A RATIONAL ECONOMIC APPROACH TO THE
PSYCHOLOGY OF CHANGE:
AMNESIA, INERTIA, AND IMPULSIVENESS

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Amnesia, Inertia, and Impulsiveness

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This paper models how imperfect memory affects the optimal continuity of policies. We examine the choices of a rational player (individual or firm) who observes previous actions but cannot remember the rationale for these actions. In a stable environment, the player rationally responds to memory loss with excess \textit{inertia}, defined as a higher probability of following old policies than would occur under full recall. In a volatile environment, the player can exhibit excess \textit{impulsiveness} (i.e., be more prone to follow new information signals). The model provides a rational explanation for some documented psychological biases, implies that habits, routines, and societal traditions should be more important in stable environments than in volatile ones, and provides other empirical implications relating memory and environmental variables to the continuity of decisions.
...the most radical revolutionary will become a conservative on the day after the revolution.


Be not the first by whom the new are tried, nor yet the last to lay the old aside.


When I was younger I always conceived of a room where all these [strategic] concepts were worked out for the whole company. Later I didn't find any such room.... The strategy [of the company] may not even exist in the mind of one man. I certainly don't know where it is written down. It is simply transmitted in the series of decisions made.

General Motors executive, quoted in Quinn (1980).

1 Introduction

The problem of memory loss permeates human choices, from the shopper trying to remember which detergent cleans better to the newly-arrived manager trying to learn the relevant aspects of his new firm's history. There has been remarkably little research on the consequences of such amnesia for economic decisions. This paper offers a rational economic model of how memory loss affects the continuity of behavior.

An individual's habits, an organization's existing policies and routines, and a society's traditions are often firmly entrenched even when the rationales are not evident. Habits and policies are often maintained and even escalated despite opposing information (see, e.g., Arkes and Blumer [1985], Ruef [1997]). Similarly, the *non*-adoption of a potential activity often continues despite the arrival of favorable information.

At the level of the firm, this paper offers a new theory of the determination of organizational continuity versus change in the face of memory loss.\(^1\) It seems plausible that firm behavior is most likely to change when a new manager arrives. However, a

\(^1\)Rumelt (1994) argues that the issue of the optimal degree of organizational inertia is central to corporate strategy research, that managers should take into account a firm's own inertia in evaluating actions, and that in practice strategic success is often due to the inertia of competitors as much as to the cleverness of the innovator.
new manager is likely to be hampered by lack of information about the source of past decisions. In some cases, a new manager may infer that there is a good rationale for past policies, even if he or she is not sure what that rationale is. If so, it may make good sense to maintain, or at least carefully consider, the status quo. For example, when RJR Nabisco CEO Louis Gerstner replaced John Akers as CEO of IBM following a period of dramatically poor performance, Gerstner surprised observers by deciding not to embark on a radical change of course.\(^2\) As an executive with a low-tech marketing background, Gerstner may have initially lacked the expertise needed to critically evaluate existing IBM policies, and thus may have been compelled to rely on these established policies.

Even executives hired specifically to effect change, and ardent social reformers, often perpetuate many of the policies of the regime they replace. Change is often limited to a small set of high-profile issues. Policy continuity is even more prevalent (if less striking) after routine losses of memory that arise from imperfect communication, incomplete records, and routine managerial transitions (for retirement, promotion, illness or other personal reasons). This paper attempts to explain behavioral continuity versus change in relation to the ordinary problems of memory loss commonly faced by individuals and firms. (Our primary focus is not upon the rare traumatic events in which managers are replaced in order to effect change and restructuring. In such situations, the forces described here are likely to apply, but other effects may be equally or more important.)

At the individual level, the analysis offers a rational explanation for psychological patterns related to cognitive dissonance and 'status quo bias.' In the psychological theory of self-attribution, people inferentially attribute reasons or motives to themselves based upon observation/recall of their own actions and experiences (Bem [1965]). In our analysis, such inferences determine whether behaviors continue or shift. An individual

\(^2\)…three months into the job, Gerstner has made it clear that he has no intention of reconstructing IBM. Instead, the man everyone saw as the Great Changemeister is determined, for the moment, to carry out a set of policies put in place by none other than the much-maligned Akers." [See "At IBM, More of the Same—Only Better? In Sales and Strategy, Louis Gerstner Is Following John Akers' Path," Business Week, 7/26/93. According to this article, Gerstner is "...still following through on Akers' two-year-old restructuring." "Lou is not rushing to make significant changes in vision," according to Gerald M. Czarnecki, IBM's new head of human resources.]
who remembers his old actions, and presumes that there must have been a good rationale for them will, under conditions that we delineate, exhibit excess inertia. Thus, our model offers a possible analytical foundation for self-attribution theory and the study of behavioral change versus continuity. It thereby offers a rational explanation for the inertial behavior that the psychological theory of cognitive dissonance seeks to explain.3 Similarly, the ‘status quo’ bias (see Samuelson and Zeckhauser (1988)), in which initial assignment of ownership causes individuals to reverse their preferences in regard to barter trades, is consistent with our theory. An individual who owns an item now may have reason to be attached to it, since it may have been selected for a good but forgotten reason in the past.4

Although inertia is common, it sometimes appears that individuals or organization are over-sensitive to new informations. Private individuals and managers are sometimes criticized for mercurial or ‘weather-vane’ decision styles. Several competing theories of organizational inertia are discussed in Section 6, but there is little work that addresses this opposite phenomenon, which we term impulsiveness.5 We offer a model to explore the conditions under which firms will be more or less responsive to new information, i.e., when inertia versus impulsiveness obtains.

The degree of behavioral continuity depends on the balance between newly arrived information and the pool of old information. It seems natural to presume that the

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3The psychological theory of cognitive dissonance holds that individuals are reluctant to abandon old beliefs and policies—a form of inertia—because doing so creates emotional discomfort. Our approach is based on individuals who are rational (apart from memory loss). However, it is possible that Nature has designed human cognition so that emotions guide us to follow policies that are optimal given limited memory. In this sense our analysis could be viewed as consistent with, rather than an alternative to, cognitive dissonance theory.

4There has been considerable effort by economists to incorporate findings from psychology to understand economic decisions; see, e.g., Akerlof and Dickens [1982] on the economic consequences of cognitive dissonance. However, economists have seldom attempted to apply decision-theoretic reasoning to explain psychological mechanisms. In this regard, see also some of the recent papers discussed further in Subsection 6.

5See, however, the discussion in Section 6 of Prendergast and Stole (1996). Rumelt’s overview opposes organizational inertia (persistence) to plasticity, a readiness to respond appropriately to environmental change. Thus, inertia (or lack of plasticity) is an obstacle to optimal behavior. This categorization implicitly excludes the possibility that firms sometimes place undue weight on new information (impulsiveness) and hence adjust behavior too readily.
effect of memory loss is to reduce the weight of old information in determining later actions. However, because of the form of the memory loss that we examine, sometimes the opposite will be the case.

Our approach is based on the premise that past actions (policies, routines) are remembered well, while past information signals are remembered poorly. Actions are more visible, salient, and memorable than underlying justifications. Information transmission, absorption, and retention is costly, so the full reasons for past decisions are often forgotten. Thus, organizations often lose access to old information even if individual managers, like elephants, never forget. A decisionmaker with such differential recall must infer past signals from the coarse summary provided by past actions. Our conclusions derive from the fact that this coarseness distorts later decisions—even when the decisionmaker optimally adjusts for the resulting information loss.

The basic argument of the paper, in simplest form, is as follows. Suppose that an initial player had adopted a policy for some time and that the new player—without access to his predecessor's information—must decide whether to adopt a similar action. The new player does not know how strongly his predecessor's information favored the original decision. Consequently, even if the new player receives an opposing signal he should rationally continue the old behavior if the presumed favorable information of his predecessor outweighs the new signal. In this circumstance, he never switches. In contrast, in a benchmark regime of perfect recall, a continuing player whose information happens to barely favor the initial project would switch after a new opposing signal. In this situation—a player with perfect memory sometimes switches while one who recalls only past decisions never switches—there is excess inertia under amnesia relative to a perfect recall regime. (Section 3 also analyzes the persistence of inertia as further

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6According to Huber (1991), "Everyday observations make clear (1) that personnel turnover creates great loss for the human components of an organization's memory; (2) that nonanticipation of future needs for certain information causes great amounts of information not to be stored... or not to be stored such that it can be easily retrieved...., and (3) that organizational members with information needs frequently do not know of the existence or whereabouts of information possessed or stored by other members."
information arrives.)

However, there are forces that can weaken the inference that should be drawn from the predecessor’s action. For example, if the gains to adopting a project change stochastically, then an old choice that was correct may become incorrect later. A continuing player with perfect recall would in some cases know that the old information was quite strong, and in such cases would follow old policies over new signals. In contrast, a new replacement player may find it rationally optimal to follow his own signal regardless of past actions. In this changing-environment scenario amnesia causes excess impulsiveness rather than inertia—an individual with poor memory of past signals is more likely to switch behaviors than one with good memory.

In sum, our model of choice under memory loss provides implications about several determinants of inertia versus impulsiveness. These include the information load an individual must process, volatility of the decision environment, how long policies and executives have been in place, and the quality of a firm’s information systems.

The remainder of this paper is structured as follows. Section 2 provides the basic three-period model. In this model a stable environment leads to excess inertia, and a volatile environment to excess impulsiveness. Section 3 extends the model to many periods, to examine the effect of varying the strength of information about the past, and to examine whether inertia or impulsiveness can persist in the long term. Section 4 discusses implications of the model based on the value and sources of memory. Section 5 compares the model to other theories of tradition, rules, and inertia. Section 7 concludes.

2 The Shadow of History: The Basic Model

We now describe a stylized model of the shadow that the past history of actions casts on future behavior after a transition, and the factors that can intensify or dispel this shadow. With imperfect memory a player cannot attune his policies perfectly to old information.
A new player observes only a coarse summary of what the previous player knew, as reflected in past actions. We will show that if past actions are highly informative they tend to outweigh new signals, so that an optimizing player rationally exhibits excess inertia. If past actions are only weakly informative, an optimizing player rationally exhibits excess impulsiveness.

We construct the simplest possible model to capture these insights. Consider a player (individual or firm) who faces a sequence of three identical decisions or 'projects.' The true value of each project, which is constant through time, can be either good (value state $\phi = G$) or bad (value state $\phi = B$), with equal ex-ante probability. Payoffs are perfectly correlated across projects, but are not observed until the end of the game. Instead, a player can only observe a single signal, $H$ or $L$, each period. If the underlying state is good, the player observes signal $H$ with probability $p$ ($p > 0.5$), and $L$ with probability $1 - p$. If the underlying state is bad, the player observes $L$ with probability $p$ and $H$ with probability $1 - p$ (see Table 1). Conditional upon the value state, the successive signals are assumed statistically independent.\(^7\)

Table 1 Here

In each period, the player chooses an action, either adopt ($A$), or reject ($R$). Net payoffs are such that the player adopts the project if state $G$ is more likely than state $B$, rejects if $B$ is more likely than $G$, and flips a fair coin when $G$ and $B$ are equally likely.\(^8\)

We compare two scenarios. In the full recall scenario, both actions and signals are retained (i.e., the initial player $I$ stays in place for all three periods and retains access to his old information). In the amnesia scenario, only actions are remembered, i.e., the new player $N$ can recall the two actions taken prior to the transition date (period 3),

\(^7\)An alternative interpretation of the model would define actions $A$ and $R$ as expanding or shrinking a project that is already in place, states $G$ and $B$ as higher and lower probabilities of getting a high payoff per period, and signals $H$ or $L$ as immediately-observed high or low payoffs. Note that information arrives each period regardless of whether the project was accepted or rejected in the previous period.

\(^8\)For expository simplicity we examine a symmetric model. This leads to the possibility of exact indifference and randomization. Qualitatively similar results apply when distributions are asymmetric.
but not the signals of his predecessor. The quality of N’s decisions is always inferior to that of a hypothetical continuing I, because I’s information about the past dominates.

Just before date 3, the underlying value of adopting versus rejecting may change. Specifically, with probability $\sigma$, the state is redrawn. If it is redrawn, with equal probability the new state is $G$ or $B$. (For simplicity, the redrawing occurs only at a single date. We conjecture that similar results would apply in a setting with a probably of environmental shifts at each date.) Thus,

$$Pr(\phi' = G | \phi = G) = 1 - \frac{\sigma}{2}.$$ 

We focus on the transitional behavior of N versus I at date 3. There are four possible action patterns for the first two dates: $AA$, $AR$, $RR$, and $RA$. $AA$ can arise either from $HH$ or $HL$, and $RR$ from either $LL$ or $LH$. In contrast, $AR$ can arise only from the single signal sequence $HL$, and $RA$ only from $LH$. So opposing actions allow N to infer perfectly that the first two signals were opposed ($HL$ or $LH$), and that a coin flip led to the second action choice. Thus, under either full recall or amnesia, posterior beliefs are based on the cancellation of the first two signals, and the date 3 player N follows his latest signal. Since this is true under both full recall and amnesia, such realizations do not contribute to inertia or impulsiveness.

Thus, we focus on realizations where I follows like actions for two dates, $AA$ or $RR$. Like actions are likely to arise from like signals. $AA$ could arise either from $HH$, or from $HL$ with a coin flip. $RR$ could arise either from $LL$, or from $LH$ with a coin flip.

We now show that when volatility is low, the probability of an action switch under amnesia is zero, whereas there is a positive probability of switch under full recall. Consider, for example, the case of a fully stable environment ($\sigma = 0$). In an amnesia scenario where the decisionmaker observes the past two actions but not signals, the new player draws a favorable inference from $AA$ (that the signals were $HH$ or $HL$). Because of a high likelihood that $AA$ came from $HH$, even if he observes $L$, he still adopts.$^9$ He

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$^9$Since $p > 1/2$, $Pr(HH|AA) = \frac{1-2p+2p^2}{1-p+p^2} > Pr(HL|AA) = \frac{p(1-p)}{1-p+p^2}$. 

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knows based on the first $A$ that the first signal was $H$. This offsets his latest $L$ signal. The second $A$ came from either an $H$ or an $L$ followed by a coin flip. The former is more likely, since a coin flip could lead to $R$ instead of $A$. Therefore, on balance it is more likely that the state is $G$ than $B$. Thus, under amnesia, $N$ always adopts. Similarly, after $RR$ he always rejects. In contrast, if the two signals were indeed $HL$ or $LH$, a continuing player $I$ with an opposing signal in the third period reverses course.

This outcome of excess inertia when environmental volatility is low is shown in the second column in Table 2. Intuitively, in a stable environment (low $\sigma$), old policies are very informative relative to new signals. It is therefore rational for an amnesiac individual to place heavy weight on the past, leading to inertia.

We now show that in a more rapidly changing environment, a full-recall player sometimes follows his old signal. (The full-recall player is more willing to put some weight on the past because it knows past signals accurately instead of having to infer them from action.) Thus, $N$ tends to overreact to new information, and is too quick to reverse old policies. In such an environment, memory improvements tend to make the player more willing to retain old policies.

As environmental volatility increases (medium $\sigma$ in Table 2), players place relatively little weight on the past (either under full recall or amnesia). At some point, the relevance of old policies to current decisions weakens just enough that an amnesiac player always follows his new signal. In contrast, at this point a continuing $I$ does not always follow his latest signal. (Specifically, if prior signals overwhelmingly favored a project, $I$ sticks with that project.) Therefore memory loss causes impulsiveness. Specifically, as volatility increases, a point is reached for $N$ ($\sigma = \sigma^N$) where a single $L$ signal outweighs the information implicit in $AA$ (which could have arisen from $HH$ or $HL$). At this point, $N$ always follows his own signal without regard to the past. In contrast, since $I$ has more refined information about the past, when he observes strongly favorable past information $HH$, he adopts even if his third signal is $L$. Thus, in this range of $\sigma$ amnesia causes impulsiveness.
Finally, for a highly unstable environment (high σ in Table 2), even past signals have weak relevance for the future, so even a full-recall player will follow the new signal. In other words, an L signal not only outweighs two AA signals but also two HH signals. Since I or N behave identically, there is neither excess inertia nor impulsiveness.

This reasoning suggests the following result:

**Proposition 1** In the model with possible environmental shifts, the critical value for σ above which the player always follows the date 3 signal is larger for the full-recall player than for amnesia player, σ^N < σ^I. Thus, if the probability of a value shift is:

1. low, σ < σ^N, under memory loss the player exhibits excess inertia;
2. intermediate, σ^N < σ < σ^I, the player exhibits excess impulsiveness;
3. high, σ^I < σ, there is neither excess impulsiveness nor inertia.

**Proof:** See Appendix A.

| Table 2 Here |

The proposition describes the conditions that promote impulsiveness versus inertia. Firms in relatively stable industries are predicted to be excessively inert, and those in more rapidly-changing industries as computing and telecommunications to exhibit impulsiveness.\(^{10}\) This is especially the case when employee job mobility (and loss of firm-specific memory) is high, e.g., as in Silicon Valley during the rise of personal computing and the internet. Part (3) of the proposition seems to be an extreme case; it is unlikely that many individuals or industries are in environments so volatile that past signals are uniformly useless.

Environmental instability causes the value of information to decay, because information about the past value state becomes less relevant for the current adoption decision. Thus, even with full recall it is no surprise that instability leads to greater change. The interesting point is that amnesia exaggerates the importance of old signals in a stable environment, and the importance of new signals in a volatile one. (Except for the unrealistic case of extreme volatility described in part (3), in which even with memory loss the first-best outcome is achieved.)

\(^{10}\)Thus, Henry Ford's view that "History is more or less bunk... We don't want tradition... the only history that is worth a tinker's damn is the history we make today," is more appropriate to the Ford of 1916 than the Ford of today.
Intuitively, amnesia coarsens information about the past, thereby rendering the player less aware of somewhat unusual circumstances. If, on the whole, the information reflected in past actions is quite informative for the future, deviation from past actions is seldom needed. It makes sense to follow past actions often, despite the cost of wrongly maintaining old policies when change is needed. Similarly, if past actions are not very informative, it is optimal to follow past actions seldom, and bear the cost of wrongly abandoning old policies when stasis is needed. Thus, coarse information causes a kind of overshooting. Conditions that, under full recall, favor low (high) weight upon past information cause excess impulsiveness (excess inertia) when memory is lost.

This suggests why certain factors intensify or diminish the shadow of the past. We have just seen that since environmental volatility weakens the informativeness of old actions for the future, high volatility tends to cause greater impulsiveness. Furthermore, we have shown in an analysis similar to the basic model that if there is only a probability each period that a player receives a meaningful signal, then even in a stable environment there can be impulsiveness. The possibility that past actions are based on relatively little information reduces the weight on them relative to a new signal (if a new signal is received). On the other hand, the next section examines a factor that can intensify the weight placed on the past, a longer action history.

3 Long-Term Effects of Amnesia

So far we have assumed a fairly short history prior to memory loss (two dates), and have examined only one decision immediately after memory loss. We have therefore allowed history to cast only a short shadow: transitional inertia versus impulsiveness. We now examine whether such behavior can persist in the long term. Furthermore, we examine how the length of the project adoption history (how well-established the existing policy is) affects long-term inertia versus impulsiveness. To examine these issues, we extend the model to many periods before and after the memory loss.
For simplicity, in this section we eliminate environmental shifts. For the reasons analyzed in the basic model, if the environment is highly volatile, decisions will be impulsive. Our focus here is on the effects of history in a stable environment. We will show that the longer an action is established by the initial player, the greater the inertia; even for fairly recently established policies there is excess inertia. Inertia now involves comparing probabilities of continuing past actions, rather than the certainties under amnesia in the basic model of Section 2.

3.1 A Multiperiod Decision Setting

This section differs from the basic model in three ways. First, it allows for any number of periods either before or after the memory loss (player transition) date. Second, the environment is stable ($\sigma = 0$). Third, for analytical convenience we allow a third action choice (see below).

In each of a number of discrete periods, the current decision-maker (individual or firm) has to choose publicly an action, either to adopt the project ($A$), reject the project ($R$), or abstain from decision ($\emptyset$). There are two possible value states, $G$ and $B$, which are constant through time. Players adopt if $G$ is more likely than $B$, reject if $B$ is more likely than $G$, and abstain if $G$ and $B$ are equally likely. Appendices B (Table 3) provides a simple payoff structure such that this behavior is optimal.

As before, the player's prior belief is that $G$ and $B$ are equally likely, and each period the player receives one private signal, $H$ or $L$. Payoffs are revealed only when the game ends. The signal structure is summarized in Table 1 of the previous subsection. Define the signal state of $I$ at time $t$ as the difference between the number of $H$ and $L$ signals that he observes, $s_t = n_H - n_L$ (in $t$ draws). For example, if I observed $HHLHL$, he is in signal state $+1$ at date 5. Given the value state ($G$ or $B$), the transition probabilities

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11 As in Section 2 (see footnote 8), our simplifying assumption of symmetry leads to the possibility of exact indifference. Introducing the (somewhat artificial) option of abstaining simplifies the algebra and discussion by eliminating randomization, tie-breaking rules or assumed asymmetry. We have derived similar results when there is no option of abstention, and distributions are asymmetric.
from signal state \( s \) to \( s + 1 \) or \( s - 1 \) are just the conditional probabilities of another \( H \) or \( L \) signal being observed. Thus, conditional on the unknown value state, the player's signal state follows a conditional Markov process. The tie value \( s_t = 0 \) is the point where the player switches from either \( A \) or \( R \) to \( \emptyset \). Appendix B derives the probability of reaching \( s_t = 0 \) given an information set.

Suppose \( I \) has been in place for \( M \) periods. For \( N \), observing only \( I \)'s past actions (adopt, reject or abstain) is equivalent to observing only whether the Markov signal-states at dates 1 through \( M \) were above, below or equal to 0. We call abstention an \textit{action switch}, since the player shifts from strongly preferring one action to being indifferent. Whenever an action switch occurs, \( N \) can perfectly infer \( I \)'s signal state to be zero at the switch date.

Since adopt and reject are symmetric, we focus on the consequences of past adoption. All computations are conditional upon having observed a sequence of exactly \( M \) uninterrupted adopt decisions in the last \( M \) periods prior to the memory loss date. This is without loss of generality, because the signal state is zero whenever there is an act of abstaining. For example, suppose there have been 100 past actions, ending with the string \( \emptyset AAAAA \). The first 95 actions are irrelevant (the abstain demonstrates 48 \( H \)'s and 48 \( L \)'s, which jointly are uninformative about the value state) for \( N \), so this history is equivalent to one where \( N = 4 \) and a string of 4 \( A \)'s has occurred. It is useful to generalize the notion of signal state to apply to either \( I \) and \( N \).

\textbf{Definition} The equivalent state \( \bar{e} \) for a new or continuing player is the minimum number of consecutive opposing a signals (\( L \)-signals for an adopting player, \( H \)-signals for a rejecting player) required for the player to switch action to abstain or beyond.

For a continuing \( I \), the equivalent state is the same as the signal state.

### 3.2 Transitional Inertia

We now compare the likelihood of action changes under amnesia (a change in players) to the full recall scenario where \( I \) remains in place. Just after a transition, \( N \) with amnesia
has an equivalent state that depends only on the past action history. We denote the equivalent state as \( \bar{e}(M, p) \) as a function of the number of past adoptions and the signal probability. The next proposition shows that a player with amnesia initially is inert (relative to a full-recall player):

**Proposition 2**  
**Transitional Inertia after Memory Loss**  
Conditional on \( M \) prior adopt decisions, a new player never switches before time \( M + \bar{e}(M, p) \) periods, whereas there is a strictly positive probability that a continuing player switches by time \( M + 2 \), where \( \bar{e}(M, p) \geq 2 \) for \( M > 2 \).

(For \( M \leq 2 \), \( N \) makes a perfect inference and consequently behaves like \( I \).) Intuitively, after a sequence of \( M \) adopts by \( I \), \( N \) does not know how strong the evidence was in favor of adopting. At best there could have been \( M \) consecutive \( H \) signals; at worst there could have been close to equal numbers of \( H \) and \( L \) signals. \( N \) will draw the statistical inference that quite likely the signal state is intermediate between these extremes. Thus, if \( M \) is sufficiently large, \( N \) believes it is unlikely \( I \) was very near the borderline, so he does not switch actions after observing just a single \( L \) signal. But a continuing \( I \) who has observed the actual sequence might have been right on the borderline. So if there was only very weak evidence supporting the current action, a continuing \( I \) switches action based on one or only a few opposing signals with positive probability. Since \( N \) is more likely to switch than a continuing \( I \), there is excess inertia. This intuition suggests that the result of transitional inertia (which was the focus of Section 2) is quite robust with respect to assumptions about signal structure and value distribution.

### 3.3 Medium-Term and Long-Term Inertia

Does inertia persist beyond the first few decisions? Sequences of signal realizations exist such that \( N \) switches when \( I \) does not, and *vice versa*.\(^{12}\) We therefore compare the
probability of an action switch over a given period of time for N versus I. Asymptotically, N exhibits excess inertia:

Proposition 3  Long-Run Inertia under Amnesia
(i) Holding constant the number \( M > 3 \) of immediately consecutive prior adoptions, as \( t \to \infty \), the probability that the new player reverses action at least once is less than or equal to the probability that the continuing player does so. (ii) Holding constant the number \( M' > 3 \) of periods before the transition (which may include A or R), as \( t \to \infty \), the probability that the new player reverses action at least once is strictly less than the probability that a continuing (full recall) player does so.

Proof: See Appendix C.

Intuitively, if the player begins with the wrong action, then with either full recall or amnesia (I or N), as information accumulates, eventually there will be a switch to the right action. In this case there is no difference. Suppose instead that the player begins with the right action. Then, under full recall, often the player begins close to the decision boundary, and switches temporarily to the wrong action along the way. This is less common under amnesia because forgetting of past signals compresses N's beliefs away from the edge of the decision boundary. Thus, we have shown that for a period immediately after the transition, and also in the asymptotic long run many periods after the transition, memory loss causes excess inertia.

Although we have no general proof, for an intermediate time period \( T \) after the switch, extensive simulations uniformly indicate that the probability that \( N \) switches is strictly below the probability that \( I \) switches for any \( M \geq 3 \). (For \( M < 3 \), \( N \) perfectly infers \( I \)'s information and so behaves identically to a continuing \( I \).) Thus, in the explored parameter ranges, inertia obtains uniformly. Figures 1-4 plot some switch probabilities for the the two scenarios with \( M = 4, 32 \) and signal accuracies \( p = 0.5001, 0.6 \).\(^{13}\) The uniformly higher probability of switch under full recall illustrates inertia in the short, medium and long terms.

\(^{13}\)Excess inertia holds uniformly when \( M = 64 \) or when \( p = 0.51, 0.75 \) or 0.9. All results have been both computed from the described formulas and confirmed in simulations (using the computed equivalent state decision rule).
So far in this paper, we have assumed that I maximizes expected profits each period. However, it may be better for the player to sacrifice short-run profit to improve future decisions. If a transition is foreseen, then I can adjust his policies as a function of his information in a sort of code to communicate with a later player who does not have access to I's information signals. Such behavior can be termed strategic signaling. One such code would require I to act in opposition to the preponderance of evidence right before the transition if he is sufficiently close to indifference. The expected cost is low since deliberate incorrect decision-making is restricted to a few periods when the project is almost a toss-up. Thus, the benefit from strategic project choice sometimes outweighs the cost. However, in practice it may be difficult to design contracts that motivate a manager to select poor projects or reject good projects in order to credibly communicate information to his successor (see also Section 4).

4 Sources and Value of Memory

Because economic analysis usually assumes perfect memory, we discuss a relatively wide-ranging set of phenomena in which memory loss effects are likely to be important. The inefficiencies just described can be eliminated if I accurately records the signal state and communicates it to N. Thus, our model is most applicable for policies for which information is hard to record, store, retrieve, and transmit. Several studies describe factors leading to imperfect individual memory (e.g., Kahneman, Slovic and Tversky (1982), Nisbett and Ross (1980), and Starbuck and Milliken (1988)) and organizational memory (e.g., Huber (1991), Walsh and Ungson (1991), Han (1997)). The fear of legal liability discourages record-keeping, implying that industries that are vulnerable to litigation should be more prone to problems of amnesia.

Amnesia in organizations is likely to be more severe when decisionmakers are stressed by a severe information load, when there is a long time lag between related decisions, for decisions that depend on confidential or legally-sensitive information, and for decisions
that are hard to foresee. The theory implies that it is these firms in which inertia and impulsiveness are likely to most important (inertia for venerable policies in stable environments, impulsiveness for recent policies in volatile environments).

4.1 Information Technology and Record-Keeping

Owing to the high turnover rate of programmers, "reverse engineering" the software developed by departed programmers is a growing problem (Peterson [1993]). Although it is easy to communicate the actions of individual programs (or major subroutines) to a new programmer, it is usually very difficult to understand the detailed rationale for specific pieces of code. Thus, it is less likely to be optimal for a new programmer than for the original programmer to make major changes in the code.\textsuperscript{14} This effect can lock a firm into an unwieldy information system.

The introduction of groupware and knowledge management systems at many firms provides natural experiments on the effects of changes in organizational memory. This software facilitates the recording, retrieval and transmission of transactions and decisions within the firm. The economic importance of electronic knowledge management is highlighted by IBM's 1995 $3.3$ billion acquisition of Lotus in order to acquire Lotus' Notes software. If effective, groupware should help improve memory and therefore reduce inertia and impulsiveness. Our analysis therefore predicts that groupware, and more generally improved knowledge management procedures, will reduce inertia for firms in stable industries, but move firms in more volatile environments toward greater inertia (less impulsiveness).

Managerial specialization and spreading of decisions in large firms have multiple affects on memory. The spreading of memory of decision rationales probably leads to a relatively steady rate of forgetting in large firms as managers depart. In contrast,

\textsuperscript{14}Subroutines and lines of code can have hidden rationales and side effects that make them difficult for a later programmer to understand, or even for the original programmer to revisit. Indeed, the movement first to procedural languages and now to object-oriented programming (see, e.g., Forbes ASAP, 12/2/96, p. 187) encourages modularization to reduce the need to make any changes to old code.
small firms run by a small entrepreneurial team are likely to have very poor memory when an executive leaves, and very good memory when no turnover occurs. So small entrepreneurial firms should usually be more agile than large bureaucratic firms, but with occasional bursts of extreme inertia or impulsiveness.

Scale economies in memory systems can create an advantage for larger firms. There are fixed costs associated with computerizing, introducing sophisticated internal accounting systems, and supporting redundant managers. This suggests that big firms should tend to be nimbler than small firms in reacting to change in the long run.

4.2 Executive Mobility and Succession

Our focus is not on traumatic involuntary replacements of managers in order to effect change. Such cases obviously involve effects outside our model. Even for more routine transitions, organizational theorists have emphasized the disruptive effects of managerial succession (e.g., Grusky [1960]). There is evidence that the announcement of voluntary CEO departures without pressure by the board is on average associated with negative abnormal stock returns (Furtado [1989]). Two trends suggest that the problems that arise from memory loss may be of great importance: the increasing tendency of large U.S. firms to hire outsider CEO's and directors, and the movement toward elimination of middle-management levels through downsizing.\textsuperscript{15}

Corporate boards of directors can be reservoirs of memory when top executives are replaced. This provides a rationale for a strong insider presence on the board. Since boards preserve memory only imperfectly, the notorious reluctance of directors to remove

\textsuperscript{15}According to The Economist (4/20/96 pp. 51-2) in an article entitled "Fire and Forget?," "Having spent the 1990s in the throes of restructuring, re-engineering and downsizing, American companies are worrying about corporate amnesia." The article provides several anecdotal examples of firms which achieved leaner workforces but failed to improve performance, apparently because of loss the information possessed by key employees. Similarly, according to Forbes ASAP, 12/4/95, "Companies that went overboard 'right-sizing' are now desperately trying to keep experienced hands to steady the ship." An article in Forbes 2/10/97 p. 64 entitled "Ford's boss-a-year plan" attributes poor performance at Ford Europe to its treatment of the top Europe position as a temporary stepping stone (ten bosses in 18 years), and consequent inertia.
poorly performing CEOs can be viewed as excess inertia. Our analysis implies that in volatile environments, impulsiveness is possible. This suggests that in contrast to the entrenchment of large corporate managers, entrepreneurs financed by venture capitalists may be removed more often than would be optimal under full recall.

Our theory implies that firms whose managers are routinely more mobile, retire earlier, and are hired from the outside have stronger inertia or impulsiveness than firms that provide lifetime employment, have late retirement age, and in which new management is groomed for the job. While employee mobility allows firms to adjust labor usage easily, it may cause undesirable memory loss and lead to inertia or impulsiveness in decision-making. Factors that might seem to help entrench the status quo—low turnover, promotion from within, and late retirement ages—by improving memory can sometimes improve the firm’s ability to adapt.

Management style also influences organizational memory. Consensus management techniques distribute memory about reasons for decisions (not just the decisions themselves) among more participants. This reduces the memory loss associated with turnover. Similarly, the benefits of functional specialization within a management team must be balanced against the memory loss when a key member departs.

Turnover should impose lower average amnesia costs on large than on small firms, owing both to steadier proportionate turnover and scale economies in information management systems. This analysis therefore implies that large firms optimally adopt policies more conducive to employee turnover. Grusky (1961) finds that turnover rates are indeed higher in larger firms.\footnote{To examine these hypotheses empirically, a researcher must be careful of a post-selection bias. A small firms that suffer a drastic memory loss will often go out of business. There is thus a tendency to compare lumbering large firms only with the up-and-coming small competitors, which biases results in favor of the popular view that small firms track environmental shifts more adeptly than large firms.}
4.3 The Value of Memory

The importance of memory loss is evidenced by the resources devoted to information preservation by means of files, personal information managers, minutes, memos, secretaries, accounting systems, expert systems, groupware,\textsuperscript{17} establishment of knowledge management systems and Chief Knowledge Officer positions for preservation of organizational memory (see, e.g., Business Week 6/10/96, p. 6, and Forbes ASAP, 8/24/98, p. 92), and deferral of employee retirement. An impressive example is the extensive cataloging activity of Ernst and Young’s 200-person Center for Business Knowledge.

Our approach implies that the extent to which individuals and firms use costly devices for improving memory depends on the value of full recall rather than recall of actions but not signals. Consider a firm that faces repeated environmental shifts, and has repeated memory losses about past signals. If the probability of a value shift is quite low, then the past history of actions is highly informative about the value state. Fairly accurate decisions can therefore be made even without recall of signals, so the value of full recall is modest. If volatility is quite high, past history of any sort has little relevance for the future, so the gain to preserving past signals is low.\textsuperscript{18}

Thus, the value of full recall is greatest for players who face intermediate environmental volatility. This is consistent with the evidence of Moorman and Miner (1997), who report evidence from 92 new product development projects that survey-based proxies for better and more dispersed organizational memory are associated with good short term financial performance of new products, but that under conditions of high environmental turbulence the positive effect of high memory dispersion disappears.

The analysis also suggests that preservation of memory is relevant for optimal term lengths and limits for public officials. Memory preservation provides one rationale for a civil service in which a newly elected executive cannot replace bureaucrats at will. It

\textsuperscript{17}Groupware technology can include several aspects, such as email, document management, workflow management, and knowledge sharing.

\textsuperscript{18}The general trend (cited earlier) among U.S. firms toward hiring outsider CEO’s since the 1960’s could, in part, be related to a more volatile business environment as markets have globalized.
also provides a reason to re-elect incumbents ("Don't change horses in mid-stream"). Our theory implies that the desirability of incumbency advantage and long tenure for officials is greatest in environments with intermediate volatility.

5 Memory Loss and Society

Even without explicit sanctions, traditions and conventions have great influence upon behavior.\textsuperscript{19} Progressives and radicals have often argued that traditional norms and rules are irrational. In this view, people should select the best policies based on the latest information about current circumstances, rather than deferring to the dead hand of the past. The same criticism can be leveled at organizational routines.

Amnesia provides a rationale for rules and traditions. Traditions reflect the hard-won experience of previous generations. Since much information available to our ancestors has been forgotten, it can be optimal to adhere to traditional behavior. Thus, Hayek (1967, chs. 4, 6) argues that there is a wisdom in the spontaneous order of traditions and rules that is often not evident to current decision-makers (also a theme of the traditionalist Edmund Burke). As expressed vividly by G.K. Chesterton (1908; ch. 4):

Tradition may be defined as an extension of the franchise. Tradition means giving votes to the most obscure of all classes, our ancestors. It is the democracy of the dead. Tradition refuses to submit to the small and arrogant oligarchy of those who merely happen to be walking about. All democrats object to men being disqualified by accident of birth; tradition objects to their being disqualified by the accident of death. Democracy tells us not to neglect a good man's opinion, even if he is our groom; tradition asks us not to neglect a good man's opinion, even if he is our father.

As with social traditions, organizational routines, bureaucratic rules and corporate culture may be ways to aid memory and constrain future behavior to be consistent with currently available information. This institutionalization of memory is especially useful when memory loss is so severe that actions, not just rationales, are likely to be forgotten. Simple codifications of wise behavior provide coarse summaries of past action-

\textsuperscript{19}Weick and Gibb\textsuperscript{11} (1971) provide experimental evidence that arbitrary traditions of group behavior sometimes persist as group composition gradually changes, without explicit sanctions or externalities.
relevant information. Such summaries encourage individuals to adhere voluntarily to the prescribed action. Safety codes and standardized alarms for poisonous materials and hazardous roads allow individuals to follow standard behavior without knowing the reasons in every instance. More importantly, conventions for proper public behavior allow individuals to interact smoothly in busy environments without attending to the behavior of bystanders (see Goffman [1971]).

While social amnesia makes it rational to stick with traditions and rules, such traditions are not without cost. Traditions and rules only coarsely summarize useful information from the past. Actual preservation of detailed information is better, because then people will know when to break with tradition. This helps explain a paradox of social change, that the rise of technologies that improve recording and preservation of the past have accelerated change. Our theory is consistent with the fact that traditions have tended to be supplanted more frequently as better mechanisms for information preservation were invented (e.g., the invention of writing, Gutenberg’s printing press, and the rise of electronic databases).

In our analysis, volatile environments eliminate inertia and can cause excess impulsiveness. The idea that environmental instability makes individuals more receptive to new behaviors is a recurring theme of the social philosopher Eric Hoffer. He observed that “...a population subjected to drastic change is a population of misfits—unbalanced, explosive, and hungry for action,” (see Hoffer [1964], Ch. 1). Hoffer argues that the emotional discomfort of a changing environment causes individuals to adopt new behaviors. In our approach individuals change as a result of weakened belief in the profitability of conventional behaviors.

Rituals and taboos are specific and extreme types of traditions that seem to lack current justification from an outsider’s perspective. According to Freud (1950), “Taboo prohibitions have no grounds and are of unknown origin. Though they are unintelligible

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20 Consider sayings such as “Don’t draw for an inside straight,” or mottos such as ‘Sell it to the sales staff,’ at Hewlett Packard, or ‘Those who implement the plans must make the plans,’ at Texas Instruments.
to us, to those who are dominated by them they are taken as a matter of course” [p. 18], and “No external threat of punishment is required, for there is an internal certainty, a moral conviction, that any violation will lead to intolerable disaster” [p. 26-7]. One explanation for the persistence of taboos is that people suspect a good rationale exists, though forgotten or preserved only in the form of crude allegories.\footnote{21}

6 Other Theories

This section compares our theory with several alternative and complementary theories. Our assumption of rationality contrasts with several theories of individual and organizational inertia and conservatism; see, e.g., Thaler (1980), and the reviews of Kuran (1988) and Rumelt (1994).\footnote{22} On the other hand, Lazear (1991) concludes that the economic approach of rational optimization has promise for explaining the behavior and institutions of labor markets and organizations.

Actual behavior is of course influenced by a combination of factors. Managers are sometimes hired specifically to bring about policy changes, which raises issues outside our model. To control for such effects, our model can be tested by focusing on the more common and routine forms of managerial turnover (e.g., retirements, voluntary departures or death rather than firing).

Mullainathan (1997), in independent work, examines individuals who apply Bayes’ rule to a selectively recalled signal history as if it were the true one. Our approach differs in two main respects. First, we examine inertia versus impulsiveness. Second, we assume that decisionmakers apply Bayes’ rule \textit{rationally} subject to the single cognitive limitation of amnesia.

\footnote{21}McNeil (1976) pp. 155-56 discusses seemingly irrational folkways, probably arrived at by trial and error, which in the light of modern medical science are recognized as protecting against deadly disease. \footnote{22}The limited ability of individuals to broadcast or absorb information (implicit in a failure of organizational memory) should be distinguished from an invalid use of information that the individual has ample opportunity to absorb (as in theories of irrational inertia).
The organizational ecology literature generally assumes that firms do not change, and focuses on how populations of firms are modified through processes of creation and selection. Hannan and Freeman [1984] justify this inertia assumption based on the tendency of organizations to institutionalize procedures to maintain reliability. In our approach, high stability is an optimal response to memory loss only in a stable environment. Their approach contrasts with an adaptionist perspective which emphasizes organizational change in response to the environment (see, e.g., Nelson and Winter [1982]). Our theory offers a middle path between these approaches by predicting a degree of inertia.

Kuran (1987) provides a model in which individual preference or social pressure cause conformity. This results in periods of stability interrupted by occasional large shifts. Our model focuses on a problem of inference rather than one of social pressure.

Dixit (1992) offers an options theory of inertia based upon the benefit of deferring costly changes until further information arrives. Dixit’s theory implies that inertia is strongest in a highly volatile environment. In our theory, volatile environmental change opposes inertia and can lead to its opposite, impulsiveness.

A literature on informational cascades derives something akin to inertia as a consequence of the rational inference by later decision-makers about the signals observed by early decision-makers (see Banerjee [1992], Bikhchandani, Hirshleifer and Welch [1992], and Welch [1992]). In these models, cascades occurs after a sufficiently large number of individuals have made decisions, and cascading individuals follow predecessors with certainty. In the model presented here, inertia or impulsiveness develops after the multiple decisions of one (or just a few) decision-makers. We examine here the more general issue of the probability that a player will follow past actions.

Some theories of inertia are based on divergence of interests between different decision-makers (see e.g., Rasmusen [1992]). For example, a manager may take actions in order to persuade observers that he has high ability (see Boot [1992], Prendergast and Stole [1996], and Zwiebel [1995]). In Zwiebel, a manager may hold to a standard project
rather than risk a superior innovative one. Prendergast and Stole's study offers a possible explanation of an apparent bias in favor of escalation of past decisions. They show that a manager concerned with others' perceptions may initially use information too aggressively, and later adhere too strongly to old decisions. Our analysis is based on information loss rather than influencing the beliefs of others, and therefore does not require that the decisionmaker be observed by others.

Our theory differs from alternative theories in its prediction of which variables favor inertia or impulsiveness. Indeed, past research has devoted little attention to the possibility of impulsiveness. The variables suggested by our model include information load, managerial mobility, managerial overlap, and the quality of information storage/retrieval systems (see, e.g., the discussion of the predicted effects of introduction of Groupware in Subsection 4.1). The contrasting predictions of our theory and the real options theory for the effects of volatility on inertia versus impulsiveness were discussed earlier. In contrast with several reputational theories, our approach implies that high managerial attachment to a project can lead to impulsiveness, not inertia, depending on the memory and environmental variables mentioned earlier. Furthermore, the analysis offers implications about which firms should find investment in retaining organizational memory most valuable (see Subsection 4.3).

7 Summary and Extensions

Traditional information economics assumes perfect recall. However, casual observation and extensive evidence from psychology suggest that memory loss frequently affects decision outcomes. This paper shows that memory loss affects economic decisions in a predictable way, and therefore has important consequences for the continuity of behavior. Since excess inertia and impulsiveness are optimal responses, intervention or legislation designed to oppose inertia or impulsiveness, such as requiring or blocking change, would, in the absence of extra information, reduce decision quality. Thus, behavior which may seem to reflect faulty cognitive strategies (e.g., escalation bias, cognitive dissonance) can
be understood as rational responses given memory constraints.

Memory loss occurs at both the individual and organizational levels. Firms can lose information both from individual managerial amnesia, and from turnover and failures in interpersonal communication and record-keeping. Risk of legal liability and desire to avoid 'paper trails' may have the side-effect of weakening internal memory. How information retention affects organizational policies is of growing importance as modern economies shift to services and information management, and as restructuring causes managerial and employee turnover.

The model is based on the premise that a new player (an individual after memory loss, or a firm under new management) can observe/recall previous actions but is unaware of the rationale for these actions. We show that memory loss not only leads to poorer decisions, but to rational adjustments to the proclivity to follow or deviate from past policies. When the environment is stable, and if a player has followed an old policy a long enough time, there is excess inertia: as new information arrives the player optimally tends to maintain old policies more under amnesia (e.g., a new player) than with full recall (e.g., a continuing initial player). Thus, the model implies that individual habits, organizational routines and cultural norms that institutionalize inertia should be more extensive and effective in stable environments than volatile ones. Under the opposite set of circumstances, i.e., when the value of old information is weak and/or decays over time, there is excess impulsiveness: an amnesiac player will optimally tend to shift policies more often than would a full-recall player.

The analysis suggests several forces which can intensify or dispel the shadow of the past. These include the age (duration) of the project, policy or activity; the speed of change of the environment; the quality of information (both new and old); the information-load; the amount of managerial and employee turnover; the quality of record-keeping and information systems; and the volatility of the decision environment.

At the individual level, the model provides a possible framework for the psycho-
logical theory of self-attribution, in which individuals draw inferences about their past information based on their past behavior. Similarly, it offers a possible explanation for the 'status quo' bias in which consumer preferences are altered by the initial assignment of ownership of goods. The theory therefore predicts when individuals will behave in a fashion consistent with the psychological theory of cognitive dissonance (inertial behavior) and when we expect to see the opposite (impulsiveness).

Although the focus of the model is on loss of memory about past signals, sometimes amnesia is so severe that past actions and signals are both lost. This possibility creates a benefit to institutionalizing past decisions through traditions, taboos, rituals, conventions, organizational routines and cultural norms. It could be argued that such institutions are designed to protect against hostile, unpredictable environments. However, our approach implies that such traditions will thrive in highly stable environments, where inertia is an optimal response to memory loss. In such environments, the institutionalization of inertia (and consequent loss of flexibility) has relatively low cost.

Memory loss is commonplace, and the problems it creates are ubiquitous. Thus, explicit analysis of how memory loss affects rational economic choices promises to enrich the theory of rational decisionmaking, and to be fruitful for a wide range of applications. We will close by mentioning three of these. First is the determination of brand loyalty of consumers, a topic of extensive empirical research in marketing. A second application is project recommendation within organizations, in which loss of information between hierarchical levels can be viewed as a kind of memory loss. Finally, a third possible application is to model the feedback from the behavior of many individuals and firms to the business environment. Our finding that environmental stability promotes inertia and instability promotes impulsiveness suggests that self-reinforcing effects can occur. A slight increase in environmental volatility may increase the optimal impulsiveness of individuals' and firms' policies. This in turn will tend to make the environment even less stable. This reason suggests that aggregate shifts in the volatility of economic choices such as investment may seem disproportionate to observable causes.
Appendix A: The Three Period Model

We show that $\sigma^N < \sigma^O$, so that the region of excess impulsiveness of the amnesiac player actually does exist. We being by outlining the argument. At the end of period 2, the difference between $H$ and $L$ signals after $HH$ or $LL$ is 2 or -2. For the amnesiac player, the sub-equivalent state after $AA$ or $RR$ is less than 2 in absolute value, because the second action may have been the result of a coin flip. By the definition of $\sigma^N$, the amnesiac player is virtually indifferent after $AA$ followed by $L$ (or $RR$ followed by $H$). $HH$ is more favorable than $AA$, so it follows that the full-recall player strongly prefers to follow the old signals after $HH$ (or $LL$). The algebraic details of the proof of the following lemma are routine, and are omitted.

**Lemma 1** (1) Let $G'$ denote the event $\phi' = G$. Then $Pr(G'|G, L) > Pr(G'|B, L)$. (2) $Pr(G|HHL) > Pr(G|AAL)$.

Intuitively, conditioning on $G$ rather than $B$ state increases the probability that the later state is $G'$; and conditioning on $HH$ is more favorable than conditioning on $AA$, which may or may not have come from $HH$.

**Proof of Proposition 1:** We must show that there exists values of $\sigma \in [0, 1)$ such that

$$Pr(G'|AAL) < \frac{1}{2} < Pr(G'|HHL).$$

Expanding the LHS and RHS terms gives

$$Pr(G'|AAL) = Pr(G'|AAL, G)Pr(G|AAL) + Pr(G'|AAL, B)Pr(B|AAL)$$

$$Pr(G'|HHL) = Pr(G'|HHL, G)Pr(G|HHL) + Pr(G'|HHL, B)Pr(B|HHL).$$

So

$$Pr(G'|AAL) = Pr(G'|L, G)Pr(G|AAL) + Pr(G'|L, B)[1 - Pr(G|AAL)]$$

$$= [Pr(G'|L, G) - Pr(G'|L, B)]Pr(G|AAL) + Pr(G'|L, B),$$

and

$$Pr(G'|HHL) = Pr(G'|L, G)Pr(G|HHL) + Pr(G'|L, B)[1 - Pr(G|HHL)]$$

$$= [Pr(G'|L, G) - Pr(G'|L, B)]Pr(G|HHL) + Pr(G'|L, B).$$

By Lemma 1, $Pr(G'|AAL) < Pr(G'|HHL)$. Consider now some $\sigma$ arbitrarily close to zero. Then

$$Pr(G'|HHL) \approx Pr(G|HHL)$$

$$= \frac{Pr(HHL|G)Pr(G)}{Pr(HHL|G)Pr(G) + Pr(HHL|B)Pr(B)}$$

$$= p.$$
So for \( p > 1/2 \) and \( \sigma > 0 \) small, the continuing I will always adopt after HHL. On the other hand, if \( \sigma \approx 1 \), then history is irrelevant, so I will always reject.

Similarly, consider \( \sigma \) close to 1. Then

\[
Pr(G'|AAL) \approx Pr(G'|L) = \frac{Pr(L|G')Pr(G')}{Pr(L|G')Pr(G') + Pr(L|B')Pr(B')}
= 1 - p.
\]

So for \( p > 1/2 \) and \( \sigma \approx 1 \), N always rejects after AAL.

If \( \sigma \approx 0 \), then the probability that the state is G given observation of AAL is

\[
Pr(G'|AAL) \approx \frac{Pr(G,AAL)}{Pr(AAL)} = \frac{\frac{1}{2}[Pr(HHL|G) + \frac{1}{2}Pr(HLL|G)]}{\frac{1}{2}[Pr(HHL|G) + \frac{1}{2}Pr(HLL|G)] + \frac{1}{2}[Pr(HHL|B) + \frac{1}{2}Pr(HLL|B)]}
= \frac{1 + p}{3}.
\]

So if \( p > 1/2 \), N always accepts after AAL.

To summarize, as \( \sigma \) increases from 0 to 1, both \( Pr(G'|AAL) \) and \( Pr(G'|HHL) \), decrease from greater than 1/2 to less than 1/2. Since \( Pr(G'|AAL) < Pr(G'|HHL) \) for all \( \sigma \in [0,1) \), it follows by continuity of these probabilities in \( \sigma \) that for all \( p > 1/2 \) there exists a value of \( \sigma \in [0,1) \) such that \( Pr(G'|AAL) < 1/2 \) and \( Pr(G'|HHL) > 1/2 \). Thus, for some \( \sigma \) N rejects after AAL while I accepts after HHL. \( \square \)

**Appendix B: The Multiperiod Model**

For notational simplicity and without loss of generality, the appendices take as given that the most recent past actions were adoptions; the analysis is symmetric for past rejections.

**Payoff Structure**

The net gains each period from adopting or rejecting the current project are summarized in Table 3. The net discounted value of the project is unknown, either \( v = 1 \) (G value state) or \( v = -1 \) (B value state), equally probable and unknown to the players. These are the payoffs obtained by adopting (undertaking the project). Rejecting the project yields a payoff of 0 in both states. Abstaining from decision generates net payoffs of \( 0.5 + \epsilon \) or \( -0.5 + \epsilon \) (\( \epsilon > 0 \) small) in states G and B respectively, which are halfway
between that from adopting and rejecting (plus $\epsilon$). With $\epsilon$ small, the player abstains if and only if he is (arbitrarily close to) indifferent between adopting and rejecting.

**Table 3 Here**

For purpose of simulation, we compute the probability that a continuing $I$ with full recall and a new $N$ with amnesia switch actions within $t$ periods of the date of potential memory loss, after just having adopted for $M$ periods.

**Preliminary Lemmas**

The probability that $I$'s action changes is the likelihood of reaching signal-state $s_t = 0$. The following two well-known lemmas about Markov processes are used in subsequent analysis. Let $\theta$ be the probability of an up move. For notational simplicity, we focus on the case of $s_0 > 0$.

**Lemma 2** Starting at position $s_0 > 0$, the probability of reaching state $s_t = 0$ exactly once at (and not before) time $t$ is

$$a(s_0, t, \theta) = \begin{cases} 
0 & \text{if } t < s_0, \text{ or } s_0 + t \text{ is odd} \\
\frac{s_0}{t} \left( \frac{t}{t+s_0} \right)^{\theta(t-s_0)/2(1-\theta)(t+s_0)/2} & \text{otherwise.}
\end{cases}$$

**Lemma 3** Starting at position $s_0 > 0$, the probability of reaching state $s_t = 0$ at least once ("absorption") by time $t$ is

$$A(s_0, t, \theta) = \begin{cases} 
0 & \text{if } t < s_0 \\
A(s_0, t-1, \theta) & \text{if } t + s_0 \text{ is odd, and } t > s_0 \\
B\left(\frac{t-s_0}{2}, t, \theta\right) + \left(\frac{1-\theta}{\theta}\right)^s_0 B\left(\frac{t-s_0}{2} - 1, t, 1 - \theta\right) & \text{otherwise,}
\end{cases}$$

where $B(\cdot)$ is the cumulative binomial distribution,

$$B(k, t, \theta) = \sum_{i=0}^{k} \binom{t}{i} \theta^i (1-\theta)^{t-i}.$$

The recursion implicit in the middle entry of $A(\cdot)$ must "bottom out" since moving backwards from an odd value for $t + s_0$ must lead to an even value.

**Deriving The Probability that the Value State is Good**

Given a sequence of $M$ adopt decisions, the probability that the value state is $G$ is

$$Pr(G|M, p) = \frac{Pr(M|G)Pr(G)}{Pr(M|G)Pr(G) + Pr(M|B)Pr(B)}.$$
We can use Lemmas 2 and 3 to compute \( Pr(M|G) \) and \( Pr(M|B) \). Any qualifying signal sequence must lead at the start to 2 adopts, and thus must start with two \( H \) signals. Then, starting at signal state 2, it must be followed by a signal sequence that does not touch state \( s = 0 \) (observable abstention) within \( M - 2 \) periods. Thus,

\[
Pr(M|G, p) = p^2[1 - A(2, M - 2, p)].
Pr(M|B, p) = (1 - p)^2[1 - A(2, M - 2, 1 - p)],
\]

and \( Pr(G|M, p) \) becomes

\[
Pr(G|M, p) = \frac{p^2[1 - A(2, M - 2, p)]}{p^2[1 - A(2, M - 2, p)] + (1 - p)^2[1 - A(2, M - 2, 1 - p)]}.
\]

Let \( A(k, t, p) \) (calculated in Appendix B) be the probability that I or N who starts in equivalent state \( k \) receives enough low signals to make an action switch (abstain or beyond) at least once within \( t \) periods.

**Calculation of Switch Probability under Full Recall**

I's optimal decision is based on the difference in the number of \( H \) and \( L \)-signals. The probability of an action switch by time \( t \) is the probability that the number of \( L \)-signals equals the number of \( H \)-signals by time \( t \). In Markov terms, this happens when the particle reaches the (absorbing) boundary 0. Let

\[
c(k; M, p) \equiv \frac{a(k, M, 1 - p)}{p^2[1 - A(2, M - 2, p)]} \quad (k \leq M).
\]

We now verify that \( c(k; M, p) \) is the conditional probability of starting in state \( k \), given \( M \) prior adopts. We first compute the posterior probability of a player being in state \( k \) given \( M \) adopts if the probability of an \( H \) signal is \( p \) (i.e., assuming for the moment that the value state is \( G \)).

The numerator of equation (2) gives the probability of being in state \( k \) and having \( M \) prior adopts. The denominator normalizes the probabilities. It exploits the fact that \( \sum_k a(k, M, 1 - p) = p^2[1 - A(2, M - 2, p)] \), because the first two signals must be \( H \), and no absorption may take place within the remaining \( M - 2 \) periods.

It follows that the probability that the initial player, having made exactly \( M \) successive adopt decisions, switches action at least once within \( t \) periods is

\[
r_t(M, t, p) \equiv Pr(G|M) \left[ \sum_{k=1}^{M} c(k; M, p)A(k, t, p) \right] + Pr(B|M) \left[ \sum_{k=1}^{M} c(k; M, 1 - p)A(k, t, 1 - p) \right],
\]

(3)
where \( c(k; M, p) \) is the probability that he is in state \( k \) given exactly \( M \) prior adoptions.

The first term on the RHS conditions on the value state being good. If the value state is good, I can be in state \( k \), for which the probability is \( c(k; M, p) \). If he starts in state \( k \), the probability that he will reach state 0 (observable action reversal) is \( A(k, t, p) \). The second term conditions on the value state being bad, and consequently, the probability of observing a low signal is not \( p \), but \( 1 - p \).

**Calculation of Switch Probabilities under Amnesia**

Since the equivalent state must be an integer, for continuous calculations it is useful to define what we call the sub-equivalent state.

**Definition** Let \( \bar{e} \) be the equivalent state. A sub-equivalent state is defined as any real number on the interval \((\bar{e} - 1, \bar{e})\), if \( \bar{e} > 0 \), and on the interval \([\bar{e}, \bar{e} + 1)\), if \( \bar{e} < 0 \).

Thus, I’s equivalent state is equal to the sub-equivalent state if the latter is an integer, and is equal to the next-larger-in-absolute-value integer if the latter has a fractional part.

A player whose equivalent state has higher absolute value is less likely to switch his action at least once within a given time period than a player in an absolutely lower state.\(^{23}\) At pre-transition-date in which an abstention occurs, there are an equal number of \( H \) and \( L \) signals, which is completely uninformative. Thus, the player can base his decision solely on the signals or actions that he observes since the latest pre-transition abstain decision. Prior to the player change, the relevant summary of past actions is just the number of immediately preceding consecutive all-adopt (or all-reject) decisions. We therefore need only show, starting with an arbitrary number \( M \) of all-adopt decisions, that \( N \) has a lower probability of switching than \( I \) within an arbitrary \( t \) periods after the switch.

Let \( I \) be the information set the player has about events prior to the date of possible player change. Let \( N \) be the signal sequence received by the player after the transition. By Bayes’ rule, the decision rule for a player is abstain if

\[
E(V|I, N) = \frac{Pr(N|G)Pr(I|G)}{Pr(N|G)Pr(I|G) + Pr(N|B)Pr(I|B)} = 1/2,
\]

reject if the expectation is less than 1/2, and adopt in the reverse case. Simplifying this expression, the condition for abstaining becomes

\[
Pr(N|G)Pr(I|G) = Pr(N|B)Pr(I|B).
\]

\(^{23}\)Proof: the set of possible signal realizations that lead the player in a higher state to switch is a subset of the set of possible signal realizations that lead the player in a lower state to switch.
Let \( k \) be the equivalent state, i.e., the extra number of \( L \)-signals over and above any \( H \)-signals that are required to bring a player who has observed \( \mathcal{I} \) back to indifference (or beyond). In our Markov structure, \( \Pr(\mathcal{N}|G) = (1 - p)^k \) and \( \Pr(\mathcal{N}|B) = p^k \), so

\[
(1 - p)^k \Pr(\mathcal{I}|G) = p^k \Pr(\mathcal{I}|B), \quad \text{or}
\]

\[
k = \frac{\log[\Pr(\mathcal{I}|G)/\Pr(\mathcal{I}|B)]}{\log[p/(1 - p)]}.
\]

(4)

Noninteger solutions for \( k \) give sub-equivalent states.

Adding one more \( H \) signal to \( \mathcal{I} \) increases \( k \) by 1 regardless of the information in \( \mathcal{I} \). After one additional \( H \) signal, \( \Pr(\mathcal{I}|G) \) is replaced by \( \Pr(\mathcal{I}|G)p \); \( \Pr(\mathcal{I}|B) \) by \( \Pr(\mathcal{I}|B)(1 - p) \). Thus, a player can act by computing his state (or state inference) based on the last abstain decision—when \( \mathcal{I} \) was publicly known to be in state 0.

Finally, we need to compute \( \Pr(\mathcal{I}|G) \) and \( \Pr(\mathcal{I}|B) \). This has already been used in the computation of \( \Pr(G|M) \). It is the probability of all sequences that lead to uniform adoption, i.e. that start with two \( H \)-signals, and are followed by sequences that do not lead to absorption within \( M - 2 \) periods. Therefore, the sub-equivalent state for \( \mathcal{N} \) who has observed successive \( M \) adopt decisions by \( \mathcal{I} \) (and no signals of his own) is as given in equation (5).

Thus, the new player's sub-equivalent state is

\[
e(\mathcal{N}, p) = 2 + \frac{\log\left[\frac{1 - A(2, M - 2, p)}{1 - A(2, M - 2, 1 - p)}\right]}{\log[p/(1 - p)]}.
\]

(5)

If the sub-equivalent state \( e \) is not an integer, it needs to be rounded up in absolute value, because an action switch does not occur until the absorbing boundary of 0 is crossed.\(^{24}\)

By standard probability calculus, the probability that a new player, having observed \( M \) successive adopt decisions, switches action within \( t \) periods after the memory transition is

\[
r_{\mathcal{N}}(M, t, p) \equiv \Pr(\mathcal{G}|M)A(e[M, p], t, p) + \Pr(\mathcal{B}|M)A(e[M, p], t, 1 - p).
\]

(6)

The first term again conditions on the probability that the value state is good. A reversal is observed if \( e[M, p] \) low signals are observed (which happens with probability \( A(e, t, p) \)). If the value state is bad, the probability of a low signal is \( 1 - p \) instead of \( p \).

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\(^{24}\)The (sub)-equivalent state is not equal to the player's expectation of the signal state, because of the player's uncertainty about the values of previous signals. For example, when the expected signal state is greater than +1, a single additional \( L \) can sometimes cause the player to revise his inference about pretransition signals in favor of \( L \) over \( H \) so much that the player switches from adopt to reject.
Proof of Proposition 2: This follows directly from equations (2) and (5). \(\square\)

Proof of Proposition 3: \(^{25}\) We begin with two lemmas.

Lemma 4 The sub-equivalent state \(e\) and the probability \(\pi\) that the value state is \(G\) are related by

\[
e = \frac{\log[\pi/(1 - \pi)]}{\log[p/(1 - p)]}.
\]

(7)

Proof: Let \(\pi_e\) be the probability that the value state is \(G\) given the sub-equivalent state expression given above. (It is easy to show that there is a one-to-one map between the two.) Taking the odds ratio of \(\pi\) and assuming that \(Pr(G) = Pr(B)\), we find that

\[
\frac{\pi_e}{1 - \pi_e} = \frac{Pr(G|e)}{Pr(B|e)} = \frac{Pr(e|G)}{Pr(e|B)}.
\]

Because the sub-equivalent state is defined in equation (4) as

\[
e = \frac{\log[Pr(e|G)/Pr(e|B)]}{\log[p/(1 - p)]},
\]

it follows that

\[
\frac{\pi_e}{1 - \pi_e} = \left(\frac{p}{1 - p}\right)^e.
\]

Solving for \(e\) gives (7). \(\square\)

The next lemma relates the probability of obtaining a reversal given that the player starts at sub-equivalent state \(e\) to the reversal probability given value state \(G\). Let \(C\) denote the event that no reversal ever occurs, let \(C'\) be the complementary event that reversal occurs, and let \((\pi_e)\) be the probability that the value state is \(G\). Then:

Lemma 5 The cumulative asymptotic probability of ever observing a reversal given an arbitrary number of past adoptions is \(Pr(C'|e) = 2 - 2\pi_e\).

Proof: Because the probability of observing a reversal as \(T \to \infty\) if I took the wrong action is 1, the probability of \(C\) is the probability that the action is correct given the (unknown) value state (so that there is a drift away from zero) times the probability of (temporarily) not reaching a state of 0. Consequently, the probability of no reversal given sub-equivalent state \(e\) is

\[
Pr(C|e) = \pi_e \left[1 - \left(\frac{1 - p}{p}\right)^e\right],
\]

\(^{25}\)This proof was pointed out to us by Larry Glosten.
where $\pi_e$ is the (above) probability that the value state is good given starting state $e$. Using the relation between $\pi_e$ and $e$ in (7), we can substitute for $e$ and use the fact that $x^a \log(y) = y^a \log(x)$ to obtain

$$Pr(C|e) = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right)^{\log(\pi_e/(1 - \pi_e)) \log(p/(1 - p))} \right] = \pi_e \left[ 1 - \left( \frac{1 - \pi_e}{\pi_e} \right)^{\log((1 - p)/p) \log(p/(1 - p))} \right] = 2\pi_e - 1. \Box$$

We now complete the proof of Proposition 3.

**I:** Its reversal behavior is the weighted sum of reversal probabilities:

$$Pr_{\text{rev}}(C|A^M) = \sum_{s=1}^{M} Pr(C|s) Pr(s|A^M),$$

where $A^M$ denotes a sequence of $M$ adopts. By Lemma 5, this expression can be simplified to

$$\sum_{s=1}^{M} (2\pi_i - 1) Pr(s|A^M) = \sum_{s=1}^{M} \left[ 2 Pr(G|s) Pr(s|A^M) - Pr(s|A^M) \right] = 2\tilde{\pi}_M - 1,$$

where $\tilde{\pi}_M$ is the probability that the value state is $G$ given $M$ pre-transition adopts.

**New Player:** $N$ starts at the next-highest integer ($\overline{e}$) following the sub-equivalent state $e$ as in (7). Consequently, the equivalent state $\overline{e}$ is the sum of the sub-equivalent state $e$ and a positive fraction ($0 < f < 1$). By Lemma 5,

$$Pr_{\text{new}}(C|\overline{e}) = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right)^{\overline{e}} \right] = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right)^{e + f} \right] = \pi_e \left[ 1 - \left( \frac{1 - p}{p} \right)^{e} \left( \frac{1 - p}{p} \right)^{f} \right]$$

$$= \pi_e \left[ 1 - \left( \frac{1 - \pi_e}{\pi_e} \right) \left( \frac{1 - p}{p} \right)^{f} \right] = (2\pi_e - 1) + (1 - \pi_e) \left[ 1 - \left( \frac{1 - p}{p} \right)^{f} \right],$$

which is greater than $2\pi_e - 1$ when $f > 0$.

Part (ii): If there are $M'$ periods prior to the memory transition, there could any number from 0 through $M'$ of consecutive immediate pre-transition adopt decisions. As before, this number is all that is relevant in determining future actions after memory loss. Conditional on this number, in some cases, both $N$ and $I$ will switch with equal probability; in other cases, $N$ will switch after $I$. Therefore the unconditional probability that $I$ switches at least once is greater than the probability that $N$ switches at least once. $\Box$
References


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