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EVALUATING THE IMPACT OF SMALL CHANGES IN ORDERED WIDTHS ON TRIM YIELDS

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Executive Summary

For some roll trim problems, a small reduction in the size of one or more of the ordered widths can have a significant favorable impact on the yield of the primary production process. Integrated producers of converted products such as corrugated boxes, folding cartons, bags, disposable diapers and business forms may achieve a significant cost saving by systematically making small changes in the roll widths ordered for their converting operations.

Haessler and Sweeney describe an efficient procedure for identifying situations in which significant yield improvements can occur. Their procedure also identifies the changes in ordered sizes that must be made to achieve this improvement. By minimizing the design costs of changing one or more of the ordered widths to improve yield, a producer can achieve the maximum possible profit improvement. An application of this technique is demonstrated with data from a producer of an extruded film used in disposable diapers.

It is well known that small changes in ordered widths can have a significant impact on trim loss or process yield for any material which is produced in wide rolls and slit to satisfy a variety of customer order sizes. For example, consider the difference in yield on a 200 inch production roll when the only ordered width is changed from 66 3/4 inches to 66 5/8 inches. Although not many real world situations are that obvious or dramatic, it still may be very worthwhile for an organization that controls both the roll production process and the sizes ordered to evaluate the impact that small changes in ordered sizes have on yield. This opportunity is open to integrated converters of all sorts of woven and non-woven products such as corrugated boxes, folding cartons, business forms, bags and diapers.

In the case of the integrated corrugated-box producer, the roll stock sizes carried by the box plants may be easily changed if there is sufficient economic incentive to do so. For other converting operations such as disposable diapers, the roll widths of the non-woven barrier may be changed only by changing the design of the final product. This may involve a significant cost which would have to be offset by improved yield at the primary production process for the change to be worthwhile.

The purpose of this paper is to demonstrate a methodology for:

1) identifying opportunities for improving primary yields by reducing ordered sizes; and
2) selecting the least costly way to achieve the yield improvement in those cases where there are alternative ways to change the ordered widths.

Only opportunities to improve yield through possible reductions in ordered widths will be considered. Although trim yield can also be improved by increasing ordered widths, this will not lead to an economic gain for an integrated firm unless it allows the converting operation to decrease operating costs or increase revenues. These situations are outside the scope of this article.

Identifying Yield Improvement Opportunities

Consider a converting operation which places orders for **m** different sizes of some product on a regular basis. For every 1000 rolls of this product ordered, let R_i be the number needed of width W_i. Clearly each R_i must relate directly to the mix in which the converted product is sold. Let TMW be the maximum usable width of the production rolls at the primary process. Although the yield improvements, we are considering will come from reductions in the ordered widths W_i, the easiest way to identify these opportunities is as follows. We hypothetically increase the maximum usable width of the primary process in small increments (INC).

Let S_k be the master roll width after the kth increment:

$$S_o = TMW$$

$$S_k = TMW + k \bullet INC$$

The advantage of proceeding in this way is that the whole range of possible yield improvements can be identified in a single pass without having to be concerned with the specific combinations of changes in the individual ordered widths W_i.

The values of k at which yield improvements occur can be identified using a Gilmore-Gomory (1, 2) algorithm for solving the following LP model.

$$Z_k = \text{Min } \Sigma_j \Sigma_k X_{jk}$$

$$\Sigma_j \Sigma_k A_{ijk} \bullet X_{jk} = R_i \quad \text{for } i = 1\text{---}m$$

$$X_{jk} \geq 0$$

where

X_{jk} - usage of pattern j at the kth increment in production roll size.

A_{ijk} - number of times roll size i appears in pattern j for the kth production roll increment.

At each increment A_{ijk} must satisfy the usual requirements for a slitting pattern:

$$\Sigma_i A_{ijk} W_i \leq S_k$$

$$A_{ijk} \geq, 0 \text{ integer.}$$

When k = 0, the best possible process yield with the current order sizes is given by

$$\Sigma_i R_i W_i / (TMW \bullet Z_o)$$

As the production roll width S_k is increased the critical question is what happens to Z_k. If Z_k remains the same, it means that there is no reduction in the number of production rolls required to produce the 1000 rolls ordered and, therefore, no economic benefit. If Z_k decreases, this represents a reduction in the number of production rolls required to produce the 1000 rolls ordered and, therefore, an improvement in yield. If TC_o is the current total annual cost of producing rolls for this converting operation, the benefit of producing with effective production roll width S_k is given by

$$TC_o(1- Z_k/Z_o)$$

Because the width of the production roll is not actually being increased, this annual cost reduction can be obtained, if and only if, the values of W_i can be reduced by a sufficient amount to make the slitting patterns associated with the nonzero X_{jk} variables feasible with a production roll width of size TMW. This issue will be discussed in more detail as part of the example which follows.

Example

Consider an integrated producer of converted products which requires 4 sizes of a single product in the following mix per 1000 rolls:

Width (inches)	Quantity
14.1	30
13.4	380
13.0	490
10.7	100
	1000

The extruder on which the non-woven product is produced has a maximum usable width of 115 inches.

The optimal LP solution to this problem requires 115.084 production rolls. As the usable width of the production roll is hypothetically increased, the solution to the LP problem will be reduced if new patterns can be found which take advantage of the hypothetical "extra" production roll width. Remember, once again, that ultimately these patterns must actually be made feasible by reducing the ordered widths W_i. Table 1 shows the initial LP solution to this problem and the LP solution at each incremental production roll width, for which yield changes occur up to a maximum production roll width of 117 inches.

Note that at a production roll width of 117 inches, only 110.873 production rolls are required to produce the 1000 ordered rolls. This is a reduction in the time required (yield improvement) to meet the requirements of this converting operation of:

$$1 - \frac{110.873}{115.084} = .0366$$

Table 1

S_K	Ordered Widths 14.1	13.4	13.0	10.7	Pattern Trim Loss	Pattern Usage
	Slitting Patterns (No. for each width)					
115.0	8	0	0	0	2.2	3.750
	0	8	0	0	7.8	32.000
	0	0	8	1	0.3	58.667
	0	6	1	2	0.2	20.667
						115.084
115.1	8	0	0	0	2.3	3.750
	0	8	0	0	7.9	26.923
	0	1	7	1	0.0	67.692
	0	6	1	2	0.3	16.154
						114.519
115.2	8	0	0	0	2.4	3.750
	0	8	0	0	8.0	25.625
	0	1	7	1	0.1	70.000
	0	7	0	2	0.0	15.000
						114.375
115.5	8	0	0	0	2.7	3.750
	0	8	0	0	8.3	19.062
	0	2	6	1	0.0	81.667
	0	7	0	2	0.3	9.167
						113.646
115.9	8	0	0	0	3.1	3.750
	0	8	0	0	8.7	9.875
	0	3	5	1	0.0	98.000
	0	7	0	2	0.7	1.000
						112.625
116.3	8	0	0	0	3.5	3.750
	0	8	0	0	9.1	8.750
	0	3	5	1	0.4	90.000
	0	4	4	1	0.0	10.000
						112.500
117.0	0	5	3	1	0.3	57.143
	0	0	9	0	0.0	21.587
	1	3	4	1	0.0	30.000
	0	2	2	6	0.0	2.143
						110.873

Changing the Ordered Widths.

In order to achieve the 3.66% yield improvement identified above, the patterns in the LP solution must be feasible with the true production roll size of 115 inches. For this to happen, the ordered widths must be reduced so that these patterns fit within the 115 inches of the production rolls produced. For example, the first slitting pattern in the solution when $S_k = 117$ is

Item Number	Number In Pattern	Size	Total Inches
1	0	14.1	0
2	5	13.4	67
3	3	13.0	39
4	1	10.7	10.7
			116.7

To produce this pattern from a production roll of 115 inches, the three ordered sizes must be reduced by a weighted total of 1.7 inches. Let D_i be the amount by which order size i is to be decreased. In order for this pattern to be feasible the following conditions must be met.

$$5D_2 + 3D_3 + 1D_4 \geq 1.7$$

Similarly the other 3 patterns generate the following restrictions.

$$9D3 \geq 2.0$$

$$1D_1 + 3D_2 + 4D_3 + 1D_4 > 2.0$$

$$2D_2 + 2D_3 + 6D_4 \geq 2.0$$

In order to achieve the maximum possible overall economic gains, the objective is to find the amount D_i by which each ordered size W_i should be decreased to satisfy the above constraints at the minimum possible cost. Let $C_i(D_i)$ be the cost of reducing width W_i by an amount D_i where $C_i(0) = 0$. The problem to be solved is

$$\min \quad C_1(D_1) + C_2(D_2) + C_3(D_3) + C_4(D_4)$$

subject to

$$\begin{aligned}
5D_2 + 3D_3 + 1D_4 &\geq 1.7 \\
9D_3 &\geq 2.0 \\
1D_1 + 3D_2 + 4D_3 + 1D_4 &\geq 2.0 \\
2D_2 + 2D_3 + 6D_4 &\geq 2.0
\end{aligned}$$

The solution to the problem will depend on the nature of the costs associated with changing the order widths.

In the simplest case, if the cost of changing a width is proportional to the amount of the change, the above problem reduces to a LP problem and can be solved easily using any standard LP package.

If there is a large fixed cost associated with making a change, the objective would be to minimize the number of widths changed. Because size 3 is the only width that appears in all 4 patterns it is the only one-change solution possible. If size 3 cannot be changed enough to meet all the restriction then size 2 or 4 could also be changed.

If the cost of reducing a width increases at an increasing rate, the cost relationship can be treated in a piecewise linear fashion and this problem can also be solved as an LP problem, using a standard package.

As a final check, the cost of making the width reductions must be economically justified by the benefits derived from the increased yield over the foreseeable life of the converted product and/or primary process.

Conclusion

The option of improving yield by changing the ordered sizes is one that is readily available to a number of integrated producers of converted products. It may also be available to producers who sell to independent converters provided the two parties are willing to work together and share the benefits of the yield improvement.

In either case, the most important step is to identify the potential yield improvement possible if the ordered widths are changed. In some cases there may not be enough potential benefit to warrant incurring the costs associated with making the necessary changes. In other cases, particularly those where the volume is significant and the yields initially are fairly low, there may be large payoffs associated with relatively small changes in the ordered widths. This paper has described an efficient procedure for determining the potential benefit of size reductions, by observing how the solution changes to an LP trim loss minimization problem as the production roll size hypothetically increases. The patterns in the optimal solution define a specific set of order width reduction relationships that must be met in order to achieve the associated yield improvement.

Literature Cited

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