

THE POWER TO INFLUENCE IN
FUTURES MARKETS

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ABSTRACT

The effects on the equilibrium price in a futures market are examined for the case when a large or dominant trader with superior information is assumed to exist. This trader influences the market price and thus alters the price signals when he transacts. If the trader accounts for his impact on the price it is shown that he will hold fewer contracts (in absolute value). In the price determination process, this results in the trader's private information set not being weighted to the same extent as it would if he were a price-taking trader with the same information. Thus, the futures price will be biased, with the direction of the bias dependent on which side of the market the large trader is on. There will be a market power premium implicit in the futures price that causes it to differ from the expected spot price, however, this premium is unrelated to the risk preferences of the market participants.

SECTION I -- INTRODUCTION

In most models describing financial markets there is an underlying assumption that the participants in the markets are small and homogeneous. Their trading activity is assumed to have an imperceptible effect on the market price. We know that this is not the case in thinly traded markets and may not be a reasonable assumption for many financial markets. Individual traders may be able to influence the price and given this, many of these traders will undoubtedly take their own price effects into account when they trade. There is anecdotal evidence of this in futures markets, where major traders carry out complicated trading strategies in order to hide their trading activity and retain their anonymity. In this paper, a price leadership model is presented in a futures market setting. Specifically, I examine the effects on the market price of allowing for a dominant or large trader who possesses superior information. This trader alters the price signals when he trades futures. He realizes this and thus must account for the effects of his activity on others.

I present the model in a non-game theoretic framework. This allows for the major points to emerge in a rather straight-forward manner. The model builds upon the framework presented by Anderson and Danthine (1981) and extends the work of Newbery (1984) and Anderson and Sundaresan (1984). Overall, I show that in the determination of the equilibrium market price the dominant trader's information set will not be weighted to the same extent to which it would if the trader was a price-taker and had no effect on the market price. In addition, because the large trader holds fewer contracts (in absolute value) less of his private information is revealed to other participants. What results from the inclusion of market power in a model of pure determination is

a systematically biased futures price. This bias is not necessarily related to any risk (or net hedging demand), but rather to the position taken by the large trader. Instead of a risk premium we have a market power premium, whereby the futures price is a biased predictor of the expected spot price. This premium can take either sign.

Much of the literature on futures markets has discussed and debated the existence of a risk premium (Blau 1944-45; Cootner 1960; Hicks 1978; Houthakker 1957; Keynes 1930). The futures price is said to differ from the expected spot price by this risk premium. The magnitude of the premium is related to the net hedging demand of commercial traders (i.e., those cash market participants who attempt to reduce their business risks in the futures market). In addition, certain other characteristics of the market and the market participants are also said to impact on the size of the premium. The risk premium is shown to be a function of such variables as the risk preferences of the participants, trader uncertainty expressed as price volatility, and opportunities to hedge given as the covariance between spot and futures prices.

An alternative hypothesis of why the expected spot price and the futures price might diverge is presented by Williams (1985). He shows that the difference between the current futures price and the expected spot price may be unrelated to risk and instead be a function of the relative transactions costs in the spot and futures markets.

This paper puts forth yet another explanation of why there might be systematic differences between the expected spot and futures prices. The results in this paper imply that the premium may be directly related to the position of the trader who possesses market power, or the power to influence prices. The price bias emerges because less than full weight is given to the information possessed by the dominant trader. In general what this implies,

is that the market price will differ depending on the composition of the market (e.g., degree of concentration or characteristics of the participants). In other words, *ceteris paribus*, we will observe a different market price when one side of the market is dominated by large price-making traders, as when compared to small price-taking traders.

The presentation is as follows. In section II a discussion of what is meant by market power will be offered. In section III the model will be presented. This model is modified in section IV where a more detailed discussion and expression of trader expectations is made. In section V the contract demand by the dominant trader is shown in its most detailed version. In section VI the equilibrium futures price is derived and the bias expressed. Finally, in section VII some conclusions are offered.

SECTION II -- WHAT IS MEANT BY STATIC MARKET POWER

Futures and other financial markets are composed of many different types of traders. These traders differ in their position sizes and wealth endowments (together these determine the individual trader's probability of ruin) and they differ in their abilities to gather private information or to filter publicly available information (we can call this their skill).

In combination with noisy prices that do not fully reveal all information, trader heterogeneity leads directly to the possibility of certain individuals or firms possessing the power to alter or distort the equilibrium price of contracts (Anderson and Sundaresan 1984; Newbery 1984). Static market power is possessed by a trader who alone can alter the equilibrium price when he trades.¹

¹I differentiate between static and dynamic market power. Dynamic market power is when a large trader induces price trends or longer term price movements. Static market power is when the trader's activity alters the equilibrium market price. See Hartzmark (1984).

The trader possessing static market power must evaluate the external effects his trading has on the price in order to maximize his expected return. He reduces his demand for contracts in an attempt to withhold a portion of his private information which the price movements would reveal to other market participants. Therefore, his private information set will not be fully weighted in the determination of the market price. This will impart a systematic bias in the futures price which is directly related to the position taken by the powerful trader. His position and the bias are also conditional on the information that the small traders possess.

For example, assume that there is a one time auction of futures contracts and that traders via recontracting must purchase and sell all of their contracts at one price. At time t , a futures contract is valued at 90 and a powerful trader with superior information knows that the price will be 100 when it matures at time $t+1$.

He will then maximize his expected profits realizing that by increasing his desired quantity of contracts by one he will bid up the purchase price for all other infra-marginal contracts. As with a monopolist producer, the trader will purchase contracts up to the point where the marginal gain from purchasing one more contract and bidding up the price is equal to zero (assuming zero transactions costs). Therefore, the equilibrium price for contracts will be less than 100.

Every time there is new information entering the futures market this auction will take place. In this way the large trader will not have time enough to gradually accumulate positions. He needs to trade in small amounts or to spread his orders among many different agents in order to retain his anonymity and not fully reveal his private information. A sophisticated trading strategy takes time and has the risk that other well-informed traders or groups of

traders will enter the market and bid the price up (or down) to the zero profit level before the large trader, who first received the information, is able to enter the market. Therefore, it is probably the case that the relevant (actual plus expected) marginal cost of purchasing additional contracts is positive and upward sloping. This will cause the large trader to withhold part of his private information by not bidding the price up to the expected level. In addition, by the time the large trader can re-enter the market most, if not all, of his private information will be revealed, or new information will have arrived and thus change the trading strategy.

This auction and recontracting is key to the model that follows. It makes sense when we have a world where information is constantly flowing into the market and thus trading intervals or auctions are continuously taking place. Another alternative method of modelling the buying behavior of the large trader is assuming that he acts as a price discriminator. In this model the trader can buy or sell small quantities of contracts at different prices. He continues to bid the market price up (or down) to the zero expected profit level. Unfortunately given the time and precision necessary to implement such a strategy it probably does not make sense in a short term trading model. In the model presented in this paper one can think of there being a large number of auctions each day, one following the other. These auctions begin when new information becomes available.

Because there are many different traders on both sides of the market, no one can possibly determine what the trading strategy of the large trader is, unless the large trader uses a system that can be detected using variables directly observable to outsiders. In addition, there is a constant turnover of traders in the market (something that appears to happen in the actual market). With this it is unlikely that the small traders will be able to learn of or

offset the behavior of the large trader. Therefore, in general we can think of this auction taking place over and over again, with a large proportion of new participants each time.

In the absence of collusion, this theory implies that when one side of the market is populated by a large group of small contract price takers, all with information set A, the price will be different than if that side is dominated by one large trader with the same information set A, who can alter price signals when he trades.¹ To use this type of static market power most effectively, a trader must possess more accurate and timely information than others. Most likely, the type of trader who would possess this type of power and have the ability to use it effectively, would be a major participant in the cash or spot market who utilizes the futures market to selectively hedge his cash position.

In the recontracting model presented below I show that traders possessing static market power reduce their contract holdings (whether short or long). The amount of the reduction depends on the response of other traders when the large trader enters the market and alters the contract price. A biased contract price results from the actions of the powerful trader, with the direction of the bias partially dependent on the position he chooses.

SECTION III -- A MODEL OF STATIC MARKET POWER

To develop a model of static market power an elementary model of expected utility maximization is used which combines aspects of both the risk-trading theory and the information-trading theories.² It is assumed that the traders

¹Obviously, as a group these price-takers have an effect on prices even though no one individual can effect prices.

²I use a model by Anderson and Danthine (1981) as the foundation for my model.

who participate in the market are risk averse. Further, an assumption is made that individual traders possess differing pieces of information and/or differing abilities to filter that information. Finally, to simplify the model there is only one price leader in the market, who by his trading activity can alter the price signals in the market. However, because there exists so much noise in the market price, not all of the price leader's or other market participants' information is fully revealed in the market price.

I use a two period model where individuals are expected utility maximizers.¹ The i^{th} individual's expected utility function can be expressed as:

$$(1) \quad E(\tilde{U}_i) = E(\tilde{W}_{i1}) - (1/2)A_i V(\tilde{W}_{i1})$$

Where A_i = Risk parameter of individual i ;

$A_i > 0$ implies a risk averse individual;

$E(\tilde{W}_{i1})$ = Expected terminal wealth of individual i ;

$V(\tilde{W}_{i1})$ = Variance of expected terminal wealth of individual i .

Where (\sim) denotes a random variable in period 0 to be realized in period 1.

Uncertain terminal wealth can be represented as:

$$(2) \quad \tilde{W}_{i1} = \tilde{p}_1^s y_{i0} - C(y_{i0}) + f_{i0}(\tilde{p}_1^f - p_0^f) + W_{i0}(1 + r_0)$$

Where

W_{i0} = Wealth set aside in period 0 to be invested at the risk free lending rate, (I shall assume $r_0 = 0$);

y_{i0} = The amount of the commodity which the individual commits in period 0 to be purchased ($y_{i0} < 0$) or sold ($y_{i0} > 0$) in period 1. (Note: there is no output uncertainty in this model);

¹I use a quadratic expected utility function which assumes that each trader's coefficient of risk aversion remains constant throughout all levels of wealth.

$C(y_{i0})$ = Net revenue or cost function¹; $C_y * Y_{i0} > 0$ and $C_{yy} * Y_{i0} \geq 0$;

\tilde{p}_1^f = The uncertain price in period 1 of a maturing futures contract purchased in period 0;

\tilde{p}_1^s = The uncertain price in period 1 of the commodity if purchased in the spot market;

p_0^f = The current or period 0 price of a futures contract maturing in period 1;

f_{i0} = Number of futures contracts purchased ($f_{i0} > 0$) or sold ($f_{i0} < 0$) in period 0 by individual i .

This specification of terminal wealth allows for great flexibility in characterizing what type of trader the i^{th} individual is (Anderson and Danthine 1981, pp. 1185-86). One can allow the trader to choose y_{i0} and f_{i0} simultaneously and then solve for the optimal y 's and f 's. However, since this model focuses on the futures market I shall assume that $y_{i0} = y_i^*$, or that y_{i0} is given and thus we solve for f_{i0} . I also assume that the large and small traders cannot influence the spot price by their choices of f_{i0} . Given this, the trader chooses f_{i0} so as to maximize expected utility. The first order condition is:

$$(3) \quad p_{i1}^f - p_0^f - A_i (f_{i0}^2 \sigma_{fi}^2 + y_i^* \sigma_{sfi}) = 0$$

If, however, the trader can influence the current price of a futures contract (p_0^f) by his decision on f_{i0} then equation (3) must be expressed as:

$$(3') \quad p_{i1}^f - p_0^f (1 + \theta_i) - A_i (f_{i0}^2 \sigma_{fi}^2 + y_i^* \sigma_{sfi}) = 0$$

Where,

¹Note that this includes purchase costs ($C(y) > 0$) incurred in period 0 or revenues from merchandising agreements committed to in period 0 ($C(y) < 0$). Mark Bagnoli has shown to me why the cost (revenue) conditions must be stated as they are. This is different from Anderson and Danthine (1981, p. 1186) and I believe correct.

$p_{i1}^f = E(\tilde{p}_{i1}^f)$ = Individual i 's expectation of the period 1 futures price;

$\sigma_{fi}^2 = V(\tilde{p}_{i1}^f)$ = The variance of individual i 's prediction error of the futures price, or individual i 's estimation of the variance distribution of the futures prices;

$\sigma_{sfi} = \text{COV}(\tilde{p}_{i1}^f, \tilde{p}_{i1}^s)$ = Covariance of the futures and spot price prediction errors, or individual i 's estimation of the covariance between spot and futures prices;

(4) $\theta_i = \frac{\partial p_0^f}{\partial f_{i0}^f} \cdot \frac{f_{i0}^f}{p_0^f}$ = The elasticity of market reponse, comparable to the inverse of the elasticity of "other traders" demand, such that:

$$\theta_i \begin{matrix} \geq 0 \\ < 0 \end{matrix} \text{ as } f_{i0}^f \begin{matrix} \geq 0 \\ < 0 \end{matrix}.$$

With equation (3') we can now solve for the trader's optimal futures position, f_i^* , or:

$$(5) \quad f_i^* \Big|_{y_{i0} = y_i^*} = \frac{[p_{i1}^f - p_0^f]}{A_i \sigma_{fi}^2} - \frac{[y_i^* \sigma_{sfi}]}{\sigma_{fi}^2} - \frac{[p_0^f \theta_i]}{A_i \sigma_{fi}^2}$$

Demand equation (5) is composed of three separate components. The first expression on the r.h.s. is the speculative component of demand, the magnitude of which is directly related to the difference between the market price for contracts and the trader's own expectation of what that price should be. If the market price is below the trader's expectation of the maturing contract price (i.e., $p_{i1}^f - p_0^f > 0$) then the trader will take a long speculative position (i.e., $f_i > 0$). Alternatively, if the market price is above the expected price then the trader takes a short speculative position (i.e., $f_i < 0$). The magnitude of the trader's speculative position is reduced as his risk aversion increases or his uncertainty increases (i.e., σ_{fi}^2 rises).

The second expression on the r.h.s. is the hedge component of demand, which is directly related to the size of the trader's cash commitment, y_i^* . The hedge demand is also positively related to the covariance between the futures and spot price. The market is considered to be a much more effective

hedging vehicle the larger this covariance is in absolute value. Finally, the hedge demand is a decreasing function of the trader's uncertainty associated with predicting futures prices. For the pure speculators, y_i^* is zero. This is justified by assuming that the costs of participating in the cash market are too high for these traders.

The last component of equation (5) represents the adjustment in contract demand due to the external effects the trader's activity has on the market price. A non-zero elasticity of market response, θ_i , alters the large trader's demand for contracts. This coefficient represents the market power the large trader possesses and as we shall see is a function of the characteristics of the other traders (e.g., number of traders, size, risk aversion) as well as the quality of their information sets. As with a monopolist producer or price leader in a goods market, the large trader will take a position that in absolute value is less than if he had no effect on the market price (i.e., when $\theta_i = 0$). The magnitude of the contract adjustment is a function of A_i and σ_{fi}^2 as well.

Notice that θ_i changes sign depending on whether the dominant trader is long or short. This switching arises because the trader's position can be either negative or positive, and this position value serves as a multiplicand when deriving the elasticity of response. Whether the trader is long or short, the relationship between the number of contracts demanded (in absolute value) and the price is positive. Thus, this derivative is always positive, however, to get the elasticity of response you must multiply by the position value which is negative if the trader is short and positive if he is long. Therefore, the sign of the elasticity of response is determined by which side of the market the large trader is on.

This elasticity is similar to the elasticity of expectations discussed by Hicks (1978, p. 205). As with Hick's concept, this measure may be unstable and and variable. Its magnitude depends on many outside factors in addition to the actions of the large trader (i.e., his trading strategy). Further, this elasticity is not related to how well the large trader predicts prices. If $p_{i1}^f = p_0^f$ the dominant speculator will not enter the market, therefore $\theta_i = 0$. When the powerful trader is a hedger, θ_i will not necessarily equal zero, even if $p_{i1}^f = p_0^f$, since the trader will effect prices via his purchase to fulfill his hedge demand.

SECTION IV -- THE FORMULATION OF EXPECTATIONS BY SMALL TRADERS

Many of the models in the literature of futures markets use a rational expectations framework. In these models it is assumed that traders act as if they possess information on the production and utility functions, the underlying price distributions, and the nature of risk. Is this operationally useful? I believe a rational expectations framework assumes away much of what is interesting in a market which processes and transmits information. Below I make some assumptions about how traders formulate their expectations. These assumptions are not critical to any of the results that follow. A variety of assumptions could have been made. What this formulation offers is a way to model the individual trader's confidence in his prediction in a clear manner. It is a method by which I build the trader characteristics directly into the the model.

I assume that traders act as if they possess limited information. The assumption that traders use naive decision rules, such as those based on past price changes or other technical aspects of the market (e.g., open interest or volume), may be more appropriate, even if these rules fail on average. Models assuming less rationality could allow for continual losses by certain groups of

traders even if expected profits are assumed to be positive. In essence there is no learning. The market is composed of a large proportion of continual losers who flow out of the market only to be replaced by other unsophisticated traders. This constant flow of traders into and out of the market is a reasonable description, and models that assume a constant stock of traders who are constantly learning do not offer an accurate description of futures markets. Their predictions about the aggregate behavior of the market are often incorrect, simply because a rational expectations framework is inappropriate.

If the turnover of naive traders is rapid enough and the pool of these traders is large enough, learning might not take place, and a "limited information rational expectations framework" of analysis of aggregate behavior may be more appropriate (Shiller 1973). The specification of how expectations are formed is crucial in analyzing the model's implications. Therefore, specification of expectation formation in my model is left as general as possible.

I assume that each trader is given a private information set (\emptyset_1) which is used to forecast \tilde{p}_{11}^s and \tilde{p}_{11}^f . I assume that there is basis risk, or that there is some positive probability that p_1^s and p_1^f will not converge. All individuals realize that a portion of their private information will be revealed to other traders through the market price when they trade. On the whole, however, I assume that each trader possesses some amount of private information which is not fully revealed in the market price. It is also assumed that the traders will use the market price to varying degrees to determine their own forecasts. In the extension of the previous model each trader has his own a priori expectation of next period's futures price conditional on the private information set which he is given:

$$E_{i0} (\tilde{p}_{i1}^f / \phi_i) = q_{i1}^f$$

After formulating an a priori price forecast, a trader enters the futures market and a tâtonnement process is begun. From this process a market price for contracts is determined. At each stage of this auction, individuals update their forecasts to obtain a posterior forecast of the futures price, where the forecast is a weighted average of one's a priori prediction and the market price. In the end this forecast can be specified as:

$$(6) \quad E_{i0} (\tilde{p}_{i1}^f / \phi_i, p_0^f) = p_{i1}^f = \lambda_i q_{i1}^f + (1 - \lambda_i) p_0^f$$

where $0 \leq \lambda_i \leq 1$

The weight, λ_i , that each trader places on his a priori estimate represents the relative confidence that he has in the precision of his own estimate. This weight can be viewed as a function of variances and covariances of spot and futures prices, or individual or market prediction errors.¹ At this point, however, I shall not attempt to describe λ_i in any greater detail.

When substituting equation (6) into equation (5) there is a slight change in the specification of the demand function:

$$(7) \quad f_i^* = \frac{\lambda_i [q_{i1}^f - p_0^f]}{A_i \sigma_{fi}^2} - \frac{\lambda_i p_0^f \theta_i}{A_i \sigma_{fi}^2} - \frac{y_i^* \sigma_{sfi}}{\sigma_{fi}^2}$$

Equation (7) shows that only when λ_i is different from zero will traders speculate in the futures market.² This formulation does not, however, eliminate trend followers from speculating. Because trend followers look at both

¹ Jaffee and Winkler (1976) presents one example of how these weights might be determined.

² If $\lambda_i = 0$ or $q_{i1}^f = p_0^f$ then a powerful speculator will not enter the market and $\theta_i = 0$. If either of these conditions held a powerful hedger would still enter the market and $\theta_i \neq 0$ for this trader.

p_0^f and past prices, they may still have a λ_i which is greater than zero since individual methods of distilling information from past prices would lead to different weightings of p_0^f in determining p_{i1}^f .

SECTION V -- CONTRACT DEMAND BY THE DOMINANT TRADER

In the analysis to follow, the theoretical model is simplified by assuming that there are three distinct types of traders. First, there are n equally able and endowed non-commercial traders who are given the information set ϕ_n .¹ Second, there are c equally able and endowed commercial traders who are given a different information set, ϕ_c . Third, there is one large (dominant) trader who trades only on his superior information set, ϕ_L . This trader's private information set includes ϕ_n and ϕ_c , therefore he has 100% confidence in his price estimate, so that $\lambda_L = 1$. This dominant trader however, takes into account the indirect effect of his activity on all other traders via his direct effect on the market price, p_0^f .

A dominant trader is one who can, by his trading activity, alter the price of a futures contract. By altering the price signals in the market the large trader can influence other traders' expectations as is shown in equation (6). The dominant trader may have achieved his position of dominance because of successful trading in the past. In this case he grew large because he was a good forecaster of price changes. On the other hand, he may have achieved dominance because of imperfections in the economic system. For example, if capital markets are not perfect and there are decreasing costs of transactions, a large initial wealth endowment may have enabled the large trader to take larger positions or have a smaller probability of ruin than an equally able,

¹Non-commercial traders are speculators in that they have no interest in spot market transactions. Commercial traders participate in both the cash and futures markets. They combine hedging and speculating.

larger positions or have a smaller probability of ruin than an equally able, less well endowed trader. This large position may offer advantages because of the ability to distort the market price or the opportunity to analyze the determinants of price movements (Hartzmark 1984). For example, by holding a substantial proportion of the positions on one side of the market the trader is better able to detect whether price movements are due to new information flowing into the market or the trading of others. Thus, a large trader may look like a good forecaster simply because he is big. However, this influence is only an advantage if the dominant trader can remain at least partially undetected. Otherwise, all his actions and thus his information will be fully revealed to all other traders.

I shall define the demand by non-commercial traders by using equation (7) and setting: $f_i^* = f_n$, $\lambda_i = \lambda_n$, $q_{i1} = q_n$, $A_i = A_n$, $\theta_i = 0$, $\sigma_{fi}^2 = \sigma_{fn}^2$, $\sigma_{sfi} = \sigma_{sfn}$ and $y_i^* = 0$. For the commercial traders I shall use: f_c , λ_c , q_c^f , A_c , σ_{fc}^2 , σ_{sfc} , $\theta_i = 0$, and $y_i^* = y_c$. The large or dominant trader's demand can be represented as:

$$(8) \quad f_L = \frac{q_L^f - p_0^f(1+\theta)}{A_L \sigma_{fL}^2} - \frac{Y_L \sigma_{sfL}}{\sigma_{fL}^2}$$

Where $\theta = \theta_L$;

$Y_L = 0$, implies a dominant non-commercial trader;

$Y_L > 0$, implies a dominant commercial trader.
 $<$

It is possible to derive expressions for θ , f_L , and p_0^f by using the demand equations of the three groups and assuming that the futures market must clear (i.e., total long positions equals total short positions.). Market clearing gives us:

$$(9) \quad 0 = f_L + \sum_{i=1}^n f_{ni} + \sum_{j=1}^c f_{cj}$$

Therefore, the large trader's demand for contracts must equal the other traders' excess supply at any given market price. So that:

$$(10) \quad f_L = - \left[\sum_{i=1}^n f_{ni} + \sum_{j=1}^c f_{cj} \right] = \text{excess supply}$$

$$\text{Where } \Sigma f_{ni} = \frac{n\lambda_n (q_n^f - p_0^f)}{A_n \sigma_{fn}^2}$$

$$\Sigma f_{cj} = \frac{c\lambda_c (q_c^f - p_0^f)}{A_c \sigma_{fc}^2} - \frac{cy_c \sigma_{sfc}}{\sigma_{fc}^2}$$

Therefore:

$$(11) \quad f_L = \frac{Y\sigma_{sfc}}{2\sigma_{fc}} - z_c (q_c^f - p_0^f) - z_n (q_n^f - p_0^f)$$

Where $Y = cy_c$ = total cash market commitments of the commercial traders;

$$z_c = \frac{c\lambda_c}{A_c \sigma_{fc}^2} = \text{characteristics of commercial traders;}$$

$$z_n = \frac{n\lambda_n}{A_n \sigma_{fn}^2} = \text{characteristics of non-commercial traders.}$$

I have included the variables z_n and z_c to simplify the expressions to follow. The z 's are coefficients describing the characteristics and skills of the two groups of traders. As the traders are more confident in their ability to predict price changes (i.e., z_i increases because λ_i increases or σ_{fi}^2 decreases) they will increase the magnitude of their positions (in absolute value). As they become more adverse to risk (i.e., z_i falls) they will decrease the magnitude of their positions. Ceteris paribus, an increase in the number of either commercial or noncommercial traders results in a rise in aggregate demand and thus a fall in excess supply.

Using equation (11) we can now derive an expression for θ or the elasticity that appears in equation (8). It is possible to show how the market price changes when the position of the large trader changes, or:

$$(12) \quad \frac{\partial p_0^f}{\partial f_L} = \frac{1}{(z_c + z_n)}$$

Equation (12) gives the inverse of the slope of the excess supply curve. As the z 's increase (i.e., λ_c , λ_n , n or c increase, or A_c , A_n , σ_{fc}^2 or σ_{fn}^2 fall) there is more demanded at each p_0^f , and thus the slope of the excess supply curve is greater or the inverse of the slope is less.

To derive θ , the market response to changes in the position of the dominant trader, we multiply equations (11) and (12) and divide by p_0^f to get:

$$(13) \quad \theta = \frac{f_L}{f} \cdot \frac{\partial p_0^f}{\partial f_L} = \left(\frac{z_c}{z_n + z_c} \right) \frac{1}{\eta_c} + \left(\frac{z_n}{z_n + z_c} \right) \frac{1}{\eta_n}$$

where

$\frac{1}{\eta_c}$ = inverse elasticity of demand for contracts by commercial traders,

$$= \frac{Y_{\sigma_{sfc}}}{z_c^f p_0^f \sigma_{fc}^2} + \frac{p_0^f - q_c^f}{p_0^f}$$

$\frac{1}{\eta_n}$ = inverse elasticity of demand for contracts by non-commercial traders,

$$= (p_0^f - q_n^f) / p_0^f$$

The sign of θ in equation (13) must be the same as the sign of f_L in equation (11) since all that has been done is to multiply equation (11) by $[1/p_0^f(z_c + z_n)]$, a positive expression. The first term of the commercial inverse elasticity is a market hedge ratio deflated by the market price times the coefficient of trader characteristics. As the hedge ratio increases, ceteris paribus, the elasticity of response increases. A greater hedge ratio indicates that there will be a larger risk premium in the market. Hedgers will be willing to pay

more because the market offers them a greater opportunity to reduce their risks. Or, they will be willing to pay more to reduce their risks (in dollar terms) if they have a greater supply of the cash commodity to hedge. With greater hedge demand there is also a greater excess supply of contracts and thus a greater elasticity of response.

As one would expect the elasticity of response is a weighted average of the commercial and non-commercial inverse elasticities of contract demand. The weights are indirectly related to the quantities demanded by the commercial and non-commercial traders. Also, like the elasticity of response faced by the large trader, the signs of the commercial and non-commercial elasticities are determined by the side of the market chosen by the trader groups. If the group is short, the elasticity is positive. If the group is long, the elasticity is negative.

If the two groups are both short, then the elasticity of response facing the large trader is positive. Therefore, the large trader holds long positions. If the inverse elasticities for the two groups of small traders are opposite in sign then the relative magnitudes of the inverse elasticities and the relative sizes of the trader characteristic coefficients will determine the sign of θ , as well as the side of the market that the large trader is on. If the elasticities of the two groups happen to be the same, then the elasticity of response of the large trader is equal to the inverse elasticities of the smaller traders. As η_c and η_n get larger the elasticity of response gets smaller. If the trader groups have infinitely elastic demands then the large trader can have no price effects.

The greater are Z_c and Z_n , ceteris paribus, the greater are the amounts demanded by the small traders and the smaller the elasticity of response of the large trader. Obviously, the relative values of the Z's depend on the

number of traders in each group, the confidence and precision of the groups' a priori price estimates, and the groups' relative risk aversion coefficients. As n goes up relative to c , or λ_n relative to λ_c , or A_c relative to A_n , or σ_{fc}^2 relative to σ_{fn}^2 , the relative importance of the non-commercial traders (measured by z_n) is increased. We can show that the sign $\frac{\partial \theta}{\partial z_n}$ is the same as the sign of $[\frac{1}{\eta_n} - \frac{1}{\eta_c}]$. Therefore the sign depends on the sides of the market taken by the individual groups, as well as the magnitudes of the inverse elasticities.

The signs of $\frac{\partial \theta}{\partial Y}$ and $\frac{\partial \theta}{\partial \sigma_{sfi}}$ are positive, since they are positively related to the inverse elasticities. As q_n^f and q_c^f increase, ceteris paribus, the inverse elasticities get smaller as does θ . As q_n^f approaches p_0^f the elasticity of the non-commercial traders goes to infinity and the related inverse elasticity goes to zero. This is not the case for the commercial traders since their demand consists of both hedge and speculative components.

Substituting the expression for θ in equation (13) into equation (8) yields:

$$(14) \quad f_L = \frac{[q_L^f - p_0^f]}{A_L \sigma_{fL}^2} - \left[\frac{p_0^f}{A_L \sigma_{fL}^2} \left(\frac{d_c}{\eta_c} + \frac{d_n}{\eta_n} \right) \right] - \frac{Y_L \sigma_{sfi}}{\sigma_{fL}^2}$$

$$\text{where } d_c = z_c / (z_c + z_n)$$

$$d_n = z_n / (z_c + z_n)$$

Thus, the large trader's contract demand has three components. The first expression on the r.h.s. is the speculative component. The final expression the hedge component if the large trader is commercial. The second expression takes into account the response of the other traders to the large trader's actions. Also note that the large trader need not know the values of every component of θ , just θ itself. The large trader may view the computation

of θ as taking place in a black box. In addition, if the characteristics of the traders in the market remain the same (not necessarily the identity of traders who participate) over time the large trader will become better able to estimate θ .

SECTION VI — PRICE DETERMINATION AND PRICE BIAS

Using the solution for the dominant trader's demand for contracts we can now solve for the market price of contracts. We shall do this for two cases: 1) when the dominant trader possesses the power to influence price; and 2) when he has no such power. After comparing the results we will be able to see the effects of market power on the bias and the premium.

To begin, we can combine the dominant trader's demand for contracts from equation (14) and the other trader groups' demands from equation (11) and substitute them into the market clearing equation (9). We then solve for the market price as a function of the trader characteristics. In the case when we have a dominant trader who does NOT influence price (i.e., $\theta = 0$) we get:

$$(15) \quad p'_0 = \frac{(q_L^f/A_L \sigma_{fL}^2) + z_c q_c + z_n q_n - H_L - H_c}{z_c + z_n + (1/A_L \sigma_{fL}^2)}$$

where H_L and H_c represent the hedge components of demand.

In the case where the dominant trader does have the power to influence the market price, we get:

$$(16) \quad p_0^* = \frac{(q_L^f/A_L \sigma_{fL}^2) + z_c q_c + z_n q_n - H_L - H_c}{z_c + z_n + [1/A_L \sigma_{fL}^2] [1 + d_c/\eta_c + d_n/\eta_n]}$$

There are a number of interesting insights that we get from equations (15) and (16). First, we can derive an explicit expression for the risk premium or the difference between the expected futures price at expiration and the current futures price. This is the risk premium as discussed in the futures literature and is directly related to the hedging components of the

contract demands. To simplify the expression, assume that all traders are homogeneous, except that certain traders hold stocks (i.e., commercials) and others do not (i.e., non-commercials). This assumption means that q_i , σ_{sfi} , σ_{fi}^2 , A_i and λ_i are the same for all traders. In addition, let us derive the risk premium in the case where the dominant trader does not have the power to influence the market price. Manipulating equation (15) we get the risk premium:

$$(17) \quad R = p'_0 - q^* = \left[\frac{-A\sigma_f^2}{\lambda(n+c) + 1} \right] (H_c + H_L)$$

where, q^* is the a priori estimate of the maturity futures price that all traders have.

In this case, the risk premium, R , (or the difference between the current price and the expected price) is a function of the total stocks that are to be hedged, the risk preferences of the participants, the numbers of participants, and the prediction skills or uncertainty in the market. Ceteris paribus, as A , σ_f^2 , H_L or H_c increase the risk premium must increase to induce traders to enter the market. As λ or the numbers of traders in either group increase the risk premium falls.

Now we want to examine the relationship between p' and p^* in equations (15) and (16) to see if there is an additional market power premium in the price or if market power offsets some of the bias due to risk premium. We also want to see what market characteristics are related to the market power. In this way we will be able to see how changes in the composition of the market can effect the market price. The ratio of p' to p^* is:

$$(18) \quad \frac{p'}{p^*} = \frac{z_c + z_n + \frac{1}{(A_L \sigma_{fL}^2)} + \frac{1}{(A_L \sigma_{fL}^2)} \left[\frac{d_c}{\eta_c} + \frac{d_n}{\eta_n} \right]}{z_c + z_n + \frac{1}{A_L \sigma_{fL}^2}}$$

It is immediately obvious that the magnitude of this ratio is related to the elasticities of demand of the trader groups. The ratio is different from one depending on whether the small traders' elasticities are positive or negative. In general, as A_L or σ_{fL}^2 rise the ratio goes towards one and the market power premium is reduced toward zero. As the elasticities of the small traders go toward infinity (in absolute value) the ratio approaches one, and again the market power premium goes to zero.

If the small traders are all short and therefore $\eta_i > 0$, then $p' > p^*$. Therefore, there is an additional premium due to the fact that the dominant trader does not bid for as many long positions as he would if he did not influence the price. If this trader is in the market at all times and the other traders are always net short, then we would see that the price rise as the contract approaches maturity.

Observing such futures price movement is normally attributed to normal backwardation. Normal backwardation is said to appear when commercial traders are net short and thus must induce long non-commercial traders into the market by paying them the risk premium. The mechanism that facilitates the transfer of funds to the non-commercial traders is a systematically increasing futures price as the contract approaches maturity. Therefore, we now have two possible theories to explain systematic price appreciation. Unfortunately, the issue of whether futures prices move systematically is not fully resolved, as there is conflicting evidence. It is also unlikely that the large trader will trade in a systematic manner, at least as observed by the academic researcher. Therefore, we may not be able to find such a market power premium unless we can observe the trading activity of the dominant trader.

One can also see from the ratio above that the composition of the market does matter or does have an effect on the market price. As the η_i 's change or

the z_1 's change the market price and premium will also change. In addition, the risk premium in equation (17) can go to zero and we can still have a market power premium implicit in the futures price. So in the case where $H_C = H_L = 0$ it need not be the case that $p_0^f = q^*$, even under the assumption of trader homogeneity. When the q_1 's differ it is also the case that the weights which weigh each trader groups' price expectations will be distorted if there is a dominant trader who can influence the market price. Also note that the market power premium may partially, fully or more than offset the risk premium, should it exist.

SECTION VII -- Conclusions

In the previous section an expression describing the market power premium was derived. The magnitude of the bias that emerges was shown to be a function of the characteristics of the small price-taking traders in the market and the trading behavior of the dominant trader. The market power premium could exist whether or not a risk premium is implicit in the futures price. It could combine with the risk premium to increase the differential between the expected spot price and the futures price or it could offset some of the bias induced by the risk premium.

The derivations also offer insight into factors that influence the price determination process. Generalizing these results, we can conclude that the composition of the market will have an effect on the price performance in the futures market. By composition it is meant, the degree of concentration of positions held or of transactions made by large traders. It can also be defined to include the make-up of the market, or who is participating (e.g., greater proportions of naive or sophisticated traders, of big or small traders, or of informed or uninformed traders). Therefore, in general, we will observe a different market price (and maybe different price volatility) when one side

of the market is composed of small, naive, price-taking traders with a given private information set as opposed to large, sophisticated, price-making traders with the same information set.

This theory indicates that if we could follow the trading activity of these large participants we would earn positive returns. Does this also mean that we should find statistically significant non-random futures price movements? Not necessarily. We may test for randomness and accept that the prices are randomly distributed. However, there still may be some systematic movements in prices that the researcher will be unable to detect. In the case of traders who can influence price, the systematic relationship would be between the positions of the large traders (an unobserved variable to the researcher that, by itself, appears random) and subsequent price movements. Combined the relationship between the positions and price changes may be highly significant. So unless these large traders use systems that are directly related to some observable market phenomenon, price movements to us will appear randomly distributed.

This has implications for the success of technical trading strategies, as well. If the technician, who uses past prices or market activity variables to predict the future, inadvertently discovers some way to identify when the large traders are transacting, he may be able to come up with a successful trading strategy by unknowingly following their activity. Again, to the outside observer everything may look random, but to the technician a systematic component may have been discovered.¹

¹Lester Telser has told me a related story of why technical trading may work when all the research scientists reject the notion. He starts by asking me the 10,000th digit of π (pi). We agree that I have a 1 in 10 chance of being correct. In fact, if you look at the sequence of digits that make-up π and test for randomness you could not reject that the sequence is random. However, there are those who have been able to come up with the system to generate the digits that compose π . We could think of them as the sophisticated technical traders who have found the systematic elements in the seemingly random sequence of numbers. The moral of the story is that what may look random may have systematic components.

It is interesting to note that there is some empirical evidence that supports the theory in this paper. In Houthakker (1957), Rockwell (1964), Chang (1985) and Hartzmark (1986) the big winners (in dollar terms) in the futures markets were observed to be the large traders. There is also support in Stewart (1949), Hieronymus (1977) and Tewles, Harlow and Stone (1977), where it is observed that the big losers are the small traders. In general, this indicates that there is some direct relationship between the positions of the large traders and the subsequent price movements. This is especially true in Hartzmark (1984, 1986) where the distribution of returns are highly skewed with the largest traders earning disproportionately large returns. Obviously, this may be related to the risk premium, to forecasting skill or to simple luck. However, one cannot reject the notion that the large traders may be earning all or a portion of their returns due to their ability to influence price in the futures market. Only future research will tell.

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