly, this does not imply that the promotion rule is inefficient for shareholders. The manager with the highest project output is still the most likely to have the highest ability. The fact that overconfident managers are more likely to get promoted does not reduce the average ability of promoted managers because the distribution of ability and overconfidence are independent.\(^{14}\)

B. Is An Overconfident Leader Good For The Firm?

Given that an overconfident manager is more likely to get promoted, the natural question is: does this benefit shareholders more than having a rational manager become the CEO? We address this question in the next two subsections with two versions of our basic model. In the first, we assume that managers are choosing risk of individual projects as in the basic model, and the CEO is setting strategy in a way that determines the overall risk class of all the projects in the firm. In the second, we make a similar assumption about the CEO's role but allow each manager to choose an effort input that affects the payoff distribution of the project he is managing. In both cases, we find that CEO overconfidence benefits the shareholders, but for somewhat different reasons.

C. Managerial Risk Choices and CEO Overconfidence:

We assume that a project can have two sources of risk. One source of risk is project-specific, the risk we have discussed in previous sections. The project manager determines the level of this risk.

\(^{14}\)It would be interesting to extend the analysis to see how the choice of riskiness and the promotion probability are affected if managerial ability and the probability of being overconfident are correlated.
In addition to this project-specific risk, there is a risk affecting all the projects available to the managers. The level of this risk is determined by the business decisions of the CEO. Only the CEO knows this firmwide risk. Other managers lack access to this information.

The risk of the project managed by manager $i$, $R_i$, consists of firmwide risk $S$ and project-specific risk $T_i$. Thus,

$$R_i = S + T_i.$$  \hfill (9)

The probability density function of the output is given by (2) in terms of the total risk of the project. We shall show how an overconfident CEO can affect the risk of projects across the firm.

Proposition 3 shows that the probability of promotion is higher for an overconfident manager than for a rational manager. Thus, the probability that the new CEO is overconfident is $\pi$, which exceeds $\theta$, the ex ante probability of a manager being overconfident in the first period. Suppose the risk choice made (strategy chosen) by a rational CEO is $S$. Then, an overconfident CEO will make a risk choice of $CS$. The managers must now choose project-specific risk so as to maximize their utility. In doing so, they believe that the firmwide risk is $S$ with probability $1-\pi$ and $CS$ with probability $\pi$.

**Proposition 4:** With rational managers and no promotion concerns in the second period, the risk of the projects chosen is $R^0$ when CEO type is known with certainty. When there is uncertainty about CEO type, the risk is less than $R^0$ if the CEO is rational and more than $R^0$ if the CEO is overconfident. Firm value is higher if the CEO is overconfident than if the CEO is rational. Expected firm value is independent of the probability $\pi$ of an overconfident CEO.

In the second period, managers have no promotion concerns, so each chooses project-specific risk to maximize his expected utility of consumption. The optimal risk from the perspective of a manager is $R^0$ (see Lemma 1). When CEO type is known with certainty, the manager adjusts project-specific risk to achieve risk $R^0$. Each manager chooses a project-specific risk of $R^0 - S$ when the CEO is rational and $R^0 - CS$ when the CEO is overconfident. However, when the probability $\pi$ of an overconfident CEO is strictly between 0 and 1, each manager chooses project-specific risk greater than
$R^0 - CS$ and less than $R^0 - S$. This achieves a tradeoff between the disutility from excess risk when risk exceeds $R^0$ in the case of an overconfident CEO and the disutility from low project return when the risk is less than $R^0$ in the case of a rational CEO. The expected risk of projects equals $R^0$, independent of $\pi$. This means expected firm value, which is linear in risk, is also independent of $\pi$.

**Proposition 5**: With rational managers competing for promotion, the risk of the projects chosen is $R^r$ when CEO type is known with certainty. When there is uncertainty about CEO type, the risk is less than $R^r$ if the CEO is rational and more than $R^r$ if the CEO is overconfident. Firm value is higher if the CEO is overconfident than if the CEO is rational. Expected firm value is higher when there is uncertainty about the type of the CEO than when the CEO type is known.

The intuition for Proposition 5 is similar to that for Proposition 4. The difference is that now, in choosing project-specific risk, managers trade off the disutility from compensation risk against the disutility from a lower mean compensation and diminished promotion chances. They choose project-specific risk between $R^r - CS$ and $R^r - S$, so the risk may be greater than or less than $R^r$ depending on the type of CEO. The uncertainty in CEO type increases expected firm value because expected risk is higher than $R^r$ and mean project output is linearly increasing in risk. The expected risk is higher than $R^r$, the risk when CEO type is known, because the probability of promotion is concave in risk, and uncertainty in risk lowers the probability of promotion. To compensate for this, managers raise expected risk.

**D. Managerial Effort Input and CEO Overconfidence**:

We now assume that the CEO’s choice of strategy means the decision to accept or reject a project class available to her. A project class is a set of numerous ex ante identical projects and is defined by the mean return and the risk of the constituent projects. If the CEO accepts the project class available, projects from this project class are allocated to different managers in the firm, each of whom chooses the effort he puts into his project.
The common second-period belief about the CEO’s ability, $A_0$, reflects her performance in the first period. The belief about the CEO’s ability is correlated with the probability that the CEO is overconfident because the effect of ability and overconfidence cannot be decoupled in the prior performance of the new CEO. In our static analysis, we ignore any interaction between the results in the two periods and assume that in the second period the probability distribution over the CEO’s ability is independent of the probability that the CEO is overconfident. The CEO’s ability is normalized so that it is uniformly distributed between $-\frac{1}{2}$ and $\frac{1}{2}$. The probability that the CEO is overconfident is $\pi$.

The project class available to the CEO is a random draw by nature, governed by a joint probability distribution on the mean return and the risk of the project class. The mean return, $m$, of the project class observed by the CEO follows the following probability distribution

$$h(m) = \begin{cases} \frac{1}{M - \overline{M}} \left[ 1 + \left( m - \frac{\overline{M} + \underline{M}}{2} \right) A_0 \right] & \text{if } \underline{M} \leq m \leq \overline{M} \\ 0 & \text{otherwise.} \end{cases}$$ \hspace{1cm} (10)

Thus, the probability distribution of the mean return for a higher-ability CEO first-order stochastically dominates the probability distribution of the mean return for a lower-ability CEO. This makes it subgame perfect for shareholders to promote the manager with the highest perceived ability at the end of the first period to be the CEO in the second period. The risk $r$, conditional on the mean return $m$, is uniformly distributed between $L(m)$ and $H(m)$ with $H(m) > L(m) > 0$, where $L$ and $H$ are strictly increasing functions. Thus, mean return and risk are positively correlated in the cross-section. After observing the project class, the CEO adopts it if its risk is “acceptable” given its mean return; this is made precise shortly.

If the CEO adopts the project class available to her, the new projects are managed along with the projects that are already available to the managers. Any value added by the new projects is

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15 A complete characterization of the effect of $\pi$, the probability of CEO overconfidence, on firm value would require a general equilibrium analysis. We have assumed that the form of the managerial compensation contract
incremental to the value added by the existing projects and there is no interaction between these projects. The only way in which existing projects affect decisions about new projects is that their characteristics are used to determine the criterion for adopting or rejecting a new project class. We take this criterion as exogenous. Since we are interested only in the incremental value added by the new projects, we ignore the value added by the existing projects by setting it to zero.

The CEO’s compensation is linear in the aggregate profit of the firm and she wants to raise her perceived ability at the end of the second period. Her combined payoff from her compensation and from her perceived ability is

$$d_1 + \delta(d_2 Y - I) + d_3 E[A_0]$$

(11)

where $d_1$, $d_2$, and $d_3$ are positive constants, $\delta$ is an indicator variable which equals 1 if a project class is adopted and 0 otherwise, $Y$ is the aggregate value of all $n$ projects if the project class is adopted, $I$ is the CEO’s cost of effort in adopting a project class, and the expectation of the CEO’s ability, $E[A_0]$ is based on the information available at the end of the second period. The CEO is risk averse and her preferences are represented by the utility function in (2) over her payoff. The CEO solves her problem to arrive at the decision rule to adopt or reject a project class. The firm is already in business and the CEO can reject a project class entirely if its risk is too high given its mean return. She adopts a project class if $r \leq s(m)$ and rejects it otherwise. This decision rule takes into account the strategy of managers conditional on the adoption of a project class, and we shall characterize $s(m)$ after discussing the managers’ responses to the CEO’s actions.

If the CEO adopts a project class, the $n$ subordinate managers get to know the return and the risk of the project class and use this information to determine their effort inputs to their individual projects. Finally, the return from each project is realized. The net value of a project is given by $y = \epsilon z$,
where $e$ is managerial effort and $z$ is a random variable whose distribution, given the mean return $m$ and the risk $r$, is\footnote{The results are qualitatively similar if we consider $y = z + e$.}

\[
f(z) = \begin{cases} 
\frac{1}{2r} [1 + k(z - m)A] & \text{if } |z - m| \leq r \\
0 & \text{if } |z - m| > r 
\end{cases}
\]  

(12)

The compensation of a manager with net project value $y$ is given by

\[ x = c_1 + c_2 y \]  

(13)

The manager chooses effort to maximize his expected utility. His utility function is

\[ U(x, e) = -\alpha (\beta - x)^2 - \delta (e - \bar{e})^2. \]  

(14)

First, we consider the case where the CEO is known to be rational, i.e., $\pi = 0$. Then the managers know that the risk of the projects announced by the CEO is correct and use this in determining their effort choices in their individual projects.

**Lemma 3:** The effort input by each manager is

\[ e = \frac{\delta \bar{e} + \alpha c_2 (\beta - c_1)m}{\delta + \alpha c_2^2 (m^2 + r^2/3)}. \]  

(15)

The effort and the value of the project are decreasing in the risk $r$. The expected net value of the project is increasing in the mean return $m$.

Higher risk reduces the expected utility from compensation for a risk-averse manager. This reduces the manager’s marginal benefit from effort, causing less effort to be provided to the project and reducing its expected value.

Now we consider the possibility that the CEO is overconfident. An overconfident CEO underestimates the risk of any project class. If the mean return of a project class is $m$ and the risk is $r$, an overconfident CEO will believe that the risk is $L(m) + (r - L(m))/C$, where $C > 1$. Thus, the risk perceived by an overconfident CEO, though incorrect, is in the feasible range of risk for the return of the project class. Rather than assuming that the managers can directly verify the true mean return and
the risk of the project class themselves, we now assume that they can only verify the return and the risk perceived by the CEO.

**Lemma 4:** An overconfident CEO is more likely to adopt a project class than is a rational CEO.

The intuition is straightforward. The CEO adopts a project class only if the risk of the project class is acceptable given its mean return. An overconfident CEO underestimates risk and is therefore more likely to find a project class acceptable. Although overconfidence causes the CEO to take decisions that reduce her expected utility, this effectively results in the CEO overcoming her risk aversion and making decisions that may actually lead to higher firm value.

**Proposition 6:** After the CEO announces the characteristics of the chosen project class, let the ex post probability that the CEO is overconfident be \( \omega \). Then, each manager inputs the effort level

\[
e = \frac{\delta \varepsilon + \alpha \varepsilon_1 (\beta - c_1) m}{\delta + \alpha \varepsilon_2 \left[ m^2 + \frac{1}{3} (1 - \omega) r^2 + \omega L(m) + (r - L(m)) C \right]^2},
\]

where \( r \) is the risk of the projects as reported by the CEO. The effort and expected project value are decreasing in the reported risk \( r \), the degree of overconfidence \( C \), and the probability \( \omega \) of the CEO being overconfident\(^{17}\). The expected project value is increasing in the mean return \( m \).

Proposition 6 is similar to Lemma 3 except for the fact that it accounts for the possibility that the CEO could be overconfident. Managers reduce their effort input if the CEO is likely to be overconfident because projects chosen by an overconfident CEO have higher risk than reported; the riskier output is a noisier signal of managerial effort, resulting in weaker effort-supply incentives.\(^{18}\)

This reduction in effort causes a reduction in expected project value. Thus, even though an

\(^{17}\) The probability \( \omega \) that the CEO is overconfident is the ex post probability, conditional on the characteristics of the adopted project class. It differs from ex ante probability \( \pi \) that the CEO is overconfident; \( \pi \) is the posterior probability that exists after a manager is promoted to CEO but before the CEO announces whether or not she will adopt a project class.

\(^{18}\) Risky compensation need not always cause the managers to reduce their effort. The exact effect depends on the way that the project value depends on risk and effort. If the project output is \( y = e + \varepsilon \), effort would be independent of risk and thus independent of \( \theta \) and \( C \). If the project output is \( y = z \) with the probability distribution as in (2) except that \( A_1 \) is replaced by \( A_1 + e_0 \), effort would be increasing in risk, and consequently increasing in \( \theta \) and \( C \). In the first case, the expected value of the project is independent of \( C \) and \( \theta \), while in the
overconfident CEO benefits the firm by overcoming her own risk aversion and adopting a new project class that a rational CEO may consider too risky, the probability of the CEO being overconfident results in reduced effort by the managers, leading to a decline in the value of adopted projects.

In some states managers are unable to distinguish between an overconfident and a rational CEO, but in some other states they can noiselessly infer that the CEO is rational. An overconfident CEO always reports low risk, so if a CEO reports sufficiently high risk for a project class, she must be rational. If a CEO reports low risk, managers use Bayesian inference to update the probability that the CEO is overconfident.

**Proposition 7:** If the adopted project class is such that \( r > L(m) + (H(m) - L(m))/C \), the CEO is rational. Otherwise, the ex post probability that the CEO is overconfident is \( \omega = C\pi(I - \pi + C\pi) \),

where \( \omega \) is greater than \( \pi \), the ex ante probability of an overconfident CEO.

The maximum risk that an overconfident CEO reports is \( \tilde{r} = L(m) + \|H(m) - L(m)\|/C \). This means that a CEO adopting a project with perceived risk greater than \( \tilde{r} \) must be rational. If a CEO adopts a project with risk less than \( \tilde{r} \), she could be either rational or overconfident. The fact that an overconfident CEO is more likely than a rational CEO to find a project with acceptable risk suggests that \( \omega \), the probability of the CEO being overconfident, conditional on the adoption of the project, is higher than \( \pi \), the unconditional probability of the CEO being overconfident.

We assume that the parameters are such that an overconfident CEO will reject some projects. An overconfident CEO is willing to adopt a project class with perceived risk of \( s(m) \) or less, whereas the maximum risk that she perceives in a project class is \( L(m) + \|H(m) - L(m)\|/C \). Thus, we assume

\[
L(m) + \|H(m) - L(m)\|/C > s(m).
\]

(17)

In this case, the CEO can never be unambiguously identified as rational or overconfident because the reported risk for adopted projects varies from \( L(m) \) to \( s(m) \) for both types of CEOs. The second case it is increasing in these two parameters. We have considered a model that yields the worst outcome in terms of effort-supply incentives -- when the CEO is likely to be overconfident.
probability that the CEO is overconfident, conditional on the adoption of a project class, is
\[ \omega = \frac{C\pi}{1 - \pi + C\pi}. \]

**Proposition 8:** When there is uncertainty about the CEO's type, expected firm value is higher with an overconfident CEO than with a rational CEO.

The intuition is as follows. A manager's effort input is determined solely by the reported mean return and risk of the adopted project class. The project value, which depends on the mean return and the effort input, is the same whether the CEO is rational or overconfident. The distribution of reported mean return and reported risk is same for both kinds of CEOs -- because they use the same criterion to accept project classes -- so the expected project value is the same, conditional on the adoption of a project. The fact that an overconfident CEO adopts projects more often than does a rational CEO and the expected incremental value of all adopted projects is positive means that expected firm value is higher with an overconfident CEO than with a rational CEO.

We now endogenize the decision rule used by the CEO to adopt or reject a project class. If the CEO accepts a project class with mean return \(m\) and risk \(r\), she will also accept a project class with the same mean return but lower risk. One reason for this is that a project class with lower risk reduces the risk of compensation for the CEO. The other reason is that managers provide higher effort to projects with lower risk; this increases the expected aggregate value of the projects and hence the expected compensation of the CEO. The risk of the project class does not affect the perceived ability of the CEO, which forms the other component of her objective. Thus, the optimal decision rule is to adopt all projects with risk less than \(s(m)\) for a given mean return \(m\).

The risk cutoff \(s(m)\) is the risk \(r\) at which the CEO's utility from adopting a project class equals her utility from rejecting it. First, we consider the case when the CEO is known to be rational, i.e., \(\pi = 0\). Using (10), the CEO's mean perceived ability conditional on the mean return of project class observed is \(\left( m - \frac{\bar{M} + \bar{m}}{2} \right)/12\). The distribution of aggregate project value is obtained by considering it as the sum of \(n\) independent random variables each of which has the probability
distribution in (12). Evaluating the CEO’s utility as a function of the payoff in (11) for the cases of adoption and rejection of the project class and equating the two we get

\[
\left\{ \beta - d_1 - \frac{d_2t}{12}\left( m - \frac{\bar{M} + M}{2} \right) + I \right\}^2 + \frac{n^2}{3} + n^2m^2
\]

\[-2\left\{ \beta - d_1 - \frac{d_2t}{12}\left( m - \frac{\bar{M} + M}{2} \right) + I \right\}d_{2nem} = \left\{ \beta - d_1 - \frac{d_2t}{12}\left( m - \frac{\bar{M} + M}{2} \right) \right\}^2,
\]

where effort \( e \) is determined according to (15). Simplifying, we get

\[
s(m)^2 = \frac{6}{d_2^2e^2n} \left\{ \beta - d_1 - \frac{d_2t}{12}\left( m - \frac{\bar{M} + M}{2} \right) \right\} \left( d_{2nem} - 1 \right) - \frac{3}{d_2^2e^2n} \left( d_{2nem} - 1 \right)^2.
\]

When there is a positive probability \( \pi \) that the CEO is overconfident, the cutoff risk also depends on the probability \( \pi \) (or equivalently ex post probability \( \omega \); see Proposition 7) and is given by

\[
(1 - \omega)s(m, \omega)^2 + \omega[L(m) + C[s(m, \omega) - L(m)]]^2
\]

\[=
\frac{6}{d_2^2e^2n} \left\{ \beta - d_1 - \frac{d_2t}{12}\left( m - \frac{\bar{M} + M}{2} \right) \right\} \left( d_{2nem} - 1 \right) - \frac{3}{d_2^2e^2n} \left( d_{2nem} - 1 \right)^2 \equiv \nu,
\]

where effort \( e \) is determined according to (16). If we use the term “volatility” for the square of risk, the left side of equation (18) gives the expected true volatility (where the expectation is across the events that the CEO is rational or overconfident) of a project class that is adopted by the CEO at the cutoff risk. Equation (18) says that this expected volatility is a constant independent of the probability \( \omega \). The reason is that the expected volatility determines the managers’ effort in (16), which in turn determines the CEO’s payoff. The cutoff perceived risk \( s(m, \omega) \) is decreasing in the probability \( \omega \) (and hence probability \( \pi \)) because the CEO lowers the risk threshold for project class adoption to account for the higher likelihood of her being overconfident and underestimating risk.

E. Comparison of the Two Models

The two models we have studied show how CEO overconfidence affects the behavior of managers and firm value. In the first model, managers choose project-specific risk that supplements
the CEO's choice of firmwide risk. In the second, the CEO adopts a project class with an acceptable risk-return combination and managers determine the effort levels they put into projects. In both cases, when managers are uncertain about whether the CEO is rational or overconfident, the shareholders are better off when the CEO is overconfident than when the CEO is rational. This happens for related but somewhat distinct reasons in the two models. In the first model, an overconfident CEO elevates risk of the projects taken by the firm. Although this lowers the expected utility of risk-averse managers, firm value increases because these projects have high expected returns. In the second model, an overconfident CEO is less likely than a rational CEO to reject positive-NPV projects because of risk aversion; this increases firm value.

The general idea then is that overconfidence enables the CEO to overcome her own risk aversion and take risky projects that are value-enhancing for the shareholders but are avoided by rational CEOs due to risk aversion. When managers cannot unambiguously identify whether the CEO is rational or overconfident, their expected-utility-maximizing choice of risk or effort incorporates this uncertainty. Thus, an overconfident CEO gets managers to choose higher effort or risk than they would have if they knew that the CEO was overconfident. An overconfident CEO is also more likely to adopt a new project class.

5. IMPLICATIONS AND EXTENSIONS

In this section we discuss implications of our analysis and then two extensions. In the implications subsection we consider the correspondence of our analysis to real-world views of leadership, new product development, and its implications for small versus large firms. In the extension subsection we analyze the implications of extending the analysis to multiple periods and the impact of CEO overconfidence on "organization culture" (a term we attempt to give some economic meaning to).

A. Correspondence to Real-World Views of Leadership

Two aspects of leadership seem to be integral to how highly-regarded real-world leaders behave. One is that they are able to inspire others to follow them voluntarily, and the other is that they
are self-confident risk takers who are able to make bold decisions. Hermelin (1998) has provided a theory of how leaders motivate others to follow. We have explained how people get promoted to positions of leadership and why overconfident people are more likely than rational people to become leaders when the assignment of leadership is based on demonstrated ability.

CEO overconfidence turns out to be a virtue for risk-neutral shareholders in our model because it helps overcome to some extent the risk aversion of managers and thus closes the “preference gap” between shareholders and managers. Despite this, when managers know that the CEO is overconfident, it hurts the shareholders.

The role of (over)confidence and its interaction with risk-taking in effective leadership is echoed in the following quotes attributed to Jack Welch, CEO of General Electric (see Lowe (1998)):

“Self-confidence is the fuel of productivity and creativity, decisiveness, and speed” (p.142) and “Shun the incremental and go for the leap” (p.100).

B. New Product Development

Leadership style often has an important bearing on how capital budgeting is conducted and new products developed within organizations. New product development involves gathering information through time and then making resource allocation decisions. An overconfident CEO will demand less information before making a decision because she will view the desired precision as having been attained with more noisy information than will a rational CEO. Since it is the amount of information that is gathered before deciding to launch a new product that determines “product development cycle time”, an organization led by an overconfident CEO will tend to bring new products to market faster than one led by a rational CEO. Associated with the shorter product development cycle times will also be greater errors in new product introductions, e.g., a larger fraction of new product launches that fail.

C. Small versus Large Firms
Will the propensity of managers to take risk be stronger or weaker in small firms? From Proposition 1 we know that the larger the number of competing managers, the higher is the risk each manager takes. Moreover, the private benefit of control, $B$, is likely to be greater in larger firms, further reinforcing this effect. Thus, the effects of competition and hence project risk choices will be higher in bigger firms.

D. Extension: Multiple Periods

Although we have derived our results using a two-period model, similar results will obtain with more time periods in any finite-horizon model as long as project choices are made in only one period. That is, we can imagine a model in which there is a CEO at the helm with $n$ managers reporting to her. The CEO will be in office for $T \in (1, \infty)$ periods. Managers make project choices at the start of the first period. The project a manager chooses yields identical and independently distributed (i.i.d.) payoffs over each of the $T$ periods that the current CEO is in place; project choice is unalterable for $T$ periods once it is made in the first period.

After $T$ payoffs have been observed on each project, there is a posterior assessment of managerial ability, based on which a new CEO is chosen. The new CEO makes a one-time choice of strategy, which is followed by a second (and final) project choice by each manager. Beyond this point, project payoffs can be observed for any finite number of time periods.

Such a structure will yield essentially the same results we have at present. What seems intractable is a model in which the CEO makes multiple (more than two) strategy choices and managers make multiple project choices.

E. Extension: Organizational Culture

What is organization culture? We have no economic theories of organization culture, but if one reads organization behavior research (see Quinn (1996), for example), it appears that organization culture can be defined as a collection of explicit rules and implicit norms that define how the organization behaves in terms of assigning employees to tasks, promoting them, firing them, and
providing guidelines for risk taking and effort choices. The obvious question in our context is: how does the CEO's type affect organization culture?

The aspect of organization culture we want to focus on is its performance orientation. An organization with a stronger performance-oriented culture is one in which employee performance is a stronger determinant of task assignments and promotions than other considerations (e.g., politics). We examine the link between the CEO and organization culture by focusing on the impact of the CEO's type on the revelation of \((a \text{ priori} \ unknown)\) managerial abilities through observed project outputs (performance measures). The reason for doing this is that in our model it is the revelation of managerial abilities that will determine the assignment of tasks and who gets promoted.

An overconfident CEO adopts riskier projects for the firm. This may happen either when the CEO determines the overall risk of all the projects or when risk is determined partially by the CEO and partially by the managers who cannot distinguish between an overconfident CEO and a rational CEO. One possible outcome of the adoption of riskier projects is better revelation of the abilities of managers through time. Suppose the probability distribution of project output is given by (2). Then the following proposition shows the effect of risk on the probability that higher managerial ability leads to higher project output.

**Proposition 9:** Consider two ability levels \(A_L\) and \(A_H\), with \(A_L < A_H\). The probability that the project output of a manager with ability \(A_H\) exceeds that of a manager with ability \(A_L\) is increasing in the common risk of the projects and is given by

\[
\frac{1}{2} + \frac{k(A_H - A_L)R}{6}.
\]

The probability distribution function (2) is such that an increase in project risk increases the range of possible project output values, and managerial ability plays a more important role in determining the likelihood of extreme outcomes. Managerial ability has little impact on project output when the risk of the project is low; all managers tend to produce moderate project outputs. When project risk is high, a manager with higher ability is more likely to get a high project output and a
manager with lower ability is more likely to get a low project output. Thus, the higher the risk of projects, the higher the probability that a more able manager outperforms a less able manager.

An overconfident CEO thus leads an organization that achieves better revelation of the abilities of managers through observed project outputs. Consequently, the organization can achieve a better matching of the abilities of different managers with the requirements of different tasks. This can further improve the precision with which managerial ability can be inferred from performance, in addition to the direct productivity benefits due to better task-ability matching. An outcome of the improved inference precision is that managerial turnover will be higher as low-ability managers are revealed and replaced faster. By the same token, high-ability managers get quicker promotions.

To an outsider looking in, it will appear that the organization with an overconfident CEO has a more performance-oriented “culture” than the one with a rational CEO. However, since our analysis hinges on managers being unsure of whether the CEO is rational or overconfident, one cannot use this result to argue for an a priori choice of a rational or an overconfident CEO.\footnote{It would be implausible to think that the Board of Directors could locate a CEO who they know is overconfident but about whom the managers are unsure.}

Although economists have paid little attention to organization culture, it is an important issue in the real world and in organizational behavior research. The dimensions of organization culture we have attempted to provide some economic meaning to are the speed with which managerial abilities are identified, managerial turnover, and ability-task matching. In our model, the CEO’s attributes have an important bearing on organization culture, as they do in practice.

6. CONCLUSION

We have developed a theory of leadership that examines the determinants of how leaders are chosen and the economic consequences of this choice mechanism. In particular, we address two basic questions: (i) if leaders (CEOs) are chosen on the basis of demonstrated ability, are overconfident agents more likely than rational agents to be appointed CEO? and (ii) is an overconfident CEO better or worse for the firm than a rational CEO?
Our basic results are as follows. First, as long as there is uncertainty about whether the agents competing to be CEO are overconfident or rational, an overconfident agent has a higher probability than a rational agent of winning the race to be CEO. Second, an overconfident CEO produces a higher firm value than a rational CEO in an ex post sense, as long as nobody knows for sure the CEO type.

In addition to these main results, our analysis also generates many ancillary results in a setting in with multiple competing managers, each of whom has some probability of being overconfident. First, the relative advantage of an overconfident manager over a rational manager in the race to be CEO is increasing in the degree of overconfidence and in the number of competing managers. Second, the level of risk chosen by competing managers is increasing in the private managerial benefit to promotion and it is decreasing in the sensitivity of compensation to output, in the sensitivity of output to managerial ability, and in the degree of overconfidence. Third, when managers know that there is some probability the CEO is overconfident, their choice of project risk and effort input are decreasing in the degree of CEO overconfidence (conditional on the CEO being overconfident) as well as in the probability that the CEO is overconfident. Fourth, managerial uncertainty about the CEO's type (rational or overconfident) benefits the shareholders in the sense that firm value is higher when such uncertainty is present than when the CEO type is known for sure to be rational or overconfident.

We have also explored the implications of our analysis for new product development and organization culture. We conclude that, compared to organizations managed by rational CEOs, those managed by overconfident CEOs will: (i) bring new products to market faster and have a greater incidence of these introductions fail and (ii) have a more performance-oriented organization culture that achieves a more efficient matching of managerial ability to tasks. Thus, we have attempted to provide economic content to one aspect of the "soft" notion of organization culture within the framework of a formal theory of leadership.

The analysis produces many testable predictions, which are summarized below:

- In the cross-section of managers, the probability that someone is overconfident increases as one moves up the leadership hierarchy in an organization.
• Firms with a relatively high degree of product innovation will also promote managers faster and have higher turnover among managers.

• Managers take higher risk in larger firms than in smaller ones.

The principal insight of our paper that overconfidence helps an agent overcome personal risk aversion and therefore make decisions that increase the likelihood of becoming a leader seems to correspond well with real-world accounts of leadership. Moreover, the result that CEO overconfidence may actually benefit the firm is consonant with the assertion that successful CEOs are often "on the lunatic fringe." We hope that this paper, along with Hermelin's (1998) insightful contribution about how leaders motivate others, will lead to future work in building a coherent economic theory of leadership.

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20 See the Jack Welch quotation prior to the Introduction.
APPENDIX

Proof of Lemma 1: Manager $i$'s utility from compensation is

$$-\alpha \left[ \beta - c_1 - c_2 \frac{k(y_i - \gamma R^b)}{12} \right]^2, \quad (A1)$$

where $R^b$ is everyone's belief about the risk of manager $i$'s project. Using the probability distribution of output $y_i$ from (2), the expected utility is

$$-\alpha \left\{ (\beta - c_1)^2 + \frac{c_2^2 k^2 R^o^2}{432} + \frac{\gamma^2 c_2^2 k^2 (R^b - R^o)^2}{144} + \frac{c_2 k \gamma (R^b - R^o)(\beta - c_1)}{6} \right\}, \quad (A2)$$

The manager chooses risk to maximize expected utility. In equilibrium, $R^o = R^b$, and the first order condition yields risk

$$R^o = \frac{36\gamma (\beta - c_1)}{c_2 k}. \quad \square$$

Proof of Lemma 2: The probability that the output of the project of any other manager is less than $y$ is $(y + \hat{R} - \gamma \hat{R})/2\hat{R}$ if $-(1+\gamma)\hat{R} \leq y \leq (1+\gamma)\hat{R}$ and $1$ if $y > (1+\gamma)\hat{R}$. Thus, the probability that manager $i$'s output exceeds that of the $n-1$ remaining managers is

$$\int_{-(1+\gamma)\hat{R}}^{(1+\gamma)\hat{R}} \frac{(1+\gamma)\hat{R} - y}{2\hat{R}}^{n-1} \frac{dy}{2\hat{R}} + \int_{(1+\gamma)\hat{R}}^{(1+\gamma)\hat{R}} \frac{dy}{2\hat{R}} = \frac{1+\gamma}{2} - \frac{\hat{R}}{R} \left( \frac{1+\gamma}{2} - \frac{1}{n} \right). \quad \square$$
Proof of Proposition 1: The expected utility of manager $i$ from compensation and from potential promotion is

$$
\begin{align*}
&-\alpha \left\{ (\beta - c_1)^2 + \frac{c_2^2 k^2 R^2}{432} + \frac{\gamma^2 c_2^2 k^2 (R^* - R)^2}{144} + c_2 k \gamma (R^* - R) (\beta - c_1) \right\} \\
&+ B \left[ \frac{1 + \gamma}{2} - \frac{R^*}{R} \left( \frac{1 + \gamma}{2} - \frac{1}{n} \right) \right]
\end{align*}
$$

where $R$ is the risk of manager $i$'s project and $R^*$ that of other managers. To maximize this with respect to $R$, we find the first-order condition and substitute the equilibrium requirement $R = R^*$ to get

$$
\frac{\alpha (\beta - c_1) c_2 k \gamma}{6} - \frac{\alpha c_2^2 k^2 R^*}{216} + \frac{B}{R^*} \left( \frac{1 + \gamma}{2} - \frac{1}{n} \right) = 0.
$$

(A3)

The solution to this equation is

$$
R^* = \frac{18 \gamma (\beta - c_1) + 6 \sqrt{9 \gamma^2 (\beta - c_1)^2 + 6 B / \alpha \left( 1 + \frac{\gamma}{2} - \frac{1}{n} \right)}}{c_2 k}
$$

This is increasing in $B/\alpha$, $n$, and $\gamma$, but decreasing in $c_2$ and $k$. □

Proof of Proposition 2: From (7), the probability of promotion for manager $i$ exceeds $1/n$, so the probability of promotion for any of the remaining $n-1$ rational managers is less than $1/n$. □

Proof of Proposition 3: Suppose manager $i$ chooses risk $R$ when other managers choose risk $R^*$. The expected utility of manager $i$ from compensation is given by (A2), with $R^e$ replaced by $(1-\theta + \theta C)R^*^e$ and $R^e$ replaced by $R$ with probability $1-\theta$ and by $CR$ with probability $\theta$. Thus, the expected managerial utility from compensation is
\[
\frac{-\alpha}{432}\left(\beta - c_i\right)^2 + \frac{c_i^2 k^2 \left(1 - \theta + \theta C^2\right)}{6} R^2 + c_2 k \gamma \left(1 - \theta + \theta C\right) \left(R'' - R\right) \left(\beta - c_i\right)
\]
\[
+ \frac{1}{144} \left(1 - \theta + \theta C^2\right) R^2 + \left(1 - \theta + \theta C \right)^2 \left(R''^2 - 2R'' R\right)
\]  

(A4)

With probability 1-\(\theta\), manager \(i\) is rational and chooses risk \(R\). The probability of promotion for manager \(i\) in this case is

\[
P_i \left[\frac{1 + \gamma + (1 - \gamma) CR''}{2 CR''}\right]^{-1} \frac{dy}{2R} = \frac{R''}{2nR}(1-\gamma)\left[\frac{\theta}{2} \left(1 - \frac{1}{C}\right) (1 - \gamma)\right]^{-1}
\]

\[
- \frac{1}{2n} \left(\frac{CR''}{R} - 1\right) (1 - \gamma) \left[\frac{\theta}{2} \left(1 - \frac{R}{CR''}\right) (1 - \gamma)\right]^{-1}
\]

\[
+ \frac{R''}{2nR} \left(1 + \gamma + \frac{C(1 - \gamma)}{C + \theta - C\theta}\right) \left[\frac{1 - \theta}{2} \left(1 - \frac{1}{C}\right) (1 + \gamma)\right]^{-1}
\]

\[
- \frac{R''}{2nR} \left(\theta (C - 1)(1 - \gamma)\right) \left[\frac{\theta}{2} \left(1 - \frac{1}{C}\right) (1 - \gamma)\right]^{-1}
\]

\[
+ \frac{1}{2n} \left(1 + \gamma + \frac{CR''}{R} \left(\frac{2}{\theta} - (1 + \gamma)\right)\right) \left[\frac{1 - \theta}{2} \left(1 - \frac{R}{CR''}\right) (1 + \gamma)\right]^{-1}
\]

\[
- \frac{R''}{2nR} \left(\frac{2C}{\theta} - (1 + \gamma)(C - 1)\right) \left[\frac{1 - \theta}{2} \left(1 - \frac{1}{C}\right) (1 + \gamma)\right]^{-1}
\]  

(A5)

With probability \(\theta\), manager \(i\) is overconfident and chooses risk \(CR\). The probability of promotion for manager \(i\) in this case is
\[ P_2 = \int \left( \frac{y + (1-y)CR''}{2CR''} \right)^{n-1} \frac{dy}{2CR} + \int \left( \frac{(1-\theta)y + (1-\gamma)R''}{2R''} + \theta y + (1-\gamma)CR'' \right)^{n-1} \frac{dy}{2CR} + \int \left( \frac{(1-\theta)y + (1-\gamma)CR''}{2CR''} \right)^{n-1} \frac{dy}{2CR} + \int \frac{(1-\gamma)CR}{2CR''} \frac{dy}{2CR} \]

\[ = \frac{R''}{2nR} \left( \frac{1-\gamma}{R'} \right) \left[ \frac{\theta}{2} \left( \frac{1-1}{C} \right) (1-\gamma) \right]^{n-1} + \frac{R''}{2nR} \left[ \frac{1+\gamma}{C} + \frac{1-\gamma}{C+\theta-C\theta} \right] \left[ \frac{1-\theta}{2} \left( \frac{1-1}{C} \right) (1+\gamma) \right]^{n-1} \]

\[ - \frac{R''}{2nR} \left[ \frac{\theta(C-1)(1-\gamma)}{C(C+\theta-C\theta)} \right] \left[ \frac{\theta}{2} \left( \frac{1-1}{C} \right) (1-\gamma) \right]^{n-1} + \frac{R''}{n\theta R} - \frac{R''}{2nR} \left[ \frac{2}{\theta} (1+\gamma) \left( \frac{1-1}{C} \right) \right] \left[ \frac{1-\theta}{2} \left( \frac{1-1}{C} \right) (1+\gamma) \right]^{n-1} + \frac{1+\gamma}{2} - \frac{1+\gamma R''}{2R} \]

(A6)

The expected utility of manager \( i \) consists of the part from compensation, given by (A4), and \( B[(1-\theta)P_1 + \theta P_2] \). The manager chooses risk \( R \) to maximize expected utility. Finding the first-order condition using (A4), (A5), and (A6) and substituting the equilibrium condition \( R=R'' \), we get

\[-\alpha \left\{ \frac{c^2k^2 [(1-\theta + \theta C^2) + 3\gamma^2 \theta (1-\theta)(C-1)^2]}{216} - \frac{c_2k \gamma (1-\theta + \theta C)(\beta-c)}{6} \right\} \]

\[ + \frac{B(1-\theta)}{2nR''} \left[ \frac{\theta}{2} \left( \frac{1-1}{C} \right) (1-\gamma) \right]^{n-1} \left( 1-\gamma \right) \left( n + \frac{\theta(C-1)}{C+\theta-C\theta} \right) \]

\[ + \left[ 1-\frac{\theta}{2} \left( \frac{1-1}{C} \right) (1+\gamma) \right]^{n-1} \left( 1+\gamma \right) \left( n-1 - \frac{C(1-\gamma)}{C+\theta-C\theta} \right) \]

\[ + \frac{B \theta}{2nR''} \left[ \frac{\theta}{2} \left( \frac{1-1}{C} \right) (1-\gamma) \right]^{n-1} \left( 1-\gamma \right) \left( \frac{1}{C+\theta-C\theta} - 1 \right) \]

\[ + \left[ 1-\frac{\theta}{2} \left( \frac{1-1}{C} \right) (1+\gamma) \right]^{n-1} \left[ \frac{2}{\theta} (1+\gamma) - \frac{1-\gamma}{C+\theta-C\theta} \right] \]

\[ + \frac{B \theta}{R'' \left( \frac{1+\gamma}{2} - \frac{1}{n\theta} \right)} = 0. \]

Solving this quadratic equation yields the solution in (8). □
Proof of Proposition 4: Suppose manager $i$ chooses project-specific risk $T$ so that risk is $S+T$ with probability $1-\pi$ and is $CS+T$ with probability $\pi$. Others believe that the expected value of risk is $R^b$. Manager $i$'s utility from consumption is given by (A1). The expected utility is similar to that in (A2) except for the fact that $R$ can be either $S+T$ or $CS+T$. Substituting this in (A2), the expected utility is

\[
\left[ (\beta - c_1)^2 + \frac{e^2k^2(1-\pi)(S+T)^2 + \pi(CS+T)^2}{432} \right]
- \alpha \left[ \frac{\gamma^2c_1k^2(R^b + (1-\pi)(S+T)^2 + \pi(CS+T)^2 - 2R^b((1-\pi+C\pi)S+T}}{144} \right]
+ \frac{c_2k\gamma(R^b - (1-\pi+C\pi)S - T)(\beta - c_1)}{6} \right] \tag{A7}
\]

The manager chooses $T$ to maximize this expectation. Substituting $R^b = (1-\pi+C\pi)S + T$ in the first-order condition yields

\[
T = \frac{36\gamma(\beta - c_1)}{c_2k} - (1-\pi+C\pi)S = R^\circ - (1-\pi+C\pi)S.
\]

When the CEO is known to be rational we have $\pi=0$, and when CEO is known to be overconfident we have $\pi=1$. In the former case $S+T = R^\circ$, and in the latter case $CS+T = R^\circ$. When there is uncertainty about CEO type but the CEO is rational, risk is $S+T < R^\circ$. When there is uncertainty about CEO type but the CEO is overconfident, risk is $CS+T > R^\circ$. The expected value of risk, $T + (1-\pi)S + \pi CS = R^\circ$, is independent of $\pi$. The expected project output equals a constant $\gamma$ times risk, so firm value is also independent of $\pi$. \qed

Proof of Proposition 5: Suppose the equilibrium project-specific risk is $T^b$, so the expected value of risk is $R^b = T^b + (1-\pi+C\pi)S$. Suppose manager $i$ contemplates choosing project-specific risk $T$. His expected utility from consumption is as in (A7) and the expected utility from potential promotion is
\[ B \left[ \frac{1 + \gamma}{2} - \left( \frac{1 - \pi(S + T^b)}{(S + T)} + \frac{\pi(CS + T^b)}{(CS + T)} \right) \left( \frac{1 + \gamma - \frac{1}{n}}{2} \right) \right]. \]

In the above, the probability of promotion differs from (5) to account for risk differences stemming from whether the CEO is rational or overconfident. Adding the two components of expected utility, maximizing with the first-order condition for \( T \) and then substituting the equilibrium condition, \( T = T^b \), we get

\[ \frac{\alpha^2 k \gamma (\beta - c_1)}{6} - \frac{\alpha^2 k^2 216}{216} \left( \frac{1 - \pi}{S + T} + \frac{\pi}{CS + T} \right) \left( \frac{1 + \gamma - \frac{1}{n}}{2} \right) = 0. \]

(A8)

When the CEO type is known with certainty, that is, \( \pi = 0 \) or \( \pi = 1 \), the above equation reduces to (A3) and the risk is \( R^* \).

When CEO type is not known and the CEO is rational, we claim that risk \( S + T < R^* \). Suppose counterfactually this is not true. Then \( S + T \geq R^* \) and \( CS + T > R^* \). Substituting in (A8) gives

\[ \frac{\alpha^2 k \gamma (\beta - c_1)}{6} - \frac{\alpha^2 k^2 216}{216} \frac{B}{R^*} \left( \frac{1 + \gamma - \frac{1}{n}}{2} \right) > 0, \]

which contradicts (A3).

When CEO type is not known and the CEO is overconfident, we claim that risk \( CS + T > R^* \). Suppose counterfactually this is not true. Then \( CS + T \leq R^* \) and \( S + T < R^* \). Substituting in (A8) gives

\[ \frac{\alpha^2 k \gamma (\beta - c_1)}{6} - \frac{\alpha^2 k^2 216}{216} \frac{B}{R^*} \left( \frac{1 + \gamma - \frac{1}{n}}{2} \right) < 0, \]

which contradicts (A3). Thus, \( CS + T > R^* \).

We now prove that the expected value of risk, \((1 - \pi)S + \pi CS + T\), is greater than \( R^* \). Suppose counterfactually this is not true. Then \((1 - \pi)S + \pi CS + T \leq R^* \). We also use the fact that the arithmetic mean always exceeds the harmonic mean to get

\[ \frac{1 - \pi}{S + T} \frac{\pi}{CS + T} > \frac{1}{(1 - \pi)(S + T) + \pi(CS + T)}. \]
Substituting these inequalities in (A8) yields
\[
0 = \frac{\alpha_3}{6} k \gamma (\beta - c_1) - \frac{\alpha_3^2}{216} k^2 \left\{ \left(1 - \pi \right) S + \pi CS + T \right\}
+ B \left\{ \frac{1 - \pi}{S + T} + \frac{\pi}{CS + T} \right\} \left( \frac{1 + \gamma}{2} - \frac{1}{n} \right)
\geq \frac{\alpha_3}{6} k \gamma (\beta - c_1) - \frac{\alpha_3^2}{216} k^2 R^* + \frac{B}{R^*} \left( \frac{1 + \gamma}{2} - \frac{1}{n} \right),
\]
which contradicts (A3). Thus, \((1 - \pi) S + \pi CS + T \leq R^*\). The mean project output is proportional to risk, so mean project output is higher when CEO type is uncertain.

Proof of Lemma 3: The probability distribution of \(z\), conditional on the manager's ability, is given by (10). Ability is uniformly distributed between \(-1/2\) and \(1/2\). Taking expectation with respect to ability, the unconditional distribution of \(z\) is uniform between \(m - r\) and \(m + r\). Thus,
\[
E[y] = e E[z] = em; \quad E[y^2] = e^2 E[m^2] = e^2 \left( m^2 + \frac{r^2}{3} \right).
\]
Using (11) and (12), the expected utility of the manager is
\[
- \alpha (\beta - c_1)^2 - \alpha c_2^2 \left( e^2 m^2 + e^2 \frac{r^2}{3} \right) + 2 \alpha (\beta - c_1) c_2 em - \delta (e - e)^2,
\] (A9)
and it is maximized at
\[
e = \frac{\delta e + \alpha c_2 (\beta - c_1) m}{\delta + \alpha c_2^2 \left( m^2 + \frac{r^2}{3} \right)}
\]
The expected firm value is
\[
E[y] = \frac{\delta em + \alpha c_2 (\beta - c_1) m^2}{\delta + \alpha c_2^2 \left( m^2 + \frac{r^2}{3} \right)},
\]
Both effort and expected firm value are decreasing functions of \(r\). The firm value is an increasing function of \(m\) using the fact that \(\beta > c_1 + c_2 em\).
Proof of Lemma 4: A rational CEO adopts a project class with mean return \( m \) if the risk does not exceed \( s(m) \) while an overconfident CEO will adopt a project class as long as the true risk does not exceed \( L(m) + C\{s(m)-L(m)\} > s(m) \). Thus, an overconfident CEO adopts project classes more often than does a rational CEO. □

Proof of Proposition 6: The expected utility of a manager choosing effort is as in (A9) but the true risk is either equal to the reported risk \( r \) (with probability \( 1-\omega \), which is the probability of a rational CEO) or \( L(m)+\{r-L(m)\}C \) (with probability \( \omega \), which is the probability of an overconfident CEO).

Thus, expected utility is

\[
-\alpha (\beta - c_1)^2 - \alpha \delta_2 e^{2} \left( m^2 + \frac{1}{3} ((1-\omega) r^2 + \omega \{L(m) + (r - L(m))C\}^2) \right) + 2\alpha (\beta - c_1) \kappa_2 em - \delta (e-e)^2
\]

and it is maximized at

\[
e = \frac{\delta \omega + \alpha \delta_2 (\beta - c_1) m}{\delta + \alpha \delta_2 \left( m^2 + \frac{1}{3} ((1-\omega) r^2 + \omega \{L(m) + (r - L(m))C\}^2) \right)}.
\]

The mean net value of the project is \( em \). Thus, managerial effort input and expected firm value are decreasing functions of \( r, C \) and \( \omega \). Expected firm value is an increasing function of \( m \). □

Proof of Proposition 7: The highest risk possible for a project class with mean return \( m \) is \( H(m) \). Thus, the highest risk an overconfident CEO will report is \( L(m) + (H(m) - L(m))/C \). If the CEO reports higher risk, she must be rational.

Suppose a project class is adopted and the reported risk is less than \( L(m) + (H(m) - L(m))/C \). Clearly, the CEO may be overconfident. Since the project class is adopted, the reported risk must also be less than \( s(m) \). Thus, the CEO may also be rational. The probability density function of reported risk for a
rational CEO is \(1/(H(m)-L(m))\) on its support \([L(m),H(m)]\), and the prior probability of a rational CEO is \(1-\pi\). When the risk is \(r\), an overconfident CEO reports \(L(m) + (r-L(m))/C\), so the probability density function of reported risk for an overconfident CEO is \(C/(H(m)-L(m))\) on its support \([L(m),L(m)+(H(m)-L(m))/C]\), and the prior probability of an overconfident CEO is \(\pi\). Thus, the posterior probability that the CEO is overconfident is

\[
\omega = \frac{\pi \times \frac{C}{H(m)-L(m)}}{(1-\pi) \times \frac{1}{H(m)-L(m)} + \pi \times \frac{C}{H(m)-L(m)}} = \frac{C\pi}{1 - \pi + C\pi}.
\]

Proof of Proposition 8: The distribution of mean return \(m\) is identical for both rational and overconfident CEOs, so it is sufficient to show that, conditional on the mean return \(m\), the expected project value is higher for an overconfident CEO. Let \(y(r)\) denote the expected project value when the reported risk is \(r\). This is well defined because the expected project value \(em\) depends on \(m\), which is fixed, and \(e\), which is determined by the manager as a function of \(m\) and \(r\). Further, \(y\) is positive whenever a project class is adopted and zero otherwise. Thus, the expected change in firm value when the CEO is rational is

\[
E[y(r)] = \frac{1}{H(m)-L(m)} \int_{L(m)}^{H(m)} y(r) dr \tag{A10}
\]

The expected change in firm value when the CEO is overconfident is

\[
E[y(r)] = \frac{1}{H(m)-L(m)} \int_{L(m)}^{L(m)+(H(m)-L(m))/C} y\left(L(m) + \frac{r-L(m)}{C}\right) dr = \frac{C}{H(m)-L(m)} \int_{L(m)}^{H(m)} y(u) du \tag{A11}
\]

Comparing the expected change in firm value in the two cases, we see that the expected firm value is higher when the CEO is overconfident. \(\square\)
Proof of Proposition 9: Let \( y_H \) and \( y_L \) be the outputs of projects of managers with abilities \( A_H \) and \( A_L \), respectively. The effect of \( \gamma \) in the distribution of project outputs in (2) is to increase the outputs by \( \gamma R \).

For the purpose of comparing the outputs, we ignore \( \gamma \) by assuming \( \gamma = 0 \).

\[
P(y_L < y) = \frac{1}{2R} \int_{-R}^{R} \left( y + kxA_L \right) dx = \frac{1}{2R} \left( (y + R) + \frac{k(y^2 - R^2)}{2} A_L \right)
\]
\[
= d_2 y^2 + d_1 y + d_0,
\]
where,
\[
d_2 = \frac{kA_L}{4R}, \quad d_1 = \frac{1}{2R}, \quad d_0 = \frac{1}{2} - \frac{kRA_L}{4}.
\]

Now,
\[
P(y_H > y_L) = \frac{1}{2R} \int_{-R}^{R} \left( d_2 y^2 + d_1 y + d_0 + kyA_H \right) dy = \left( d_2 + d_1 kA_H \right) \frac{R^3}{3} + d_0.
\]

Substituting for \( d_2, d_1 \) and \( d_0 \), and simplifying, we get
\[
P(y_H > y_L) = \frac{1}{2} + \frac{k(A_H - A_L)}{6} R.
\]
\[ \square \]
References


Table 1: List of mathematical symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>Number of managers</td>
</tr>
<tr>
<td>$U$</td>
<td>Manager's utility</td>
</tr>
<tr>
<td>$x$</td>
<td>Manager's compensation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Sensitivity of managerial utility to compensation</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Determines concavity of managerial utility to compensation</td>
</tr>
<tr>
<td>$y_i$</td>
<td>Output of manager $i$'s project</td>
</tr>
<tr>
<td>$f_i$</td>
<td>Probability density function of output of manager $i$'s project</td>
</tr>
<tr>
<td>$A_i$</td>
<td>Ability of manager $i$</td>
</tr>
<tr>
<td>$k$</td>
<td>Sensitivity of density function of output to managerial ability</td>
</tr>
<tr>
<td>$R_i$</td>
<td>Risk of manager $i$'s project</td>
</tr>
<tr>
<td>$R_{\text{max}}$</td>
<td>Highest possible risk of a project</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Sensitivity of mean project output to project risk</td>
</tr>
<tr>
<td>$c_1$</td>
<td>Fixed part of manager's compensation</td>
</tr>
<tr>
<td>$c_2$</td>
<td>Sensitivity of manager's compensation to his perceived ability</td>
</tr>
<tr>
<td>$S$</td>
<td>CEO's strategy or firmwide risk choice</td>
</tr>
<tr>
<td>$\lambda_0$</td>
<td>CEO's ability</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Equilibrium risk without promotion concerns</td>
</tr>
<tr>
<td>$B$</td>
<td>Net increase in utility of a manager due to promotion</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Probability of promotion of manager $i$</td>
</tr>
<tr>
<td>$R^*$</td>
<td>Equilibrium risk with promotion concerns</td>
</tr>
<tr>
<td>$C$</td>
<td>Degree of overconfidence</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Probability that any manager is overconfident</td>
</tr>
<tr>
<td>$R^{**}$</td>
<td>Equilibrium risk with promotion concerns and possibility of overconfidence</td>
</tr>
<tr>
<td>$T_i$</td>
<td>Project-specific risk</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Ex ante probability that the new CEO (in second period) is overconfident</td>
</tr>
<tr>
<td>$m$</td>
<td>Mean return of a project class</td>
</tr>
<tr>
<td>$h$</td>
<td>Probability density function of mean return of project class</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$M$</td>
<td>Lower bound on mean return of project class</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Upper bound on mean return of project class</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Indicator variable for adoption of a project class by the CEO</td>
</tr>
<tr>
<td>$Y$</td>
<td>Aggregate value of all projects</td>
</tr>
<tr>
<td>$z$</td>
<td>Ratio of a project's value to its manager's effort</td>
</tr>
<tr>
<td>$r$</td>
<td>Risk of a project class</td>
</tr>
<tr>
<td>$L$</td>
<td>Lower bound on the risk of project class</td>
</tr>
<tr>
<td>$H$</td>
<td>Upper bound on the risk of project class</td>
</tr>
<tr>
<td>$s$</td>
<td>Cutoff ratio of risk to mean return for adoption of a project class</td>
</tr>
<tr>
<td>$y$</td>
<td>Net value of project</td>
</tr>
<tr>
<td>$e$</td>
<td>Effort input by any manager</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Sensitivity of manager's disutility to effort</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Probability that the CEO is overconfident conditional on adopted project class</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>Maximum risk that an overconfident CEO reports</td>
</tr>
<tr>
<td>$\bar{v}$</td>
<td>Average volatility of adopted project class adopted at cutoff risk</td>
</tr>
<tr>
<td>$R^b$</td>
<td>Common belief about project risk</td>
</tr>
</tbody>
</table>
- Each of the n managers chooses risk of his/her project.
- Project outputs are observed.
- Managers are compensated based on the output of their projects.
- The manager with the highest project output becomes the new CEO.
- The new CEO chooses strategy for the firm.
- The managers in turn choose their project choices (project-specific risk or effort input).

- Second period project outputs are observed.
- Managers are compensated based on the output of their projects.

Figure 1